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Discovering fuzzy association rules using fuzzy partition methods

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Abstract

Fuzzy association rules described by the natural language are well suited for the thinking of human subjects and will help to increase the flexibility for supporting users in making decisions or designing the fuzzy systems. In this paper, a new algorithm named fuzzy grids based rules mining algorithm (FGBRMA) is proposed to generate fuzzy association rules from a relational database. The proposed algorithm consists of two phases: one to generate the large fuzzy grids, and the other to generate the fuzzy association rules. A numerical example is presented to illustrate a detailed process for finding the fuzzy association rules from a specified database, demonstrating the effectiveness of the proposed algorithm. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

Relational databases have been widely used in data processing and support of business operations, and their size has grown rapidly. For the activities of decision-making and market prediction, knowledge discovery from a database is very important for providing necessary information to a business. Association rules are one of the ways of representing knowledge, having been applied to analyze market baskets to help managers realize which items are likely to be bought at the same time [1,2]. For example, rule $\{P\} \Rightarrow \{Q\}$ represents that if a customer bought P , then he should buy Q at the same time. The left-hand side of ' \Rightarrow ' is the antecedence of rule, and the right-hand side is the consequence. We call $\{P\}$ and $\{Q\}$ itemsets. Two important parameters are required to generate effective association rules; one is *support* and the other is *confidence* [2]. Support is the number of transactions with all the items in the rule; and confidence is the ratio of the number of transactions with all the items in the rule to the number of transactions with just the items in the condition [1]. Hence, the support of $\{P, Q\}$ can be described as (Number of transactions which

contains both P and Q)/(Number of transactions in the database); and the confidence of $\{P\} \Rightarrow \{Q\}$ can be described as (Number of transactions which contains both P and Q)/(Number of transactions contains P).

Generally, there are two phases for mining association rules [3]. In the first phase, we first find all the large itemsets. The supports of large itemsets are larger than the minimal supports specified by users. If there are k items in a large itemset, then we call it a large k -itemset. We can find that a subset of a large itemset must also be large. Subsequently, we use the large itemsets generated in the first phase to generate effective association rules. If the confidence of an association rule is larger than or equal to the minimum confidence specified by users, then it is effective. The key work for finding the association rules is to find all the large itemsets.

Initially, Agrawal et al. [4] proposed a method to find the large itemsets. Subsequently, Agrawal et al. [5] also proposed the Apriori algorithm. However, these algorithms must scan a database many times to find the large itemsets. Moreover, when they generated a candidate itemset, the apriori-gen function must have wasted much time to check if its subsets are large or not.

Wur and Leu [6] proposed the Boolean algorithm to scan a database only once, not wasting much time reading data from disk. Moreover, it used Boolean operations (AND, OR and XOR) on the table structure

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they proposed to generate the large itemsets and the association rules. For discovering association rules, it seems that Boolean algorithm works more efficient than other algorithms.

By partitioning quantitative attributes, Srikant and Agrawal [7] proposed the partial completeness to be the criterion for finding association rules. Fukuda et al. [8] also proposed concepts of optimized association rules. In the rule representation, the consequent part was required to be a fixed value, and the antecedent part was composed of one or two quantitative attributes. However, it seems that such representations were restricted.

A clustering method named CLIQUE was further proposed by Agrawal et al. [9]. In general, data clusters were distributed in the feature subspaces which were constructed by some quantitative attributes, and CLIQUE could efficiently find the subspaces where data clusters were really distributed. To do this, CLIQUE divided each quantitative attribute into many partitions with equal length, and viewed each partition as a candidate 1-itemset. Therefore, a k -itemset ($k \geq 1$) is an itemset that consisted of k partitions distributed in k quantitative attributes. Finally, large k -itemsets could be found. In comparison with other clustering methods, including c -means [1] and BIRCH [10], we can find that CLIQUE worked more efficient because it cannot directly find clusters that were constructed by all quantitative attributes.

The well-known methods (i.e. partial completeness, optimized association rules and CLIQUE) we have mentioned above, divided the quantitative attributes into many crisp partitions. There were no intersections between the partitions. However, crisp partitions may be unreasonable for some situations. For example, if we tried to divide the range (170, 180 cm) of the attribute ‘height’ into two partitions, then the separable point was not different between 175.01 and 174.99 cm. Hence, intersection between any of the neighborhood partitions can be promised. Moreover, we considered that the fuzzy association rules described by the natural language are well suited for the thinking of human subjects and will help to increase the flexibility for users in making decisions or designing the fuzzy systems. The fuzzy partition methods are thus used to find the fuzzy association rules.

In this paper, an effective algorithm named fuzzy grids based rules mining algorithm (FGBRMA) is proposed. For the proposed algorithm, both quantitative and categorical attributes are divided into various linguistic values. Large fuzzy grids and effective fuzzy association rules can be determined by the proposed fuzzy support and the fuzzy confidence, respectively. Like Boolean algorithm, FGBRMA uses the proposed table structures to generate both large fuzzy grids and fuzzy association rules. It seems that the proposed algorithm is also an efficient algorithm because it also scans a database only once and applies Boolean

operations on the proposed table structures to generate both large fuzzy grids and fuzzy association rules.

In the following sections, the cases for fuzzy partitioning in quantitative and qualitative attributes are introduced in Section 2. In Section 3, the definitions of the fuzzy support and the fuzzy confidence are proposed. We present the proposed algorithm in Section 4. In Section 5, a numerical example is presented to illustrate a detailed process for finding the fuzzy association rules from a specified database relation, demonstrating the effectiveness of the proposed algorithm. Discussions and conclusions are presented in Sections 6 and 7, respectively.

2. Fuzzy partition method

Notations used in this paper are stated as follows:

- K : prespecified number of linguistic values in a linguistic variable;
- d : number of attributes of a database relation, where $1 \leq d$;
- k : dimension of a fuzzy grid, where $1 \leq k \leq d$;
- $A_{K,i_m}^{x_m}$: i_m th linguistic value of K various linguistic value defined in x_m , where $1 \leq i_m \leq K$;
- $\mu_{K,i_m}^{x_m}$: membership function of $A_{K,i_m}^{x_m}$;
- t_p : p th tuple of a database relation, where $t_p = (t_{p_1}, t_{p_1}, \dots, t_{p_d})$ and $p \geq 1$.

Fuzzy set was proposed by Zadeh [16], and the division of the features into various linguistic values has been widely used in pattern recognition and fuzzy inference. From this, various results have been proposed, such as application to pattern classification by Ishibuchi et al. [10–12], the fuzzy rules generated by Wang and Mendel [13], and methods for partitioning feature space were also discussed by Sun [14] and Bezdek [15].

In this paper, we view each attribute as a linguistic variable, and the variables are divided into various linguistic values. A linguistic variable is a variable whose values are linguistic words or sentences in a natural language [17–20]. For example, the values of the linguistic variable ‘Age’ may be ‘close to 30’ or ‘very close to 50’, and referred to as linguistic values. Triangular membership functions are used for each linguistic value defined in each quantitative attribute for simplicity. Hence, each linguistic value is a fuzzy number, which is a fuzzy subset in the universe of discourse that is both convex and normal [20,21].

The cases for fuzzy partitioning in quantitative and categorical attributes are introduced in Sections 2.1 and 2.2, respectively.

2.1. Fuzzy partitioning in quantitative attributes

A quantitative attribute can be divided into K various linguistic values ($K = 2, 3, 4, \dots$). For example, for

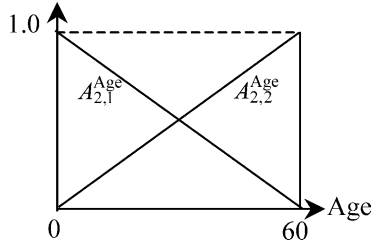


Fig. 1. $K = 2$ for 'Age'.

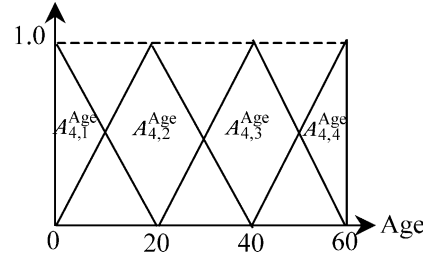


Fig. 3. $K = 4$ for 'Age'.

the attribute 'Age' (range from 0 to 60), we describe $K = 2$, $K = 3$ and $K = 4$ in Figs. 1–3, respectively. Moreover, A_{K,i_m}^{Age} can be used to represent a candidate 1-dim fuzzy grid. Then, μ_{K,i_m}^{Age} can be represented as follows:

$$\mu_{K,i_m}^{Age}(x) = \max\{1 - |x - a_{i_m}^K|/b^K, 0\} \quad (1)$$

where

$$a_{i_m}^K = mi + (ma - mi) \cdot (i_m - 1) / (K - 1) \quad (2)$$

$$b^K = (ma - mi) / (K - 1) \quad (3)$$

where ma is the maximum value of the attribute's domain, and mi is the minimum value. It is clear that $ma = 60$ and $mi = 0$ for 'Age'. Generally, A_{K,i_m}^{Age} can be described in a linguistic sentence such as:

$$A_{K,1}^{Age} : \text{young, and below } 60/(K - 1) \quad (4)$$

$$A_{K,K}^{Age} : \text{old, and above } [60 - 60/(K - 1)] \quad (5)$$

$$A_{K,i_m}^{Age} : \text{close to } (i_m - 1) \cdot [60 - 60/(K - 1)], \text{ and} \\ \text{between } (i_m - 2) \cdot [60 - 60/(K - 1)] \text{ and} \\ i_m \cdot [60 - 60/(K - 1)], \text{ for } 1 < i_m < K \quad (6)$$

A high-dimensional fuzzy grid can be further generated. For example, if we divide both 'Age' (x_1) and 'Salary' (x_2) into three linguistic values, then a feature space can be divided into 3×3 2-dim fuzzy grids, as shown in Fig. 4. For the shaded 2-dim fuzzy grid shown in Fig. 4, we can use a 2-dim fuzzy grid whose linguistic value is $A_{3,1}^{Age} \times A_{3,3}^{Salary}$ to stand for it. This concept is similar to the k -itemset used in CLIQUE [9].

2.2. Fuzzy partitioning in qualitative attributes

Qualitative attributes of a relational database have a finite number of possible values, with no ordering among values

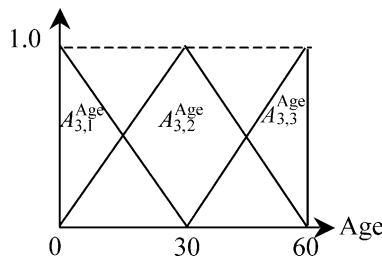


Fig. 2. $K = 3$ for 'Age'.

(e.g. sex, color) [11]. If the distinct attribute values are n' (n' is finite), then this attribute can only be partitioned by n' linguistic values. For example, the linguistic sentence of each linguistic value defined in 'Sex' can be stated as follows:

$$A_{2,1}^{Sex} : \text{male} \quad (7)$$

$$A_{2,2}^{Sex} : \text{female} \quad (8)$$

Each linguistic value distributed in either quantitative or categorical attributes is viewed as a candidate 1-dim fuzzy grid. The subsequent task is how to use these candidate 1-dim fuzzy grids to generate the other large fuzzy grids and fuzzy association rules. As we have mentioned above, the definitions of fuzzy support and the fuzzy confidence must be proposed.

3. Determine large fuzzy grids

After all candidate 1-dim fuzzy grids have been generated, we need to determine how to find the other large fuzzy grids and fuzzy association rules. The model for generating fuzzy association rules is described in Fig. 5, from which we can see that large fuzzy grids and fuzzy association rules are generated by phases I and II, respectively.

Suppose each linguistic variable, x_m , is divided into K various linguistic values. Given a candidate k -dim fuzzy grid, say $A_{K,i_1}^{x_1} \times A_{K,i_2}^{x_2} \times \dots \times A_{K,i_{k-1}}^{x_{k-1}} \times A_{K,i_k}^{x_k}$, the degree which t_p belongs to this fuzzy grid can be computed as $\mu_{K,i_1}^{x_1}(t_{p_1}) \cdot \mu_{K,i_2}^{x_2}(t_{p_2}) \cdot \dots \cdot \mu_{K,i_{k-1}}^{x_{k-1}}(t_{p_{k-1}}) \cdot \mu_{K,i_k}^{x_k}(t_{p_k})$.

To check whether this fuzzy grid is to be large or not, we define its fuzzy support $FS(A_{K,i_1}^{x_1} \times A_{K,i_2}^{x_2} \times \dots \times A_{K,i_{k-1}}^{x_{k-1}} \times A_{K,i_k}^{x_k})$

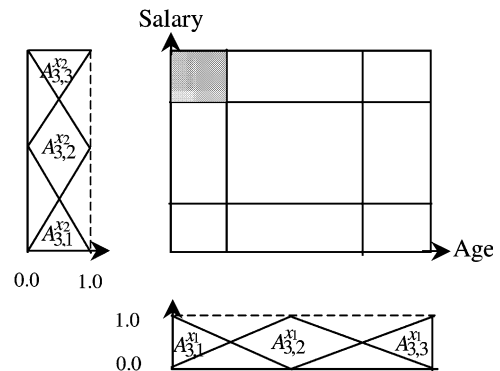


Fig. 4. Both 'Age' and 'Salary' are divided into three linguistic values.

as follows:

$$FS(A_{K,i_1}^{x_1} \times A_{K,i_2}^{x_2} \times \dots \times A_{K,i_{k-1}}^{x_{k-1}} \times A_{K,i_k}^{x_k}) = \left[\sum_{p=1}^n \mu_{K,i_1}^{x_1}(t_{p_1}) \cdot \mu_{K,i_2}^{x_2}(t_{p_2}) \cdot \dots \cdot \mu_{K,i_{k-1}}^{x_{k-1}}(t_{p_{k-1}}) \cdot \mu_{K,i_k}^{x_k}(t_{p_k}) \right] / n \quad (9)$$

When $FS(A_{K,i_1}^{x_1} \times A_{K,i_2}^{x_2} \times \dots \times A_{K,i_{k-1}}^{x_{k-1}} \times A_{K,i_k}^{x_k})$ is larger than or equal to the user-specified minimum fuzzy support (i.e. min FS), we can say that $A_{K,i_1}^{x_1} \times A_{K,i_2}^{x_2} \times \dots \times A_{K,i_{k-1}}^{x_{k-1}} \times A_{K,i_k}^{x_k}$ is a large k -dim fuzzy grid. For any two large grids, say $A_{K,i_1}^{x_1} \times A_{K,i_2}^{x_2} \times \dots \times A_{K,i_{k-1}}^{x_{k-1}} \times A_{K,i_k}^{x_k}$ and $A_{K,i_1}^{x_1} \times A_{K,i_2}^{x_2} \times \dots \times A_{K,i_{k-1}}^{x_{k-1}} \times A_{K,i_k}^{x_k} \times A_{K,i_{k+1}}^{x_{k+1}}$, since $\mu_{A_{K,i_1}^{x_1} \times A_{K,i_2}^{x_2} \times \dots \times A_{K,i_{k-1}}^{x_{k-1}} \times A_{K,i_k}^{x_k} \times A_{K,i_{k+1}}^{x_{k+1}}}(t_p) \leq \mu_{A_{K,i_1}^{x_1} \times A_{K,i_2}^{x_2} \times \dots \times A_{K,i_{k-1}}^{x_{k-1}} \times A_{K,i_k}^{x_k}}(t_p)$ from Eq. (9), $A_{K,i_1}^{x_1} \times A_{K,i_2}^{x_2} \times \dots \times A_{K,i_{k-1}}^{x_{k-1}} \times A_{K,i_k}^{x_k} \times A_{K,i_{k+1}}^{x_{k+1}} \subseteq A_{K,i_1}^{x_1} \times A_{K,i_2}^{x_2} \times \dots \times A_{K,i_{k-1}}^{x_{k-1}} \times A_{K,i_k}^{x_k}$ thus holds. It is clear that any subset of a large fuzzy grid must also be large.

The general form of a fuzzy association rule, say R , can be formulated as follows:

$$\text{Rule } R: A_{K,i_1}^{x_1} \times A_{K,i_2}^{x_2} \times \dots \times A_{K,i_\beta}^{x_\beta} \Rightarrow A_{K,i_{\beta+1}}^{x_{\beta+1}} \times A_{K,i_{\beta+2}}^{x_{\beta+2}} \times \dots \times A_{K,i_{\alpha-1}}^{x_{\alpha-1}} \times A_{K,i_\alpha}^{x_\alpha} \quad (10)$$

with $FC(R)$, for $1 \leq \alpha, \beta \leq d$

where $FC(R)$ is the fuzzy confidence of the above-mentioned rule. The left-hand side of ‘ \Rightarrow ’ is the antecedent part of R , and the right-hand side is the consequent part.

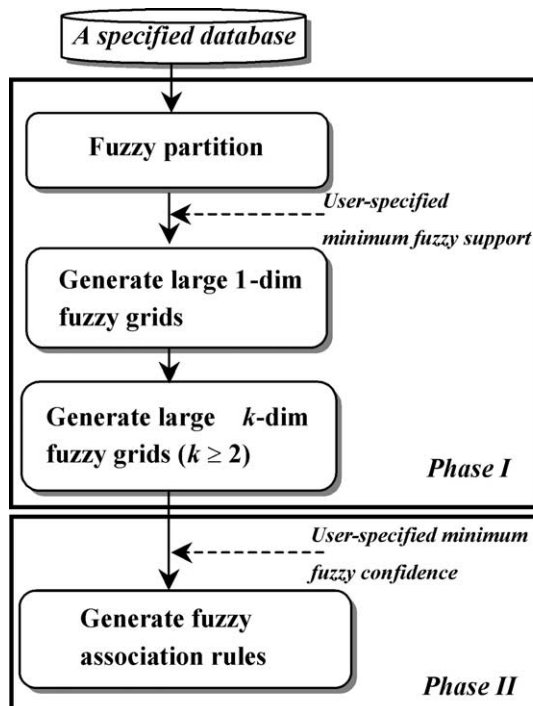


Fig. 5. The model for generating fuzzy association rules.

The linguistic description of this rule is that: if x_1 is $A_{K,i_1}^{x_1}$ and x_2 is $A_{K,i_2}^{x_2}$ and...and x_β is $A_{K,i_\beta}^{x_\beta}$, then $x_{\beta+1}$ is $A_{K,i_{\beta+1}}^{x_{\beta+1}}$ and $x_{\beta+2}$ is $A_{K,i_{\beta+2}}^{x_{\beta+2}}$ and...and x_α is $A_{K,i_\alpha}^{x_\alpha}$.

Since R is generated by two large fuzzy grids (i.e. $A_{K,i_1}^{x_1} \times A_{K,i_2}^{x_2} \times \dots \times A_{K,i_\beta}^{x_\beta} \times A_{K,i_{\beta+1}}^{x_{\beta+1}} \times \dots \times A_{K,i_{\alpha-1}}^{x_{\alpha-1}} \times A_{K,i_\alpha}^{x_\alpha}$ and $A_{K,i_1}^{x_1} \times A_{K,i_2}^{x_2} \times \dots \times A_{K,i_\beta}^{x_\beta}$), we define its fuzzy confidence as follows:

$$FC(R) = FS(A_{K,i_1}^{x_1} \times A_{K,i_2}^{x_2} \times \dots \times A_{K,i_\beta}^{x_\beta} \times A_{K,i_{\beta+1}}^{x_{\beta+1}} \times \dots \times A_{K,i_{\alpha-1}}^{x_{\alpha-1}} \times A_{K,i_\alpha}^{x_\alpha}) / FS(A_{K,i_1}^{x_1} \times A_{K,i_2}^{x_2} \times \dots \times A_{K,i_\beta}^{x_\beta}) \quad (11)$$

If its fuzzy confidence is larger or equal to min FC, then it is effective. The minimum fuzzy confidence is also user-specified. Here, we give a simple example to demonstrate how we generate the large 1-dim fuzzy grids.

3.1. Example

As shown in Fig. 6, the quantitative attribute ‘Age’ denoted by x_1 was divided into three various linguistic values. These three candidate 1-dim fuzzy grids could be described as follows:

- $A_{3,1}^{\text{Age}}$: young
- $A_{3,2}^{\text{Age}}$: medium
- $A_{3,3}^{\text{Age}}$: old

The degrees which t_p ($1 \leq p \leq 12$) belong to $A_{3,1}^{\text{Age}}$, $A_{3,2}^{\text{Age}}$ and $A_{3,3}^{\text{Age}}$ are shown in Table 1. We can thus compute the fuzzy support of these three candidate 1-dim fuzzy grids:

$$FS(A_{3,1}^{\text{Age}}) = \sum_{p=1}^{12} \mu_{3,1}^{\text{Age}}(t_{p_1}) / 12 = 0.275 \quad (12)$$

$$FS(A_{3,2}^{\text{Age}}) = \sum_{p=1}^{12} \mu_{3,2}^{\text{Age}}(t_{p_1}) / 12 = 0.475 \quad (13)$$

$$FS(A_{3,3}^{\text{Age}}) = \sum_{p=1}^{12} \mu_{3,3}^{\text{Age}}(t_{p_1}) / 12 = 0.250 \quad (14)$$

If the user-specified minimum fuzzy confidence is 0.30, then only $A_{3,2}^{\text{Age}}$ is the large 1-dim fuzzy grid.

In the subsequent section, an effective algorithm named FGBRMA is proposed to discover the fuzzy association rules.

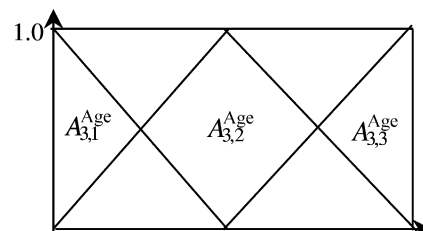


Fig. 6. Attribute ‘Age’ is divided into three various linguistic values.

Table 1
The degrees which t_p ($1 \leq p \leq 12$) belong to $A_{3,1}^{Age}$, $A_{3,2}^{Age}$, $A_{3,3}^{Age}$

t_p	$A_{3,1}^{Age}$	$A_{3,2}^{Age}$	$A_{3,3}^{Age}$
t_1	0.95	0.05	0.00
t_2	0.00	0.90	0.10
t_3	0.45	0.55	0.00
t_4	0.00	0.10	0.90
t_5	0.00	1.00	0.00
t_6	0.00	0.20	0.80
t_7	0.90	0.10	0.00
t_8	0.00	0.80	0.20
t_9	0.35	0.65	0.00
t_{10}	0.00	0.95	0.05
t_{11}	0.65	0.35	0.00
t_{12}	0.00	0.05	0.95

4. Fuzzy grids based rules mining algorithm

For the proposed algorithm, a table structure named FGTTFS is implemented to generate large fuzzy grids. This table consists of the following substructures:

- (a) Fuzzy grids table (FG): each row represents a fuzzy grid, and each column represents a linguistic value $A_{K,i_m}^{X_m}$.
- (b) Transaction table (TT): each column represents a tuple t_p , while each element records the membership degree to which t_p belongs to the corresponding fuzzy grid.
- (c) Column FS: stores the fuzzy support corresponding to the fuzzy grid in FG.

An initial tabular FGTTFS is shown as Table 2 as an example, from which we can see that there are two tuples t_1 and t_2 , and two attributes x_1 and x_2 in a given relation. Both x_1 and x_2 are divided into two linguistic values. Since each row of FG is a bits string consisting of 0 and 1, we can apply Boolean operations on FG[u] (i.e. the u th row of FG) = (FG[$u,1$], FG[$u,2$], FG[$u,3$], FG[$u,4$]) and FG[v] (i.e. the v th row of FG) = (FG[$v,1$], FG[$v,2$], FG[$v,3$], FG[$v,4$]) to generate some desired results. For example, FG[1] OR FG[3] = (1, 0, 0, 0) OR (0, 0, 1, 0) = (1, 0, 1, 0), corresponding to a candidate 2-dim fuzzy grid $A_{2,1}^{x_1} \times A_{2,1}^{x_2}$, is generated. Then, $FS(A_{2,1}^{x_1} \times A_{2,1}^{x_2}) = TT[1] \cdot TT[3] = [\mu_{2,1}^{x_1}(t_1) \cdot \mu_{2,2}^{x_2}(t_1) + \mu_{2,1}^{x_1}(t_2) \cdot \mu_{2,1}^{x_2}(t_2)]/2$ is obtained to compare with the min FS. However, any two linguistic values defined in the same candidate k -dim fuzzy grid ($k \geq 2$). Therefore, both (1, 1, 0, 0) and (0, 0, 1, 1) are invalid.

Generally, a candidate k -dim fuzzy grid, say $A_{K,i_1}^{x_1} \times A_{K,i_2}^{x_2} \times \dots \times A_{K,i_{k-1}}^{x_{k-1}} \times A_{K,i_k}^{x_k}$, is derived by joining two large ($k - 1$)-dim fuzzy grids (i.e. $A_{K,i_1}^{x_1} \times A_{K,i_2}^{x_2} \times \dots \times A_{K,i_{k-2}}^{x_{k-2}} \times A_{K,i_{k-1}}^{x_{k-1}}$, and $A_{K,i_1}^{x_1} \times A_{K,i_2}^{x_2} \times \dots \times A_{K,i_{k-2}}^{x_{k-2}} \times A_{K,i_{k-1}}^{x_{k-1}}$) and these two large grids share ($k - 2$) linguistic values. For example, we can use $A_{3,2}^{x_1} \times A_{3,2}^{x_2}$ and $A_{3,2}^{x_1} \times A_{3,3}^{x_3}$ to generate a candidate 3-dim fuzzy grid $A_{3,2}^{x_1} \times A_{3,1}^{x_2} \times A_{3,3}^{x_3}$ because $A_{3,2}^{x_1} \times A_{3,1}^{x_2}$ and

Table 2
Initial tabular FGTTFS

Fuzzy grids	FG				TT		FS
	$A_{2,1}^{x_1}$	$A_{2,2}^{x_1}$	$A_{2,1}^{x_2}$	$A_{2,2}^{x_2}$	t_1	t_2	
$A_{2,1}^{x_1}$	1	0	0	0	$\mu_{2,1}^{x_1}(t_1)$	$\mu_{2,1}^{x_1}(t_2)$	FS($A_{2,1}^{x_1}$)
$A_{2,2}^{x_1}$	0	1	0	0	$\mu_{2,2}^{x_1}(t_1)$	$\mu_{2,2}^{x_1}(t_2)$	FS($A_{2,2}^{x_1}$)
$A_{2,1}^{x_2}$	0	0	1	0	$\mu_{2,1}^{x_2}(t_1)$	$\mu_{2,1}^{x_2}(t_2)$	FS($A_{2,1}^{x_2}$)
$A_{2,2}^{x_2}$	0	0	0	1	$\mu_{2,2}^{x_2}(t_1)$	$\mu_{2,2}^{x_2}(t_2)$	FS($A_{2,2}^{x_2}$)

$A_{3,2}^{x_1} \times A_{3,3}^{x_3}$ share the linguistic value $A_{3,2}^{x_1}$. However, $A_{3,2}^{x_1} \times A_{3,1}^{x_2} \times A_{3,3}^{x_3}$ can also be generated by joining $A_{3,2}^{x_1} \times A_{3,1}^{x_2}$ to $A_{3,1}^{x_1} \times A_{3,3}^{x_3}$. This implies that we must select one of many possible combinations to avoid redundant computations. The method we adopt here is that if there exist integers $e_1, e_2, \dots, e_{k-1}, e_k$ where $1 \leq e_1 < e_2 < \dots < e_{k-1} < e_k \leq d$, such that $FG[u, e_1] = FG[u, e_2] = \dots = FG[u, e_{k-2}] = FG[u, e_{k-1}] = 1$ and $FG[v, e_1] = FG[v, e_2] = \dots = FG[v, e_{k-2}] = FG[v, e_k] = 1$, where FG[u] and FG[v] correspond to large ($k - 1$)-dim fuzzy grids, then FG[u] and FG[v] can be paired to generate a candidate k -dim fuzzy grid.

On the other hand, we still apply Boolean operations to obtain the antecedent part and consequent part of each rule. For example, if there exists FG[u] = (1, 0, 0, 0) and FG[v] = (1, 0, 0, 1) corresponding to large fuzzy grids L_u and L_v , where $L_v \subset L_u$, respectively; then the antecedent part $A_{2,1}^{x_1}$ and the consequent part $A_{2,1}^{x_2}$ of one rule, say R , can be obtained by computing (FG[u] AND FG[v]) (i.e. (1, 0, 0, 0)) and (FG[u] XOR FG[v]) (i.e. (0, 0, 1, 0)), respectively. $FC(R) = FS(A_{2,1}^{x_1} \times A_{2,1}^{x_2})/FS(A_{2,1}^{x_1})$ is further obtained to compare with the min FC to determine whether R is effective or not. FGBRMA is described as follows.

Algorithm. Fuzzy grids based rules mining algorithm

Input: a. a specified database; b. the user-specified minimum fuzzy support; c. and the user-specified minimum fuzzy confidence.

Output: phase I: Generate large fuzzy grids; phase II: Generate effective fuzzy association rules.

Method:

Phase I. Generate large fuzzy grids

Step 1. Perform the fuzzy partition

Step 2. Scan the database, and construct the initial table FGTTFS

Step 3. Generate large 1-dim fuzzy grids

3-1. Set $k = 1$ and eliminate the rows of initial FGTTFS corresponding to the candidate 1-dim fuzzy grids which are not large.

3-2. Reconstruct FGTTFS.

Step 4. Generate large k -dim fuzzy grids

Set $k + 1$ to k . If there is only one ($k - 1$)-dim fuzzy grid, then go to Step 5.

Table 3
Relation EMP

t_p	Age	Married	Numcars	Income	Career
t_1	19	N	0	20 000	Student
t_2	35	Y	1	50 000	Teacher
t_3	23	N	1	33 000	Engineer
t_4	33	Y	1	35 000	Engineer
t_5	45	Y	1	50 000	Trader
t_6	56	Y	1	45 000	Trader
t_7	18	N	0	25 000	Student
t_8	20	N	1	30 000	Engineer
t_9	33	Y	1	35 000	Engineer
t_{10}	35	Y	1	45 000	Trader

For any two unpaired rows, $FGTFS[u]$ and $FGTFS[v]$ ($u \neq v$), corresponding to large $(k - 1)$ -dim fuzzy grids do

4-1. From $(FG[u] \text{ OR } FG[v])$ that corresponds to a candidate k -dim fuzzy grid c , if any two linguistic values are defined in the same linguistic variable, then discard c and skip Steps 4-2, 4-3 and 4-4. That is, c is invalid.

4-2. If $FG[u]$ and $FG[v]$ do not share $(k - 2)$ linguistic terms, then discard c and skip Steps 4-3 and 4.4. That is, c is invalid.

4-3. If there exist integers $1 \leq e_1 < e_2 < \dots < e_k$ such that $(FG[u] \text{ OR } FG[v])[e_1] = (FG[u] \text{ OR } FG[v])[e_2] = \dots = (FG[u] \text{ OR } FG[v])[e_k] = 1$, then compute $(TT[e_1] \cdot TT[e_2] \cdot \dots \cdot TT[e_k])$ and the fuzzy support fs of c .

4-4. Add $(FG[u] \text{ OR } FG[v])$ to table FG, $(TT[e_1] \cdot TT[e_2] \cdot \dots \cdot TT[e_k])$ to TT and fs to FS when fs is larger than or equal to the min FS; otherwise, discard c .

End

Table 4
An initial FG obtained from EMP

Fuzzy grids	FG										
	$A_{3,1}^{Age}$	$A_{3,2}^{Age}$	$A_{3,3}^{Age}$	$A_{2,1}^{Married}$	$A_{2,2}^{Married}$	$A_{2,1}^{Numcars}$	$A_{2,2}^{Numcars}$	$A_{3,1}^{Income}$...	$A_{4,3}^{Career}$	$A_{4,4}^{Career}$
$A_{3,1}^{Age}$	1	0	0	0	0	0	0	0	...	0	0
$A_{3,2}^{Age}$	0	1	0	0	0	0	0	0	...	0	0
$A_{3,3}^{Age}$	0	0	1	0	0	0	0	0	...	0	0
$A_{2,1}^{Married}$	0	0	0	1	0	0	0	0	...	0	0
$A_{2,2}^{Married}$	0	0	0	0	1	0	0	0	...	0	0
$A_{2,1}^{Numcars}$	0	0	0	0	0	1	0	0	...	0	0
$A_{2,2}^{Numcars}$	0	0	0	0	0	0	1	0	...	0	0
$A_{3,1}^{Income}$	0	0	0	0	0	0	0	1	...	0	0
$A_{3,2}^{Income}$	0	0	0	0	0	0	0	0	...	0	0
$A_{3,3}^{Income}$	0	0	0	0	0	0	0	0	...	0	0
$A_{4,1}^{Career}$	0	0	0	0	0	0	0	0	...	0	0
$A_{4,2}^{Career}$	0	0	0	0	0	0	0	0	...	0	0
$A_{4,3}^{Career}$	0	0	0	0	0	0	0	0	...	1	0
$A_{4,4}^{Career}$	0	0	0	0	0	0	0	0	...	0	1

Step 5. Check whether or not any large k -dim fuzzy grid is generated

If any large k -dim fuzzy grid is generated, then repeat by going to Step 4, else continue to execute the phase II. It is noted that that the final FGTTFS only stores large fuzzy grid.

Phase II: Generate effective fuzzy association rules

For two unpaired rows, $FG[u]$ and $FG[v]$ ($u < v$), corresponding to large fuzzy grids L_u and L_v respectively do

Step 1. Generate the antecedent part of the rule

1-1. Let $temp$ be the number of nonzero elements in $(FG[u] \text{ AND } FG[v])$.

1-2. If the number of nonzero elements in $FG[u]$ is equal to $temp$, then $L_v \subset L_u$ is hold, and the antecedent part of one rule, say R , is generated as L_u ; otherwise skip Steps 2 and 3.

Step 2. Generate the consequence of the rule

Use $(FG[u] \text{ XOR } FG[v])$ to obtain the consequent part of R .

Step 3. Check or not whether rule R can be generated

$FC(R) = FS(L_v)/FS(L_u)$

If $FC(R) \geq \min FC$, then R is effective.

End

It should be noted that the design of FGBRMA follows that of the Apriori algorithm. In Section 5, a numerical example is used to demonstrate the effectiveness of FGBRMA.

5. Numerical example

A database relation EMP with 10 tuples t_p ($1 \leq p \leq 10$) is shown as Table 3. The purpose is to employ FGBRMA to

Table 7
Reconstruction of TTFS

TT										FS
t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	
0.6333	0.8333	0.7667	0.9000	0.5000	0.1333	0.6000	0.6667	0.9000	0.8333	0.6767
0.0000	1.0000	0.0000	1.0000	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000	0.6000
1.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1.0000	1.0000	0.0000	0.0000	0.4000
0.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0000	1.0000	1.0000	1.0000	0.8000
0.2222	0.4444	0.8000	0.8889	0.4444	0.6667	0.4444	0.6667	0.8889	0.6667	0.6133
0.0000	0.0000	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	1.0000	0.0000	0.4000
0.0000	0.0000	0.0000	0.0000	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.3000

Table 8
FG with large 2D fuzzy grids

Fuzzy grids	FG						
	$A_{3,2}^{Age}$	$A_{2,1}^{Married}$	$A_{2,2}^{Married}$	$A_{2,2}^{Numcars}$	$A_{3,2}^{Income}$	$A_{4,3}^{Career}$	$A_{4,4}^{Career}$
$A_{3,2}^{Age} \times A_{2,1}^{Married}$	1	1	0	0	0	0	0
$A_{3,2}^{Age} \times A_{2,2}^{Numcars}$	1	0	0	1	0	0	0
$A_{3,2}^{Age} \times A_{3,2}^{Income}$	1	0	0	0	0	0	0
$A_{3,2}^{Age} \times A_{4,3}^{Career}$	1	0	0	0	0	1	0
$A_{2,1}^{Married} \times A_{2,2}^{Numcars}$	0	1	0	1	0	0	0
$A_{2,1}^{Married} \times A_{3,2}^{Income}$	0	1	0	0	0	0	0
$A_{2,1}^{Married} \times A_{4,4}^{Career}$	0	1	0	0	0	0	1
$A_{2,2}^{Numcars} \times A_{3,2}^{Income}$	0	0	0	1	1	0	0
$A_{2,2}^{Numcars} \times A_{4,3}^{Career}$	0	0	0	1	0	1	0
$A_{2,2}^{Numcars} \times A_{4,4}^{Career}$	0	0	0	1	0	0	1
$A_{3,2}^{Income} \times A_{4,3}^{Career}$	0	0	0	0	1	1	0

Table 9
TTFS with large 2D fuzzy grids

TT										FS
t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	
0.0000	0.8333	0.0000	0.9000	0.5000	0.1333	0.0000	0.0000	0.9000	0.8333	0.4100
0.0000	0.8333	0.7667	0.9000	0.5000	0.1333	0.0000	0.6667	0.9000	0.8333	0.5533
0.1407	0.3703	0.6134	0.8000	0.2222	0.0889	0.2666	0.4445	0.8000	0.5556	0.4302
0.0000	0.0000	0.7667	0.9000	0.0000	0.0000	0.0000	0.6667	0.9000	0.0000	0.3233
0.0000	1.0000	0.0000	1.0000	1.0000	1.0000	0.0000	0.0000	1.0000	1.0000	0.6000
0.0000	0.4444	0.0000	0.8889	0.4444	0.6667	0.0000	0.0000	0.8889	0.6667	0.4000
0.0000	0.0000	0.0000	0.0000	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.3000
0.0000	0.4444	0.8000	0.8889	0.4444	0.6667	0.0000	0.6667	0.8889	0.6667	0.5467
0.0000	0.0000	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	1.0000	0.0000	0.4000
0.0000	0.0000	0.0000	0.0000	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.3000
0.0000	0.0000	0.8000	0.8889	0.0000	0.0000	0.0000	0.6667	0.8889	0.0000	0.3245

$A_{3,2}^{Age} \times A_{2,2}^{Numcars}$ share $A_{3,2}^{Age}$ and no any two linguistic values are defined in the same linguistic variable, therefore, we are sure that c is valid. The fuzzy support (i.e. 0.3277) of $A_{3,2}^{Age} \times A_{2,1}^{Married} \times A_{2,2}^{Numcars}$ can be further obtained by computing $(TT[1] \cdot TT[2] \cdot TT[4]) = (0.0000, 0.6944, 0.0000, 0.8100, 0.2500, 0.0178, 0.0000, 0.0000, 0.8100, 0.6944)$. Since 0.3277 is larger than 0.3, $A_{3,2}^{Age} \times A_{2,1}^{Married} \times A_{2,2}^{Numcars}$ is inserted to FGTTFS. Large 3-dim fuzzy grids are also sequentially inserted to the bottom of FGTTFS. Those large

3-dim fuzzy grids that can be inserted to FGTTFS are shown as Tables 10 and 11.

- Generate large 4-dim fuzzy grids

It is clear that no any large 4-dim fuzzy grid can be generated, we thus stop to generate large fuzzy grids and continue to execute the phase II.

Phase II: Generate effective fuzzy association rules

When all large fuzzy grids are generated, fuzzy association rules can be easily generated. If the minimum

Table 10
FG with large 3D fuzzy grids

Fuzzy grids	FG						
	$A_{3,2}^{Age}$	$A_{2,1}^{Married}$	$A_{2,2}^{Married}$	$A_{2,2}^{Numcars}$	$A_{3,2}^{Income}$	$A_{4,3}^{Career}$	$A_{4,4}^{Career}$
$A_{3,2}^{Age} \times A_{2,1}^{Married} \times A_{2,2}^{Numcars}$	1	1	0	1	0	0	0
$A_{3,2}^{Age} \times A_{2,2}^{Numcars} \times A_{3,2}^{Income}$	1	0	0	1	1	0	0
$A_{3,2}^{Age} \times A_{2,2}^{Numcars} \times A_{4,3}^{Career}$	1	0	0	1	0	1	0
$A_{2,1}^{Married} \times A_{2,2}^{Numcars} \times A_{3,2}^{Income}$	0	1	0	1	1	0	0
$A_{2,1}^{Married} \times A_{2,2}^{Numcars} \times A_{4,4}^{Career}$	0	1	0	1	0	0	1
$A_{2,2}^{Numcars} \times A_{3,2}^{Income} \times A_{4,3}^{Career}$	0	0	0	1	1	1	0

Table 11
TTFS with large 3D fuzzy grids

TT										FS
t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	t_9	t_{10}	
0.0000	0.8333	0.0000	0.9000	0.5000	0.1333	0.0000	0.0000	0.9000	0.8333	0.4100
0.0000	0.3703	0.6134	0.8000	0.2222	0.0889	0.0000	0.4445	0.8000	0.5556	0.3895
0.0000	0.0000	0.7667	0.9000	0.0000	0.0000	0.0000	0.6667	0.9000	0.0000	0.3233
0.0000	0.4444	0.0000	0.8889	0.4444	0.6667	0.0000	0.0000	0.8889	0.6667	0.4000
0.0000	0.0000	0.0000	0.0000	1.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.3000
0.0000	0.0000	0.8000	0.8889	0.0000	0.0000	0.0000	0.6667	0.8889	0.0000	0.3245

fuzzy confidence is 0.75, then fuzzy association rules with individual fuzzy confidences that are extracted from EMP are shown as Table 12. Obviously, the larger minimum fuzzy confidence, the smaller number of fuzzy association rules.

The main purpose of this numerical example is to demonstrate the effectiveness and usefulness of FGBRMA. In fact, the meaning of the fuzzy terms can be changed by linguistic hedge, as discussed in Section 6.

6. Discussions and analysis

In this paper, we propose the FGBRMA. As we have explained above, FGBRMA consists of two phases: one to generate the large fuzzy grids, and the other to generate the fuzzy association rules. It seems that the proposed algorithm is an efficient algorithm since it scans a database only once and applies Boolean operations on tables to generate large fuzzy grids and fuzzy association rules.

However, some significant topics must be discussed as follows.

6.1. Use the linguistic hedge to change the meaning of the fuzzy terms

The meaning of the linguistic values of a quantitative attribute, say x_m , can be changed by a linguistic hedge [20,21], such as ‘very’ or ‘more or less’. For example,

$$(A_{K,i_m}^{x_m})' = \text{very } A_{K,i_m}^{x_m} = (A_{K,i_m}^{x_m})^2 \tag{15}$$

$$(A_{K,i_m}^{x_m})'' = \text{more or less } A_{K,i_m}^{x_m} = (A_{K,i_m}^{x_m})^{1/2} \tag{16}$$

The membership functions, shown as Fig. 7, of $(A_{K,i_m}^{x_m})'$ and $(A_{K,i_m}^{x_m})''$ are $[\mu_{K,i_m}^{x_m}(x)]^2$ and $[\mu_{K,i_m}^{x_m}(x)]^{1/2}$, respectively.

It seems that these use of linguistic hedge will provide usefully linguistic values, which will make the fuzzy association rules discovered from the database more flexible for the users.

Table 12
Fuzzy association rules from EMP

Fuzzy association rules	FC
$A_{3,2}^{Age} \Rightarrow A_{2,2}^{Numcars}$	0.8176
$A_{4,3}^{Career} \Rightarrow A_{3,2}^{Age}$	0.8083
$A_{2,1}^{Married} \Rightarrow A_{2,2}^{Numcars}$	1.0
$A_{2,2}^{Numcars} \Rightarrow A_{2,1}^{Married}$	0.75
$A_{4,4}^{Career} \Rightarrow A_{2,1}^{Married}$	1.0
$A_{3,2}^{Age} \Rightarrow A_{2,2}^{Numcars}$	0.8914
$A_{4,3}^{Career} \Rightarrow A_{2,2}^{Numcars}$	1.0
$A_{4,4}^{Career} \Rightarrow A_{2,2}^{Numcars}$	1.0
$A_{4,3}^{Career} \Rightarrow A_{3,2}^{Age}$	0.8113
$A_{3,2}^{Age} \times A_{2,1}^{Married} \Rightarrow A_{2,2}^{Numcars}$	0.7993
$A_{2,1}^{Married} \times A_{3,2}^{Age} \Rightarrow A_{2,2}^{Numcars}$	1.0
$A_{2,1}^{Married} \times A_{4,4}^{Career} \Rightarrow A_{2,2}^{Numcars}$	1.0
$A_{2,2}^{Numcars} \times A_{4,4}^{Career} \Rightarrow A_{2,1}^{Married}$	1.0
$A_{2,2}^{Numcars} \times A_{4,3}^{Career} \Rightarrow A_{3,2}^{Age}$	0.8113
$A_{3,2}^{Age} \times A_{4,3}^{Career} \Rightarrow A_{2,2}^{Numcars}$	1.0

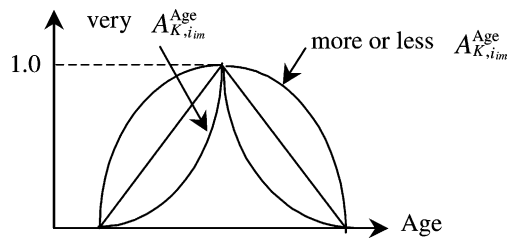


Fig. 7. Linguistic hedge for $A_{K,im}^{Age}$.

6.2. Define different number of linguistic values in each quantitative attribute

The number of linguistic values defined in each quantitative attribute need not be equal to K . For example, 'Age' can be divided into three fuzzy sets, and 'Income' can be divided into four fuzzy sets. Thus, a fuzzy association rule such as $A_{3,2}^{Age} \times A_{4,1}^{Income} \Rightarrow A_{5,3}^{Height}$ may be generated. In fact, decision makers may specify possible linguistic values for one quantitative attribute by using their preferences or domain knowledge.

6.3. Other topics

We do not restrict the shapes of the membership functions defined in the quantitative attributes. That is, trapezoid functions can also be used. In Lin and Lee [22], and Jang [23], the adjustment of the membership functions by learning from examples was proposed. Therefore, it seems to be possible to refine the membership functions of linguistic values by using various machine learning techniques.

On the other hand, database mining problems involving classification can be viewed within a common framework of rule discovery [24]. Based on FGBRMA, it is feasible to develop effective algorithms to discover the fuzzy classification rules.

7. Conclusions

Fuzzy association rules described by the natural language are well suitable for the thinking of human subjects. Thus, fuzzy association rules will be helpful to increase the flexibility for the users in making any decisions or designing the fuzzy systems. Furthermore, it seems that the goal of knowledge acquisition can be achieved for users by checking the fuzzy classification rules. Therefore, finding fuzzy association rules is necessary.

In this paper, the definitions of fuzzy support and fuzzy confidence for determining large fuzzy grids and effective fuzzy association rules, respectively, are proposed. An effective algorithm named FGBRMA consisting of two phases is further proposed. From the numerical example

described in Section 5, we can see that the proposed algorithm is effective and useful for finding fuzzy association rule.

As we have mentioned in discussions, we can also use various partition methods or the linguistic hedge to make the fuzzy association rules discovered from the database more flexible to decision makers.

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