Adaptive Fuzzy Sliding Mode Controller for Linear Systems With Mismatched Time-Varying Uncertainties

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***Abstract*—A new design approach of an adaptive fuzzy sliding mode controller (AFSMC) for linear systems with mismatched time-varying uncertainties is presented in this paper. The coefficient matrix of the sliding function can be designed to satisfy a sliding coefficient matching condition provided time-varying uncertainties are bounded. With the sliding coefficient matching condition satisfied, an AFSMC is proposed to stabilize the uncertain system. The parameters of the output fuzzy sets in the fuzzy mechanism are on-line adapted to improve the performance of the fuzzy sliding mode control system. The bounds of the uncertainties are not required to be known in advance for the presented AFSMC. The stability of the fuzzy control system is guaranteed and the system is shown to be invariant on the sliding surface. Moreover, the chattering around the sliding surface in the sliding mode control can be reduced by the proposed design approach. Simulation results are included to illustrate the effectiveness of the proposed AFSMC.**

I. Introduction

SINCE the time-varying uncertainties (structured or unstructured) are inevitable in many practical linear systems, the control of the linear systems with time-varying uncertainties has been an important research topic in the engineering area [15], [19]. In the past several decades, the variable structure with sliding mode has been effectively applied to control the systems with uncertainties because of the intrinsic nature of robustness of the variable structure with sliding mode [2], [14], [18], [22]. When the system reaches the sliding mode, the system with variable structure control is insensitive to the external disturbances and the variations of the plant parameters [16]. Moreover, the variable structure system can be invariant to the uncertainties in many cases [10]. However, the sliding mode control suffers from the problem of chattering, which is caused by the high-speed switching of the controller output in order to establish a sliding mode. The undesirable chattering may excite the high-frequency system response [3], [7] and result in unpredictable instabilities. Furthermore, most of the uncertain systems with the traditional sliding mode control techniques require that the uncertainties satisfy the matching conditions. This would limit the application of the sliding mode control.

The fuzzy techniques [6] have been widely applied to the systems with uncertainties. Recently, researchers have utilized the fuzzy techniques together with the sliding mode control for many engineering control systems. Hwang *et al.* designed a fuzzy controlled low-pass filter to smooth the output from a sliding mode controller (SMC) [12]. In [21], the sliding mode control schemes with fuzzy system are proposed with the uncertain system function is approximated by the fuzzy system . The fuzzy control rules based on the sliding function and the derivative of the sliding function are constructed in [11]. It can be seen that fuzzy techniques have been incorporated with the sliding mode control as the fuzzy sliding mode control to alleviate the chattering in the pure sliding mode control [8], [13], [17], [20]. Also, the utilization of the fuzzy techniques can release the limitation on the known bounds of uncertainties which is required for the traditional SMC [5]. However, the determination of the parameters in the fuzzy SMC is not trivial. The complexity of the fuzzy SMC is increased quickly as the number of sliding functions increases.

In this paper, a new adaptive fuzzy sliding mode controller (AFSMC) for linear systems with mismatched time-varying uncertainties is proposed. The coefficient matrix of the sliding function can be designed to satisfy a sliding coefficient matching condition [1] provided time-varying uncertainties are bounded. With the sliding coefficient matching condition satisfied, a stability guaranteed SMC can be constructed if the necessary information of the uncertainties is assumed to be available. Since the assumptions may be too limited in the real application, a fuzzy SMC is proposed based on the analysis of the system characteristics to release some of the limitations. The parameters of the output fuzzy sets in the fuzzy SMC are on-line adapted with the approach in [9] extended to minimize the decreasing rate of the square of sliding function for the case with multi-dimensional sliding function vectors. The on-line parameter adaptive process simplifies the design of the fuzzy sliding mode control system. The stability of the fuzzy control system is guaranteed and the system is shown to be invariant on the sliding surface. Moreover, the chattering around the sliding surface in the sliding mode control can be reduced by the proposed design approach. Simulation results are included to illustrate the effectiveness of the proposed AFSMC.

The remainder of this paper is organized as follows. The system model considered in this paper and the sliding coef-

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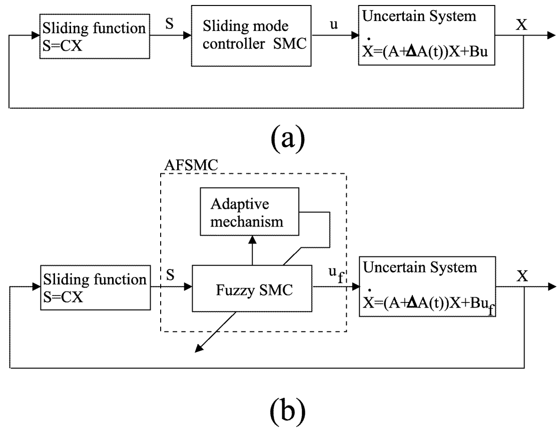

Fig. 1. Block diagrams of (a) the sliding mode control system and (b) the adaptive fuzzy sliding mode control system.

ficient matching condition are described in Section II. The reaching mode of the SMC is discussed in Section III. The AFSMC is designed in Section IV. Also in Section IV, the sliding mode of the proposed AFSMC is guaranteed. Moreover, the characteristics of the adaptive fuzzy sliding mode control system on the sliding surface are also described in Section IV. In Section V, simulation results of the illustrative examples are presented. Finally, conclusions are presented in Section VI.

II. The System Model Description

In this section, the uncertain linear system with the mismatched uncertainties satisfying the sliding coefficient matching condition [1] is described. Let the state equation of the linear system with the only mismatched time-varying uncertainties [4], $\Delta A(t)$ be

$$\dot{x}(t) = (A + \Delta A(t))x(t) + Bu(t) \tag{1}$$

where the state vector is $x(t) \in R^n$ and the control input vector is $u(t) \in R^m$. The constant matrices A, B are assumed to be known with proper dimensions. Furthermore, it is assumed that (A, B) is controllable and B has full rank. With a nonsingular state transformation matrix T, (1) is transformed into the regular form

$$\begin{aligned}\dot{z}(t) &= \begin{bmatrix} \dot{z}_1(t) \\ \dot{z}_2(t) \end{bmatrix} = T\dot{x}(t) \\ &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} + \begin{bmatrix} \Delta A_1(t) \\ \Delta A_2(t) \end{bmatrix} \\ &\quad \times z(t) + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u(t) \end{aligned} \tag{2}$$

where

$$\begin{aligned} &z_1(t) \in R^{(n-m)*1}, \quad z_2(t) \in R^{m*1} \\ &\Delta A_1(t) \in R^{(n-m)*n} \\ &\Delta A_2(t) \in R^{m*n} \quad \text{and} \quad B_2 \in R^{m*m}. \end{aligned}$$

It can be easily seen that $\Delta A_2(t) = B_2 \xi(t)$ satisfies the classical matching condition, since the matrix B_2 is nonsingular.

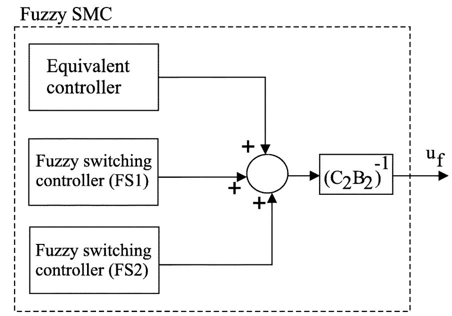

Fig. 2. Stucture of fuzzy sliding mode controller.

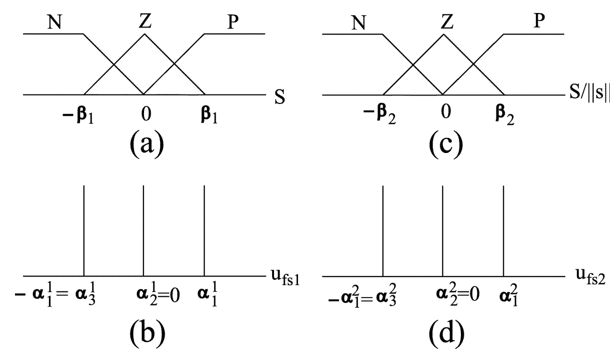

Fig. 3. Input and output membership functions.

The mismatched time-varying uncertainty $\Delta A_1(t)$ is assumed, as in [15], to have the structure

$$\Delta A_1(t) = DF(t)E \tag{3}$$

with the constant matrices D, E and the uncertain matrix $F(t) \in R^{r_1 \times r_2}$ satisfying

$$F^T(t)F(t) \leq I$$

where I is the corresponding identity matrix. Moreover, the rank of E needs to satisfy

$$\text{rank}(E) = \max\{\text{rank}(\Delta A_1(t))\}.$$

The matrices $D, F(t)$ do not have zero column vectors or zero row vectors. As in our previous paper [1], the sliding surface is designed to be

$$S = Cz(t) = [C_1 \quad C_2]z(t) = 0 \tag{4}$$

with C_2B_2 being an invertible matrix. In the following, the sliding coefficient matching condition is reviewed and the system considered in this paper is defined.

Definition 1: (Sliding Coefficient Matching Condition [1]): For the uncertainty $\Delta A_1(t) = DF(t)E$ and the sliding function $S = Cz(t)$, the sliding coefficient matching condition is defined as

$$\text{rank}[C^T] = \text{rank}[C^T \,|\, E^T]. \tag{5}$$

That is

$$E = E_a C. \tag{6}$$

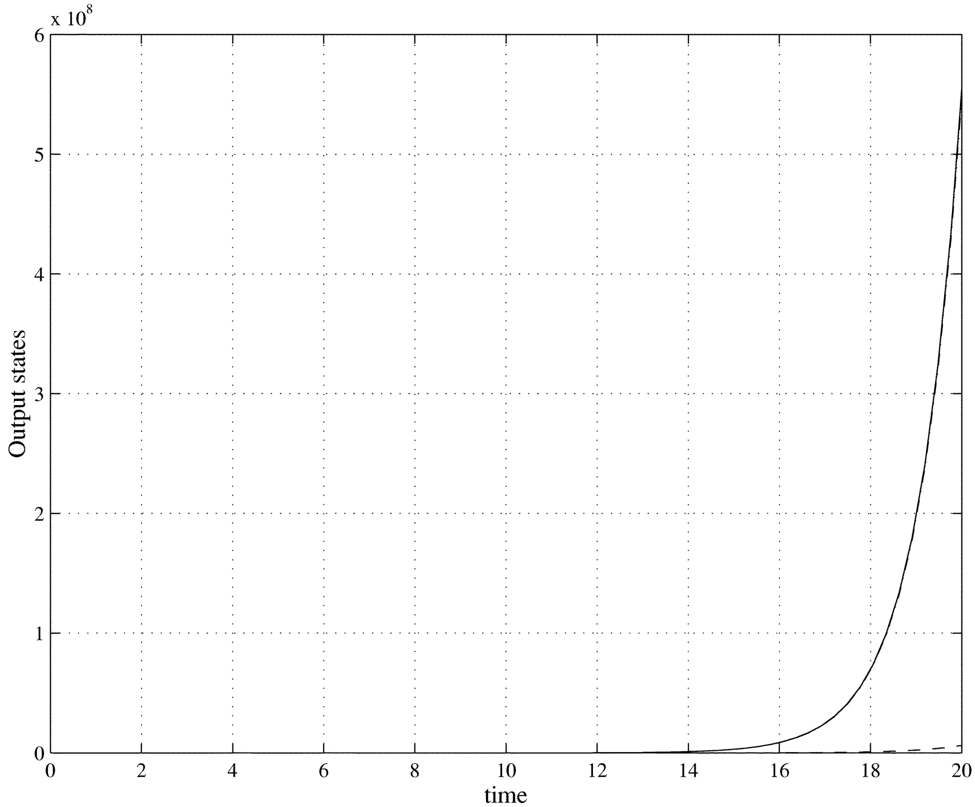

Fig. 4. States of the open-loop system: x_1 (solid line), x_2 (dash-dot line), and x_3 (dashed line).

Definition 2: (SCMCS System): For a sliding mode control system with the state equation in (2) and the sliding function $S = Cz(t)$, if the mismatched uncertainty $\Delta A_1(t) = DF(t)E, (F(t)^T F(t) \leq I)$ satisfies the sliding coefficient matching condition, then the sliding mode control system is called a sliding coefficient matched control system (SCMCS).

III. The Reaching Mode of the Sliding Mode Control System

It is known that the sliding mode of the uncertain system with an SMC [in Fig. 1(a)] is guaranteed if

$$S^T \dot{S} < 0, \quad S \neq 0.$$

Lemma 1 [15]: If $F(t)^T F(t) \leq I$, then

$$2x^T F(t) y \leq x^T x + y^T y; \quad \forall x, y \in R^n.$$

■

With Lemma 1 applied, the sliding mode reaching condition of a system SCMCS becomes

$$\begin{aligned} S^T \dot{S} &= S^T C \dot{z}(t) \\ &= S^T (C_1 (A_{11} z_1(t) + A_{12} z_2(t) + \Delta A_1(t) z(t)) \\ &\quad + C_2 (A_{21} z_1(t) + A_{22} z_2(t) \\ &\quad + \Delta A_2(t) z(t) + B_2 u(t))) \\ &= S^T (C_1 (A_{11} z_1(t) + A_{12} z_2(t) \\ &\quad + DF(t) E_a C z(t)) + C_2 (A_{21} z_1(t) + A_{22} z_2(t) \\ &\quad + B_2 \xi(t) z(t) + B_2 u(t))) \\ &\leq S^T (C_1 (A_{11} z_1(t) + A_{12} z_2(t)) + C_2 (A_{21} z_1(t) \\ &\quad + A_{22} z_2(t))) + 1/2 \left(S^T C_1 D D^T C_1^T S \right. \\ &\quad \left. + S^T E_a^T E_a S \right) + S^T C_2 B_2 \xi(t) z(t) + S^T C_2 B_2 u(t). \end{aligned} \tag{7}$$

Since $\Delta A_2(t)$ satisfies the traditional matching condition, the assumption

$$\|\xi(t) z(t)\| \leq \rho, \quad \rho \text{ is a constant scalar}$$

is made as usual [10]. Furthermore, we can assume that

$$\begin{cases} S^T C_2 B_2 \xi(t) z(t) \leq \frac{\rho S^T C_2 B_2 B_2^T C_2^T S}{\|B_2^T C_2^T\| \|S\|}, & \text{if } S \neq 0 \\ S^T C_2 B_2 \xi(t) z(t) = 0, & \text{if } S = 0. \end{cases}$$

Therefore

$$\begin{aligned} S^T \dot{S} &\leq S^T (C_1 (A_{11} z_1(t) + A_{12} z_2(t)) + C_2 (A_{21} z_1(t) \\ &\quad + A_{22} z_2(t))) + 1/2 \left(S^T C_1 D D^T C_1^T S \right. \\ &\quad \left. + S^T E_a^T E_a S \right) + \frac{\rho S^T C_2 B_2 B_2^T C_2^T S}{\left\| B_2^T C_2^T \right\| \|S\|} \\ &\quad + S^T C_2 B_2 u(t), \quad S \neq 0 \\ &\leq S^T (C_1 (A_{11} z_1(t) + A_{12} z_2(t)) + C_2 (A_{21} z_1(t) \\ &\quad + A_{22} z_2(t))) \\ &\quad + S^T P S + \frac{S^T Q S}{\|S\|} + S^T C_2 B_2 u(t), \quad S \neq 0 \end{aligned} \tag{8}$$

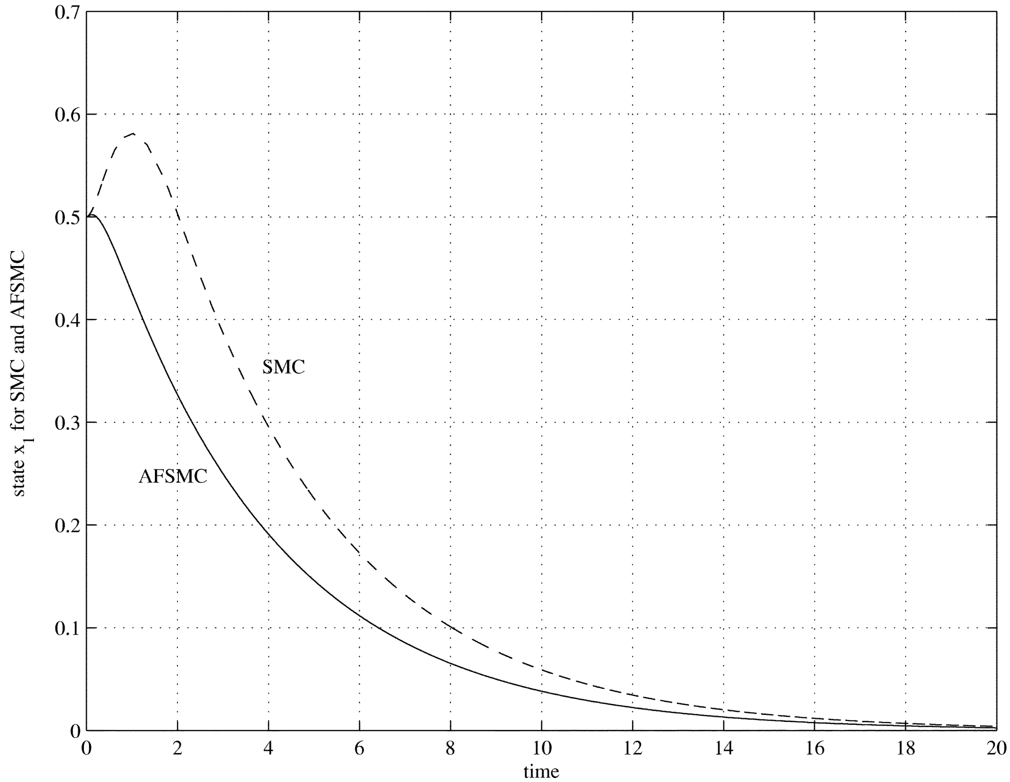

Fig. 5. Performance comparison for state x_1 of the uncertain system with SMC and AFSMC.

where

$$P = 1/2\left(C_1 D D^T C_1^T + E_a^T E_a\right)$$

and

$$Q = \frac{\rho C_2 B_2 B_2^T C_2^T}{\left\|B_2^T C_2^T\right\|}.$$

Since P and Q are Hermitian and positive semidefinite, it is reasonable to assume that

$$P < \gamma I \quad \text{and} \quad Q < \epsilon I \tag{9}$$

where I is the identity matrix with the corresponding dimension. Thus, unlike the exactly known matrices D, E required in [1], the sliding mode can be guaranteed in Theorem 1 with only the known bounds of P and Q.

Theorem 1: For an uncertain system SCMCS, if the output of the SMC is designed to be

$$\begin{aligned} u(t) = &-(C_2B_2)^{-1}(C_1(A_{11}z_1(t) + A_{12}z_2(t)) \\ &+ C_2(A_{21}z_1(t) + A_{22}z_2(t))) \\ &- (C_2B_2)^{-1}(\gamma S) - (C_2B_2)^{-1}\left(\frac{\epsilon S}{\|S\|}\right) \\ &- (C_2B_2)^{-1}(k_1 \operatorname{sgn}(S) + k_2 S) \end{aligned} \tag{10}$$

then

$$\begin{aligned} S^T\dot{S} &< S^T(-k_1 \operatorname{sgn}(S) - k_2 S) \\ &< 0; \quad k_1, k_2 > 0; \quad k_1, k_2 \in R \end{aligned} \tag{11}$$

and the sliding mode for the variable structure control of the linear uncertain system is guaranteed. ∎

It can be seen from Theorem 1 that the bounds of the mismatched uncertainty ΔA_1 and the matched uncertainty ΔA_2 are necessary to be available in order to design the SMC to make the system SCMCS approach the sliding mode. To alleviate the difficulties in the design of SMC for the system SCMCS, the AFSMC is designed in the next section.

IV. Adaptive Fuzzy Sliding Mode Controller (AFSMC)

The block diagram of the uncertain system with the AFSMC is shown in Fig. 1(b). The AFSMC is a fuzzy SMC with an adaptive mechanism to adjust the parameters in the fuzzy SMC. As in Fig. 2, the fuzzy SMC is designed to have an equivalent controller and two fuzzy switching controllers, $\mathrm{FS}_1, \mathrm{FS}_2$. The output u_f of the fuzzy SMC is defined as $(C_2B_2)^{-1}$ times the sum of the equivalent control u_{fe} and the outputs $u_{\mathrm{fs1}}, u_{\mathrm{fs2}}$ of the fuzzy switching controllers $\mathrm{FS}_1, \mathrm{FS}_2$, i.e.,

$$u_f = (C_2B_2)^{-1}(u_{\mathrm{fe}} + u_{\mathrm{fs1}} + u_{\mathrm{fs2}})$$

with

$$\begin{aligned} u_{\mathrm{fe}} = &-(C_1(A_{11}z_1(t) + A_{12}z_2(t)) \\ &+ C_2(A_{21}z_1(t) + A_{22}z_2(t))). \end{aligned}$$

1) Fuzzy Switching Controllers: Let the input and switching output variable S, u_{fs1} of the fuzzy switching controller FS_1 be simply partitioned into fuzzy sets N (negative), Z (zero), and P (positive). The triangular-type input membership functions and the membership functions for output fuzzy singletons are shown in Fig. 3(a) and (b). From (8), it is easy to see that in order to

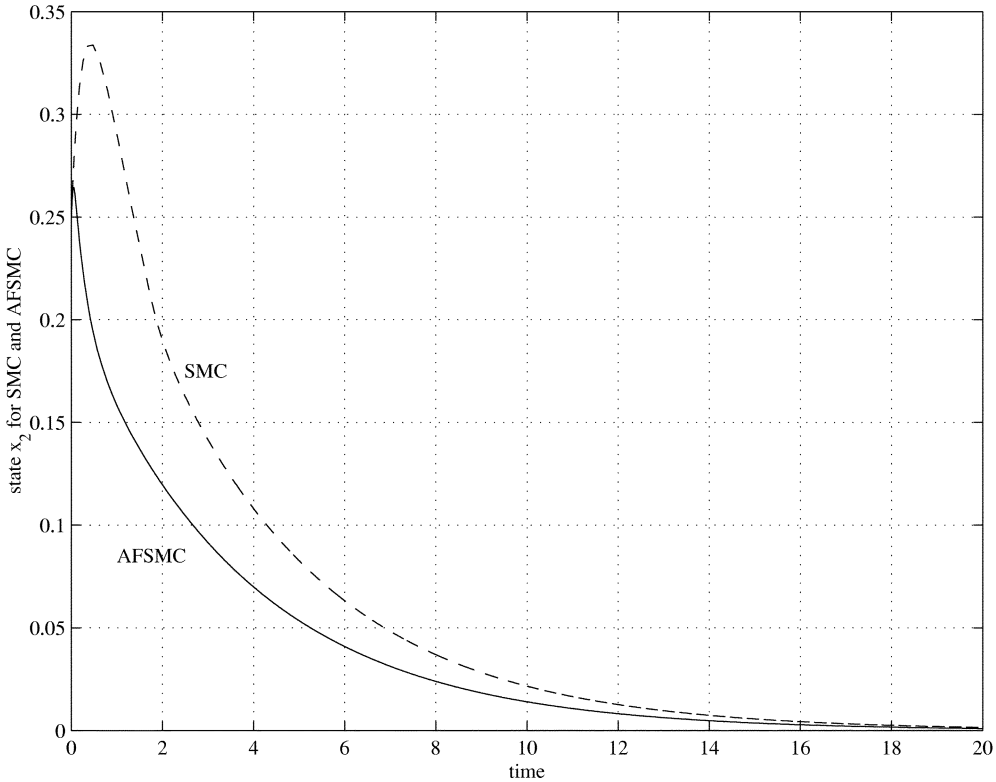

Fig. 6. Performance comparison for state x_2 of the uncertain system with SMC and AFSMC.

make $S^T\dot{S} < 0$ to guarantee the sliding mode of SCMCS, the fuzzy rules can be derived as the following.

1) If S is N, then u_{fs1} is P.
2) If S is Z, then u_{fs1} is Z.
3) If S is P, then u_{fs1} is N.

With the centroid defuzzification technique, the switching output u_{fs1} of the fuzzy SMC is calculated as

$$u_{\mathrm{fs1}} = \sum_{i=1}^{3} g_i^1 F_i^1(S) \left(\mathrm{diag}\left(\sum_{i=1}^{3} F_i^1(S)\right)\right)^{-1} \quad (12)$$

where $F_i^1 \in \{P, Z, N\}, i = 1, 2, 3$ are input membership functions, $g_i^1, i = 1, 2, 3$ are diagonal matrices with the corresponding values of output fuzzy singletons on the diagonal cells, and $\mathrm{diag}(V)$ is a matrix function to generate a diagonal matrix with the elements of the vector V on the corresponding diagonal cells. Note that the denominator of u_{fs1} in (12) is equal to an identity matrix, since the triangular membership functions are designed as in Fig. 3(a). Thus, u_{fs1} can be simplified as

$$u_{\mathrm{fs}_1} = \sum_{i=1}^{3} g_i^1 F_i^1(S). \quad (13)$$

For the switching controller FS_2, the input variable is defined as $(S/\|S\|)$. Similar to the design of the fuzzy switching controller FS_1, the fuzzy rules for the switching controller FS_2 are the following.

1) If $(S/\|S\|)$ is N, then u_{fs2} is P.
2) If $(S/\|S\|)$ is Z, then u_{fs2} is Z.
3) If $(S/\|S\|)$ is P, then u_{fs2} is N.

The input and output membership functions for FS_2 are shown in Fig. 3(c) and (d). Thus, the output of the switching controller FS_2 is

$$u_{\mathrm{fs}_2} = \sum_{i=1}^{3} g_i^2 F_i^2 \left(\frac{S}{\|S\|}\right). \quad (14)$$

2) On-Line Adaptation Mechanism: To simplify the design of the fuzzy switching controllers, g_i^1 in (13) and g_i^2 in (14), $i = 1, 2, 3$ are considered to be a scalar (times an identity matrix). As in Fig. 3, the values of the output fuzzy singletons are specified to be symmetrical to zero $(\alpha_3^1 = -\alpha_1^1, \alpha_3^2 = -\alpha_1^2)$. Furthermore, it is known that the outputs of the fuzzy switching controllers are zero when the system is in the sliding mode $(S = 0)$. Thus, the outputs of the fuzzy switching functions FS_1 and FS_2 can be simplified as

$$u_{\mathrm{fs}_1} = \alpha_1^1 \mathrm{diag}(\mathrm{sign}(S)) F^1(S)$$

and

$$u_{\mathrm{fs}_2} = \alpha_1^2 \mathrm{diag}(\mathrm{sign}(S)) F^2 \left(S\|S\|\right) \quad (15)$$

where $F^1(S), F^2(S/\|S\|)$ are column vectors with the membership values of the corresponding fuzzy sets of the elements in vectors S and $(S/\|S\|)$, respectively. Note that α_1^1 and α_1^2 are negative. The parameter $\alpha_1^1(\alpha_1^2)$ of the fuzzy switching controller $\mathrm{FS}_1(\mathrm{FS}_2)$ is on-line adjusted. To minimize the reaching rate of the sliding mode with respect to $\alpha_1^1(\alpha_1^2)$, the adaptive laws are

$$\begin{aligned}\delta\alpha_1^1 &= -\eta_1 \frac{\partial S^T \dot{S}}{\partial \alpha_1^1} \\ &= -\eta_1 S^T \mathrm{diag}(\mathrm{sign}(S)) F^1(S)\end{aligned}$$

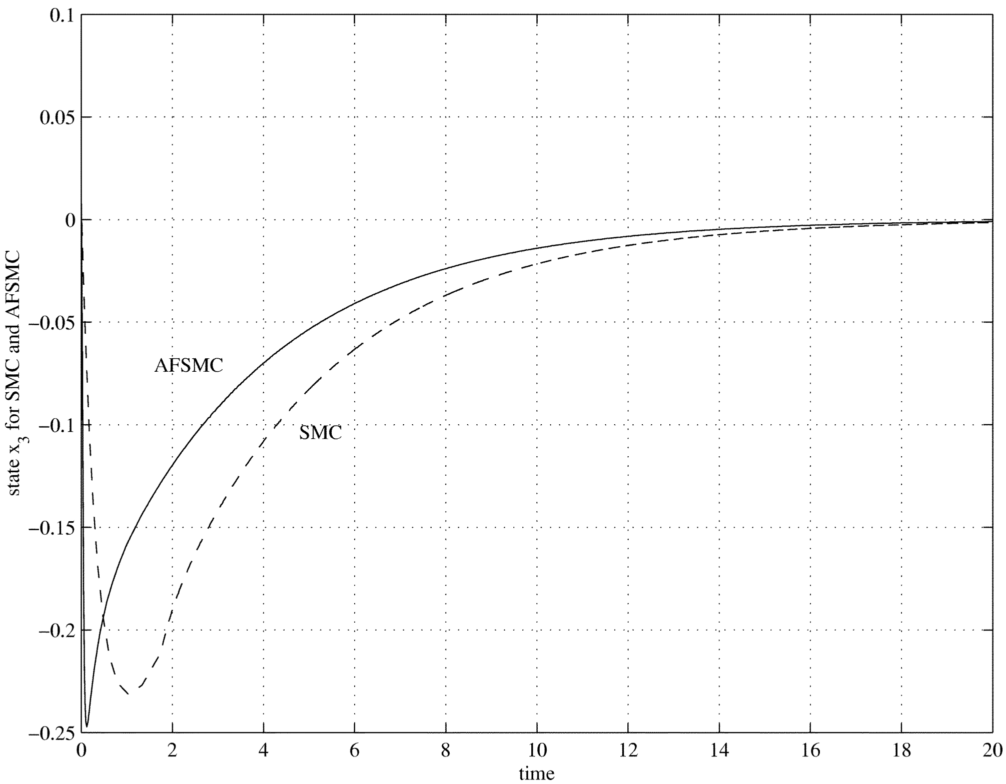

Fig. 7. Performance comparison for state x_3 of the uncertain system with SMC and AFSMC.

and

$$\begin{aligned}\delta\alpha_1^2 &= -\eta_2 \frac{\partial S^T \dot{S}}{\partial \alpha_1^2} \\ &= -\eta_2 S^T \mathrm{diag}(\mathrm{sign}(S)) F^2\left(\frac{S}{\|S\|}\right)\end{aligned} \tag{16}$$

where η_1, η_2 denotes positive learning rate. From (16), it can be easily seen that $\delta\alpha_1^1$ and $\delta\alpha_1^2$ are always negative when S is not zero and $\delta\alpha_1^1 = \delta\alpha_1^2 = 0$ when $S = 0$. This adaptation makes the system reach the sliding mode quicker when the sliding function S is getting smaller, and the system performance is then improved. Moreover, the chattering can be alleviated with this adaptation mechanism since α_1^1 and α_1^2 are small when the sliding function S is large at the beginning of the system operation. With this proposed AFSMC, the sliding mode of the system SCMCS is guaranteed, as described in Theorem 2.

Theorem 2: The uncertain system SCMCS with the AFSMC can have its sliding mode guaranteed.

Proof: The output of the AFSMC is

$$\begin{aligned}u_f &= (C_2B_2)^{-1}(u_{\mathrm{fe}} + u_{\mathrm{fs1}} + u_{\mathrm{fs2}}) \\ &= -(C_2B_2)^{-1}(C_1(A_{11}z_1(t) + A_{12}z_2(t)) \\ &\quad + C_2(A_{21}z_1(t) + A_{22}z_2(t))) \\ &\quad + (C_2B_2)^{-1}\left(\alpha_1^1 \mathrm{diag}(\mathrm{sign}(S))F^1(S)\right. \\ &\quad \left. + \alpha_1^2 \mathrm{diag}(\mathrm{sign}(S))F^2\left(\frac{S}{\|S\|}\right)\right).\end{aligned} \tag{17}$$

Then, the reaching rate of the sliding mode is

$$\begin{aligned}S^T\dot{S} &< S^T\left(PS + \alpha_1^1 \mathrm{diag}(\mathrm{sign}(S))F^1(S)\right) \\ &\quad + S^T\left(\frac{QS}{\|S\|} + \alpha_1^2 \mathrm{diag}(\mathrm{sign}(S))F^2\left(\frac{S}{\|S\|}\right)\right), \quad S \neq 0.\end{aligned} \tag{18}$$

It is known that if $|S| > \beta_1$ then $F^1(S) = 1$ (see Fig. 3). Since $P < \gamma I$

$$\max(|PS|) < \gamma \max(|S|).$$

Furthermore, because α_1^1 is getting more and more negative with the adaptive law in (16), the term $S^T(PS + \alpha_1^1 \mathrm{diag}(\mathrm{sign}(S))F^1(S))$ is going to be negative when

$$\left|\alpha_1^1\right| > \gamma \max(|S|).$$

Likewise, if $F^2(S/\|S\|) = 1$, the second term in (18) becomes negative when

$$\left|\alpha_1^2\right| > \epsilon \max\left(\left|\frac{S}{\|S\|}\right|\right).$$

Thus, the reaching rate of the sliding mode is negative in the case of $|S| > \beta_1$ and $|S/\|S\|| > \beta_2$. If $|S| \leq \beta_1$, then $\mathrm{diag}(\mathrm{sign}(S))F^1(S) = (S/\beta_1)$. α_1^1 is becoming more negative by following the adaptive law, and finally $|\alpha_1^1/\beta_1|$ will be larger than γ. In this case

$$S^T\left(PS + \alpha_1^1 \mathrm{diag}(\mathrm{sign}(S))F^1(S)\right) = S^T\left(P + \frac{\alpha_1^1}{\beta_1}\right)S < 0.$$

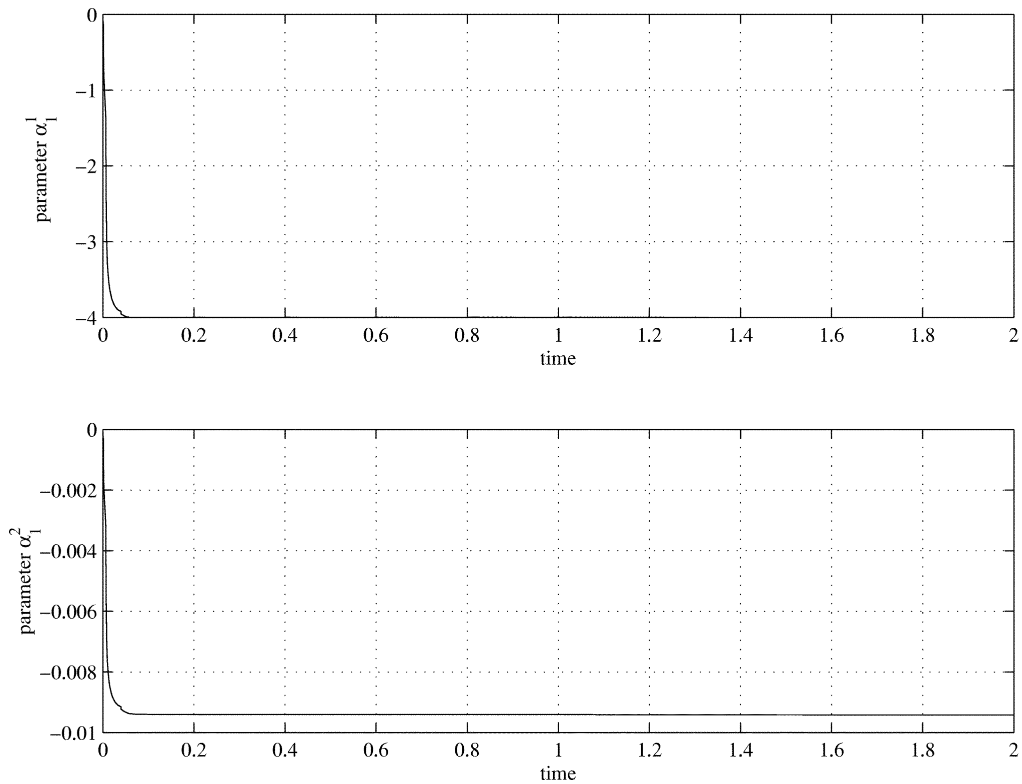

Fig. 8. Parameters α_1^1 and α_1^2.

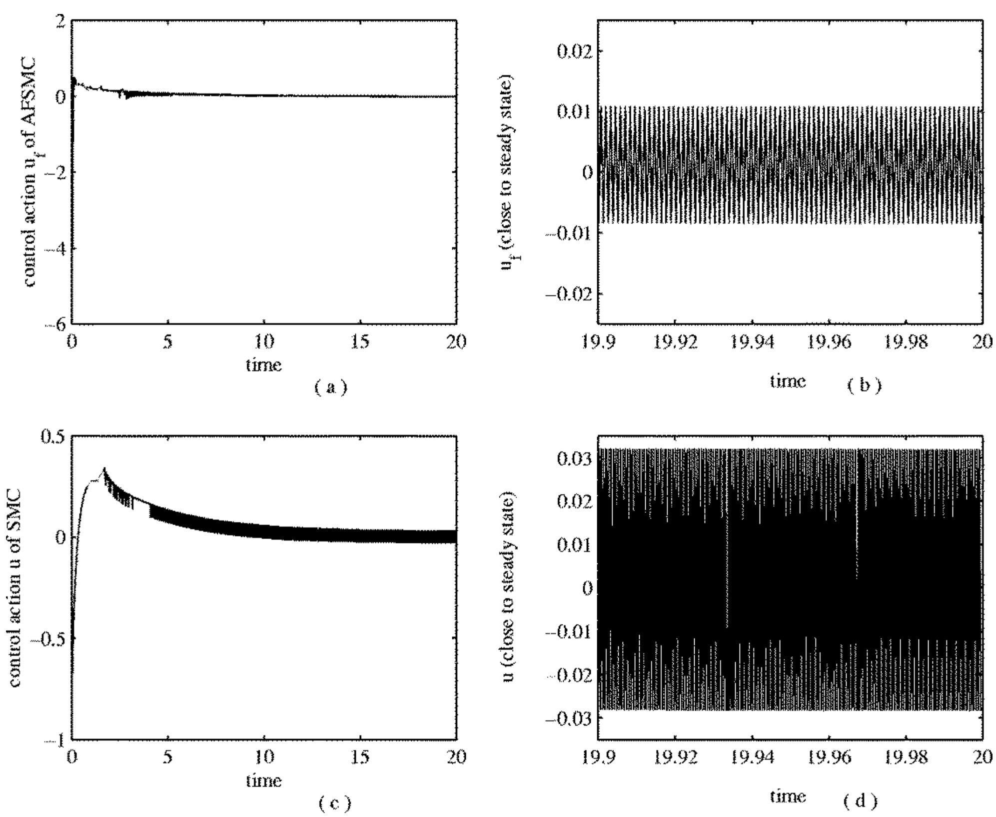

Fig. 9. (a) Control action of AFSMC. (b) Control action of AFSMC (close to steady state). (c) Control action of SMC. (d) Control action of SMC (close to steady state).

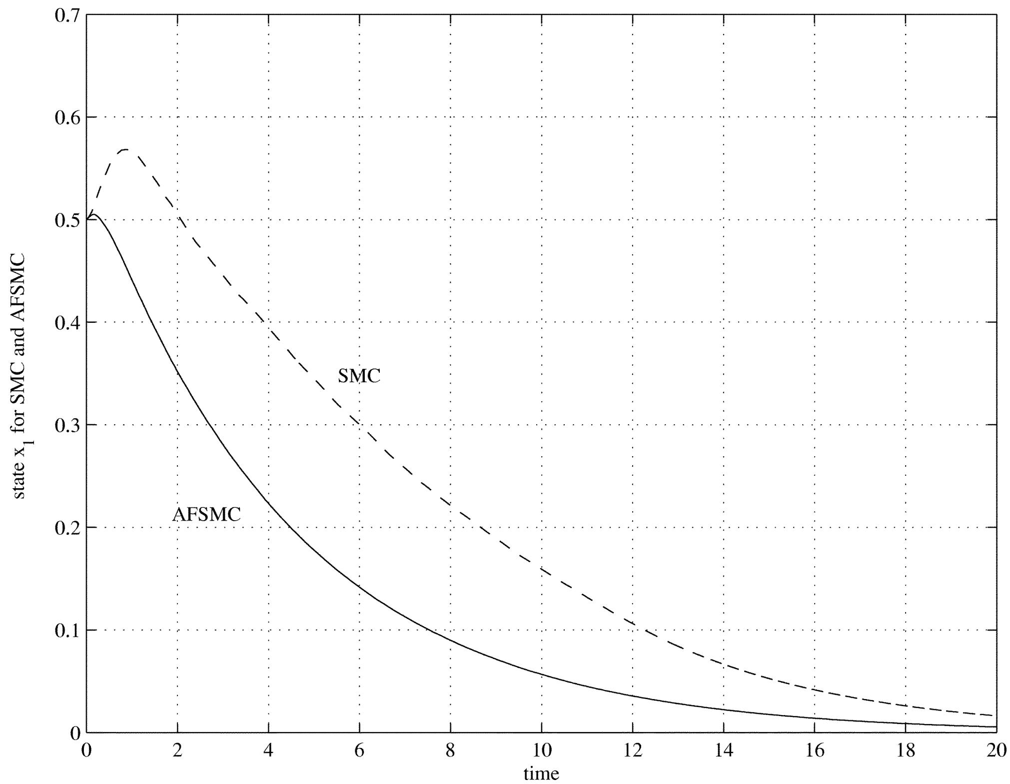

Fig. 10. Performance comparison for state x_1 of the uncertain system with SMC and AFSMC.

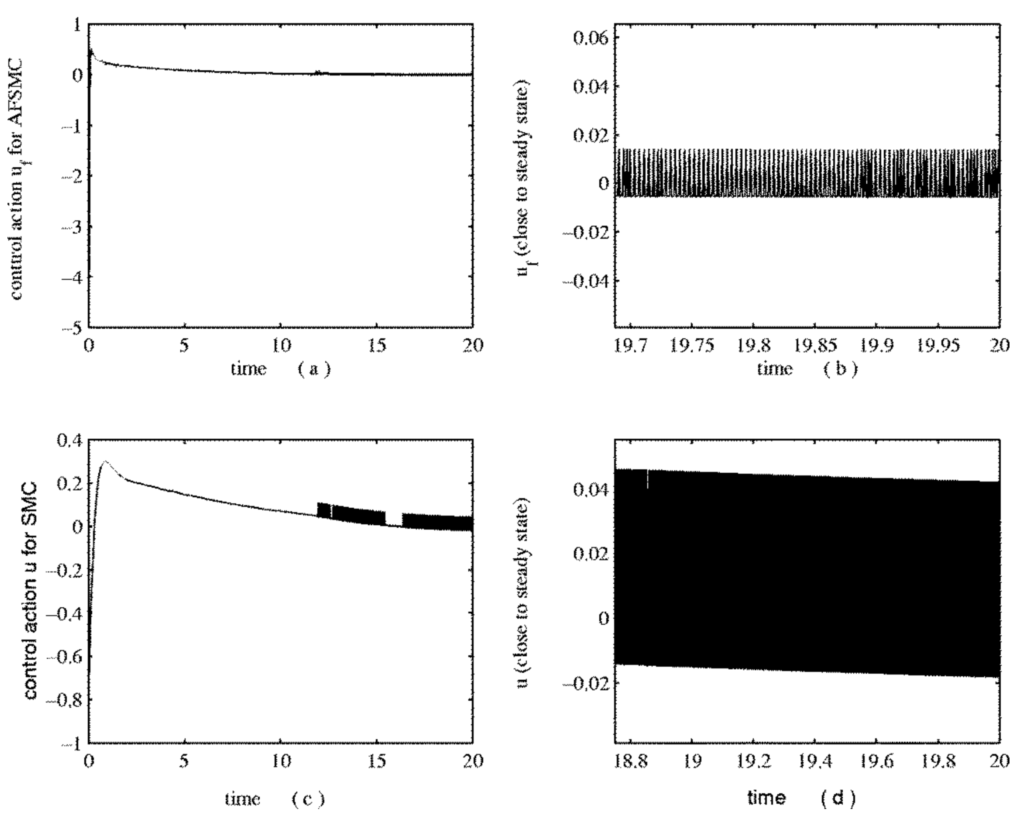

Fig. 11. (a) Control action of AFSMC. (b) Control action of AFSMC (close to steady state). (c) Control action of SMC. (d) Control action of SMC (close to steady state).

With the same idea, the second term in (18) can become neagtive when $|S/\|S\|| \leq \beta_2$ and $|\alpha_1^2/\beta_2| > \epsilon$. Again, $S^T\dot{S} < 0$ and the sliding mode of the uncertain system SCMCS is guaranteed while $|S| \leq \beta_1$ and $|S/\|S\|| \leq \beta_2$. It can be easily proven with the same approach that the sliding mode of the uncertain system SCMCS using the AFSMC is guaranteed in other cases. ■

Note that even the bounds γ and ϵ are used for the proof of the guarantee of sliding mode, γ and ϵ are not used for the design of the proposed AFSMC for SCMCS. Therefore, the requirement of the uncertainty bounds is no longer necessary for AFSMC.

3) Characteristics of the SCMCS With AFSMC on the Sliding Surface: In Theorem 3, the invariant characteristic of the SCMCS with an AFSMC is described.

Theorem 3: A linear uncertain system SCMCS (see Section II) with an AFSMC is invariant with respect to time-varying uncertainties on the sliding surface. ■

With the same approach as in [1], the proof of Theorem 3 can be derived. In order to have the adaptive fuzzy sliding control system be asymptotically stable on the sliding surface, the eigenvalues of the matrix

$$A_{11} - A_{12}C_2^{-1}C_1$$

need to be all negative. This condition can be satisfied with the proper selection of the sliding coefficient matrix C [1]. The illustrative examples are included in the next section to demonstrate the effectiveness of the adaptive fuzzy sliding controllers designed here.

V. Simulation Results

Let the uncertain linear system be described by (1) with

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 1 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

and

$$\Delta A(t) = \begin{bmatrix} 0 & 0.06\sin(0.1t) & 0.06\sin(0.1t) \\ 0 & 0.03\cos(0.1t) & 0.03\cos(0.1t) \\ 0 & 0 & 0.1\cos(0.1t) \end{bmatrix}. \quad (19)$$

From (19), it is easy to see that $\Delta A(t)$ is a mismatched time-varying uncertainty, and the mismatched uncertainty $\Delta A_1(t)$ can be represented as

$$\begin{aligned} \Delta A_1(t) &= DF(t)E \\ &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix} * \begin{bmatrix} \sin(0.1t) \\ \cos(0.1t) \end{bmatrix} * [0 \quad 0.3 \quad 0.3] \end{aligned} \quad (20)$$

where

$$F(t)^T F(t) = \sin^2(0.1t) + \cos^2(0.1t) \leq 1.$$

Thus, the sliding coefficient matrix C can be designed to be

$$C = [0 \quad 1 \quad 1]$$

to satisfy the sliding coefficient matching condition. Simulation results are provided for the open-loop system, closed-loop sliding mode control system, and adaptive fuzzy sliding mode control system, respectively. Fig. 4 illustrates that the open-loop uncertain system with initial conditions

$$x(0) = [0.5 \quad 0.25 \quad 0.01]^T$$

is unstable.

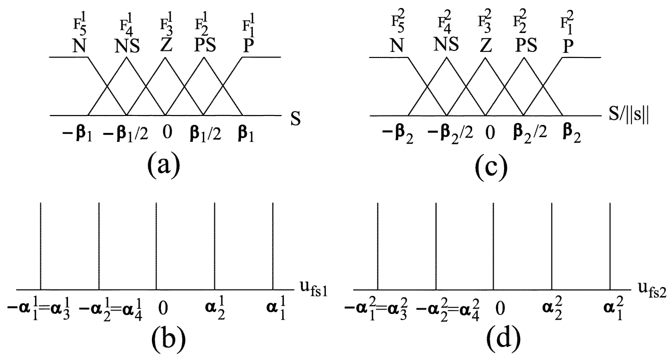

Fig. 12. Membership functions for AFSMC with five rules.

A. Sliding Mode Controller (SMC)

Since

$$1/2\left(C_1DD^TC_1^T + E_a^TEa\right) \leq 0.05$$

and

$$\frac{\rho C_2B_2B_2^TC_2^T}{\left\|B_2^TC_2^T\right\|} \leq 0.03 \quad (21)$$

the control action $u(t)$ is designed to be

$$\begin{aligned} u(t) = &-(C_2B_2)^{-1}(C_1(A_{11}z_1(t) + A_{12}z_2(t)) \\ &+ C_2(A_{21}z_1(t) + A_{22}z_2(t))) \\ &- (C_2B_2)^{-1}(\gamma S) - (C_2B_2)^{-1}\left(\frac{\epsilon S}{\|S\|}\right) \\ &- (C_2B_2)^{-1}(k_1sgn(S) + k_2S) \\ = &-[1 \quad 0]z_1 - 4z_2 - 0.05S - \frac{.03S}{\|S\|} - S \end{aligned} \quad (22)$$

with $S = Cz, k_2 = 1$, and $k_1 = 0$. Note that if $k_1 > 0$, then the chattering of the control action $u(t)$ will be increased. Fig. 5 illustrates how the unstable system is stabilized when the sliding mode control is applied.

B. Adaptive Fuzzy Sliding Mode Controller (AFSMC)

The output of the AFSMC is

$$\begin{aligned} u_f &= (C_2B_2)^{-1}(u_{\text{fe}} + u_{\text{fs1}} + u_{\text{fs2}}) \\ &= -[1 \quad 0]z_1 - 4z_2 + \alpha_1^1\text{diag}(\text{sign}(S))F^1(S) \\ &\quad + \alpha_1^2\text{diag}(\text{sign}(S))F^2\left(\frac{S}{\|S\|}\right) \end{aligned} \quad (23)$$

with the adaptive laws

$$\begin{aligned} \delta\alpha_1^1 &= -.3S^T\text{diag}(\text{sign}(S))F^1(S) \\ \delta\alpha_1^2 &= -.0007S^T\text{diag}(\text{sign}(S))F^2\left(S/\|S\|\right). \end{aligned} \quad (24)$$

Note that the learning rates are selected with very small values as the initial learning rates. Then, the learning rates are increased to reduce the overshoot and the rising time of the control system. If (the control action is too large and is over the limit) or (the chattering of the control action is large) then the learning rates are decreased. Since the parameters β_1 of the input membership functions [see Fig. 3(a)] has the effect of a boundary layer, β_1 is assigned to be a small number $\beta_1 = 0.1$. Also, $\beta_2 = 1$

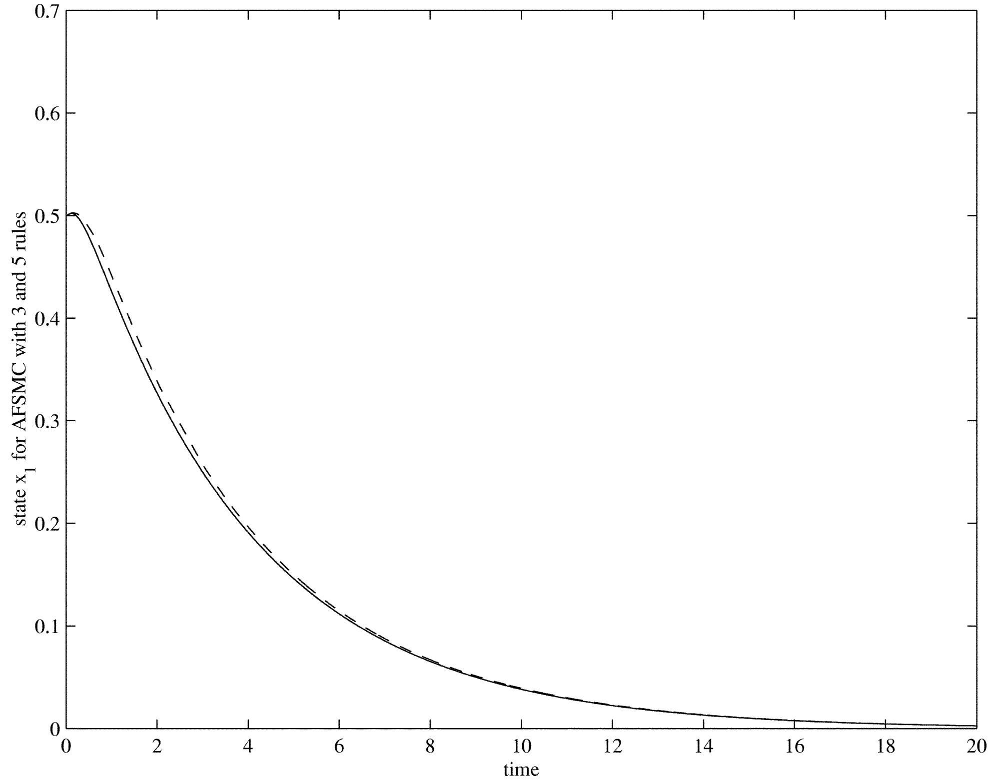

Fig. 13. Performance of the AFSMC with three rules (solid line) and five rules (dashed line).

is designed because $\max(S/\|S\|) = 1$. The performance comparisons for each state (x_1, x_2, x_3) of the uncertain system with SMC and AFSMC are provided in Figs. 5–7. In Fig. 5, it is shown that the uncertain system with AFSMC has better performance than the uncertain system with the SMC in the sense of smaller undershoot and shorter rising time. The parameters α_1^1 and α_1^2 are indicated in Fig. 8 to empirically conform so that the sliding mode is guaranteed. The control actions u_f and u of AFSMC and SMC are illustrated in Fig. 9(a) and (c). The control actions close to the steady state for the system with AFSMC and SMC are emphasized in Fig. 9(b) and (d). It can be seen in Fig. 9(b) and (d) that the chattering is reduced for the uncertain system with the AFSMC. Moreover, it is easy to verify that the reduced system matrix on the sliding surface is

$$A_{11} - A_{12}C_2^{-1}C_1 = \begin{bmatrix} -1 & 2 \\ 1 & -3 \end{bmatrix}$$

which is stable.

C. Simulations With Another Type of Uncertainties

Let $v_1(t)$ and $v_2(t)$ be two uncorrelated and uniformly distributed random processes with $|v_i(t)| \le 1, i = 1, 2$. Then, another type of uncertainty matrix $\Delta A(t)$

$$\Delta A(t) = \begin{bmatrix} 0.06v_1(t) & 0.06v_1(t) & 0.06v_1(t) \\ 0.03v_2(t) & 0.03v_2(t) & 0.03v_2(t) \\ 0 & 0 & 0.1\cos(0.1t) \end{bmatrix} \tag{25}$$

is adopted for simulations. Likewise, $\Delta A(t)$ is easy to be shown as a mismatched time-varying uncertainty, and the mismatched uncertainty $\Delta A_1(t)$ can be partitioned as

$$\begin{aligned} \Delta A_1(t) &= DF(t)E \\ &= \begin{bmatrix} 0.4 & 0 \\ 0 & 0.2 \end{bmatrix} \cdot \begin{bmatrix} 0.5v_1(t) \\ 0.5v_2(t) \end{bmatrix} \cdot [0.3 \quad 0.3 \quad 0.3] \end{aligned} \tag{26}$$

where

$$F(t)^T F(t) = 0.25v_1(t)^2 + 0.25v_2(t)^2 \le 1.$$

From (26), the sliding coefficient matrix C can be designed to have

$$C = [1 \quad 1 \quad 1]$$

and the sliding coefficient matching condition is satisfied. Then, the control action $u(t)$ for SMC becomes

$$u(t) = -[0 \quad 2]z_1 - 4z_2 - 0.145S - \frac{0.03S}{\|S\|} - S.$$

With the same adaptive laws in (24), α_1^1 and α_1^2 in AFSMC are adjusted to be -3.9945 and -0.0096, respectively. The performance comparison for state x_1 between SMC and AFSMC is shown in Fig. 10, and the control actions of SMC and AFSMC are shown in Fig. 11. From Figs. 10 and 11, AFSMC is seen to have better performance than SMC.

D. An Example of AFSMC With More Than Three Rules

To indicate the effect of the number of fuzzy rules on the performance of the AFSMC, an example is implemented. In this example, the input and output spaces of the fuzzy switching controllers are partitioned as in Fig. 12. Thus, five rules are constructed for each switching controller. With the adaptive laws

$$\begin{aligned}\delta\alpha_1^1 &= -.3S^T\mathrm{diag}(\mathrm{sign}(S))F_1^1(S)\\ \delta\alpha_2^1 &= -.3S^T\mathrm{diag}(\mathrm{sign}(S))F_2^1(S)\\ \delta\alpha_1^2 &= -.0007S^T\mathrm{diag}(\mathrm{sign}(S))F_1^2\left(\frac{S}{\|S\|}\right)\\ \delta\alpha_2^2 &= -.0007S^T\mathrm{diag}(\mathrm{sign}(S))F_2^2\left(\frac{S}{\|S\|}\right)\end{aligned} \tag{27}$$

the parameters are adjusted to the following values:

$$[\alpha_1^1 \quad \alpha_2^1 \quad \alpha_1^2 \quad \alpha_2^2] = [-4.03 \quad -0.1453 \quad -0.0098 \quad 0].$$

For simplicity, only the performance comparison of the state x_1 between the AFSMC with three and five rules is shown in Fig. 13. From Fig. 13, it can be seen that the performance of AFSMC with more rules is not necessarily better. Moreover, the forms of the membership values are more complicated for the AFSMC with more than three rules. Thus, to showing the guaranty of the sliding mode of the AFSMC with more than three rules is not as easy as in Section IV. However, we would not claim that the AFSMC with three rules is the best structure that AFSMC can have.

VI. Conclusion

In this paper, an AFSMC is proposed for the linear systems with mismatched time-varying uncertainties. The sliding coefficient matching condition is provided. The sliding mode of the uncertain system with the proposed AFSMC is guaranteed. The requirement of the available uncertainty bounds for the design of the traditional SMC is not necessary for the AFSMC. The system is shown to be invariant and stable on the sliding surface when the new matching condition is matched. Furthermore, the chattering around the sliding surface for the adaptive fuzzy sliding mode control is reduced. Simulation results are included to illustrate the effectiveness of the proposed SMC.

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