Influences of noise on the occurrence of period doubling in distributed-feedback laser diodes under direct-current modulation

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The influences of Langevin noise on period doubling (PD) of a strongly modulated distributed-feedback laser have been investigated. The onset of PD was confirmed to be reduced through the use of the rate equations as a model. The threshold values of PD were examined in terms of driving frequency and rf power and have been compared with the measurements of PD.

Introduction

The behavior of nonlinear instabilities in semiconductor laser diodes has received much attention since the discovery of scaling constants in routes to chaos. Nonlinear behaviors observed during direct-current (dc) modulation have included the period-doubling (PD) route, the quasiperiodic route, and bistability.[1–7] These investigations have been conducted not only for theoretical study but also for practical interest, especially in the area of analog modulation in fiber communication. Considerable interest thus exists in being able to determine precisely the onset points for these nonlinear instabilities. The nonlinear rate equations governing the interrelationship between carrier density and photon density have been previously used in the semiclassical approach for predicting these instabilities.[7] Despite great efforts the important role of noise in the nonlinear instability has not yet been explored in detail. The output of a cw diode laser actually exhibits a large amplitude fluctuation, with a frequency centered at the relaxation oscillation resonance.[2,5] These fluctuations have arisen from the quantum nature of spontaneous emission and cannot be eliminated in real diode lasers. It has been indicated numerically that noise can reduce the threshold of the first PD in a specific case.[8] In this article, the behavior of PD is intensively investigated on a commercially available ridge-guide distributed-feedback laser. The results are also checked by numerical calculation, with extra consideration of parasitic effects in a practical situation.

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Stochastic Rate Equations

Single-mode rate equations are used, taking into account the Langevin noise, for the sake of investigating the dynamics. The photon density, S, and carrier density, N, can be written as[4,9]

$$\frac{dN}{dt} = \frac{I(t)}{eV} - \frac{N}{\tau_e} - A(1 - \epsilon_{nl}S)(N - N_0)S + F_n(t)/V, \quad (1)$$

$$\frac{dS}{dt} = \Gamma A(1 - \epsilon_{nl}S)(N - N_0)S - \frac{S}{\tau_p} + \frac{\Gamma\beta N}{\tau_e} + F_s(t)/V, \quad (2)$$

where e is the electron charge, V is the active volume, τ_e and τ_p are the respective electron and photon lifetimes, A is the gain constant, N_0 is the carrier density for transparency, Γ is the confinement factor, β is the spontaneous emission, and ϵ_{nl} is the phenomenologically nonlinear gain-suppression factor. The driving current, containing dc and ac terms, can be expressed as $I(t) = I_{dc} + I_{ac}\sin(2\pi f_1 t)$. For convenience the strength of ac current is considered in terms of decibels. With the diode connected through a bias, T, to the 50-Ω rf signal generator, the amplitude I_{ac} related to decibels can be expressed as $I_{ac}(\mathrm{mA}) \cong 12.65 \times 10^{\mathrm{dBM}/20}$ under the approximation that the dynamical resistor of the laser diode is much less than 50 Ω. F_s and F_n are Langevin noise sources with means of zero that arise from spontaneous emission and from the discrete nature of the carrier generation and recombination, respectively. The responses of Eqs. (1) and (2) are calculated here with the fourth-order Runge–Kutta algorithm.

The noises F_s and F_n are assumed to be Gaussian random processes. Under the Markovian assumptions, the autocorrelation and cross-correlation functions of F_s and F_n are proportional to Dirac delta functions and can be expressed as[9]

$$\langle F_n(t)F_n(t')\rangle = V_n^2\delta(t - t'), \tag{3}$$

$$\langle F_s(t)F_s(t')\rangle = V_s^2\delta(t - t'), \tag{4}$$

$$\langle F_n(t)F_s(t')\rangle = \gamma_0 V_n V_s \delta(t - t'), \tag{5}$$

where V_n and V_s are the respective variances of F_s and F_n and γ_0 is the correlation coefficient and is equal to 1 for the single-mode case. The variances can be expressed, according to Marcuse,[9] as

$$V_n^2 = I/e + NV/\tau_e + A(1 + \epsilon_{nl}S)(N + N_0)SV, \tag{6}$$

$$V_s^2 = SV/\tau_p + \Gamma A(1 + \epsilon_{nl}S)(N + N_0)SV + \Gamma\beta NV/\tau_e, \tag{7}$$

$$V_n V_s = -[\Gamma\beta NV/\tau_e + \Gamma A(1 + \epsilon_{nl}S)(N + N_0)SV]. \tag{8}$$

The stochastic functions F_s and F_n in any particular time interval Δt can be treated in computation as $F_s = V_s X_s/\Delta t^{1/2}$ and $F_n = V_n X_n/\Delta t^{1/2}$, where X_s and X_n are the Gaussian random variables with zero mean and unit variance and are generated within the computer. The time interval, Δt, is chosen to be $\Delta t = T/128$, where T is the period of the driving current, to ensure that the input noise spectrum is approximately white within the $2f$ that is of interest in the simulation. The calculation has been extended to 2148 cycles, and the first 100 cycles are discarded to remove transients. The power spectra are calculated with a fast Fourier transform with 4096 points. The power spectra are further determined by averaging over 64 spectral components for the sake of improving the accuracy. This yields an error of less than approximately 1 dB.

Having the proper values for the coefficients in Eqs. (1) and (2) is essential to make a comparison with the real experiments. Considering the parasitic effects is also necessary. The laser diode used in the experiment is an InGaAsP–InP distributed-feedback ridge waveguide type with a wavelength at 1.55 μm (STC LYC-M2-11). Its threshold current, I_{th}, at 20 °C is 36.5 mA. The relaxation oscillation frequency, f_r, varies between 1 and 4 GHz for bias current I_b between 37 and 60 mA. The major contributions in the parasitic parameters, below 3 GHz, come from the resistance, R_s, in series with the active region and the shunt capacitance, C_s, between the metal contact. The parameter values are as follows: $N_0 = 1.0 \times 10^{24}/\text{m}^3$, $V = 0.2 \times 5 \times 250$ μm, $\Gamma = 0.3$, $A = 3.2 \times 10^{-12}\ \text{m}^3/\text{s}$, $\tau_p = 1$ ps, $\tau_e = 2.27$ ns, $\beta = 2 \times 10^{-4}$, $\epsilon_{nl} = 6.7 \times 10^{-23}\ \text{m}^3$ with a bias current of 39 mA[10]. The time constant, R_sC_s, is equal to approximately 140 ps with −3 dB frequency at 1.137 GHz. The parameters have been satisfactorily confirmed by checking the second-harmonic distortions.[10] The thresholds of PD are examined in the Results section by using these parameters and comparing the calculated results with the experimental results.

Results

The modulated signal has been known from previous research to be easily degraded with a bias near I_{th}. The first onset of PD with a bias $I_{dc} = 39$ mA near the threshold is thus investigated here. The corresponding relaxation oscillation frequency is $f_r \cong 1.3558$ GHz. The dynamics without noise and parasitic effects in Eqs. (1) and (2) are first intensively traced. The onset of PD is identified as an emergence of half-subharmonic $f_1/2$ in the spectrum of photon density. The PD phenomenon is unfortunately not observed with an ac signal strength below +10 dBm. The situation is totally different whenever the Langevin noises are taken into account. Because of the existence of noise, the onset is identified here as the emergence of the half-subharmonic signal from out of the noise level. Without an rf signal, only the intrinsic noise bump exists around f_r in the output spectrum, as shown in curve (a) of Fig. 1. The noise level increased, with the peak frequency shifting toward a lower frequency, as modulation was increased with frequency f_1 between f_r and $2f_r$. The symptom of PD occurs when the maximum of the bump is tuned to approach the subharmonic component $f_1/2$. The typical cases are shown in curves (b) and (c) of Fig. 1, with rf power equal to 2 and 5 dBm, respectively, at $f = 1.5$ GHz. However, no subharmonic signal is observed when the noise terms in Eqs. (1) and (2) are removed, as illustrated in curve (d). This implies that the occurrence of PD can be enhanced by noise.

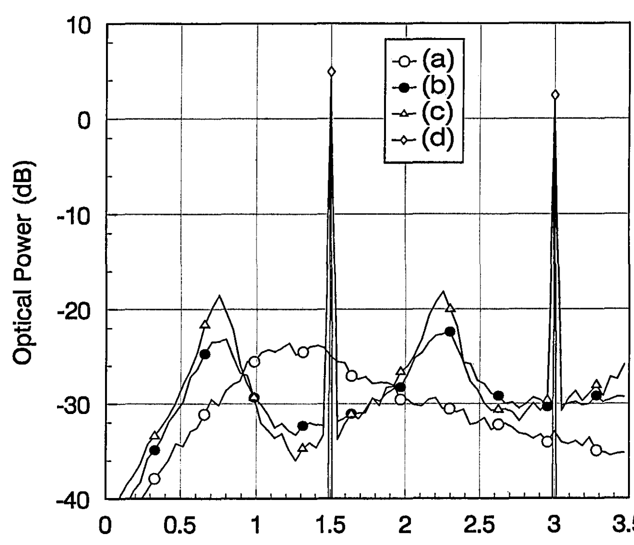

Fig. 1. Calculated Fourier spectra with $f_1 = 1.5$ GHz and (a) no rf power, (b) rf power = −2 dBm, (c) rf power = 5 dBm, and (d) rf power = 5 dBm but without noise perturbation at $f_1 = 1.5$ GHz.

The notable enhancement occurs when the external frequency is set close to $2f_r$.

The dynamic properties of this ac-pumped laser diode are next inspected for further exploration of the basis of the enhancement effect. The laser system described by Eqs. (1) and (2) can actually be viewed as an ac-pumped amplifier with a pumping frequency at f_1, in which the Langevin noise is the input signal. Any possible nonlinear instability has been indicated to have a related noise precursor in the frequency axis, where the gain factor is significant.[11] The gain factor, $\Delta S(f)/\Delta I_{ac}(f)$, is examined in this observed case so as to test the instability of PD. The gain factor used as a function of frequency at various rf powers is shown in Fig. 2. The length of time should be long enough when performing the fast Fourier transform calculation so as to avoid numerical error. The time period used here is equal to 512 cycles. Without rf power at f_1, only one gain maximum exists, as shown in curve (a) of Fig. 2. A sharp gain peak at $f_1/2$ then becomes more and more pronounced because of its nonlinear nature, and the relaxation bump shifts toward $f_1/2$ as rf power is increased. The shift of the relaxation frequency bump is indicated in Ref. 7 to be approximately dependent on the square of I_{ac}. This is because the larger photon density can deplete more of the carrier density as the modulation current is increased. It requires more time in recovering the carrier density and consequently lowers the relaxation frequency. The relaxation bump is viewed here as being able to be tuned to coincide with subharmonic gain peak either by continuously increasing the ac pump or by setting the driving frequency near $2f_r$. The diode evidently functions as an amplifier with two extrema overlapping in the frequency axis at half the excitation frequency[11] [curves (b) and (c)]. In that case, the subharmonic gain peak becomes prominent, and the broadband Langevin noises are filtered and amplified to exhibit the phenomenon of PD. Note that the level of the gain factor around the subharmonic is shown in curve (c) of Fig. 2 to decrease after the occurrence of PD. This accounts for the suppressed noise level near the subharmonic, shown in curve (c) of Fig. 1 once the half-subharmonic component emerged.

The measured results confirm the predictions mentioned above. The demonstration of PD with frequency at 2.2 GHz near $2f_r$ is demonstrated in Figs. 3(a)–(c). Only a small amount of rf power is required for generation of the PD. However, the noise-enhanced effect dies out whenever the excitation frequency is too far away from two times the relaxation frequency. Hence more rf power is required to reach the thresholds. We show a series of measured spectra obtained by scanning the frequency at a fixed rf power of 1.9 dBm in Figs. 3(d)–3(f) to confirm this idea; only Fig. 3(e) with frequency at 1.85 GHz near $2f_r$ reveals the PD. The predicted threshold in terms of rf power and frequency is shown in curve (a) of Fig. 4, which has a U shape, with the lowest value occurring around $2f_r$. However, the measured value as shown in curve (b) appears as a V shape, with the dip frequency not occurring at $2f_r$. Parasitic effects, represented by the value R_sC_s, should notably be taken into account as compared with the real experiment. These two elements actually function as a

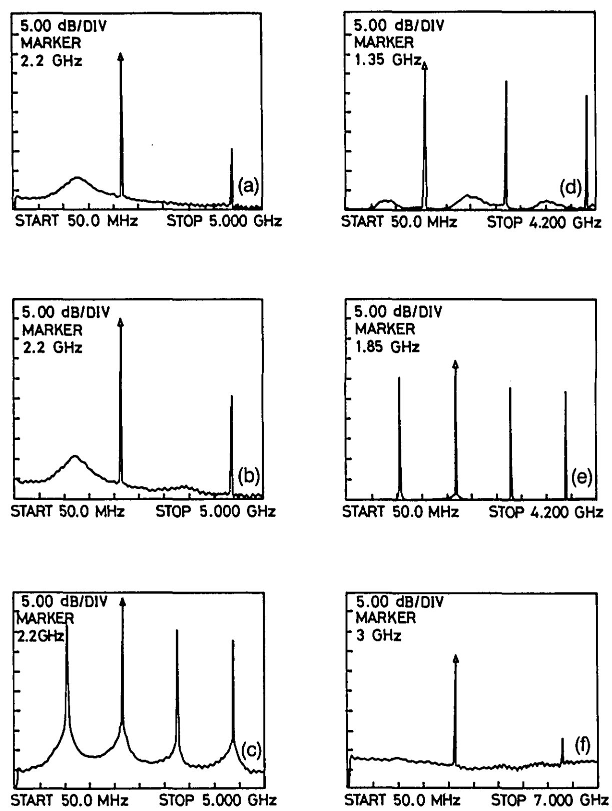

Fig. 3. Measured spectra of light output at various driving frequencies and currents with (a) f_1 = 2.2 GHz and I_{ac} = −10.7 dBm, (b) f_1 = 2.2 GHz and I_{ac} = −6.55 dBm, (c) f_1 = 2.2 GHz and I_{ac} = 1.9 dBm, (d) f_1 = 1.35 GHz and I_{ac} = +1.9 dBm, (e) f_1 = 1.85 GHz and I_{ac} = +1.9 dBm, and (f) f_1 = 3.0 GHz and I_{ac} = +1.9 dBm.

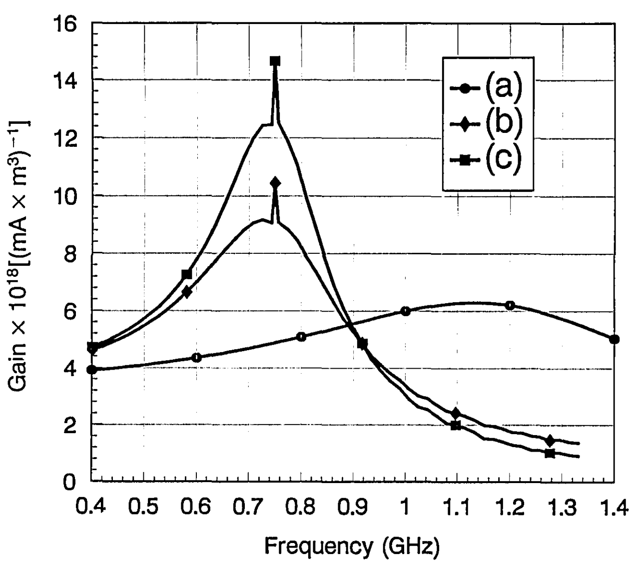

Fig. 2. Calculated gain response of the ac-pumped laser diode at various pumping strengths: (a) no rf power, (b) rf power = −2 dBm, and (c) rf power = 5 dBm with f_1 = 1.5 GHz.

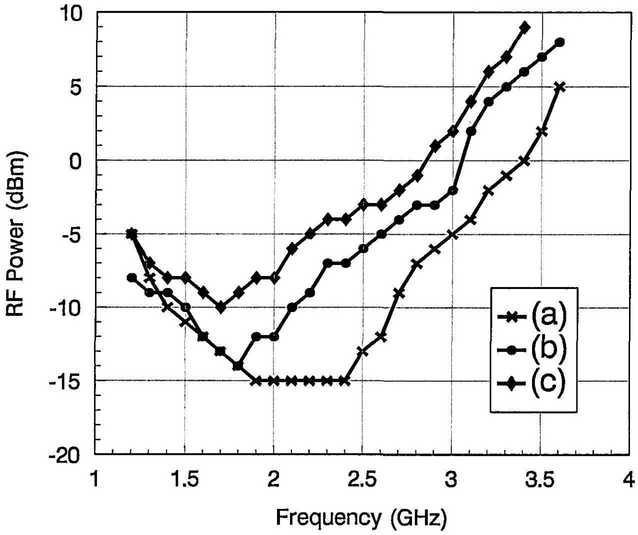

Fig. 4. Two-dimensional state diagram describing the onset of PD in the rf power and frequency space for I_b = 39 mA. Curve (a) is the calculated result with noise and without R_sC_s, curve (b) is measured results, and curve (c) is with noise and R_sC_s.

low-pass filter and shunt a part of the ac current without flowing into the active region whenever the frequency is above the 3-dB corner value. Hence the threshold current $I_{ac'}$ in the real experiment should be larger than that predicted above. The relationship between $I_{ac'}$ and I_{ac} in Eq. (1) can be expressed as $I_{ac'} \cong I_{ac}[(1 + R_s/50)^2 + (2\pi f R_s C_s)^2]^{1/2}$, based on the assumption that the impedance of the active region is much less than R_s. Accordingly the required threshold values are raised, as illustrated in curve (c) of Fig. 4, which seems to be in good agreement with the measured threshold shown in curve (b) of Fig. 4.

Conclusions

The effects of noise on the nonlinear instability of PD in deeply modulated semiconductor lasers were extensively investigated. The onset of the first PD has been indicated here to be reduced by the presence of noise, especially when the excitation frequency was approximately two times the relaxation frequency. The noise bump inherently appearing around the relaxation frequency has been confirmed to be an important precursor for the occurrence of PD. The low-pass effect of parasitic elements on the threshold curve was indicated. All the relevant results observed have been anticipated here. This study provides a key quantitative reference for most practical applications in the experiments.

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