Decoupling of degenerate positive-norm states in Witten's string field theory

Hsien-Chung Kao*
Department of Physics, Tamkang University, Tamsui, Taiwan, Republic of China
and Department of Physics, National Taiwan Normal University, Taipei, Taiwan, Republic of China

Jen-Chi Lee†
Department of Electrophysics, National Chiao Tung University, Hsinchu, Taiwan, Republic of China
(Received 17 December 2002; published 21 April 2003)

We show that the degenerate positive-norm physical propagating fields of the open bosonic string can be gauged to the higher rank fields at the same mass level. As a result, their scattering amplitudes can be determined from those of the higher spin fields. This phenomenon arises from the existence of two types of zero-norm states with the same Young representations as those of the degenerate positive-norm states in the old covariant first quantized (OCFQ) spectrum. This is demonstrated by using the lowest order gauge transformation of Witten's string field theory (WSFT) up to the fourth massive level (spin-five), and is found to be consistent with conformal field theory calculation based on the first quantized generalized sigma-model approach. In particular, on-shell conditions of zero-norm states in the OCFQ stringy gauge transformation are found to correspond, in a one-to-one manner, to the background ghost fields in off-shell gauge transformation of WSFT. The implication of decoupling of scalar modes on Sen's conjectures is also briefly discussed.

DOI: 10.1103/PhysRevD.67.086003 PACS number(s): 11.25.Sq, 11.25.Db, 11.25.Hf

I. INTRODUCTION

It was pointed out more than ten years ago by Gross [1] that, in addition to the strong coupling regime, the most important nonperturbative regime of string theory is the high-energy stringy ($\alpha'\to\infty$) behavior of the theory. It is in this regime that the theory becomes very different from point particle field theory. Among many interesting stringy behaviors, it was believed that an infinite broken gauge symmetry gets restored at an energy much higher than the Planck energy. Moreover, this symmetry is powerful enough to link different string scattering amplitudes and, in principle, can be used to express all string amplitudes in terms of, say, the dilaton amplitude.

Instead of studying stringy scattering amplitudes [2], one alternative to explicitly derive the stringy symmetry is to use the generalized worldsheet sigma-model approach. In this approach, one uses conformal field theory to calculate the equations of motion for massive string background fields in the lowest order weak field approximation, but valid to all orders in α'. The weak field approximation is thus the appropriate approximation scheme to study high-energy symmetry of the string. An infinite set of on-shell stringy gauge symmetry is then derived by requiring the decoupling of both types of zero-norm physical states in the old covariant first quantized (OCFQ) spectrum [3]. In particular, all physical propagating states at each fixed mass level are found to form a large gauge multiplet. This begins to show up at the second massive level (spin-three). Moreover, it was remarkable to discover that [4] the degenerate positive-norm physical propagating fields of the third massive level of the open bosonic string can be gauged to the higher rank fields by the existence of zero-norm states with the same Young representations. It was also shown [5] that the scattering amplitudes of these degenerate positive-norm states can be expressed in terms of those of higher spin states at the same mass level through massive Ward identities. The subtlety of the scalar state pointed out in Ref. [5] will be resolved at the end of Sec. II. This phenomenon begins to show up at the third massive level (spin-four) and was argued to be a sigma-model of $n+1$ loop results for the nth massive level. These stringy phenomena seem to be closely related to the results in Ref. [1]. In fact, an infinite number of linear relations between the string tree-level scattering amplitudes of different string states, similar to those claimed in Ref. [1], can be derived by making use of an infinite number of zero-norm states [5]. To claim that the decoupling phenomenon persist for general higher levels, it would be very important a priori to see whether one can rederive it from the second quantized off-shell Witten string field theory (WSFT) [6].

Recently there is a revived interest in WSFT, mainly due to Sen's conjecture on tachyon condensation on D-brane [7]. It becomes more and more clear that a second quantized field theory of string is unavoidable, especially when one wants to study higher string modes. Thus, a cross check by both first and second quantized approaches of any reliable string theory result would be of great importance. Unfortunately, most of the recent researches on string field theory were confined to the scalar modes on identification of nonperturbative string vacuum [8]. Our aim in this paper is to consider the gauge transformation of all string modes with any spin and in arbitrary gauge [9]. We will first prove the decoupling phenomenon of the third massive level of open bosonic string claimed in Ref. [4] by WSFT. The result is then generalized to the fourth massive level by both the first and the second quantized approaches. This paper is organized as follows. In Sec. II we first summarize the previous results obtained in the first quantized approach. In Sec. III we explic-

*Email address: hckao@phy.ntnu.edu.tw
†Email address: jcclee@cc.nctu.edu.tw

TABLE I. The OCFQ spectrum of open bosonic string.

Mass level	Positive-norm states	Zero-norm states
$m^2=-2$	$\bullet$	
$m^2=0$	$\ydiagram{1}$	$\bullet$ (singlet)
$m^2=2$	$\ydiagram{2}$	$\ydiagram{1}$, $\bullet$
$m^2=4$	$\ydiagram{3}$, $\ydiagram{1,1,1}$	$\ydiagram{2}$, $2\times\ydiagram{1}$, $\bullet$
$m^2=6$	$\ydiagram{3}$, $\ydiagram{2,1}$, $\ydiagram{2}$, $\bullet$	$\ydiagram{3}$, $\ydiagram{1,1}$, $2\times\ydiagram{2}$, $3\times\ydiagram{1}$, $2\times\bullet$

itly calculate the lowest order gauge transformation level by level up to the third massive level in WSFT, and compare them with those of the first quantized approach. Some important observations will be made for ghost fields in WSFT and zero-norm states in the OCFQ spectrum. The transformation will be separated into the matter and the ghost field parts in WSFT. The matter part is found to be consistent with the previous calculation [5] based on the old covariant string field gauge transformation of Banks and Peskin [10]. The ghost part is argued to correspond to the lifting of on-shell (including on-mass-shell, gauge, and traceless) conditions of zero-norm states in the OCFQ calculation. Section IV is devoted to the fourth massive level. Both first and second quantized calculations are new and will be presented in Sec. IV. A brief conclusion is made in Sec. V. The lengthy gauge transformation of ghost fields for level four will be collected in the Appendix.

II. OLD COVARIANT FIRST QUANTIZED APPROACH

The old covariant quantization is one of the three standard quantization schemes of string. In addition to the physical positive-norm propagating modes, there exist two types of physical zero-norm states in the bosonic open string spectrum [11]. They are as follows:

$$\text{Type I:}\quad L_{-1}|\chi\rangle,\quad \text{where } L_m|\chi\rangle=0,\ m\geqslant 1,\quad L_0|\chi\rangle=0, \tag{1}$$

$$\text{Type II:}\quad (L_{-2}+\tfrac{3}{2}L_{-1}^2)|\tilde{\chi}\rangle,$$

$$\text{where } L_m|\tilde{\chi}\rangle=0,\ m\geqslant 1,\ (L_0+1)|\tilde{\chi}\rangle=0. \tag{2}$$

While type I states have zero-norm at any spacetime dimension, type II states have zero-norm *only* at $D=26$. Their existence turns out to be important in the following discussion. The explicit forms of these zero-norm states have been calculated and their Young tabulation, together with positive-norm states, up to the third massive level, are listed in Table I. Note that zero-norm states are not included in the light-cone quantization.

It was demonstrated in the first order weak field approximation that for each zero-norm state there corresponds an on-shell gauge transformation for the positive-norm background field ($\alpha'\equiv\frac{1}{2}$) [3]:

$$m^2=0,\quad \delta A_\mu=\partial_\mu\theta, \tag{3a}$$

$$\partial^2\theta=0; \tag{3b}$$

$$m^2=2,\quad \delta B_{\mu\nu}=\partial_{(\mu}\theta_{\nu)}, \tag{4a}$$

$$\partial^\mu\theta_\mu=0,\quad (\partial^2-2)\theta_\mu=0; \tag{4b}$$

$$\delta B_{\mu\nu}=\tfrac{3}{2}\partial_\mu\partial_\nu\theta-\tfrac{1}{2}\eta_{\mu\nu}\theta, \tag{5a}$$

$$(\partial^2-2)\theta=0; \tag{5b}$$

$$m^2=4,\quad \delta C_{\mu\nu\lambda}=\partial_{(\mu}\theta_{\nu\lambda)}, \tag{6a}$$

$$\partial^\mu\theta_{\mu\nu}=\theta^\mu_\mu=0,\quad (\partial^2-4)\theta_{\mu\nu}=0; \tag{6b}$$

$$\delta C_{\mu\nu\lambda}=\tfrac{5}{2}\partial_{(\mu}\partial_\nu\theta^1_{\lambda)}-\eta_{(\mu\nu}\theta^1_{\lambda)}, \tag{7a}$$

$$\partial^\mu\theta^1_\mu=0,\quad (\partial^2-4)\theta^1_\mu=0; \tag{7b}$$

$$\delta C_{\mu\nu\lambda}=\tfrac{1}{2}\partial_{(\mu}\partial_\nu\theta^2_{\lambda)}-2\eta_{(\mu\nu}\theta^2_{\lambda)},$$

$$\delta C_{[\mu\nu]}=9\partial_{[\mu}\theta^2_{\nu]}, \tag{8a}$$

$$\partial^\mu\theta^2_\mu=0,\quad (\partial^2-4)\theta^2_\mu=0; \tag{8b}$$

$$\delta C_{\mu\nu\lambda}=\tfrac{3}{5}\partial_\mu\partial_\nu\partial_\lambda\theta-\tfrac{1}{5}\eta_{(\mu\nu}\partial_{\lambda)}\theta, \tag{9a}$$

$$(\partial^2-4)\theta=0. \tag{9b}$$

These symmetry transformations can be explicitly shown to be symmetries of the equation of motion for massive background fields, which were calculated in Ref. [3]. A complete 2D sigma-model renormalization group analysis for the first massive level was done in Refs. [12,13]. It was noted that the traceless condition of the equation of motion, which was included in Refs. [3], can only be obtained by requiring quantum Weyl invariance at a linearized *two*-loop approximation. The symmetries considered in Ref. [12,13] corresponding to adding worldsheet total derivative terms to the effective Lagrangian, however, turn out to be only the subset of symmetries calculated in Ref. [3]. The complete set of symmetries generated by two types of zero-norm states considered in this section include some nontotal derivative terms, e.g., Eq. (5).

In the above equations, A, B, C are positive-norm background fields, θ's represent zero-norm background fields, and $\partial^2\equiv\partial^\mu\partial_\mu$. There are on-mass-shell, gauge, and traceless conditions on the transformation parameters θ's, which will correspond to Becchi-Rouet-Stora-Tyutin (BRST) ghost fields in a one-to-one manner in WSFT, as will be discussed in the next section. Equation (3) is of course the usual on-shell gauge transformation, and Eq. (5) is the first residual stringy gauge symmetry. Note that θ^1_μ and θ^2_μ in Eqs. (7) and (8) are some linear combination of the original type I and type II vector zero-norm states calculated by Eqs. (1) and (2). It is interesting to see that Eq. (8) implies that the two second massive level modes $C_{\mu\nu\lambda}$ and $C_{[\mu\nu]}$ form a larger gauge multiplet [3]. This is a generic feature for higher massive level and had also been justified from S matrix point of view [14]. One might want to generalize the calculation to the second order weak field to see the intermass level symmetry. This however suffers from the so-called nonperturbative nonrenormalizability of 2D σ model, and one is forced to introduce infinite number of counter terms to preserve the worldsheet conformal invariance [15].

Instead of calculating the stringy gauge symmetry at level $m^2=6$, we will only concentrate on the equation of motion. It was discovered that an even more interesting phenomenon begins to show up at this mass level. Take the energy-momentum tensor on the worldsheet boundary in the first order weak field approximation to be of the following form:

$$\begin{aligned}T(\tau)=&-\tfrac{1}{2}\eta_{\mu\nu}\partial_\tau X^\mu\partial_\tau X^\nu+D_{\mu\nu\alpha\beta}\partial_\tau X^\mu\partial_\tau X^\nu\partial_\tau X^\alpha\partial_\tau X^\beta\\&+D_{\mu\nu\alpha}\partial_\tau X^\mu\partial_\tau X^\nu\partial_\tau^2 X^\alpha+D^0_{\mu\nu}\partial_\tau^2 X^\mu\partial_\tau^2 X^\nu\\&+D^1_{\mu\nu}\partial_\tau X^\mu\partial_\tau^3 X^\nu+D_\mu\partial_\tau^4 X^\mu,\end{aligned}\tag{10}$$

where τ is worldsheet time, $X\equiv X(\tau)$. This is the most general worldsheet coupling in the generalized σ model approach consistent with vertex operator consideration [16]. The conditions to cancel all q number worldsheet conformal anomalous terms correspond to cancelling all kinds of loop divergences [13] up to the four loop orders in the 2D conformal field theory. It is easier to use $T\cdot T$ operator-product calculation and the conditions read [4]

$$2\partial^\mu D_{\mu\nu\alpha\beta}-D_{(\nu\alpha\beta)}=0,\tag{11a}$$

$$\partial^\mu D_{\mu\nu\alpha}-2D^0_{\nu\alpha}-3D^1_{\nu\alpha}=0,\tag{11b}$$

$$\partial^\mu D^1_{\mu\nu}-12D_\nu=0,\tag{11c}$$

$$3D_{\mu\nu\alpha}{}^{\mu}+\partial^\mu D_{\nu\alpha\mu}-3D^1_{(\nu\alpha)}=0,\tag{11d}$$

$$D_{\mu\nu}{}^{\mu}+4\partial^\mu D^0_{\mu\nu}-24D_\nu=0,\tag{11e}$$

$$2D_{\mu\nu}{}^{\nu}+3\partial^\nu D^1_{\mu\nu}-12D_\mu=0,\tag{11f}$$

$$2D^{0\mu}_{\mu}+3D^{1\mu}_{\mu}+12\partial^\mu D_\mu=0,\tag{11g}$$

$$(\partial^2-6)\phi=0.\tag{11h}$$

Here, ϕ represents all background fields introduced in Eq. (10). It is now clear through Eqs. (11b) and (11d) that both $D^0_{\mu\nu}$ and $D^1_{(\mu\nu)}$ can be expressed in terms of $D_{\mu\nu\alpha\beta}$ and $D_{\mu\nu\alpha}$. $D^1_{[\mu\nu]}$ can be expressed in terms of $D_{\mu\nu\alpha\beta}$ and $D_{\mu\nu\alpha}$ by Eq. (11b). Equations (11a) and (11c) imply that $D_{(\mu\nu\alpha)}$ and D_μ can also be expressed in terms of $D_{\mu\nu\alpha\beta}$ and mixed-symmetric $D_{\mu\nu\alpha}$. Finally, Eqs. (11e)–(11g) are the gauge conditions for $D_{\mu\nu\alpha\beta}$ and mixed-symmetric $D_{\mu\nu\alpha}$ after substituting $D^0_{\mu\nu}$, $D^1_{\mu\nu}$, and D_μ in terms of $D_{\mu\nu\alpha\beta}$ and mixed-symmetric $D_{\mu\nu\alpha}$. The remaining scalar particle has automatically been gauged to higher rank fields since Eq. (10) is already the most general form of the background-field coupling. This means that the degenerate spin-two and scalar positive-norm states can be gauged to the higher rank fields $D_{\mu\nu\alpha\beta}$ and mixed-symmetric $D_{\mu\nu\alpha}$ in the first order weak field approximation. In fact, for instance, it can be explicitly shown [5] that the scattering amplitude involving the positive-norm spin-two state can be expressed in terms of those of spin-four and mixed-symmetric spin-three states due to the existence of a type I and a type II spin-two zero-norm states. The subtlety of the scalar state scattering amplitude pointed out in Ref. [5] can be resolved in the following way. Take a representative of the scalar state to be [16]

$$\begin{aligned}:\Bigg[&-\left(\eta_{\mu\nu}+\frac{13}{3}k_\mu k_\nu\right)\partial_z^2X^\mu\ \partial_z^2X^\nu\\&-i\left(\frac{20}{9}k_\mu k_\nu k_\rho+\frac{2}{3}k_\mu\eta_{\nu\rho}+\frac{13}{3}k_\rho\eta_{\mu\nu}\right)\partial_zX^\mu\ \partial_zX^\nu\ \partial_z^2X^\rho\\&+\left(\frac{23}{81}k_\mu k_\nu k_\rho k_\sigma+\frac{32}{27}k_\mu k_\nu\eta_{\rho\sigma}+\frac{19}{18}\eta_{\mu\nu}\eta_{\rho\sigma}\right)\\&\times\partial_zX^\mu\ \partial_zX^\nu\ \partial_zX^\rho\ \partial_zX^\sigma\Bigg]e^{ikX(z)}:\ .\end{aligned}$$

It turns out that one cannot gauge away the first term in the above equation by using the two scalar zero-norm states. However, we have already known the amplitude corresponding to $\partial_z^2X^\mu\ \partial_z^2X^\nu$ are fixed by those of the spin-four and mixed-symmetric spin-three states. The totally symmetric spin-three amplitude corresponding to the totally symmetric spin-three part of the second term, $\partial_zX^{(\mu}\ \partial_zX^\nu\ \partial_z^2X^{\rho)}$, can be fixed by the spin-four amplitude due to the existence of the totally symmetric spin-three zero-norm state. As a result, the scalar state scattering amplitude is again fixed by the amplitudes of spin-four and mixed-symmetric spin-three states. Although all the four-point amplitudes considered in Ref. [5] contain three tachyons, the argument can be easily generalized to more general amplitudes. This is very different from the analysis of lower massive levels where all positive-norm states have independent scattering amplitudes. Presumably, this decoupling phenomenon comes from the ambiguity in defining positive-norm states due to the existence of zero-norm states in the same Young representations. We will justify this decoupling by WSFT in the next section. Finally, one expects this decoupling to persist even if one includes the higher order corrections in the weak field approximation, as there will be even stronger relations between the background fields order by order through iteration.

III. WITTEN'S STRING FIELD THEORY APPROACH

It would be much more convincing if one can rederive the stringy phenomena discussed in the previous section from WSFT. Not only can one compare the first quantized string with the second quantized string, but also the old covariant quantized string with the BRST quantized string. Although the calculation is lengthy, the results, as we shall see, are still controllable by utilizing the results from first quantized approach in Sec. II. There exist important consistency checks of first quantized string results from WSFT in the literature, e.g., the rederivation of Veneziano and Kubo-Nielson amplitudes from WSFT [17]. In some stringy cases, calculations can only be done in the string field theory approach. For example, the recently developed pp wave string amplitudes can only be calculated in the light-cone string field theory [18]. Sen's recent conjectures of tachyon condensation on D-brane again were mostly justified by the string field theory. Therefore, a consistent check by both first and second quantized approaches of any reliable string results would be of great importance.

The infinitesimal gauge transformation of WSFT is

$$\delta\Phi = Q_B\Lambda + g_0(\Phi*\Lambda - \Lambda*\Phi). \tag{12}$$

To compare with our first quantized results in Sec. II, we only need to calculate the first term on the right-hand side (rhs) of Eq. (12). Up to the second massive level, Φ and Λ can be expressed as

$$\begin{aligned}\Phi = [&\phi(x) + iA_\mu(x)\alpha_{-1}^\mu + \alpha(x)b_{-1}c_0 - B_{\mu\nu}(x)\alpha_{-1}^\mu\alpha_{-1}^\nu \\ &+ iB_\mu(x)\alpha_{-2}^\mu + i\beta_\mu(x)\alpha_{-1}^\mu b_{-1}c_0 + \beta^0(x)b_{-2}c_0 \\ &+ \beta^1(x)b_{-1}c_{-1} - iC_{\mu\nu\lambda}(x)\alpha_{-1}^\mu\alpha_{-1}^\nu\alpha_{-1}^\lambda \\ &- C_{\mu\nu}(x)\alpha_{-2}^\mu\alpha_{-1}^\nu + iC_\mu(x)\alpha_{-3}^\mu \\ &- \gamma_{\mu\nu}(x)\alpha_{-1}^\mu\alpha_{-1}^\nu b_{-1}c_0 + i\gamma_\mu^0(x)\alpha_{-1}^\mu b_{-2}c_0 \\ &+ i\gamma_\mu^1(x)\alpha_{-1}^\mu b_{-1}c_{-1} + i\gamma_\mu^2(x)\alpha_{-2}^\mu b_{-1}c_0 + \gamma^0(x)b_{-3}c_0 \\ &+ \gamma^1(x)b_{-2}c_{-1} + \gamma^2(x)b_{-1}c_{-2}]c_1|k\rangle,\end{aligned} \tag{13}$$

$$\begin{aligned}\Lambda = [&\epsilon^0(x)b_{-1} - \epsilon_{\mu\nu}^0(x)\alpha_{-1}^\mu\alpha_{-1}^\nu b_{-1} + i\epsilon_\mu^0(x)\alpha_{-1}^\mu b_{-1} \\ &+ i\epsilon_\mu^1(x)\alpha_{-2}^\mu b_{-1} + i\epsilon_\mu^2(x)\alpha_{-1}^\mu b_{-2} + \epsilon^1(x)b_{-2} \\ &+ \epsilon^2(x)b_{-3} + \epsilon^3(x)b_{-1}b_{-2}c_0]|\Omega\rangle,\end{aligned} \tag{14}$$

where Φ and Λ are restricted to ghost numbers 1 and 0, respectively, and the BRST charge is

$$Q_B = \sum_{n=-\infty}^{\infty} L_{-n}^{\mathrm{matt}}c_n + \sum_{m,n=-\infty}^{\infty}\frac{m-n}{2}:c_mc_nb_{-m-n}: - c_0. \tag{15}$$

The transformations one gets for each mass level are the following:

$$m^2=0, \quad \delta A_\mu = \partial_\mu\epsilon^0, \tag{16a}$$

$$\delta\alpha = \tfrac{1}{2}\partial^2\epsilon^0; \tag{16b}$$

$$m^2=2, \quad \delta B_{\mu\nu} = -\partial_{(\mu}\epsilon_{\nu)}^0 - \tfrac{1}{2}\epsilon^1\eta_{\mu\nu}, \tag{17a}$$

$$\delta B_\mu = -\partial_\mu\epsilon^1 + \epsilon_\mu^0, \tag{17b}$$

$$\delta\beta_\mu = \tfrac{1}{2}(\partial^2-2)\epsilon_\mu^0, \tag{17c}$$

$$\delta\beta^0 = \tfrac{1}{2}(\partial^2-2)\epsilon^1, \tag{17d}$$

$$\delta\beta^1 = -\partial^\mu\epsilon_\mu^0 - 3\epsilon^1; \tag{17e}$$

$$m^2=4, \quad \delta C_{\mu\nu\lambda} = -\partial_{(\mu}\epsilon_{\nu\lambda)}^0 - \tfrac{1}{2}\epsilon_{(\mu}^2\eta_{\nu\lambda)}, \tag{18a}$$

$$\delta C_{[\mu\nu]} = -\partial_{[\nu}\epsilon_{\mu]}^1 - \partial_{[\mu}\epsilon_{\nu]}^2, \tag{18b}$$

$$\delta C_{(\mu\nu)} = -\partial_{(\nu}\epsilon_{\mu)}^1 - \partial_{(\mu}\epsilon_{\nu)}^2 + 2\epsilon_{\mu\nu}^0 - \epsilon^2\eta_{\mu\nu}, \tag{18c}$$

$$\delta C_\mu = -\partial_\mu\epsilon^2 + 2\epsilon_\mu^1 + \epsilon_\mu^2, \tag{18d}$$

$$\delta\gamma_{\mu\nu} = \tfrac{1}{2}(\partial^2-4)\epsilon_{\mu\nu}^0 - \tfrac{1}{2}\epsilon^3\eta_{\mu\nu}, \tag{18e}$$

$$\delta\gamma_\mu^0 = \tfrac{1}{2}(\partial^2-4)\epsilon_\mu^2 + \partial_\mu\epsilon^3, \tag{18f}$$

$$\delta\gamma_\mu^1 = -2\partial^\nu\epsilon_{\nu\mu}^0 - 2\epsilon_\mu^1 - 3\epsilon_\mu^2, \tag{18g}$$

$$\delta\gamma_\mu^2 = \tfrac{1}{2}(\partial^2-4)\epsilon_\mu^1 - \partial_\mu\epsilon^3, \tag{18h}$$

$$\delta\gamma^0 = \tfrac{1}{2}(\partial^2-4)\epsilon^2 - \epsilon^3, \tag{18i}$$

$$\delta\gamma^1 = -\partial^\mu\epsilon_\mu^2 - 4\epsilon^2 - 2\epsilon^3, \tag{18j}$$

$$\delta\gamma^2 = -2\partial^\mu\epsilon_\mu^1 - 5\epsilon^2 + 4\epsilon^3 + \epsilon_\mu^{0\ \mu}. \tag{18k}$$

It is interesting to note that Eq. (16b) corresponds to the lifting of the on-mass-shell condition in Eq. (3b). Meanwhile, Eqs. (17c) and (17d) correspond to the on-mass-shell condition in Eqs. (5b) and (4b), and Eq. (17e) corresponds to the gauge condition in Eq. (4b). Similar correspondence applies to level $m^2=4$. Equations (18e), (18f), (18h), and (18i) correspond to the on-mass-shell conditions in Eqs. (6b), (7b), (8b), and (9b). Equation (18g), (18j), and (18k) correspond to the gauge conditions in Eqs. (6b), (7b), and (8b). The traceless condition in Eq. (6b) corresponds to the trace part of Eq. (18e). Also, only zero-norm state transformation parameters appear on the rhs of matter transformation A, B, C, and all ghost transformations correspond, in a one-to-one manner, to the lifting of on-shell conditions (including on-mass-shell, gauge, and traceless conditions) in the OCFQ approach. These important observations simplify the demonstration of decoupling of degenerate positive-norm states at higher mass levels, $m^2=6$ and $m^2=8$ more specifically, in WSFT, as will be discussed in the rest of this paper.

For $m^2=4$, it can be checked that only $C_{\mu\nu\lambda}$ and $C_{[\mu\nu]}$ are dynamically independent and they form a gauge multiplet, which is consistent with the result of the first quantized calculation presented in Sec. II.

We now show the decoupling phenomenon for the third massive level $m^2=6$, in which Φ and Λ can be expanded as

$$\Phi_4=[D_{\mu\nu\alpha\beta}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}a^{\beta}_{-1}-iD_{\mu\nu\alpha}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-2}-D^{0}_{\mu\nu}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-2}-D^{1}_{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-3}+iD_{\mu}(x)\alpha^{\mu}_{-4}$$
$$-i\xi_{\mu\nu\alpha}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}b_{-1}c_0-\xi^{0}_{\mu\nu}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-1}b_{-1}c_0-\xi^{1}_{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}b_{-2}c_0-\xi^{2}_{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}b_{-1}c_{-1}$$
$$+i\xi^{0}_{\mu}(x)\alpha^{\mu}_{-3}b_{-1}c_0+i\xi^{1}_{\mu}(x)\alpha^{\mu}_{-2}b_{-2}c_0+i\xi^{2}_{\mu}(x)\alpha^{\mu}_{-1}b_{-3}c_0+i\xi^{3}_{\mu}(x)\alpha^{\mu}_{-2}b_{-1}c_{-1}+i\xi^{4}_{\mu}(x)\alpha^{\mu}_{-1}b_{-2}c_{-1}$$
$$+i\xi^{5}_{\mu}(x)\alpha^{\mu}_{-1}b_{-1}c_{-2}+\xi^{0}(x)b_{-4}c_0+\xi^{1}(x)b_{-3}c_{-1}+\xi^{2}(x)b_{-2}c_{-2}+\xi^{3}(x)b_{-1}c_{-3}+\xi^{4}(x)b_{-2}b_{-1}c_{-1}c_0]c_1|k\rangle, \tag{19}$$

$$\Lambda_4=[-i\epsilon^{0}_{\mu\nu\alpha}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}b_{-1}-\epsilon^{1}_{\mu\nu}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-1}b_{-1}-\epsilon^{2}_{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}b_{-2}+i\epsilon^{3}_{\mu}(x)\alpha^{\mu}_{-3}b_{-1}+i\epsilon^{4}_{\mu}(x)\alpha^{\mu}_{-2}b_{-2}$$
$$+i\epsilon^{5}_{\mu}(x)\alpha^{\mu}_{-1}b_{-3}+i\epsilon^{6}_{\mu}(x)\alpha^{\mu}_{-1}b_{-2}b_{-1}c_0+\epsilon^{4}(x)b_{-4}+\epsilon^{5}(x)b_{-3}b_{-1}c_0+\epsilon^{6}(x)b_{-2}b_{-1}c_{-1}]|\Omega\rangle. \tag{20}$$

The transformations for the matter part are the following:

$$\delta D_{\mu\nu\alpha\beta}=-\partial_{(\beta}\epsilon^{0}_{\mu\nu\alpha)}-\tfrac{1}{2}\epsilon^{2}_{(\mu\nu}\eta_{\alpha\beta)}, \tag{21a}$$

$$\delta D_{\mu\nu\alpha}=-\partial_{(\mu}\epsilon^{1}_{|\alpha|\nu)}-\partial_{\alpha}\epsilon^{2}_{\nu\mu}+3\epsilon^{0}_{\mu\nu\alpha}-\tfrac{1}{2}\epsilon^{4}_{\alpha}\eta_{\nu\mu}-\epsilon^{5}_{(\mu}\eta_{\nu)\alpha}, \tag{21b}$$

$$\delta D^{1}_{[\mu\nu]}=-\partial_{[\mu}\epsilon^{3}_{\nu]}-\partial_{[\nu}\epsilon^{5}_{\mu]}+2\epsilon^{1}_{[\nu\mu]}, \tag{21c}$$

$$\delta D^{1}_{(\mu\nu)}=-\partial_{(\mu}\epsilon^{3}_{\nu)}-\partial_{(\nu}\epsilon^{5}_{\mu)}+2\epsilon^{1}_{(\nu\mu)}+2\epsilon^{2}_{\mu\nu}-\epsilon^{4}\eta_{\mu\nu}, \tag{21d}$$

$$\delta D^{0}_{\mu\nu}=-\partial_{(\mu}\epsilon^{4}_{\nu)}+\epsilon^{1}_{(\mu\nu)}-\tfrac{1}{2}\epsilon^{4}\eta_{\mu\nu}, \tag{21e}$$

$$\delta D_{\mu}=-\partial_{\mu}\epsilon^{4}+3\epsilon^{3}_{\mu}+2\epsilon^{4}_{\mu}+\epsilon^{5}_{\mu}. \tag{21f}$$

It can be checked from Eqs. (21) that only $D_{\mu\nu\alpha\beta}$ and mixed-symmetric $D_{\mu\nu\alpha}$ cannot be gauged away, which is consistent with the result of the first quantized approach in Sec. II. That is, the spin-two and scalar positive-norm physical propagating modes can be gauged to $D_{\mu\nu\alpha\beta}$ and mixed-symmetric $D_{\mu\nu\alpha}$. In fact, $D_{\mu\nu\alpha}$, $D^{1}_{[\mu\nu]}$, $D^{1}_{(\mu\nu)}$, $D^{0}_{\mu\nu}$, and D_{μ} can be gauged away by $\epsilon^{0}_{\mu\nu\lambda}$, $\epsilon^{1}_{[\mu\nu]}$, $\epsilon^{1}_{(\mu\nu)}$, $\epsilon^{2}_{\mu\nu}$, and one of the vector parameters, say, ${\epsilon_{\mu}}^{3}$. The rest, ϵ^{4}_{μ}, ϵ^{5}_{μ}, and ϵ^{4} are gauge artifacts of $D_{\mu\nu\alpha\beta}$ and mixed-symmetric $D_{\mu\nu\alpha}$.

The transformations for the ghost part are the following:

$$\delta\xi_{\mu\nu\alpha}=\tfrac{1}{2}(\partial^2-6)\epsilon^{0}_{\mu\nu\alpha}-\tfrac{1}{2}\epsilon^{6}_{(\mu}\eta_{\nu\alpha)}, \tag{22a}$$

$$\delta\xi^{0}_{[\mu\nu]}=\tfrac{1}{2}(\partial^2-6)\epsilon^{1}_{[\mu\nu]}-\partial_{[\mu}\epsilon^{6}_{\nu]}, \tag{22b}$$

$$\delta\xi^{0}_{(\mu\nu)}=\tfrac{1}{2}(\partial^2-6)\epsilon^{1}_{(\mu\nu)}-\partial_{(\mu}\epsilon^{6}_{\nu)}+\epsilon^{5}\eta_{\mu\nu}, \tag{22c}$$

$$\delta\xi^{1}_{\mu\nu}=\tfrac{1}{2}(\partial^2-6)\epsilon^{2}_{\mu\nu}+\partial_{(\nu}\epsilon^{6}_{\mu)}, \tag{22d}$$

$$\delta\xi^{2}_{\mu\nu}=-3\partial^{\alpha}\epsilon^{0}_{\mu\nu\alpha}-2\epsilon^{1}_{(\mu\nu)}-3\epsilon^{2}_{\mu\nu}-\tfrac{1}{2}\epsilon^{6}\eta_{\mu\nu}, \tag{22e}$$

$$\delta\xi^{0}_{\mu}=\tfrac{1}{2}(\partial^2-6)\epsilon^{3}_{\mu}-\partial_{\mu}\epsilon^{5}+\epsilon^{6}_{\mu}, \tag{22f}$$

$$\delta\xi^{1}_{\mu}=\tfrac{1}{2}(\partial^2-6)\epsilon^{4}_{\mu}-\epsilon^{6}_{\mu}, \tag{22g}$$

$$\delta\xi^{2}_{\mu}=\tfrac{1}{2}(\partial^2-6)\epsilon^{5}_{\mu}+\partial_{\mu}\epsilon^{5}-\epsilon^{6}_{\mu}, \tag{22h}$$

$$\delta\xi^{3}_{\mu}=-\partial^{\nu}\epsilon^{1}_{\mu\nu}-\partial_{\mu}\epsilon^{6}-3\epsilon^{3}_{\mu}-3\epsilon^{4}_{\mu}, \tag{22i}$$

$$\delta\xi^{4}_{\mu}=2\partial^{\nu}\epsilon^{2}_{\mu\nu}+\partial_{\mu}\epsilon^{6}-2\epsilon^{4}_{\mu}-4\epsilon^{5}_{\mu}-2\epsilon^{6}_{\mu}, \tag{22j}$$

$$\delta\xi^{5}_{\mu}=-2\partial^{\nu}\epsilon^{1}_{\nu\mu}-3\epsilon^{3}_{\mu}-5\epsilon^{5}_{\mu}+4\epsilon^{6}_{\mu}+3{\epsilon_{\mu\nu}}^{0\nu}, \tag{22k}$$

$$\delta\xi^{0}=\tfrac{1}{2}(\partial^2-6)\epsilon^{4}-2\epsilon^{5}, \tag{22l}$$

$$\delta\xi^{1}=-\partial^{\mu}\epsilon^{5}_{\mu}-5\epsilon^{4}-2\epsilon^{5}-\epsilon^{6}, \tag{22m}$$

$$\delta\xi^{2}=-2\partial^{\mu}\epsilon^{4}_{\mu}-6\epsilon^{4}-3\epsilon^{6}+{\epsilon_{\mu}}^{2\mu}, \tag{22n}$$

$$\delta\xi^{3}=-3\partial^{\mu}\epsilon^{3}_{\mu}-7\epsilon^{4}+6\epsilon^{5}+5\epsilon^{6}+2{\epsilon_{\mu}}^{1\mu}, \tag{22o}$$

$$\delta\xi^{4}=\tfrac{1}{2}(\partial^2-6)\epsilon^{6}+\partial^{\mu}\epsilon^{6}_{\mu}+4\epsilon^{5}. \tag{22p}$$

There are nine on-mass-shell conditions, which contains a symmetric spin-three, an antisymmetric spin-two, two symmetric spin-two, three vectors and two scalar fields, and seven gauge conditions, which amount to 16 equations in Eq. (22). This is consistent with counting from zero-norm states listed in the table. Three traceless conditions read from zero-norm states correspond to the three equations involving ${\delta\xi_{\mu\nu}}^{\nu}$, ${\delta\xi_{\mu}^{0}}^{\mu}$, ${\delta\xi_{\mu}^{1}}^{\mu}$, which are contained in Eqs. (22a), (22c), and (22d).

It is important to note that the transformation for the matter parts, Eqs. (18a)–(18d) and Eqs. (21a)–(21f), are the same as the calculation [5] based on the chordal gauge transformation of free covariant string field theory constructed by Banks and Peskin [10]. The chordal gauge transformation can be written in the following form:

$$\delta\Phi[X(\sigma)] = \sum_{n>0} L_{-n}\Phi_n[X(\sigma)], \tag{23}$$

where $\Phi[X(\sigma)]$ is the string field and $\Phi_n[X(\sigma)]$ are gauge parameters, which are functions of $X[\sigma]$ only and free of ghost fields. This is because the pure ghost part of Q_B in Eq. (15) does not contribute to the transformation of matter background fields. It is interesting to note that the rhs of Eq. (23) is in the form of off-shell spurious states [11] in the OCFQ approach. They become zero-norm states on imposing the physical and on-shell state condition.

Finally, it can be shown that the number of scalar zero-norm states at nth massive level ($n \geqslant 3$) is at least the sum of those at $(n-2)$th and $(n-1)$th massive levels. So positive-norm scalar modes at nth level, if they exist, will be decoupled according to our decoupling conjecture. The decoupling of these scalars has important implication on Sen's conjectures on the decay of open string tachyon. Since all scalars on D-brane including tachyon get a nonzero VEV in the false vacuum, they will decay together with tachyon and disappear eventually to the true closed string vacuum. As the scalar states together with the higher tensor states form a large gauge multiplet at each mass level, and its scattering amplitudes are fixed by the tensor fields, these tensor fields of open string (D25-brane) will accompany the decay process. This means that the whole D-brane could disappear to the true closed string vacuum! The mechanism could provide a hint to solve the so-called $U(1)$ problem [19] in Sen's conjectures. A further study is in progress.

IV. THE FOURTH MASSIVE LEVEL

We will use both the first and the second quantized approaches to test the decoupling conjecture for the fourth massive level $m^2 = 8$.

A. The first quantized calculation

The positive-norm physical propagating fields can be found in Ref. [20]. Their Young tabulations are the following:

$$\ydiagram{5}\,,\ \ydiagram{3,1}\,,\ \ydiagram{2,1}\,,\ \ydiagram{3}\,,\ \ydiagram{1,1}\,,\ \ydiagram{1}\,. \tag{24}$$

The Young tabulations of zero-norm states can then be shown to be

$$\ydiagram{4}\,,\ \ydiagram{2,1}\,,\ 2\times\ydiagram{3}\,,\ 2\times\ydiagram{1,1}\,,\ 4\times\ydiagram{2}\,,\ 5\times\ydiagram{1}\,,\ 3\times\bullet. \tag{25}$$

Note that the two representations $\ydiagram{2,1}$ in Eq. (24) and $\ydiagram{2,1}\,'$ in Eq. (25) are different. One corresponds to $\alpha_{-1}^{\mu}\alpha_{-2}^{\nu}\alpha_{-2}^{\lambda}$ and the other corresponds to $\alpha_{-1}^{\mu}\alpha_{-1}^{\nu}\alpha_{-3}^{\lambda}$ or vice versa. So, one expects that the last three states in Eq. (24) can be gauged to the higher rank fields. The most general worldsheet coupling consistent with vertex operator consideration is

$$\begin{aligned}
T(\tau) = & -\tfrac{1}{2}\eta_{\mu\nu}\partial_\tau X^\mu \partial_\tau X^\nu \\
& + E_{\mu\nu\lambda\alpha\beta}\partial_\tau X^\mu \partial_\tau X^\nu \partial_\tau X^\lambda \partial_\tau X^\alpha \partial_\tau X^\beta \\
& + E_{\mu\nu\lambda\alpha}\partial_\tau X^\mu \partial_\tau X^\nu \partial_\tau X^\lambda \partial_\tau^2 X^\alpha \\
& + E^0_{\mu\nu\lambda}\partial_\tau X^\mu \partial_\tau X^\nu \partial_\tau^3 X^\lambda + E^1_{\mu\nu\lambda}\partial_\tau X^\mu \partial_\tau^2 X^\nu \partial_\tau^2 X^\lambda \\
& + E^0_{\mu\nu}\partial_\tau X^\mu \partial_\tau^4 X^\nu + E^1_{\mu\nu}\partial_\tau^2 X^\mu \partial_\tau^3 X^\nu + E_\mu \partial_\tau^5 X^\mu .
\end{aligned} \tag{26}$$

After a lengthy calculation, the condition to cancel all worldsheet q number anomalies are as follows:

$$5\,\partial^\mu E_{\mu\nu\lambda\alpha\beta} - 2E_{(\nu\lambda\alpha\beta)} = 0, \tag{27a}$$

$$\partial^\mu E^0_{\mu\nu} - 20E_\nu = 0, \tag{27b}$$

$$\partial^\mu E_{\mu\nu\lambda\alpha} - 12E^0_{\nu\lambda\alpha} - 8E^1_{\nu\lambda\alpha} = 0, \tag{27c}$$

$$\partial^\mu E^0_{\mu\nu\lambda} - 6E^0_{\nu\lambda} - E^1_{\nu\lambda} = 0, \tag{27d}$$

$$\partial^\mu E^1_{\mu\nu\lambda} - 6E^1_{(\nu\lambda)} = 0, \tag{27e}$$

$$20E^{\mu}{}_{\mu\nu\lambda\alpha} + \partial^\mu E_{\nu\lambda\alpha\mu} - 12E^0_{(\nu\lambda\alpha)} = 0, \tag{28a}$$

$$E^{0\mu}{}_{\mu\nu} + 4\,\partial^\mu E^1_{\mu\nu} - 120E_\nu = 0, \tag{28b}$$

$$E^{\mu}{}_{\mu\nu\lambda} + 8\,\partial^\mu E^1_{\nu\lambda\mu} - 48E^0_{\nu\lambda} - 12E^1_{\lambda\nu} = 0, \tag{28c}$$

$$E^{\mu}{}_{\nu\lambda\mu} + \partial^\mu E^0_{\nu\lambda\mu} - 4E^0_{(\nu\lambda)} = 0, \tag{29a}$$

$$E^{1\mu}{}_{\mu\nu} + 12\partial^\mu E^1_{\nu\mu} - 240E_\nu = 0, \tag{29b}$$

$$3E^{0\mu}{}_{\nu\mu} + E^{1}{}_{\nu\mu}{}^{\mu} + 6\,\partial^\mu E^0_{\nu\mu} - 30E_\nu = 0, \tag{30}$$

$$2E^{0\mu}{}_{\mu} + E^{1}{}_{\mu}{}^{\mu} + 10\partial^\mu E_\mu = 0, \tag{31}$$

$$(\partial^2 - 8)\phi = 0. \tag{32}$$

Here, ϕ again represents all background fields introduced in Eq. (26). Equations (27a)–(27e) are extracted from $1/(\tau - \tau')^3$ anomalous terms in the operator product calculation; similarly, Eqs. (28a)–(28c), (29a)–(29b), (30), and (31) are extracted from $1/(\tau-\tau')^4$, $1/(\tau-\tau')^5$, $1/(\tau-\tau')^6$, and $1/(\tau-\tau')^7$ anomalous terms, respectively. It can be carefully checked, as one did for the third massive level, that only $E_{\mu\nu\lambda\alpha\beta}$ and mixed-symmetric $E_{\mu\nu\lambda\alpha}$ and $E^1_{\mu\nu\lambda}$ (or $E^0_{\mu\nu\lambda}$) corresponding to the first three Young representations in Eq. (24) are dynamically independent as the conjecture has claimed. The last three states in Eq. (24) again can be gauged to the first three states due to the existence of zero-norm states with the same Young representations in Eq. (25).

B. WSFT calculation

Φ and Λ can be expanded at this massive level as

$$\begin{aligned}\Phi_5=[&iE_{\mu\nu\lambda\alpha\beta}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}a^{\lambda}_{-1}a^{\beta}_{-1}+E_{\mu\nu\alpha\beta}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}a^{\beta}_{-2}-iE^{0}_{\mu\nu\alpha}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-31}-iE^{1}_{\mu\nu\alpha}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-2}\alpha^{\alpha}_{-2}\\
&-E^{0}_{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-4}-E^{1}_{\mu\nu}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-3}+iE_{\mu}(x)\alpha^{\mu}_{-5}+\zeta_{\mu\nu\alpha\beta}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}\alpha^{\beta}_{-1}b_{-1}c_{0}-i\zeta^{0}_{\mu\nu\alpha}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}b_{-1}c_{0}\\
&-i\zeta^{1}_{\mu\nu\alpha}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}b_{-2}c_{0}-i\zeta^{2}_{\mu\nu\alpha}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}b_{-1}c_{-1}-\zeta^{0}_{\mu\nu}(x)\alpha^{\mu}_{-3}\alpha^{\nu}_{-1}b_{-1}c_{0}-\zeta^{1}_{\mu\nu}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-2}b_{-1}c_{0}\\
&-\zeta^{2}_{\mu\nu}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-1}b_{-2}c_{0}-\zeta^{3}_{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}b_{-3}c_{0}-\zeta^{4}_{\mu\nu}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-1}b_{-1}c_{-1}-\zeta^{5}_{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}b_{-2}c_{-1}\\
&-\zeta^{6}_{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}b_{-1}c_{-2}+i\zeta^{0}_{\mu}(x)\alpha^{\mu}_{-4}b_{-1}c_{0}+i\zeta^{1}_{\mu}(x)\alpha^{\mu}_{-3}b_{-2}c_{0}+i\zeta^{2}_{\mu}(x)\alpha^{\mu}_{-2}b_{-3}c_{0}+i\zeta^{3}_{\mu}(x)\alpha^{\mu}_{-1}b_{-4}c_{0}\\
&+i\zeta^{4}_{\mu}(x)\alpha^{\mu}_{-3}b_{-1}c_{-1}+i\zeta^{5}_{\mu}(x)\alpha^{\mu}_{-2}b_{-2}c_{-1}+i\zeta^{6}_{\mu}(x)\alpha^{\mu}_{-1}b_{-3}c_{-1}+i\zeta^{7}_{\mu}(x)\alpha^{\mu}_{-2}b_{-1}c_{-2}+i\zeta^{8}_{\mu}(x)\alpha^{\mu}_{-1}b_{-2}c_{-2}\\
&+i\zeta^{9}_{\mu}(x)\alpha^{\mu}_{-1}b_{-1}c_{-3}+i\zeta^{10}_{\mu}(x)\alpha^{\mu}_{-1}b_{-2}b_{-1}c_{-1}c_{0}+\zeta^{0}(x)b_{-5}c_{0}+\zeta^{1}(x)b_{-4}c_{-1}+\zeta^{2}(x)b_{-3}c_{-2}+\zeta^{3}(x)b_{-2}c_{-3}\\
&+\zeta^{4}(x)b_{-1}c_{-4}+\zeta^{5}(x)b_{-3}b_{-1}c_{-1}c_{0}+\zeta^{6}(x)b_{-2}b_{-1}c_{-2}c_{0}]c_{1}|k\rangle,\end{aligned} \tag{33}$$

$$\begin{aligned}\Lambda_5=[&\epsilon^{0}_{\mu\nu\alpha\beta}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}\alpha^{\beta}_{-1}b_{-1}-i\epsilon^{1}_{\mu\nu\alpha}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}b_{-1}-i\epsilon^{2}_{\mu\nu\alpha}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}\alpha^{\alpha}_{-1}b_{-2}-\epsilon^{3}_{\mu\nu}(x)\alpha^{\mu}_{-3}\alpha^{\nu}_{-1}b_{-1}\\
&-\epsilon^{4}_{\mu\nu}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-2}b_{-1}-\epsilon^{5}_{\mu\nu}(x)\alpha^{\mu}_{-2}\alpha^{\nu}_{-1}b_{-2}-\epsilon^{6}_{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}b_{-3}-\epsilon^{7}_{\mu\nu}(x)\alpha^{\mu}_{-1}\alpha^{\nu}_{-1}b_{-2}b_{-1}c_{0}+i\epsilon^{7}_{\mu}(x)\alpha^{\mu}_{-4}b_{-1}\\
&+i\epsilon^{8}_{\mu}(x)\alpha^{\mu}_{-3}b_{-2}+i\epsilon^{9}_{\mu}(x)\alpha^{\mu}_{-2}b_{-3}+i\epsilon^{10}_{\mu}(x)\alpha^{\mu}_{-1}b_{-4}+i\epsilon^{11}_{\mu}(x)\alpha^{\mu}_{-2}b_{-2}b_{-1}c_{0}+i\epsilon^{12}_{\mu}(x)\alpha^{\mu}_{-1}b_{-3}b_{-1}c_{0}\\
&+i\epsilon^{13}_{\mu}(x)\alpha^{\mu}_{-1}b_{-2}b_{-1}c_{-1}+\epsilon^{7}(x)b_{-5}+\epsilon^{8}(x)b_{-4}b_{-1}c_{0}+\epsilon^{9}(x)b_{-3}b_{-2}c_{0}+\epsilon^{10}(x)b_{-3}b_{-1}c_{-1}\\
&+\epsilon^{11}(x)b_{-2}b_{-1}c_{-2}]|\Omega\rangle.\end{aligned} \tag{34}$$

The transformations for the matter part are

$$\delta E_{\mu\nu\lambda\alpha\beta}=-\partial_{(\beta}\epsilon^{0}_{\mu\nu\lambda\alpha)}+\tfrac{1}{2}\epsilon^{2}_{(\lambda\alpha\beta}\eta_{\mu\nu)}, \tag{35a}$$

$$\begin{aligned}\delta E_{\mu\nu\alpha\beta}=&-\partial_{(\mu}\epsilon^{1}_{|\beta|\alpha\nu)}-\partial_{\beta}\epsilon^{2}_{\alpha\mu\nu}+4\epsilon^{0}_{\mu\nu\alpha\beta}-\tfrac{1}{2}\epsilon^{5}_{\beta(\nu}\eta_{\alpha\mu)}\\&-\epsilon^{6}_{(\alpha\mu}\eta_{\nu)\beta},\end{aligned} \tag{35b}$$

$$\begin{aligned}\delta E^{0}_{\mu\nu\alpha}=&-\partial_{(\mu}\epsilon^{3}_{|\alpha|\nu)}-\partial_{\alpha}\epsilon^{6}_{\nu\mu}+2\epsilon^{1}_{\alpha\nu\mu}+3\epsilon^{2}_{\alpha\nu\mu}-\tfrac{1}{2}\epsilon^{7}_{\alpha}\eta_{\nu\mu}\\&-\epsilon^{9}_{(\mu}\eta_{\nu)\alpha},\end{aligned} \tag{35c}$$

$$\begin{aligned}\delta E^{1}_{\mu\nu\alpha}=&-\partial_{\mu}\epsilon^{4}_{\nu\alpha}-\partial_{(\alpha}\epsilon^{5}_{\nu)\mu}+2\epsilon^{1}_{(\alpha\nu)\mu}-\epsilon^{8}_{(\alpha}\eta_{\nu)\mu}\\&-\tfrac{1}{2}\epsilon^{9}_{\mu}\eta_{\nu\alpha},\end{aligned} \tag{35d}$$

$$\delta E^{0}_{[\mu\nu]}=-\partial_{[\mu}\epsilon^{7}_{\nu]}-\partial_{[\nu}\epsilon^{9}_{\mu]}+3\epsilon^{3}_{[\nu\mu]}+2\epsilon^{5}_{[\nu\mu]}, \tag{35e}$$

$$\delta E^{1}_{[\mu\nu]}=-\partial_{[\mu}\epsilon^{7}_{\nu]}-\partial_{[\nu}\epsilon^{8}_{\mu]}+\epsilon^{3}_{[\nu\mu]}+\epsilon^{5}_{[\mu\nu]}, \tag{35f}$$

$$\begin{aligned}\delta E^{0}_{(\mu\nu)}=&-\partial_{(\mu}\epsilon^{7}_{\nu)}-\partial_{(\nu}\epsilon^{9}_{\mu)}+3\epsilon^{3}_{(\nu\mu)}+2\epsilon^{5}_{(\nu\mu)}+2\epsilon^{6}_{\nu\mu}\\&-\epsilon^{7}\eta_{\mu\nu},\end{aligned} \tag{35g}$$

$$\begin{aligned}\delta E^{1}_{(\mu\nu)}=&-\partial_{(\mu}\epsilon^{7}_{\nu)}-\partial_{(\nu}\epsilon^{8}_{\mu)}+\epsilon^{3}_{(\nu\mu)}+2\epsilon^{4}_{\nu\mu}+\epsilon^{5}_{(\mu\nu)}\\&-\epsilon^{7}\eta_{\mu\nu},\end{aligned} \tag{35h}$$

$$\delta E_{\mu}=-\partial_{\mu}\epsilon^{7}+7\epsilon^{7}_{\mu}+2\epsilon^{8}_{\mu}+\epsilon^{9}_{\mu}. \tag{35i}$$

Again these are the same as the calculation by Eq. (23). All background fields except $E_{\mu\nu\lambda\alpha\beta}$ and mixed-symmetric $E_{\mu\nu\lambda\alpha}$ and $E^{1}_{\mu\nu\lambda}$ (or $E^{0}_{\mu\nu\lambda}$) can be either gauged away or gauged to $E_{\mu\nu\lambda\alpha\beta}$, $E_{\mu\nu\lambda\alpha}$, and $E^{1}_{\mu\nu\lambda}$ (or $E^{0}_{\mu\nu\lambda}$) by zero-norm states. This is consistent with the result of the first quantized approach presented in Sec. IV A. The transformation for the ghost part is very lengthy and is given in the Appendix. There are 18 on-mass-shell conditions, which contain a spin-four, a mixed-symmetric spin-three, two symmetric spin-three, two antisymmetric spin-two, four symmetric spin-two, five vector and three scalar fields, and 15 gauge conditions. It is again consistent with counting the number of zero-norm states listed in Eq. (25).

V. CONCLUSION

We have explicitly shown that the degenerate positive-norm states at the third and fourth massive levels of bosonic open string theory can be gauged to the higher rank fields at the same mass level. This means that the scattering amplitudes of these degenerate positive-norm states can be expressed in terms of those of higher spin states at the same mass level through massive Ward identities. This is demonstrated by using both the OCFQ string and WSFT. We have compared the on-shell conditions of zero-norm states in the OCFQ stringy gauge transformation to the background ghost fields in off-shell gauge transformation of WSFT. This important observation makes the lengthy calculations in both the first and the second quantized approaches controllable and more importantly provides a double consistency check of our results. The interesting stringy behaviors discussed in this paper and those in Refs. [1,2] seem to imply that there must exist enormous exotic high-energy properties of string theory, which remained to be uncovered. One interesting application of the decoupling of higher scalar modes is the decay of tensor fields on D-brane into the true closed string vacuum in Sen's conjectures.

It is straightforward to generalize our calculation to closed string theory for the first quantized approach presented in Secs. II and IV A. Another way to generalize to the closed string case is to make use of the simple relation between closed and open string amplitudes in Ref. [21]. A reliable second quantized closed string field theory may help uncover more high-energy stringy properties.

ACKNOWLEDGMENTS

The authors would like to thank Pei-ming Ho for stimulating discussions, which help clarify some of the points in the paper. The authors also thank Miao Li for helpful conversations. This work was supported in part by the National Science Council, the Center for Theoretical Physics at National Taiwan University, the National Center for Theoretical Sciences, and the CosPA project of the Ministry of Education, Taiwan, Republic of China.

APPENDIX

Gauge transformation for background ghost fields of the fourth massive level are as follows:

$$\delta\zeta_{\mu\nu\alpha\beta}=\tfrac{1}{2}(\partial^2-8)\epsilon^0_{\mu\nu\alpha\beta}-\tfrac{1}{2}\epsilon^7_{(\mu\nu}\eta_{\alpha\beta)}, \tag{A1}$$

$$\delta\zeta^0_{\mu\nu\alpha}=\tfrac{1}{2}(\partial^2-8)\epsilon^1_{\mu\nu\alpha}-\partial_\mu\epsilon^7_{\nu\alpha}-\tfrac{1}{2}\epsilon^{11}_\mu\eta_{\nu\alpha}-\epsilon^{12}_{(\nu}\eta_{\alpha)\mu}, \tag{A2}$$

$$\delta\zeta^1_{\mu\nu\alpha}=\tfrac{1}{2}(\partial^2-8)\epsilon^2_{\mu\nu\alpha}+\partial_{(\mu}\epsilon^7_{\nu\alpha)}, \tag{A3}$$

$$\delta\zeta^2_{\mu\nu\alpha}=-4\partial^\beta\epsilon^0_{\mu\nu\alpha\beta}-2\epsilon^1_{(\mu\nu\alpha)}-3\epsilon^2_{\mu\nu\alpha}-\tfrac{1}{2}\epsilon^{13}_{(\mu}\eta_{\nu\alpha)}, \tag{A4}$$

$$\delta\zeta^0_{[\mu\nu]}=\tfrac{1}{2}(\partial^2-8)\epsilon^3_{[\mu\nu]}-\partial_{[\mu}\epsilon^{12}_{\nu]}, \tag{A5}$$

$$\delta\zeta^2_{[\mu\nu]}=\tfrac{1}{2}(\partial^2-8)\epsilon^5_{[\mu\nu]}+\partial_{[\nu}\epsilon^{11}_{\mu]}, \tag{A6}$$

$$\delta\zeta^4_{[\mu\nu]}=-2\partial^\alpha\epsilon^1_{[\mu\nu]\alpha}-\partial_{[\mu}\epsilon^{13}_{\nu]}-3\epsilon^3_{[\mu\nu]}-3\epsilon^5_{[\mu\nu]}, \tag{A7}$$

$$\delta\zeta^0_{(\mu\nu)}=\tfrac{1}{2}(\partial^2-8)\epsilon^3_{(\mu\nu)}-\partial_{(\mu}\epsilon^{12}_{\nu)}+2\epsilon^7_{\mu\nu}-\epsilon^8\eta_{\mu\nu}, \tag{A8}$$

$$\delta\zeta^1_{\mu\nu}=\tfrac{1}{2}(\partial^2-8)\epsilon^4_{\mu\nu}-\partial_{(\mu}\epsilon^{11}_{\nu)}-\tfrac{1}{2}\epsilon^8\eta_{\alpha\beta}, \tag{A9}$$

$$\delta\zeta^2_{(\mu\nu)}=\tfrac{1}{2}(\partial^2-8)\epsilon^5_{(\mu\nu)}+\partial_{(\nu}\epsilon^{11}_{\mu)}-2\epsilon^7_{\mu\nu}-\epsilon^9\eta_{\mu\nu}, \tag{A10}$$

$$\delta\zeta^3_{\mu\nu}=\tfrac{1}{2}(\partial^2-8)\epsilon^6_{\mu\nu}+\partial_{(\mu}\epsilon^{12}_{\nu)}-\epsilon^7_{\mu\nu}+\tfrac{1}{2}\epsilon^9\eta_{\mu\nu}, \tag{A11}$$

$$\delta\zeta^4_{(\mu\nu)}=-2\partial^\alpha\epsilon^1_{(\mu\nu)\alpha}-\partial_{(\mu}\epsilon^{13}_{\nu)}-3\epsilon^3_{(\mu\nu)}-4\epsilon^4_{\mu\nu}-3\epsilon^5_{(\mu\nu)}-\epsilon^{10}\eta_{\mu\nu}, \tag{A12}$$

$$\delta\zeta^5_{\mu\nu}=-3\partial^\alpha\epsilon^2_{\mu\nu\alpha}+\partial_{(\mu}\epsilon^{13}_{\nu)}-2\epsilon^5_{(\mu\nu)}-4\epsilon^6_{\mu\nu}-2\epsilon^7_{\mu\nu}, \tag{A13}$$

$$\delta\zeta^6_{\mu\nu}=-2\partial^\alpha\epsilon^1_{\alpha\mu\nu}+6\epsilon^0_{\mu\nu\alpha\beta}\eta^{\alpha\beta}-3\epsilon^3_{(\mu\nu)}-5\epsilon^6_{\mu\nu}+4\epsilon^7_{\mu\nu}-\tfrac{1}{2}\epsilon^{11}\eta_{\mu\nu}, \tag{A14}$$

$$\delta\zeta^0_\mu=\tfrac{1}{2}(\partial^2-8)\epsilon^7_\mu-\partial_\mu\epsilon^8+2\epsilon^{11}_\mu+\epsilon^{12}_\mu, \tag{A15}$$

$$\delta\zeta^1_\mu=\tfrac{1}{2}(\partial^2-8)\epsilon^8_\mu-\partial_\mu\epsilon^9-2\epsilon^{11}_\mu, \tag{A16}$$

$$\delta\zeta^2_\mu=\tfrac{1}{2}(\partial^2-8)\epsilon^9_\mu+\partial_\mu\epsilon^9-\epsilon^{12}_\mu, \tag{A17}$$

$$\delta\zeta^3_\mu=\tfrac{1}{2}(\partial^2-8)\epsilon^{10}_\mu+\partial_\mu\epsilon^8-2\epsilon^{12}_\mu, \tag{A18}$$

$$\delta\zeta^4_\mu=-\partial^\nu\epsilon^3_{\mu\nu}-\partial_\mu\epsilon^{10}-4\epsilon^7_\mu-3\epsilon^8_\mu+\epsilon^{13}_\mu, \tag{A19}$$

$$\delta\zeta^5_\mu=-\partial^\nu\epsilon^5_{\mu\nu}-3\epsilon^8_\mu-4\epsilon^9_\mu-2\epsilon^{11}_\mu-\epsilon^{13}_\mu, \tag{A20}$$

$$\delta\zeta^6_\mu=-2\partial^\nu\epsilon^6_{\mu\nu}+\partial_\mu\epsilon^{10}-2\epsilon^9_\mu-5\epsilon^{10}_\mu-2\epsilon^{12}_\mu-\epsilon^{13}_\mu, \tag{A21}$$

$$\delta\zeta^7_\mu=-4\partial^\nu\epsilon^4_{\mu\nu}-\partial_\mu\epsilon^{11}-4\epsilon^7_\mu-5\epsilon^9_\mu+4\epsilon^{11}_\mu+\epsilon^1_{\mu\nu\alpha}\eta^{\nu\alpha}, \tag{A22}$$

$$\delta\zeta^8_\mu=-2\partial^\nu\epsilon^5_{\nu\mu}+\partial_\mu\epsilon^{11}-3\epsilon^8_\mu-6\epsilon^{10}_\mu-3\epsilon^{13}_\mu+3\epsilon^2_{\mu\nu\alpha}\eta^{\nu\alpha}, \tag{A23}$$

$$\delta\zeta^9_\mu=-3\partial^\nu\epsilon^3_{\mu\nu}-4\epsilon^7_\mu-7\epsilon^{10}_\mu+6\epsilon^{12}_\mu+5\epsilon^{13}_\mu+4\epsilon^1_{\nu\alpha\mu}\eta^{\nu\alpha}, \tag{A24}$$

$$\delta\zeta^{10}_\mu=\tfrac{1}{2}(\partial^2-8)\epsilon^{13}_\mu+2\partial^\nu\epsilon^7_{\mu\nu}+2\epsilon^{11}_\mu+4\epsilon^{12}_\mu, \tag{A25}$$

$$\delta\zeta^0=\tfrac{1}{2}(\partial^2-8)\epsilon^7-3\epsilon^8-\epsilon^9, \tag{A26}$$

$$\delta\zeta^1=-\partial^\mu\epsilon^{10}_\mu-6\epsilon^7-2\epsilon^8-2\epsilon^{10}, \tag{A27}$$

$$\delta\zeta^2=-\partial^\mu\epsilon^9_\mu-7\epsilon^7-4\epsilon^9-3\epsilon^{10}-\epsilon^{11}+\epsilon^6_{\mu\nu}\eta^{\mu\nu}, \tag{A28}$$

$$\delta\zeta^3=-3\partial^\mu\epsilon^8_\mu-8\epsilon^7+6\epsilon^9-4\epsilon^{11}+2\epsilon^5_{\mu\nu}\eta^{\mu\nu}, \tag{A29}$$

$$\delta\zeta^4=-4\partial^\mu\epsilon^7_\mu-9\epsilon^7+8\epsilon^8+7\epsilon^{10}+6\epsilon^{11}+3\epsilon^3_{\mu\nu}\eta^{\mu\nu}+4\epsilon^4_{\mu\nu}\eta^{\mu\nu}, \tag{A30}$$

$$\delta\zeta^5=\tfrac{1}{2}(\partial^2-8)\epsilon^{10}+\partial^\mu\epsilon^{12}_\mu+5\epsilon^8+3\epsilon^9, \tag{A31}$$

$$\delta\zeta^6=\tfrac{1}{2}(\partial^2-8)\epsilon^{11}+2\partial^\mu\epsilon^{11}_\mu+6\epsilon^8-5\epsilon^9-\epsilon^7_{\mu\nu}\eta^{\mu\nu}. \tag{A32}$$

There are 18 on-mass-shell conditions and 15 gauge conditions in Eqs. (A1)–(A32), which are consistent with counting from number of zero-norm states listed in Eq. (25). Note that there are two irreducible components in Eq. (A2).

[1] D.J. Gross, Phys. Rev. Lett. **60**, 1229 (1988); J.J. Atick and E. Witten, Nucl. Phys. **B310**, 291 (1988); R. Hagedorn, Nuovo Cimento A **56**, 1027 (1968).
[2] D.J. Gross and P. Mende, Phys. Lett. B **197**, 129 (1987); Nucl. Phys. **B303**, 407 (1988).
[3] J.C. Lee, Phys. Lett. B **241**, 336 (1990); J.C. Lee and B. Ovrut, Nucl. Phys. **B336**, 222 (1990).
[4] J.C. Lee, Phys. Rev. Lett. **64**, 1636 (1990).
[5] J.C. Lee, Z. Phys. C **63**, 351 (1994).
[6] E. Witten, Nucl. Phys. **B268**, 253 (1986).
[7] A. Sen, Int. J. Mod. Phys. A **14**, 4061 (1999).
[8] V.A. Kostelecky and S. Samuel, Phys. Lett. B **207**, 169 (1988); Nucl. Phys. **B336**, 263 (1990).
[9] Ian Ellwood and Washington Taylor, ‘‘Gauge invariance and tachyon condensation in open string field theory,’’ hep-th/0105156.
[10] T. Banks and M. Peskin, Nucl. Phys. **B264**, 513 (1986).
[11] M.B. Green, J.H. Schwarz, and E. Witten, *Superstring Theory* (Cambridge University Press, Cambridge, England, 1987), Vol. I.
[12] I.L. Buchbinder, V.A. Krykhtin, and V.D. Pershin, Phys. Lett. B **348**, 68 (1995); I.L. Buchbinder, E.S. Fradkin, S.L. Lyakhovich, and V.D. Pershin, *ibid.* **304**, 239 (1993).
[13] J.M.F. Labastida and Maria A.H. Vozmediano, Nucl. Phys. **B312**, 308 (1989).
[14] J.C. Lee, Prog. Theor. Phys. **91**, 353 (1994).
[15] S.R. Das and B. Sathiapalan, Phys. Rev. Lett. **56**, 2664 (1986); **57**, 1511 (1986).
[16] S. Weinberg, Phys. Lett. **156B**, 309 (1985); Ryu Sasaki and Itaru Yamanaka, *ibid.* **165B**, 283 (1985).
[17] S. Giddings, Nucl. Phys. **B278**, 242 (1986); R. Bluhm and S. Samuel, *ibid.* **B323**, 337 (1989).
[18] M. Spradlin and A. Volovich, Phys. Rev. D **66**, 086004 (2002).
[19] E. Witten, J. High Energy Phys. **19**, 9812 (1998); A. Sen, *ibid.* **27**, 9912 (1999).
[20] J.L. Manes and M.A.H. Vozmediano, Nucl. Phys. **B326**, 271 (1989).
[21] H. Kawai, D.C. Lewellen, and S.-H.H. Tye, Nucl. Phys. **B269**, 1 (1986).