Self-Learning Fuzzy Sliding-Mode Control for Antilock Braking Systems

Chih-Min Lin and Chun-Fei Hsu

***Abstract*—The antilock braking system (ABS) is designed to optimize braking effectiveness and maintain steerability; however, the ABS performance will be degraded in the case of severe road conditions. In this study, a self-learning fuzzy sliding-mode control (SLFSMC) design method is proposed for ABS. The SLFSMC ABS will modulate the brake torque for optimum braking. The SLFSMC system is comprised of a fuzzy controller and a robust controller. The fuzzy controller is designed to mimic an ideal controller and the robust controller is designed to compensate for the approximation error between the ideal controller and the fuzzy controller. The tuning algorithms of the controller are derived in the Lyapunov sense; thus, the stability of the system can be guaranteed. Also, the derivation of the proposed SLFSMC ABS does not need to use a vehicle-braking model. Simulations are performed to demonstrate the effectiveness of the proposed SLFSMC ABS in adapting to changes for various road conditions.**

***Index Terms*—Adaptive law, antilock braking system (ABS), fuzzy approximator, fuzzy control (FC), global stability, sliding-mode control.**

I. Introduction

A MOTOR vehicle has large amount of kinetic energy as it is driven. When the brakes are applied, the kinetic energy of the vehicle is dissipated as heat energy within the brake disks, and between the wheels and the pavement. In order to implement an advanced vehicle control system that provides desired safety and vehicle motion, an antilock braking system (ABS) should maintain brake torque to provide maximum wheel traction force. Moreover, there are difficulties in designing an ABS because the vehicle-braking dynamics are nonlinear and there are unknown environmental parameters. Currently, most commercial ABSs are based on a look-up tabular approach. These tables are calibrated through iterative laboratory experiments and engineering field tests. Therefore, these systems are not adaptive and issues such as robustness are not addressed. Many different control methods for ABS have been developed and research on improved control methods is continuing [1]–[6]. However, most of these approaches require system models, and some of them cannot achieve satisfactory performance under the changes of various road conditions.

Fuzzy control (FC) using linguistic information can model the qualitative aspects of human knowledge with several advantages such as robustness, universal approximation theorem and rule-based algorithm [7]. FC has been proposed to tackle the problem of ABS for the unknown environmental parameters [2], [3], [5]. However, the large amount of the fuzzy rules makes the analysis complex. Some researchers have proposed fuzzy control design methods based on the sliding-mode control (SMC) scheme. These approaches are referred to as fuzzy sliding-mode control (FSMC) design methods [8], [9]. Since only one variable is defined as the fuzzy input variable, the main advantage of the FSMC is it requires fewer fuzzy rules than FC does. Moreover, the FSMC system has more robustness against parameter variation [9]. Although FC and FSMC are both effective methods, their major drawback is that the fuzzy rules should be previously tuned by time-consuming trial-and-error procedures. To tackle this problem, adaptive fuzzy control (AFC) based on the Lyapunov synthesis approach has been extensively studied [10], [11]. With this approach, the fuzzy rules can be automatically adjusted to achieve satisfactory system response by an adaptive law.

The motivation of this study is to propose a self-learning fuzzy sliding-mode control (SLFSMC) design method for ABS. In the proposed SLFSMC system, a fuzzy controller is the main tracking controller, which is used to mimic an ideal controller; and a robust controller is derived to compensate for the difference between the ideal controller and the fuzzy controller. The SLFSMC has the advantages that it can automatically adjust the fuzzy rules, similar to the AFC, and can reduce the fuzzy rules, similar to the FSMC. Moreover, an error estimation mechanism is investigated to observe the bound of approximation error. All parameters in SLFSMC are tuned in the Lyapunov sense, thus, the stability of the system can be guaranteed. Finally, two simulation scenarios are examined and a comparison between a SMC, an FSMC, and the proposed SLFSMC is made.

II. Formulation of ABS

The dynamic equations of ABS are the result of Newton's law applied to the wheels and the vehicle [2]. The vehicle dynamic is determined by summing the total forces applied to the vehicle during a braking operation to obtain

$$\dot{V}_v(t) = \frac{-1}{M_v}\left[4F_t(t) + B_v V_v(t) + F_\theta(\theta)\right] \tag{1}$$

where $V_v(t)$ is the velocity of the vehicle; M_v is the mass of the vehicle; B_v is the vehicle viscous friction; $F_t(t)$ is the tractive force, and $F_\theta(\theta)$ is the force applied to the car which results from a vertical gradient in the road so that

$$F_\theta(\theta) = M_v g \sin(\theta) \tag{2}$$

where θ is the angle of inclination of the road, and g is the gravitational acceleration constant. The tractive force $F_t(t)$ is given by

$$F_t(t) = \mu(\lambda) N_v(\theta) \tag{3}$$

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where $N_v(\theta)$ is the nominal wheel reaction force applied to the wheel, and $\mu(\lambda)$ is the coefficient of friction. For this model, assume that the vehicle has four wheels and the weight of the vehicle is evenly distributed between these wheels. Then, $N_v(\theta)$ can be expressed by

$$N_v(\theta) = \frac{M_v g}{4}\cos(\theta). \tag{4}$$

Since the term $V_v(t)/R_w$ is the angular velocity of the vehicle with respect to the wheel angular velocity, denote this quantity as

$$w_v(t) = \frac{V_v(t)}{R_w} \tag{5}$$

where R_w is the radius of the wheel. The wheel dynamic is determined by summing the rotational torques which are applied to the wheel to obtain

$$\dot{w}_w(t) = \frac{1}{J_w}\left[-T_b(t) - B_w w_w(t) + T_t(t)\right] \tag{6}$$

where $w_w(t)$ is the angular velocity of the wheel, J_w is the rotation inertia of the wheel, B_w is the viscous friction of the wheel, $T_b(t)$ is the braking torque, and $T_t(t)$ is the torque generated due to slip between the wheel and the road surface. In general, $T_t(t)$ is a function of the force $F_t(t)$ expressed between the wheel and the road surface as

$$T_t(t) = R_w F_t(t). \tag{7}$$

The control objective of ABS is to regulate wheel slip to maximize the coefficient of friction between the wheel and the road for any given road surface. In general, the coefficient of friction μ during a braking operation can be described as a function of the slip λ, which is defined as

$$\lambda(t) = \frac{w_v(t) - w_w(t)}{w_v(t)}. \tag{8}$$

The tractive forces have been measured in [2], [12]. The results of these experiments were approximated for dry asphalt, wet asphalt, and icy roads, as shown in Fig. 1 [2]. In addition to the tractive forces, the wheel must also generate a lateral force to direct the vehicle. Like the tractive forces, the later force is also dependent on the slip ratio. The lateral force will be decreased as the slip is increased, which is also shown in Fig. 1 [12]. Thus, the steering ability will be decreased as the slip is increased.

During braking, the wheel slip has been defined as (8). Taking the time derivative, it is revealed that

$$\dot{\lambda} = \frac{(1-\lambda)\dot{w}_v - \dot{w}_w}{w_v}. \tag{9}$$

Substituting (1), (5) and (6) into (9), it is obtained that

$$\dot{\lambda} = F_p(\lambda, t) + G_p u(t) \tag{10}$$

where

$$F_p(\lambda, t) = \left(\frac{4F_t + B_v R_w w_v + F_\theta}{M_v R_w w_v}\right)\lambda + \frac{\frac{-(4F_t + B_v R_w w_v + F_\theta)}{M_v R_w} - \frac{(-B_w W_w + T_t)}{J_w}}{w_v}$$

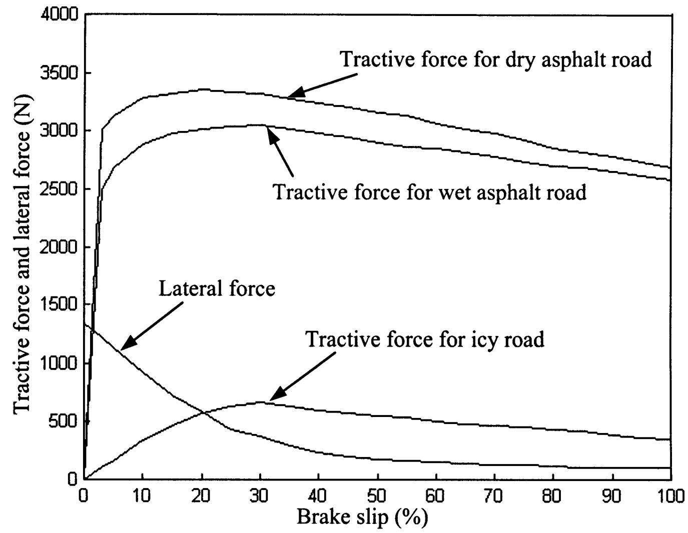

Fig. 1. Tractive forces and lateral force versus slip ratio.

is a nonlinear dynamic function; $G_p = 1/J_w$ is a control gain which is a positive constant, and $u(t) = T_b(t)/w_v$ is a control effort. Assume that the parameters of the system are well known and rewriting (10), it can represent the nominal model as

$$\dot{\lambda} = F_n(\lambda, t) + G_n u(t) \tag{11}$$

where $F_n(\lambda, t)$ and G_n are the nominal values of $F_p(\lambda, t)$ and G_p, respectively. The system parameters in the nominal condition are measured at $\mu(\lambda) = 0.9$. If the uncertainties occur, then the controlled system can be modified as

$$\begin{aligned}\dot{\lambda} &= [F_n(\lambda, t) + \Delta F(\lambda, t)] + [G_n + \Delta G]u(t)\\ &= F_n(\lambda, t) + G_n u(t) + w\end{aligned} \tag{12}$$

where $\Delta F(\lambda, t)$ and ΔG denote the system uncertainties; w is referred to as the lump uncertainty and is defined as $w = \Delta F(\lambda, t) + \Delta G u(t)$ with the assumption $|w| \le W$, in which W is a positive constant. The design of ABS is based on the assumption that the vehicle and wheel speeds are both available. Some techniques used a wheel speed sensor to measure the wheel speed and used an estimation algorithm to estimate the vehicle speed [6]. In recent years, some vehicle companies such as BMW company has used both vehicle and wheel speed sensors to measure vehicle and wheel speeds, respectively [13].

III. Sliding-Mode Control System

The control objective is to find a control law so that the slip can track the desired trajectory $\lambda_d(t)$. Define the tracking error as follows:

$$\lambda_e(t) = \lambda_d(t) - \lambda(t) \tag{13}$$

where $\lambda(t)$ is the output and $\lambda_d(t)$ is the reference trajectory, which is specified by the command input $\lambda_c(t)$ followed by a reference model. Then, define a sliding surface as

$$s(t) = \lambda_e(t) + k_1\int_0^t \lambda_e(\tau)\,d\tau \tag{14}$$

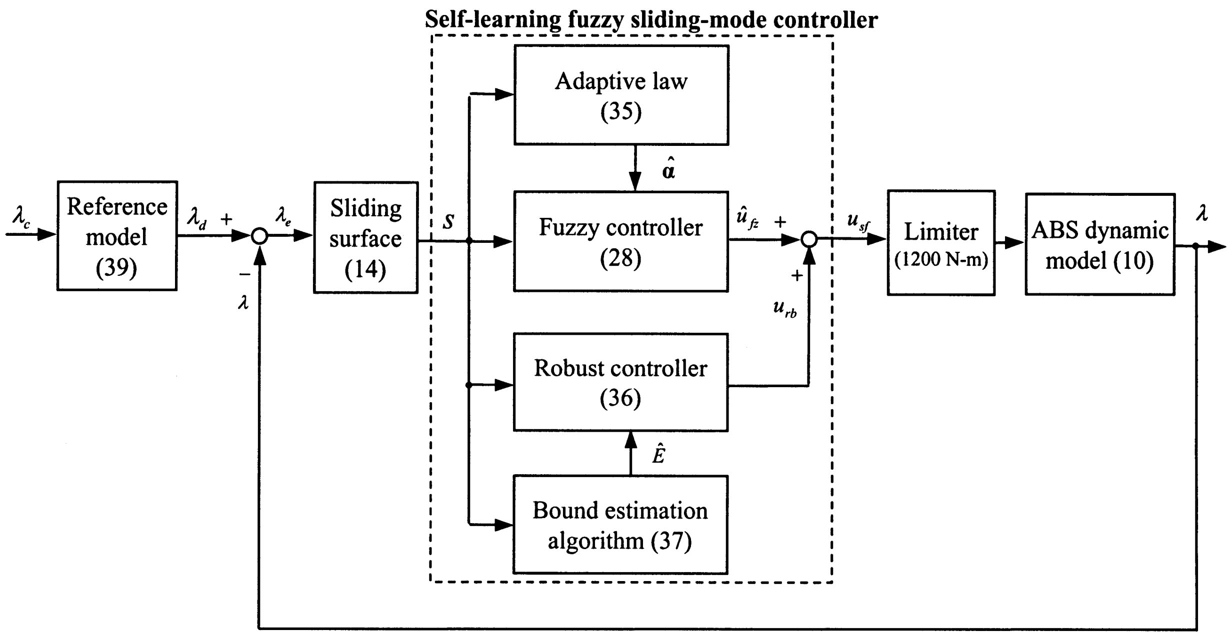

Fig. 2. SLFSMC ABS.

where k_1 is a positive constant. The sliding-mode control law is defined as [6]

$$u_{sm}(t) = u_{eq}(t) + u_{ht}(t) \tag{15}$$

where the equivalent controller $u_{eq}(t)$ is represented as

$$u_{eq}(t) = G_n^{-1}\left[-F_n(\lambda, t) + \dot{\lambda}_d(t) + k_1\lambda_e(t)\right] \tag{16}$$

and the hitting controller $u_{ht}(t)$ is designed to dispel the uncertainty as

$$u_{ht}(t) = G_n^{-1}[W\,\mathrm{sgn}(s(t))] \tag{17}$$

in which $\mathrm{sgn}(.)$ is a sign function. Substituting (15), (16), and (17) into (12), it is revealed that

$$\dot{\lambda}_e(t) + k_1\lambda_e(t) = -w - W\,\mathrm{sgn}(s(t)) = \dot{s}(t). \tag{18}$$

Then, choose a Lyapunov function as

$$V_1 = \tfrac{1}{2}s^2(t). \tag{19}$$

Differentiating (19) with respect to time and using (18), it is obtained that

$$\begin{aligned}\dot{V}_1 &= s(t)\dot{s}(t) = -s(t)w - |s(t)|W\\ &\le |s(t)|\,|w| - |s(t)|W\\ &= -|s(t)|(W - |w|) \le 0.\end{aligned} \tag{20}$$

In summary, the SMC system presented in (15) can guarantee the stability in the Lyapunov sense even under parameter variations. However, to satisfy the existence condition, a large uncertainty bound W should be chosen. In this case, the controller usually results in large and chattering control efforts. The chattering phenomena is undesirable in most applications. To reduce the chattering phenomenon, the sign function in (17) is replaced by a hyperbolic tangent function in this paper.

IV. SLFSMC SYSTEM

In designing an SMC system, it is necessary to know the system models and to find the inverse form of inertia term in system dynamics. However, the accurate mathematical models are always difficult to formulate or even not available. To solve these problems, an SLFSMC system shown in Fig. 2 is proposed to control the ABS.

A. Fuzzy Sliding-Mode Control System

Assume that there are m rules in a fuzzy rule base and each of them has the following form:

$$\text{Rule } i\text{: If } s \text{ is } \tilde{S}_i \text{ Then } u \text{ is } \alpha_i \tag{21}$$

where s is the input variable of the fuzzy system; u is the output variable of the fuzzy system; $\tilde{S}_i$ are the triangular-type membership functions; and α_i are the singleton control actions for $i = 1, 2, \ldots, m$. The defuzzification of the FSMC output is accomplished by the method of center-of-gravity [7]

$$u = \sum_{i=1}^{m} w_i \times \alpha_i \Big/ \sum_{i=1}^{m} w_i \tag{22}$$

where w_i is the firing weight of the ith rule. Equation (22) can be rewritten as

$$u = \boldsymbol{\alpha}^T\boldsymbol{\xi} \tag{23}$$

where $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \ldots, \alpha_m]^T$ is a parameter vector and $\boldsymbol{\xi} = [\xi_1, \xi_2, \ldots, \xi_m]^T$ is a regressive vector with ξ_i defined as

$$\xi_i = w_i \Big/ \sum_{i=1}^{m} w_i. \tag{24}$$

For the conventional FSMC, the control actions α_i should be previously assigned through a lot of trials to achieve satisfactory control performance. In the following, the self-learning algorithm will be proposed to tune these control actions on line.

B. SLFSMC System With Bound Estimator

Assume that the parameters of the system (10) are well known, then an ideal controller can be obtained as

$$u^*(t) = G_p^{-1}\left[-F_p(\lambda, t) + \dot{\lambda}_d(t) + k_2\lambda_e(t)\right]. \qquad (25)$$

Substituting (25) into (10) gives

$$\dot{\lambda}_e(t) + k_2\lambda_e(t) = 0. \qquad (26)$$

If k_2 is chosen to correspond to the coefficients of a Hurwitz polynomial, it is thus implying that $\lim_{t\to\infty}\lambda_e(t) = 0$. Since the system parameters may be unknown or perturbed, the ideal controller $u^*(t)$ can not be precisely implemented. Therefore, by the universal approximation theorem [10], there exists an optimal fuzzy controller $u_{fz}^*(s, \boldsymbol{\alpha}^*)$ in the form of (23) such that

$$u^*(t) = u_{fz}^*(s, \boldsymbol{\alpha}^*) + \varepsilon = \boldsymbol{\alpha}^{*T}\boldsymbol{\xi} + \varepsilon \qquad (27)$$

where ε is the approximation error and is assumed to be bounded by $|\varepsilon| \le E$. Employing a fuzzy controller $\hat{u}_{fz}(s, \hat{\boldsymbol{\alpha}})$ to approximate $u^*(t)$ as

$$\hat{u}_{fz}(s, \hat{\boldsymbol{\alpha}}) = \hat{\boldsymbol{\alpha}}^T\boldsymbol{\xi} \qquad (28)$$

where $\hat{\boldsymbol{\alpha}}$ is the estimated values of $\boldsymbol{\alpha}^*$. The control law for the developed SLFSMC is assumed to take the following form:

$$u_{sf}(t) = \hat{u}_{fz}(s, \hat{\boldsymbol{\alpha}}) + u_{rb}(s) \qquad (29)$$

where the fuzzy controller $\hat{u}_{fz}$ is designed to approximate the ideal controller $u^*(t)$ and the robust controller u_{rb} is designed to compensate for the difference between the ideal controller and the fuzzy controller. By substituting (29) into (10), it is revealed that

$$\dot{\lambda} = F_p(\lambda, t) + G_p[\hat{u}_{fz}(s, \hat{\boldsymbol{\alpha}}) + u_{rb}(s)]. \qquad (30)$$

Multiplying (25) with G_p, added to (30) and using (13) and (14), the error equation governing the system can be obtained as follows:

$$\dot{\lambda}_e(t) + k_2\lambda_e(t) = G_p(u^* - \hat{u}_{fz} - u_{rb}) = \dot{s}(t). \qquad (31)$$

Define $\tilde{u}_{fz} = u^* - \hat{u}_{fz}$, $\tilde{\boldsymbol{\alpha}} = \boldsymbol{\alpha}^* - \hat{\boldsymbol{\alpha}}$ and use (27), then it is obtained that

$$\tilde{u}_{fz} = \tilde{\boldsymbol{\alpha}}^T\boldsymbol{\xi} + \varepsilon. \qquad (32)$$

Define a Lyapunov function as

$$V_2\left(s(t), \tilde{\boldsymbol{\alpha}}, \tilde{E}\right) = \frac{1}{2}s^2(t) + \frac{G_p}{2\eta_1}\tilde{\boldsymbol{\alpha}}^T\tilde{\boldsymbol{\alpha}} + \frac{G_p}{2\eta_2}\tilde{E}^2 \qquad (33)$$

where $\tilde{E}(t) = E - \hat{E}(t)$, $\hat{E}(t)$ is the estimation of the approximation error bound, and η_1 and η_2 are positive constants. Differentiating (33) with respect to time and using (31) and (32), it is obtained that

$$\begin{aligned}\dot{V}_2&\left(s(t), \tilde{\boldsymbol{\alpha}}, \tilde{E}\right)\\ &= s(t)\dot{s}(t) + \frac{G_p}{\eta_1}\tilde{\boldsymbol{\alpha}}^T\dot{\tilde{\boldsymbol{\alpha}}} + \frac{G_p}{\eta_2}\tilde{E}\dot{\tilde{E}}\\ &= s(t)G_p(\tilde{\boldsymbol{\alpha}}^T\boldsymbol{\xi} + \varepsilon - u_{rb}) + \frac{G_p}{\eta_1}\tilde{\boldsymbol{\alpha}}^T\dot{\tilde{\boldsymbol{\alpha}}} + \frac{G_p}{\eta_2}\tilde{E}\dot{\tilde{E}}\\ &= G_p\tilde{\boldsymbol{\alpha}}^T\left(s(t)\boldsymbol{\xi} + \frac{\dot{\tilde{\boldsymbol{\alpha}}}}{\eta_1}\right) + s(t)G_p(\varepsilon - u_{rb}) + \frac{G_p}{\eta_2}\tilde{E}\dot{\tilde{E}}.\end{aligned} \qquad (34)$$

For achieving $\dot{V}_2 \le 0$, the adaptive laws of the SLFSMC are chosen as

$$\dot{\hat{\boldsymbol{\alpha}}} = -\dot{\tilde{\boldsymbol{\alpha}}} = \eta_1 s(t)\boldsymbol{\xi} \qquad (35)$$

$$u_{rb} = \hat{E}\,\mathrm{sgn}(s(t))\,\mathrm{sgn}(G_p) = \hat{E}\,\mathrm{sgn}(s(t)) \qquad (36)$$

$$\dot{\hat{E}}(t) = -\dot{\tilde{E}}(t) = \eta_2|s(t)|\,\mathrm{sgn}(G_p) = \eta_2|s(t)| \qquad (37)$$

then (34) can be rewritten as

$$\begin{aligned}\dot{V}_2\left(s(t), \tilde{\boldsymbol{\alpha}}, \tilde{E}\right) &= \varepsilon s(t)G_p - E|s(t)|\,|G_p|\\ &\le -|s(t)|\,|G_p|(E - |\varepsilon|) \le 0.\end{aligned} \qquad (38)$$

In summary, the SLFSMC is presented in (29), where $\hat{u}_{fz}$ is given in (28) with the parameters $\hat{\boldsymbol{\alpha}}$ adjusted by (35) and u_{rb} is given in (36) with the parameter $\hat{E}$ adjusted by (37). By applying these adaptive laws, the SLFSMC system can be guaranteed to be stable.

V. Simulation Results

For simulations, the parameters of the ABS used in this study are $M_v = 4 \times 342$ kg, $B_v = 6$ Ns, $J_w = 1.13$ Nms^2, $R_w = 0.33$ m, $B_w = 4$ Ns, and $g = 9.8$ m/s^2 [2]. From Fig. 1, it is seen that the tractive forces for different road conditions are maximized near $\lambda = 20\%$, so the slip command λ_c is chosen as 0.2. Moreover, a reference model is chosen as

$$\dot{\lambda}_d(t) = -10.0\lambda_d(t) + 10\lambda_c(t). \qquad (39)$$

And, the maximum braking torque is limited at 1200 N-m. Due to the fact that the wheel and vehicle velocity are nearly zero at low speeds, the magnitude of slip tends to infinity as the vehicle speed approaches zero. Therefore, simulations are conducted up to the point where the vehicle is slowed to approximate to 5 m/s.

The SMC design method described in Section III is simulated for the ABS. The simulation results for dry asphalt using SMC are shown in Fig. 3(a)–(c) with $k_1 = 100$ and $W = 25$. The simulations for transition between various road conditions using SMC are shown in Fig. 3(d)–(f). The simulation results illustrate that the SMC can achieve satisfactory control performance for dry asphalt; however, it fails to perform well under extreme road transition to the icy road. For comparison, an FSMC using the fuzzy-rule-based algorithm is applied for ABS control [2], [3], [5]. The simulation results for dry asphalt using FSMC are shown in Fig. 4(a)–(c). The simulations for transition between various road conditions using FSMC are shown in Fig. 4(d)–(f). From Fig. 4(c), it is seen that the slip tracking can

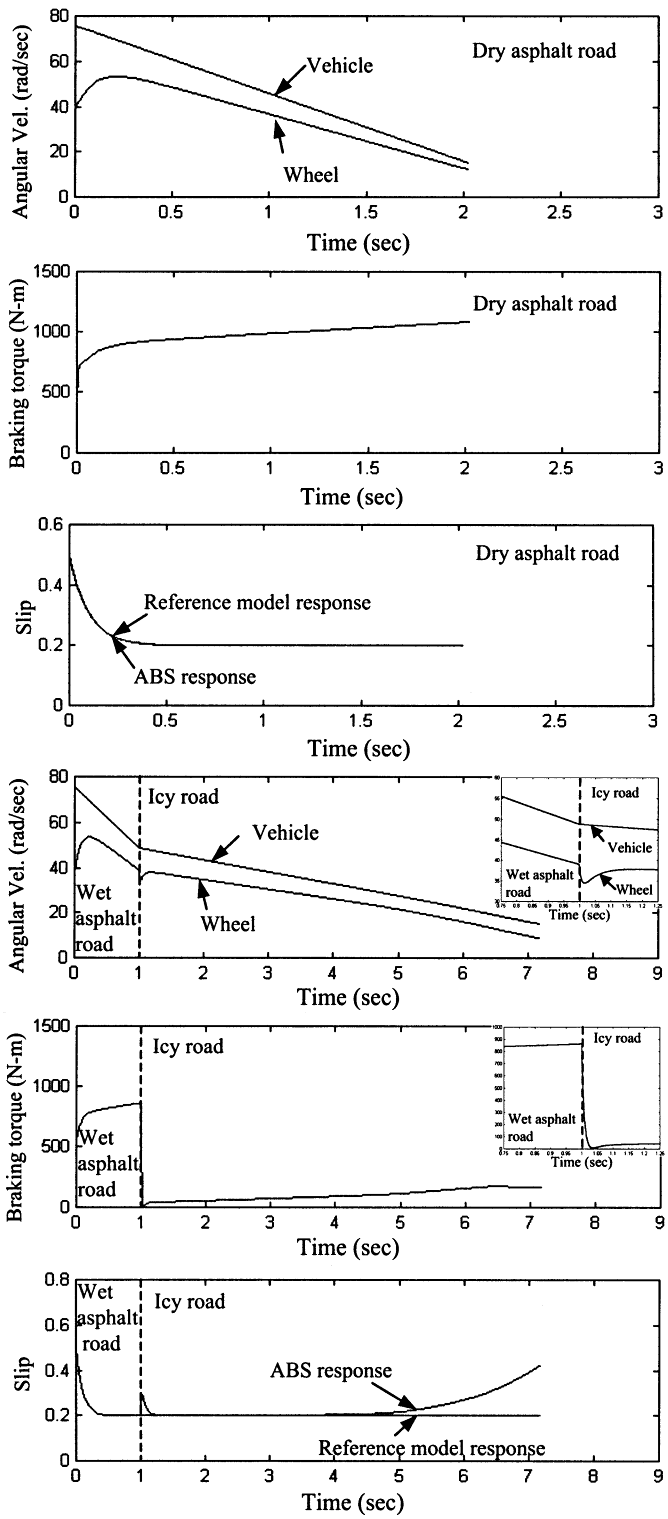

Fig. 3. Simulation results of sliding-mode control ABS.

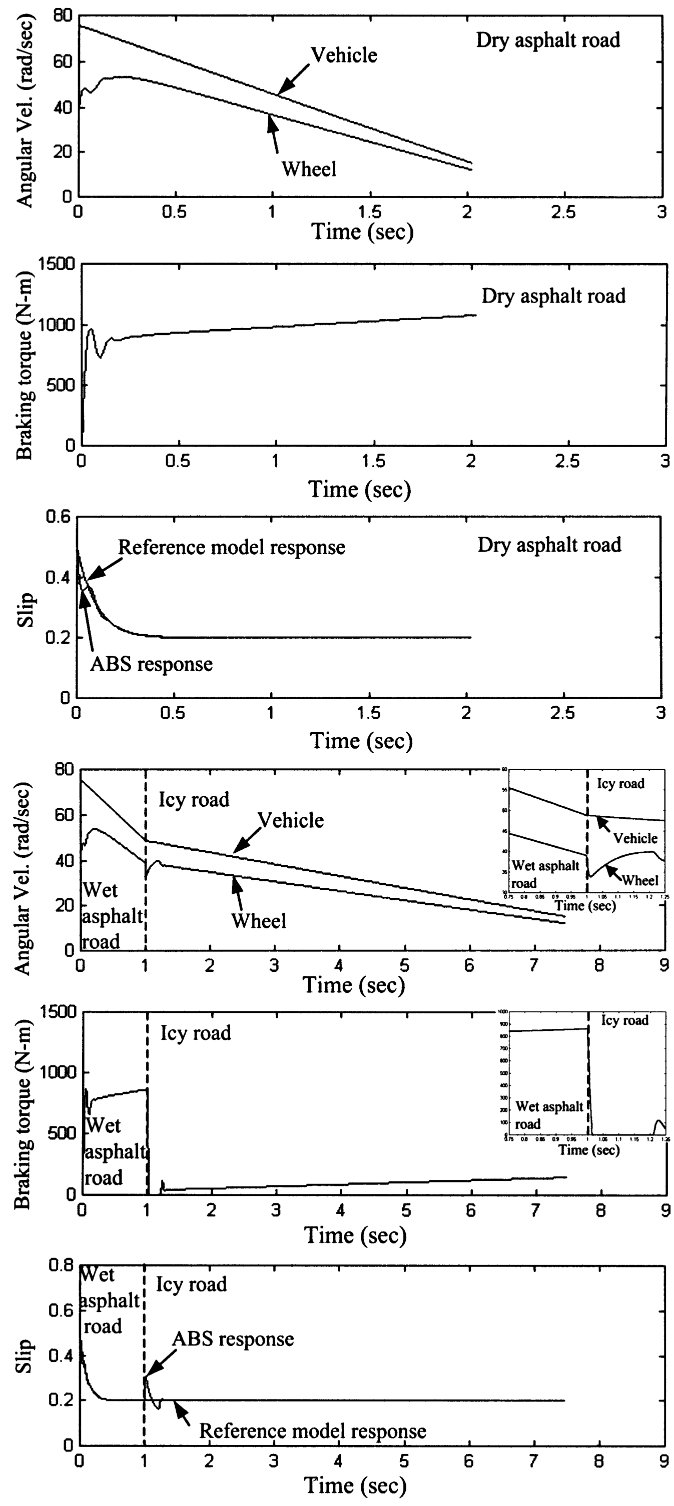

Fig. 4. Simulation results of FSMC ABS.

be achieved after two seconds for the dry asphalt; and there exists transient response for the road transition in Fig. 4(f). Moreover, the fuzzy rules must be determined by prior time-consuming trial-and-error procedures. To tackle this problem, the proposed SLFSMC system was applied for the ABS. The parameters of the SLFSMC system are selected as $k_2 = 100$, $\eta_1 = 50$, and $\eta_2 = 1$. The simulation results for dry asphalt using SLFSMC are shown in Fig. 5(a)–(c). The simulations for transition between various road conditions using SLFSMC are shown in Fig. 5(d)–(f). Comparing Fig. 5(c) and (f) with Fig. 4(c) and (f), respectively, it is seen that the SLFSMC can achieve faster tracking convergence than FSMC for both dry asphalt and road transition cases. Besides these advantages, it should be emphasized that the derivation of the SLFSMC ABS does not need to use the model in (10), and the fuzzy rules are learned from the developed adaptive laws.

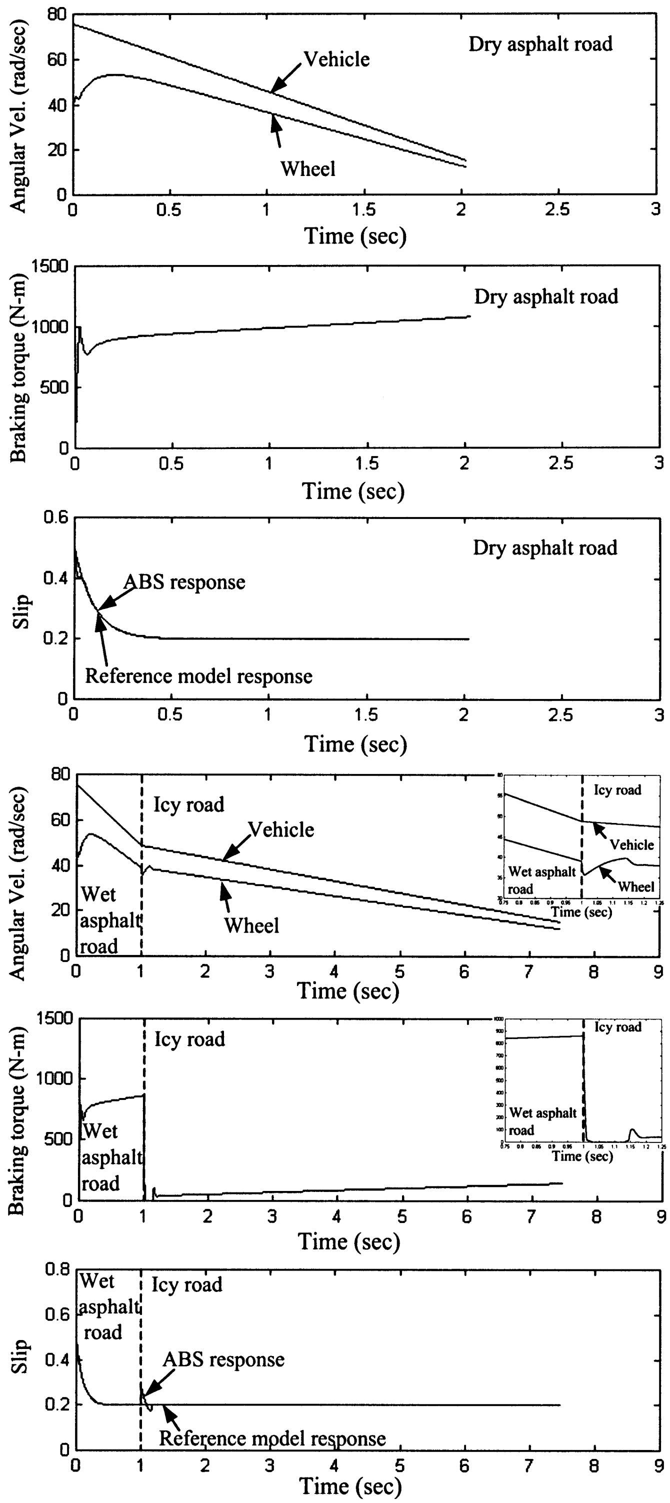

Fig. 5. Simulation results of SLFSMC ABS.

VI. Conclusion

In this study, an SLFSMC system was proposed to control an ABS. This paper has successfully demonstrated that the SLFSMC system can achieve favorable tracking and robust performance with regard to various road condition and road transition in simulation. The tuning algorithms of SLFSMC are derived in the Lyapunov sense, thus, the stability of the control system can be guaranteed. Compared with sliding-mode control and FSMC ABS, it is seen that the SLFSMC ABS can achieve the best tracking performance for various road conditions.

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