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On the equivalence between magnetic-field-induced phase transitions in the integer quantum Hall effect

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Abstract

Magnetic-field-induced phase transitions in the two-dimensional electron system in a AlGaAs/InGaAs/GaAs heterostructure are studied. Two kinds of magnetic-field-induced phase transitions, plateau–plateau (P–P) and insulator–quantum Hall conductor (I–QH) transitions, are observed in the integer quantum Hall effect regime at high magnetic fields. In the P–P transition, both the semicircle law and the universality of critical conductivities are broken and we do not observe the universal scaling. However, the P–P transition can still be mapped to the I–QH transition by the Landau-level addition transformation, and as the temperature decreases the critical points of these two transitions appear at the same temperature. Our observations indicate that the equivalence between P–P and I–QH transitions can be found by the suitable analysis even when some expected universal properties are invalid.

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Keywords: D. Phase transitions; D. Quantum Hall effect; D. Scaling

Magnetic-field-induced phase transitions observed in a two-dimensional electron system (2DES) have attracted much attention [1–15]. Applying a magnetic field B perpendicular to a two-dimensional system, in the quantum Hall (QH) regime we can observe two types of magnetic-field-induced phase transitions, the plateau–plateau (P–P) and insulator–quantum Hall conductor (I–QH) transitions [1–3]. The former is observed between two quantum Hall states while the latter is observed between the insulating and quantum Hall states. These two types of magnetic-field-induced phase transitions, in fact, are expected to be of the same universal class [1,2]. All the magnetic-field-induced phase transitions should follow the scaling with the universal critical exponent $\kappa = 0.42 \pm 0.04$ [4,5] and obey the semicircle law [6–8]. According to the scaling, every magnetic-field-induced phase transition shrinks to a critical point as the temperature T approaches zero [1,4,8]. The universality of the critical conductivities, i.e. the longitudinal and Hall conductivities σ_{xx} and σ_{xy} at the critical points, is also expected [2,9]. In the integer quantum Hall effect (IQHE), both P–P and I–QH transitions occur as the Fermi energy passes through the extended states of a Landau band as B changes [1,2]. It was shown that in the IQHE, we can map the P–P transition to the I–QH transition by

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performing the Landau-level addition transformation to subtract the contributions of the Landau bands in which all states are below the Fermi energy and are filled [1,2,8].

It was shown by Shahar et al. [2] that P–P and I–QH transitions are equivalent, and we can see the equivalence by examining the semicircle law, reflection symmetry, universal scaling, and the universality of critical conductivities. However, while the universality of κ is expected and can be taken as a quantitative evidence for the equivalence between magnetic-field-induced phase transitions [1,5,14,15], there are reports on the deviation of the critical exponent κ [10, 11]. In fact, Shahar et al. [12] later showed that magnetic-field-induced phase transitions may not have the expected scaling behaviors. The validity of the semicircle law and the universality of critical conductivities are also questionable [8,13,16]. These unexpected results could be due to the inhomogeneity [14], mixing between Landau bands [16,17], small spin-splitting [8], or finite temperatures [14,15]. In the high-mobility two-dimensional systems, we also need to take into account the nonlocal effects due to the suppression of interchannel scattering between edge and bulk current-carrying channels [18]. Actually the effective strength of the disorder potential experienced by a two-dimensional system can change with the sweeping magnetic field [1]. At low B, it was shown that the merge of extended states of different Landau bands could induce I–QH transitions which are inconsistent with the global phase diagram (GPD) of the quantum Hall effect [19–22]. In fact, most realistic two-dimensional systems at low B reveal little about the magnetic-field-induced phase transitions because the localization length can be much larger than the sample sizes and the inelastic scattering length [1,23]. Recently, there are reports on the revival of the universality [24] and on how to subtract inhomogeneity effects near the critical points [14].

To further examine the equivalence between magnetic-field-induced phase transitions in the IQHE, we report the study on the IQHE of the 2DES in a AlGaAs/InGaAs/GaAs heterostructure. At high fields, we observed the P–P transition between $\nu = 1$ and 2 quantum Hall states and the I–QH transition from $\nu = 1$ quantum Hall state to the insulating state, where ν presents the filling factor. For convenience, we denote the observed P–P and I–QH transitions as the 1–2 and 0–1 transitions. The number '0' denotes the insulating state while the numbers '1' and '2' denote the $\nu = 1$ and 2 quantum Hall states, respectively [1, 2]. In our study, expected properties such as the semicircle law, the universal scaling, and the universality of critical conductivities are not observed in the 1–2 transition. By suitable analysis, however, we can still find evidences to support the equivalence between magnetic-field-induced phase transitions in the IQHE.

The sample used for this study is an AlGaAs/InGaAs/GaAs heterostructure. Fig. 1 shows its structure, in which the 2DES is located in the $In_{0.15}Ga_{0.85}As$ quantum well. The Hall pattern was made by the standard lithography and etching processes, and the Au/Ge alloy was alloyed into

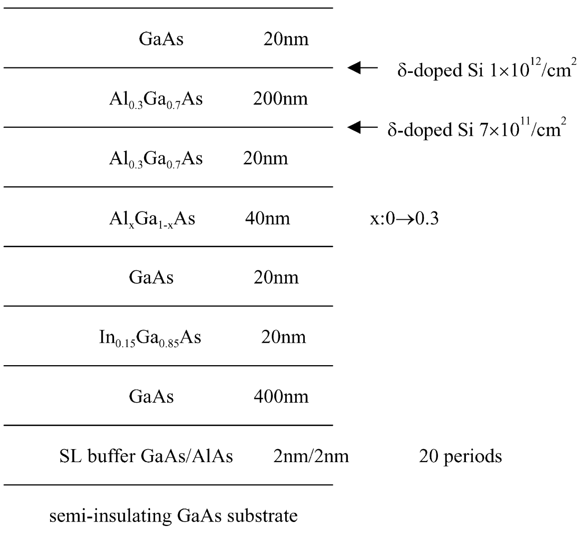

Fig. 1. The sample structure.

contact regions in N_2 atmosphere to form ohmic contacts. Magnetotransport measurements were performed with a 0–15 T superconductor magnet. We used a top-loading He^3 refrigerator for the temperature $T = 0.3$–1.6 K and a He^4 refrigerator for $T = 2.4$–4.2 K. The low-frequency AC lock-in technique was used with a current $I = 0.1$ μA to measure the longitudinal and Hall voltages V_{xx} and V_{xy}, and we can obtain the longitudinal and Hall resistivities ρ_{xx} and ρ_{xy} by $\rho_{xx} = kV_{xx}/I$ and $\rho_{xy} = V_{xy}/I$, respectively. Here $k = w/l$ with l as the distance between two probes to measure V_{xx} and w as the width of the current channel. From the Shubnikov-de Haas oscillations observed in ρ_{xx} at low B, as shown in Fig. 2, the carrier concentration $n = 1.3 \times 10^{11}\ \mathrm{cm}^{-2}$. The classical mobility $\mu_c = 1.0 \times 10^5\ \mathrm{cm^2/V\,s}$ from $\rho_{xx} = 1/ne\mu_c$ at $B = 0$. In

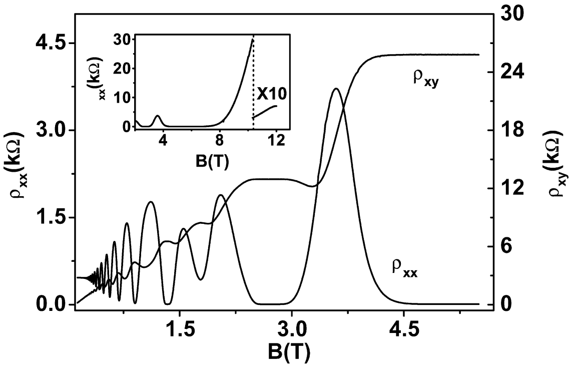

Fig. 2. The curves of the longitudinal and Hall resistivities ρ_{xx} and ρ_{xy} at the temperature $T = 0.58$ K when the magnetic field $B = 0$–5.5 T. At lower B we can observe Shubnikov-de Haas oscillations while at higher B we can observe quantum Hall states of the filling factors $\nu = 1$ and 2. The inset shows the curve of ρ_{xx} at $T = 0.58$ K when $B = 2$–12 T. When $B > 7$ T, ρ_{xx} increases rapidly and can exceed $h/e^2 \sim 25.8$ kΩ.

Fig. 2, at high B we can see $\nu = 1$ and 2 quantum Hall states in which $\rho_{xx} \to 0$ and $\rho_{xy} \to h/\nu e^2$. As shown in the inset of Fig. 2, ρ_{xx} increases monotonically as B increases and can exceed $h/e^2 \sim 25.8\ \mathrm{k}\Omega$ when $B > 10$ T, and the sample enters the insulating state [2]. Since the insulating state and the quantum Hall states of $\nu = 1$ and 2 are observed in the B sweep, we can study the 0–1 and 1–2 transitions to examine the equivalence between magnetic-field-induced phase transitions in the IQHE.

Transforming ρ_{xx} and ρ_{xy} to the longitudinal and Hall conductivities $\sigma_{xx} = \rho_{xx}/(\rho_{xx}^2 + \rho_{xy}^2)$ and $\sigma_{xy} = \rho_{xy}/(\rho_{xx}^2 + \rho_{xy}^2)$, we can plot the temperature-driven flow line diagram [13,25] for the 1–2 transition in the σ_{xy}–σ_{xx} plane, as shown in Fig. 3(a). Each solid line corresponds to a temperature-driven flow line at a magnetic field. At $B = 3.4$ T the temperature-driven flow line tends to a fixed point Q marked by the arrow as the temperature T decreases, so the critical magnetic field equals 3.4 T in the 1–2 transition [8,13]. At the critical magnetic field the filling factor $\nu = 1.59$. When

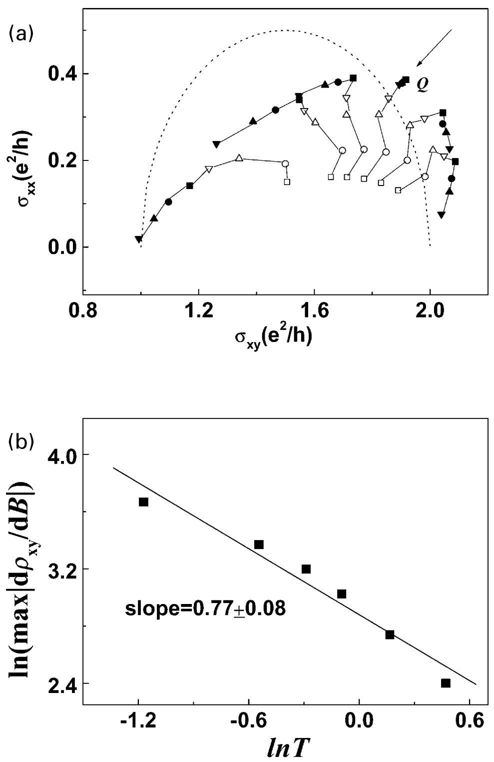

Fig. 3. (a) The temperature-driven flow line diagram for the 1–2 P–P transition. Each solid line corresponds to a temperature-driven flow line $(\sigma_{xy}(T), \sigma_{xx}(T))$ at a magnetic field B, and the dot line presents the semicircle $(\sigma_{xy} - 1.5e^2/h)^2 + \sigma_{xx}^2 = (0.5e^2/h)^2$. The open squares, circles, up triangles, and down triangles present $(\sigma_{xy}, \sigma_{xx})$ at the temperature $T = 4$, 2.4, 1.6, 1.2 K and the solid squares, circles, up triangles, and down triangles present $(\sigma_{xy}, \sigma_{xx})$ at $T = 0.91$, 0.75, 0.58, and 0.31 K, respectively. At low temperatures, the flow line corresponding to $B = 3.4$ T approaches the point $Q = (1.9e^2/h, 0.39e^2/h)$ marked by the arrow. (b) The line $\ln(\max|\mathrm{d}\rho_{xy}/\mathrm{d}B|) - \ln T$ for the 1–2 transition.

$T > 0.91$ K, the flow line corresponding to the critical magnetic field deviates from the point Q and hence the critical point is valid only for $T \leq 0.91$ K. The point Q, however, is located at $(1.9e^2/h, 0.39e^2/h)$ rather than at the expected point $(1.5e^2/h, 0.5e^2/h)$ and the universality of critical conductivities is not valid. In Fig. 3(a), in fact, at low temperatures the temperature-driven flow lines do not flow along the semicircle

$$\left(\sigma_{xy} - \frac{3}{2}\frac{e^2}{h}\right)^2 + \sigma_{xx}^2 = \left(\frac{1}{2}\frac{e^2}{h}\right)^2 \qquad (1)$$

and the semicircle law is not obeyed. Fig. 3(b) shows the curve $\ln(\max|\mathrm{d}\rho_{xy}/\mathrm{d}B|) - \ln T$ for the 1–2 transition when $T = 0.31$–1.6 K. If the universality of the scaling is valid, $\max|\mathrm{d}\rho_{xy}/\mathrm{d}B| \propto T^{-\kappa}$ with the critical exponent $\kappa = 0.42 \pm 0.04$ and hence the curve $\ln(\max|\mathrm{d}\rho_{xy}/\mathrm{d}B|) - \ln T$ must be a straight line with the slope close to 0.42 [5,10]. However, in Fig. 3(b) the slope of the line $\ln(\max|\mathrm{d}\rho_{xy}/\mathrm{d}B|) - \ln T$ is 0.77 ± 0.08, deviating from the expected value 0.42. Hence we do not observe the universal scaling, which could be due to that the temperature is not low enough [14,15] or that the universality of the scaling is broken [10–12]. Therefore, in the 1–2 transition the semicircle law and the universality of the critical conductivities are broken and the expected universal scaling is not observed.

But not all expected properties are broken in the 1–2 transition. In Fig. 3(a) the directions of temperature-driven flow lines are different on the left and right hand sides of the point Q, and Q serves as the unique unstable point in the temperature-driven flow line diagram for such a transition as expected [8,13,25]. To further study the 1–2 transition, we can perform the Landau-level addition transformation [1,2]. In a P–P transition in the IQHE, such a transformation can be used to subtract the contributions of Landau bands in which all states are filled [2]. Since there is only a filled Landau band in the 1–2 transition, we can define the effective conductivities σ_{xy}^t and σ_{xx}^t as

$$\sigma_{xy}^t = \sigma_{xy} - e^2/h. \qquad (2)$$

$$\sigma_{xx}^t = \sigma_{xx}, \qquad (3)$$

to subtract the contribution of this filled Landau band. The corresponding effective longitudinal resistivity [2,14]:

$$\rho_{xx}^t \equiv \sigma_{xx}^t/[(\sigma_{xx}^t)^2 + (\sigma_{xy}^t)^2]. \qquad (4)$$

Fig. 4 shows the curves of ρ_{xx}^t with respect to B when $T = 0.31$–0.91 K. Just as ρ_{xx} in the 0–1 transition, in fact, ρ_{xx}^t is temperature-independent at the critical magnetic field $B = 3.4$ T [2]. In addition, as T decreases the effective longitudinal resistivity ρ_{xx}^t decreases when $B < 3.4$ T, but increases when $B > 3.4$ T. So in ρ_{xx}^t the critical magnetic field separates two regions with different temperature-dependences. Therefore, the 1–2 transition is mapped to the I–QH transition by the Landau-level addition transformation. Our observations indicate that in the IQHE different magnetic-field-induced transitions can be related by the

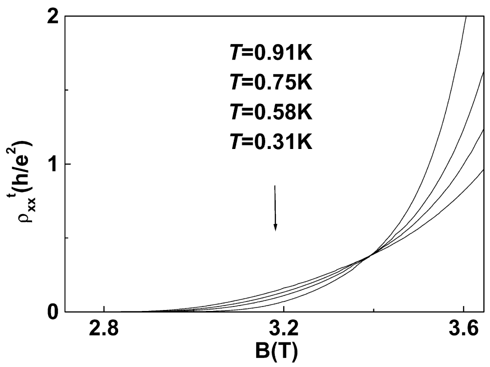

Fig. 4. The curves of effective longitudinal resistivity $\rho_{xx}^t(B)$ obtained by performing Landau-level addition transformation on the 1–2 transition at the temperature $T = 0.91$, 0.75, 0.58, 0.31 K. At the critical magnetic field $B = 3.4$ T, ρ_{xx}^t is T-independent.

Landau-level addition transformation even when the semicircle law and the universality of the critical conductivities are broken and the universal scaling is not observed.

We also observed the 0–1 transition and hence can compare such a transition to the 1–2 transition. Fig. 5 shows the curves of $\rho_{xx}(B)$ at different temperatures near $B = 9.8$ T. As T decreases from 0.91 to 0.31 K, ρ_{xx} increases and the 2DES is in the insulating state when $B > 9.8$ T, but decreases and the 2DES is in the quantum Hall state of $\nu = 1$ when $B < 9.8$ T. In the same temperature range, ρ_{xx} is temperature-independent at $B = 9.8$ T and hence in the 0–1 transition the critical magnetic field is 9.8 T, at which the filling factor $\nu = 0.55$. When $T > 0.91$ K, in Fig. 5 there is no well-defined temperature-independent point and hence the critical point of the 0–1 transition appears at 0.91 K as T decreases. At low temperatures, $\rho_{xx} = 20\ \mathrm{k}\Omega$ while $\rho_{xx}^t = 10.5\ \mathrm{k}\Omega$ at the critical magnetic fields of the 0–1 and 1–2 transitions, respectively. Therefore, unlike Shahar et al. [2], we cannot see the equivalence between the 1–2 and 0–1 transitions from the values of ρ_{xx}^t and ρ_{xx} at the critical magnetic fields. However, in Figs. 3(a) and 5 both the critical points of the 1–2 and 0–1 transitions appear at 0.91 K when T decreases, which provides the quantitative evidence for the equivalence between these two transitions in our study.

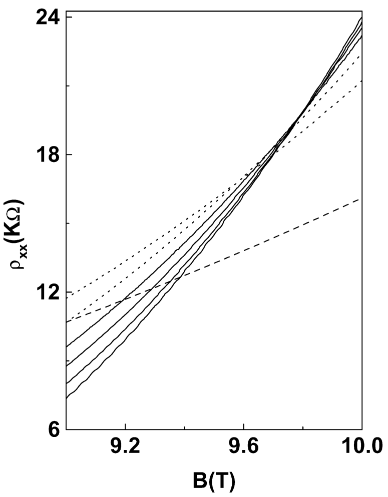

Fig. 5. The traces of the longitudinal resistivity $\rho_{xx}(B)$ at the temperature $T = 0.31$, 0.58, 0.75, 0.91, 1.18, 1.6, and 2.4 K when the magnetic field B is close to 9.8 T. The solid lines present $\rho_{xx}(B)$ when $T = 0.31$–0.91 K, and we can see that ρ_{xx} is T-independent at $B = 9.8$ T in this temperature range. The dotted lines present $\rho_{xx}(B)$ when $T = 1.18$ and 1.6 K and the dashed line corresponds to $T =$ 2.4 K. The longitudinal resistivity ρ_{xx} increases as T decreases at any $B > 9.8$ T.

In conclusion, we have performed magnetotransport measurements on the 2DES in a AlGaAs/InGaAs/GaAs system. At high fields, the 1–2 P–P and the 0–1 I–QH transitions are observed, and we can examine the equivalence between magnetic-field-induced phase transitions in the IQHE. In our study, in the 1–2 transition the semicircle law and the universality of critical conductivities are both broken and the universal scaling is not observed. However, not all expected features are broken. In the σ_{xy}–σ_{xx} plane the directions of the temperature-driven flow lines are unstable near the critical point of the 1–2 transition as expected, and we can still perform the Landau-level addition transformation to map the 1–2 transition to the insulator–quantum Hall conductor transition. The critical points of the 1–2 and 0–1 transitions, in fact, appear at the same temperature as the temperature decreases, which provides the quantitative evidence for the equivalence between these two transitions. Our observations show that the equivalence between magnetic-field-induced phase transitions can be found even when the semicircle law and the universality of critical conductivities are broken and the universal scaling is not observed.

Acknowledgements

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