Polaron effect on the binding energy of a hydrogenic impurity in a quantum well

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We have studied the polaron effect on the binding energy of a hydrogenic impurity in a quantum-well structure. The interactions of an electron with both the confined bulk phonon and the interface phonon are taken into account. The competition between these two phonon modes is discussed. We have also extended the calculation for the case of a finite quantum well. It is found that the polaronic correction becomes bigger as the potential barrier gets higher.

I. INTRODUCTION

In recent years many theoretical and experimental investigations have been performed on the issue of the "hydrogenic" binding of an electron to a donor impurity in a semiconductor quantum well or heterostructure. Bastard[1] proposed a variational approach to calculate the binding energies of donor levels in terms of the well thickness and impurity positions in the quantum well assuming the potential barrier at the interface to be infinitely high. Mailhiot, Chang, and McGill,[2] Greene and Bajaj,[3] and Liu and Quinn[4] extended the work to calculate the binding energies of the ground state and several excited states of hydrogenic donors in GaAs-$Ga_{1-x}Al_xAs$ quantum well with finite barrier height. Later, Chaudhuri[5] and Lane and Greene[6] studied the hydrogenic impurity states in multiple-quantum-well structures. The effects of a finite-width barrier upon the binding energies are discussed. Recently, the anisotropic effect[7] and the quantum-confined Stark effect[8–10] on the hydrogenic impurities in the quantum well have also been reported.

Since the III-V materials used in producing typical quantum-well structures are polar in nature, an electron weakly bound to a hydrogenic impurity in this system will interact with the longitudinal-optical phonons of the host semiconductor and tend to increase the donor binding energy. In the case of a bulk semiconductor, this polaron effect is reasonably well understood on the basis of the Fröhlich model.[11] The situation in a confined geometry, such as a quantum well, is considerably less clear. In the past several years, a number of works have been carried out to investigate the polaron effect in a quantum well system. Ercelebi and Tomak[12] studied the effect of electron-phonon coupling on the binding energies of a hydrogenic impurity in a GaAs-$Ga_{1-x}Al_xAs$ quantum well and found that the correction becomes sizable as the electron gets more deeply bound. Later, Degani and Hipolito[13] calculated the polaron effect on the ground state as a function of electron density. The screening effect of the impurity potential is discussed. Mason and Das Sarma[14] have calculated the phonon-induced shift in the impurity binding energy due to electron-phonon interaction in a two-dimensional quantum-well system. It is found that the polaron shifts in donor energy levels are of the order of 10% in the GaAs-$Ga_{1-x}Al_xAs$ system. However, in all the calculations mentioned above, the electron-phonon interaction is taken to be of Fröhlich type as in the bulk case. The phonon confinement effect and the interface phonon are ignored. This is an oversimplification; indeed, several experiments[15–20] carried out recently indicate that the optical phonon in the GaAs-$Ga_{1-x}Al_xAs$ quantum-well system is confined, i.e., the z-polarized (z perpendicular to the surface) optical phonons are equivalent to those vibrations in the infinite crystal whose wave vector is given by $m\pi/L$, where L is the thickness of the well and m is an integer. The series of phonons labeled by m are termed confined phonons and also sometimes called "folded" phonons. It has been pointed out in various investigations that phonon confinement effects lead to important modifications in the transport properties. Therefore in studying the polaron effect in a quantum-well structure, the confined bulk phonon should be taken into account. There is, however, another type of phonon which is also quite important in the quantum-well system. This is the interface mode. Recently, the interaction of an electron with the interface phonon mode has been theoretically established for two-dimensional semiconductor heterojunctions.[21,22] Degani and Farias[23] studied the exciton problem in an AlAs/GaAs system and found that the interface phonon has significant effect on the exciton binding energy. Experimental observations of these interface modes are reported by Sood *et al.*[17] in backscattering Raman spectra, by Lambin *et al.*[24] in high-resolution electron-energy-loss spectra, by Meynadier *et al.*[25] in Raman scattering of high order at the resonance with the lowest optical transition, and by Gammon, Merlin, and Markoc[26] in magnetic-field-enhanced Raman scattering. The purpose of this paper is to investigate the polaron effect on the hydrogenic impurity states in a quantum well by including both the interactions of the electron with the confined bulk phonon and the interface phonon. The competition between these two phonon modes and the correction on the binding energy of the hydrogenic impurity in the GaAs-$Ga_{1-x}Al_xAs$ quantum well will be studied.

We shall also extend the calculation to the case of the

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finite quantum well. It has been pointed out recently[12,13] that as the binding energy increases the localization of an electron becomes more pronounced, and this in turn increases the importance of electron-phonon coupling. Thus the polaron effect is expected to be smaller for the finite potential barrier where the binding would be weaker. However, this important behavior has been only briefly speculated upon in previous works, and no calculation was given since they considered only the case of the infinite quantum well.

II. THEORY

Let us consider a hydrogenic donor impurity atom located in a quantum well of thickness L. Within the framework of the effective-mass approximation, the Hamiltonian of this system contains five parts: electronic; free interface phonon (IO); electron–IO-phonon interaction; free bulk optical phonon (BO); and electron–BO-phonon interaction.

$$H=H_e+H_{\rm IO}+H_{e-{\rm IO}}+H_{\rm BO}+H_{e-{\rm BO}} \ . \tag{1}$$

The electronic Hamiltonian operator H_e can be expressed as

$$H_e=\frac{p^2}{2m}-\frac{e^2}{\epsilon_0[\rho^2+(z-z_i)^2]^{1/2}}+V(z) \ , \tag{2}$$

where $\rho^2=x^2+y^2$ is the distance in the layer plane measured from the impurity site, and z_i is the coordinate of the impurity site along the superlattice axis. We first assume, for the sake of simplicity, that the barrier potential $V(z)$ has infinite height.

$$V(z)=\begin{cases}\infty \ , & |z|>L/2 \\ 0 \ , & |z|<L/2 \ .\end{cases} \tag{3}$$

Later, we shall extend the calculations for the finite quantum well. $H_{\rm IO}$ and $H_{e-{\rm IO}}$ are the Hamiltonians for the free interface phonon and its interaction with the electron, respectively:

$$H_{\rm IO}=\sum_{q}\hbar\omega_s s_{\mathbf{q}}^{+}s_{\mathbf{q}} \ , \tag{4}$$

$$H_{e-{\rm IO}}=\sum_{\mathbf{q}}\Gamma_q(e^{-q|z-L/2|}+e^{-q|z+L/2|})(e^{-\mathbf{q}\cdot\boldsymbol{\rho}}s_{\mathbf{q}}^{+}+{\rm H.c.}) \ , \tag{5}$$

where $s_{\mathbf{q}}^{+}(s_{\mathbf{q}})$ denotes the creation (annihilation) operator for the IO phonon with the two-dimensional wave vector $\mathbf{q}$. For a heterostructure, the frequencies of the interface modes, $\omega_{\pm}$ are determined by $\epsilon_1(\omega)=-\epsilon_2(\omega)$, with GaAs as medium 1 and $Ga_{1-x}Al_xAs$ as medium 2. The frequency-dependent dielectric function is given as

$$\epsilon(\omega)=\epsilon_\infty+\frac{\epsilon_s-\epsilon_\infty}{1-\omega^2/\omega_T^2} \ , \tag{6}$$

where ϵ_s and ϵ_∞ are the static and high-frequency dielectric constants, respectively. ω_T is the transverse optical-phonon frequency. The electron–IO-phonon interaction strength Γ_q is defined as[21]

$$\Gamma_q=\frac{\hbar\omega_s}{\sqrt{qA}}[2\pi\alpha_s(\hbar/2m\omega_s)^{1/2}]^{1/2} \ , \tag{7a}$$

where A is the area of the surface and the electron–IO-phonon coupling constant

$$\alpha_s=\frac{e^2}{2\pi\epsilon_s}\left[\frac{m}{2\hbar^3\omega_s}\right]^{1/2}\frac{1}{\beta_1^{-1}(\omega_s)+\beta_2^{-1}(\omega_s)} \ , \tag{7b}$$

with

$$\beta(\omega)=\left[\frac{1}{\epsilon_\infty}-\frac{1}{\epsilon_s}\right]\frac{\omega_L^2}{\omega^2}\left[\frac{\omega^2-\omega_T^2}{\omega_L^2-\omega_T^2}\right]^2 \ . \tag{7c}$$

The Hamiltonians $H_{\rm BO}$ and $H_{e-{\rm BO}}$ for the free BO phonon and the electron–BO-phonon interaction can be written as

$$H_{\rm BO}=\sum_{\mathbf{k}}\hbar\omega_L b_{\mathbf{k}}^{+}b_{\mathbf{k}} \ , \tag{8}$$

$$H_{e-{\rm BO}}=\sum_{\mathbf{k}}W_k\sin(k_z z)(e^{-i\mathbf{k}_\rho\cdot\boldsymbol{\rho}}b_{\mathbf{k}}^{+}+{\rm H.c.}) \ , \tag{9}$$

where $\hbar\omega_L$ is the BO-phonon energy. $b_{\mathbf{k}}^{+}$ and $b_{\mathbf{k}}$ are, respectively, the creation and annihilation operators for the BO phonon with wave vector $\mathbf{k}=(\mathbf{k}_\rho,k_z)$. For the confined bulk phonon, only limited values for the z wave vector are allowed, given by $k_z=n\pi/L$, where n is an integer, since these phonons are confined within the quantum well. Recently Ren, Chu, and Chang[27] studied the anisotropy of the optical phonon and the interface mode in a superlattice, and found that the interface mode and the confined mode with zero node ($n=1$) do not coexist in a quantum well for a fixed value of in-plane wave vector $\mathbf{q}$. Therefore, the electron–BO-phonon interaction for the $n=1$ mode should be excluded to avoid doubled counting. The interaction strength W_k is given as

$$W_k=\frac{\hbar\omega_L}{k\sqrt{V}}[4\pi\alpha_b(\hbar/2m\omega_L)^{1/2}]^{1/2} \ . \tag{10}$$

Here V is the volume and the electron–BO-phonon coupling constant $\alpha_b=e^2[(1/\epsilon_\infty)-(1/\epsilon_s)](m/2\hbar^3\omega_L)^{1/2}$.

We shall use the variational method proposed by Lee, Low, and Pines[28] to treat our problem. In order to calculate the ground-state energy, we choose a trial wave function as

$$|\Psi\rangle=|\phi_e\rangle|N_{\mathbf{q}}N_{\mathbf{k}}\rangle \ , \tag{11}$$

where $|\phi_e\rangle$ is the electronic part of the wave function for the impurity in the infinite quantum well, which is taken the form as used in Bastard model:[1]

$$|\phi_e\rangle=\begin{cases}N\cos(k_1 z)\exp\left\{-\frac{1}{\lambda}[\rho^2+(z-z_i)^2]^{1/2}\right\}, & |z|<L/2 \\ 0 \ , & |z|>L/2 \ .\end{cases} \tag{12}$$

N is the normalization constant, λ is a variational parameter, and $k_1=\pi/L$. $|N_{\mathbf{q}}N_{\mathbf{k}}\rangle$ is the phonon wave function, with $N_{\mathbf{q}}$ and $N_{\mathbf{k}}$ the number of IO and BO phonons, respectively. In the low-lying state, $|N_{\mathbf{q}}N_{\mathbf{k}}\rangle$ can be taken as $|O_{\mathbf{q}}O_{\mathbf{k}}\rangle$, the phonon vacuum state.

First, we make the unitary transformation

$$U_1=\exp\left[i\left[\mathbf{P}_\rho-\sum_{\mathbf{q}}s_{\mathbf{q}}^+s_q\hbar\mathbf{q}-\sum_{\mathbf{k}}b_{\mathbf{k}}^+b_{\mathbf{k}}\hbar\mathbf{k}_\rho\right]\cdot\boldsymbol{\rho}\right], \tag{13}$$

where $\mathbf{P}_\rho$ is the two-dimensional momentum defined as

$$\mathbf{P}_\rho=\mathbf{p}_\rho+\sum_{\mathbf{q}}\hbar\mathbf{q}s_{\mathbf{q}}^+s_{\mathbf{q}}+\sum_{\mathbf{k}}\hbar\mathbf{k}_\rho b_{\mathbf{k}}^+b_{\mathbf{k}}\ . \tag{14}$$

The Hamiltonian is transformed into

$$\begin{aligned}H_1&=U_1^+HU_1\\&=\frac{P_z^2}{2m}+V(z)+\frac{1}{2m}\left[\mathbf{P}_\rho-\sum_{\mathbf{q}}\hbar\mathbf{q}s_{\mathbf{q}}^+s_{\mathbf{q}}-\sum_{\mathbf{k}}\hbar\mathbf{k}_\rho b_{\mathbf{k}}^+b_{\mathbf{k}}\right]^2\\&\quad+\hbar\omega_s\sum_{\mathbf{q}}s_{\mathbf{q}}^+s_{\mathbf{q}}+\sum_{\mathbf{q}}\Gamma_q(s_{\mathbf{q}}^++s_{\mathbf{q}})\\&\quad+\hbar\omega_L\sum_{\mathbf{k}}b_{\mathbf{k}}^+b_{\mathbf{k}}+\sum_{\mathbf{k}}W_k(b_{\mathbf{k}}^++b_{\mathbf{k}})\ .\end{aligned} \tag{15}$$

In terms of the second Lee-Low-Pines transformation,

$$U_2=\exp\left[\sum_{\mathbf{q}}(F_qs_{\mathbf{q}}^++F_q^*s_{\mathbf{q}})+\sum_{\mathbf{k}}(G_kb_{\mathbf{k}}^++G_k^*b_{\mathbf{k}})\right]\ . \tag{16}$$

The Hamiltonian finally becomes

$$H'=U_2^+H_1U_2\ . \tag{17}$$

Taking the expectation value of H' with the trial wave function given in Eq. (11), i.e.,

$$E=\langle\phi_e;O_{\mathbf{q}}O_{\mathbf{k}}|H'|\phi_e;O_{\mathbf{q}}O_{\mathbf{k}}\rangle\ . \tag{18}$$

The variational parameters F_q and G_k are determined by the variational conditions $\delta E/\delta F_q=\delta E/\delta G_k=0$, which yield

$$F_q=-\overline{\Gamma}_q\left[\hbar\omega_s+\frac{\hbar^2q^2}{2m}+\frac{\hbar\mathbf{P}_\rho\cdot\mathbf{q}}{m}(\eta_s+\eta_b^{-1})\right]^{-1}, \tag{19a}$$

$$G_k=-\overline{W}_k\left[\hbar\omega_L+\frac{\hbar^2k_\rho^2}{2m}+\frac{\hbar\mathbf{P}_\rho\cdot\mathbf{k}_\rho}{m}(\eta_s+\eta_b^{-1})\right]^{-1}. \tag{19b}$$

Here η_s and η_b are defined by

$$\sum_{\mathbf{q}}\hbar\mathbf{q}F_q^2=\eta_s\mathbf{P}_\rho\ , \tag{20a}$$

$$\sum_{\mathbf{k}}\hbar\mathbf{k}_\rho G_k^2=\eta_b\mathbf{P}_\rho \tag{20b}$$

and

$$\begin{aligned}\overline{\Gamma}_q=\Gamma q\langle\phi_e|&\exp(-q|z-L/2|)\\&+\exp(-q|z+L/2|)|\phi_e\rangle\ ,\end{aligned} \tag{21a}$$

$$\overline{W}_k=W_k\langle\phi_e|\sin(k_zz)|\phi_e\rangle\ . \tag{21b}$$

With the trial wave function $|\phi_e\rangle$ given in Eq. (12), the closed forms of $\overline{\Gamma}_q$ and $\overline{W}_k$ can be obtained. Substituting Eqs. (19a) and (19b) into Eq. (18), the ground-state energy E is obtained as

$$\begin{aligned}E=&\frac{\hbar^2k_1^2}{2m}+\frac{e^2\pi N^2\lambda^2}{2\epsilon_s}\left[1+\frac{\cos(2k_1z_i)}{1+k_1^2\lambda^2}-\frac{k_1^2\lambda^2}{1+k_1^2\lambda^2}\cosh\left[\frac{2z_i}{\lambda}\right]\exp\left[-\frac{L}{\lambda}\right]\right]\\&+\frac{\hbar^2}{2m\lambda^2}[1-(\eta_s+\eta_b)^2]-\sum_{\mathbf{q}}\overline{\Gamma}_{\mathbf{q}}^2\left[\hbar\omega_s+\frac{\hbar^2q^2}{2m}+\frac{\hbar^2q}{m\lambda}(\eta_s+\eta_b^{-1})\right]^{-1}\\&-\sum_k\overline{W}_k^2\left[\hbar\omega_L+\frac{\hbar^2k_\rho^2}{2m}+\frac{\hbar^2k_\rho}{m\lambda}(\eta_s+\eta_b^{-1})\right]^{-1}.\end{aligned} \tag{22}$$

For slow electrons, Eqs. (19) and (20) yield

$$\eta_s=\frac{2x_s}{1+2x_s+2x_b}\ , \tag{23a}$$

$$\eta_b=\frac{2x_b}{1+2x_s+2x_b}\ , \tag{23b}$$

where

$$x_s=\frac{m}{P_\rho^2}\sum_{\mathbf{q}}\overline{\Gamma}_q^2\left[\frac{\hbar\mathbf{P}_\rho\cdot\mathbf{q}}{m}\right]^2\left[\hbar\omega_s+\frac{\hbar^2q^2}{2m}\right]^{-3}, \tag{24a}$$

$$x_b=\frac{m}{P_\rho^2}\sum_{\mathbf{k}}\overline{W}_k^2\left[\frac{\hbar\mathbf{P}_\rho\cdot\mathbf{k}_\rho}{m}\right]^2\left[\hbar\omega_L+\frac{\hbar^2k_\rho^2}{2m}\right]^{-3}. \tag{24b}$$

The binding energy of the hydrogenic impurity is given by

$$E_B=E_0-E\ , \tag{25}$$

where E_0 is the ground-state energy for an infinite quantum well without impurity. Minimizing E with respect to λ, we obtain the binding energy as a function of the well width L and impurity position z_i.

Next, we shall consider the case of finite quantum well. The potential well is given as

$$V(z)=\begin{cases}0\ , & |z|<L/2\\ V_0\ , & |z|>L/2\ .\end{cases} \tag{26}$$

We assume that the band-gap discontinuity[29,30] in the

GaAs-$Ga_{1-x}Al_xAs$ heterostructure is distributed about 40% on the valence band and 60% on the conduction band with the total band-gap difference ΔE_g between GaAs and $Ga_{1-x}Al_xAs$ given as a function of the Al concentration $x < 0.4$ as[31] ΔE_g (eV) $= 1.247x$.

It is well known that when the potential well is finite, the bound-state energies cannot be solved for as simply as in the case of the infinite quantum well. One has to solve the transcendental equations. We shall use a general perturbative method proposed by Lee and Mei[32] to treat the finite potential-well problems. It is found that this method is sufficiently accurate when one is interested only in the low-lying bound states. Recently, this approach has been applied to study the anisotropic effect and Stark effect in the quantum-well structures, and successful results have been obtained.[7,10] In this method the bound-state energies of the finite quantum well can be approximated by the eigenenergies of the infinite quantum well with a perturbative correction which depends on the inverse square root of the potential height. Unlike the wave function of the infinite quantum well which must vanish at the hard wall, the actual wave function will penetrate a distance δ into the soft wall, therefore increasing the size of the wave function by a fraction of $2\delta/L$. However, this increase could obviously be achieved by simply shifting the infinite wall farther away from the original position by a distance δ. As the potential height is large compared with the eigenenergies of the low-lying states, the penetration depth δ is given by

$$\delta = \left[\frac{2m(V_0 - E^{(0)})}{\hbar^2} \right]^{-1/2}, \tag{27}$$

where $E^{(0)}$ represents the ground-state energy of the corresponding unperturbed infinite quantum well. Therefore, the hydrogenic impurity state in a finite quantum well with a well width L can be considered the same as that in an infinite quantum well with a broadened well width $L + 2\delta$.

III. RESULTS AND DISCUSSION

A. Infinite quantum well

We first give the results of the infinite quantum well for the GaAs-$Ga_{1-x}Al_xAs$ system. It is convenient to define the effective Bohr radius $a^* = \hbar^2\epsilon_0/me^2$ and the effective Rydberg $R^* = e^2/2\epsilon_0 a^*$. The material parameters[33] used in the calculation are given in Table I. Figure 1 shows the variations of the binding energies as a function of layer thickness for the on-center and on-edge impurities.

TABLE I. The material parameters for GaAs and $Ga_{1-x}Al_xAs$ used in the calculations.

	x	ϵ_0	ϵ_∞	ω_L (cm^{-1})	ω_T (cm^{-1})
GaAs		12.9	10.9	293	269
$Ga_{1-x}Al_xAs$	0.14	12.04	10.57	285	267
	0.30	11.18	10.16	278	265
	0.36	10.89	10.04	275	264

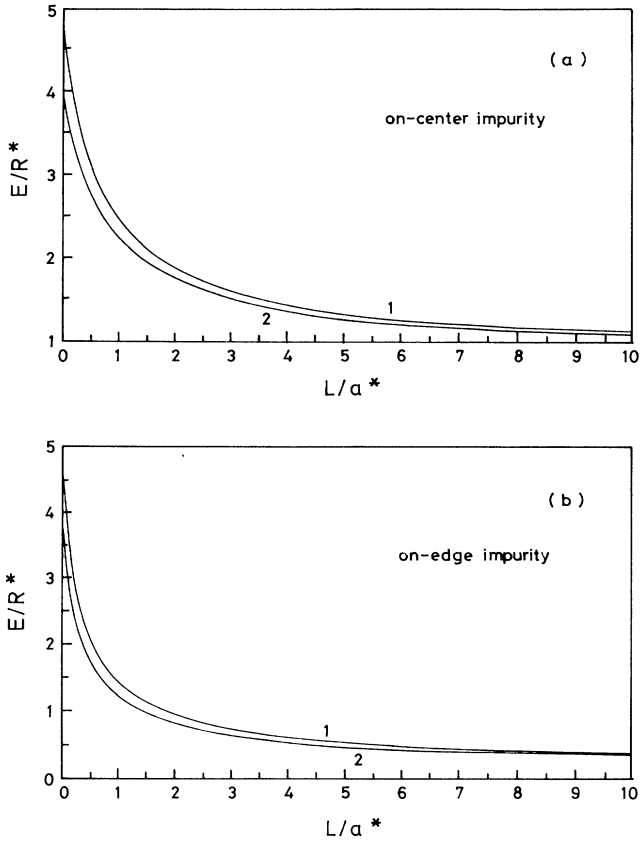

FIG. 1. Reduced binding energy E/R^* as a function of the reduced well width L/a^* for (a) the on-center impurity and (b) the on-edge impurity. The curve 1 (2) corresponds to the case with (without) electron-phonon interactions.

The results without the electron-phonon interactions are also shown in the figure for comparison. It can be seen that the polaron effects on the binding energies are important. For the on-center impurity the effects are about 22% for the small well widths, and reduce to about 1.8% as the well width increases to $L/a^* = 20$, and eventually approaches the value of the bulk. For the on-edge impurity, the polaron effect is larger and ranges from about 7% in the bulk limit to about 23% for very narrow well size. Recently Ercelebi and Tomak[12] have calculated the polaron effect for the on-center impurity, and obtained about 3% in the bulk case and 15% for the small well size. Mason and Das Sarma[14] also studied this problem and obtained a polaronic shift of 1.6% for large well thickness. As we have mentioned before, in these previous calculations the phonon confinement effects is ignored and the interface phonon mode is also neglected. This is of course an oversimplification. In our calculations, both the phonon confinement and the interface phonon are taken into account. Therefore, our result seems to be more reliable than the others. We have also studied the competition between the confined bulk phonon and the interface phonon modes. Figure 2 gives the variation of binding energy as a function of well width, including only the confined bulk phonon or the interface phonon. It is

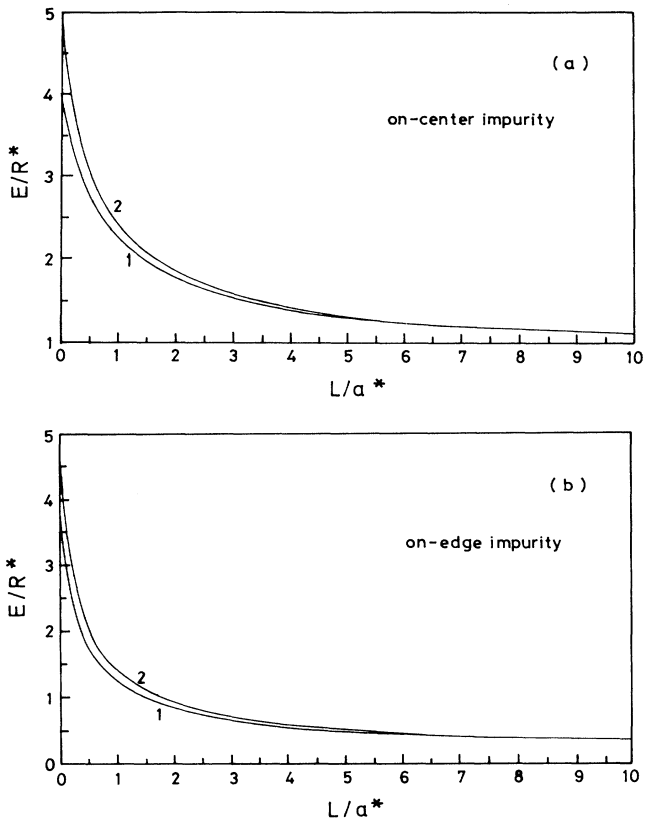

FIG. 2. Reduced binding energy E/R^* as a function of the reduced well width L/a^* with the confined bulk phonon (curve 1) or the interface phonon (curve 2) for an impurity located (a) at the center or (b) on the edge of the well.

found that the competition between these two modes indicates that in the case of small well thickness the interface phonon plays the dominant role and the confined bulk phonon makes little contribution. As the well width increases, the interface phonon contribution decreases and eventually the bulk phonon becomes the important one as $L > 10a^*$. This is in accordance with our expectation. Since the bulk phonon is confined in the longitudinal direction (i.e., the z axis) there should be no bulk phonon effect as the well width L approaches zero. It is worth to note that this result is different from the previous calculations where the usual Fröhlich interaction was used and the phonon confinement effect was neglected so that the main contribution still came from the bulk phonon even as $L \rightarrow 0$. In our calculations, it is the interface phonon which makes the main contribution as the well width gets smaller. Tatham *et al.*[34] have recently reported a significant increase in the relaxation rate for a very narrow well (25 Å), which we believe is responsible for the interface modes.

B. Finite quantum well

In this section, we present the results for the case of the finite potential well. The TO and LO frequencies for $Ga_{1-x}Al_xAs$ have been obtained with a different Al concentration x from the Kramers-Kronig dispersion analysis of the infrared reflectivity spectra.[33] Table I lists the values that are used in our calculations. We have used the general perturbative method of Lee and Mei[32] to study the finite quantum-well problem. In order to check the validity of this method for our problem, a preliminary calculation is first performed for the case without the

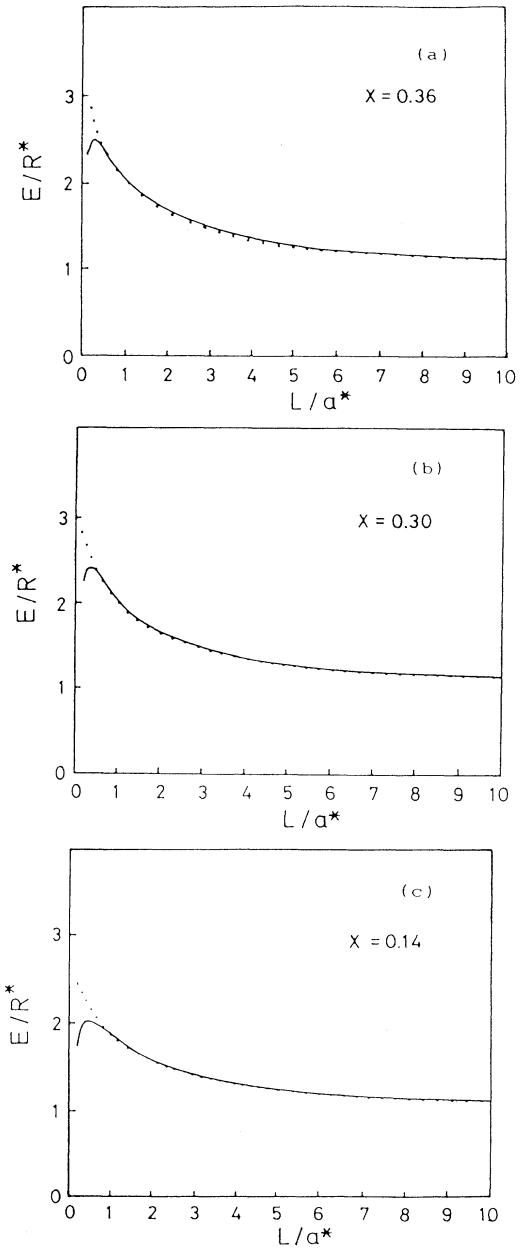

FIG. 3. The variation of the reduced binding energy E/R^* without the electron-phonon interactions as a function of the well width L/a^* using the general perturbative method (dotted line) and the exact method of Liu and Quinn (solid line) for several values of the barrier heights (a) $V_0(x=0.36)$, (b) $V_0(x=0.30)$, and (c) $V_0(x=0.14)$. The impurity is at the center of the finite quantum well.

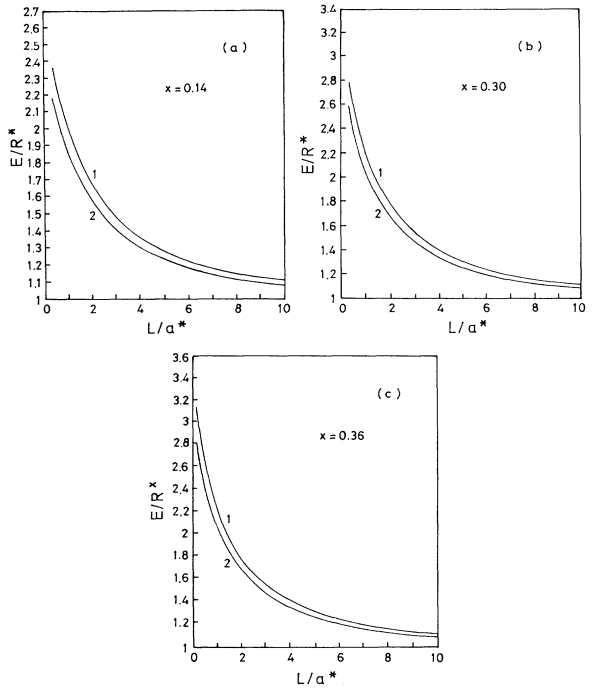

FIG. 4. Reduced binding energy E/R^* as a function of the reduced well width L/a^* for an impurity located at the center in the finite quantum well. Curve 1 (2) corresponds to the case with (without) electron-phonon interactions for (a) $V_0(x=0.14)$, (b) $V_0(x=0.30)$, and (c) $V_0(x=0.36)$.

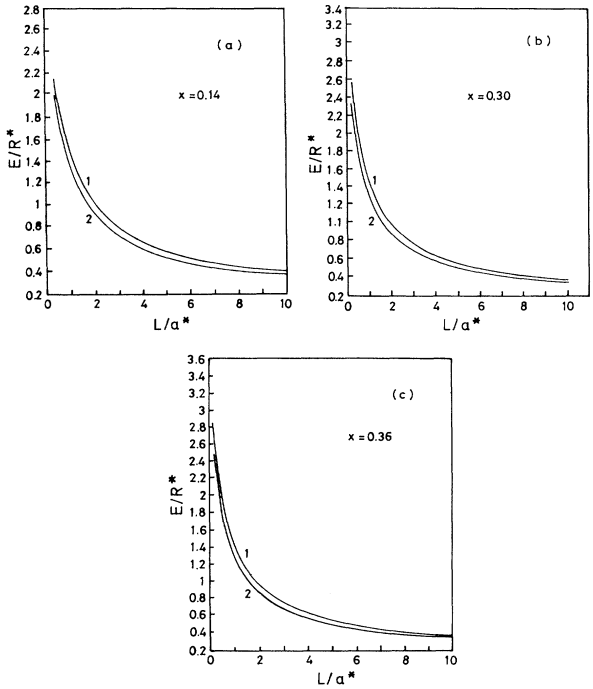

FIG. 5. Same as in Fig. 5 but for the impurity located on the edge of the well.

electron-phonon interaction, and the results are compared to those of the exact calculation using the method of Liu and Quinn[4] as shown in Fig. 3. It is clearly seen that the agreement is quite good even as the well width reduces to $L/a^* \sim 0.3$ for the case of $x=0.36$, $L/a^* \sim 0.4$ for $x=0.30$, and $L/a^* \sim 0.65$ for $x=0.14$. Therefore, our method is reasonably applicable to the problem when the well size is greater than the above regions. This method is then used to study the case with the electron-phonon interactions taken into account.

We have calculated the binding energy as a function of well thickness for different potential barrier heights V_0 corresponding to $x=0.14$, 0.30, and 0.36. The results are shown in Fig. 4 for the on-center impurity and in Fig. 5 for the on-edge impurity. Figure 6 gives the correction of binding energy due to the electron-phonon interactions. Our results show that the polaronic shift is also quite important for the case of the finite quantum well. For the GaAs-$Ga_{0.7}Al_{0.3}As$ system ($x=0.3$), which corresponds to the potential-well height $V_0=36R^*$, the polaronic shift ranges from about 13% for small well size to 2.8% for the bulk limit. It is interesting to note from Fig. 6 that the polaron effect becomes bigger as the potential barrier gets higher. This is because the higher the barrier the larger the binding energy, the localization of the electron becomes more pronounced and thus increases the importance of the electron-phonon interaction. As we have pointed out before, this important behavior has only been briefly speculated upon in previ-

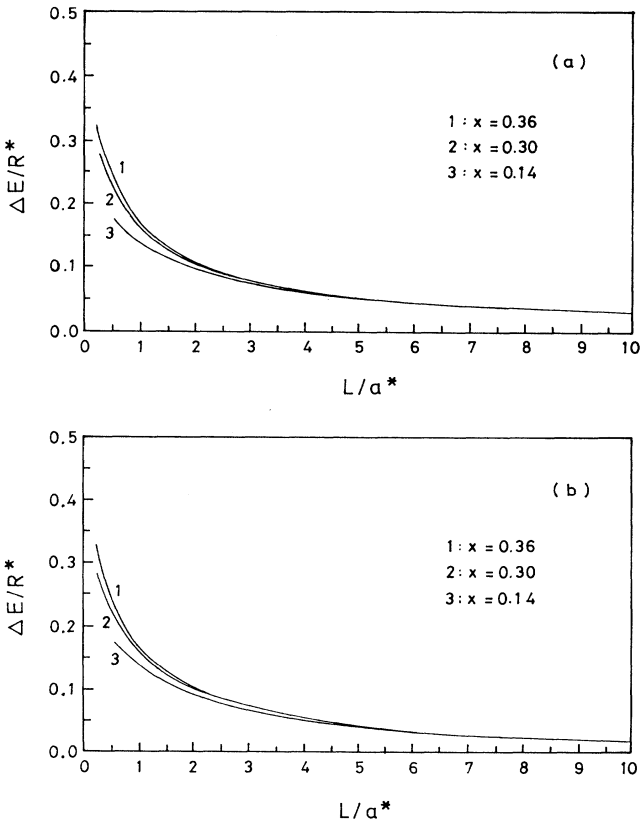

FIG. 6. The shifts of binding energy due to the electron-phonon interactions for (a) on-center and (b) on-edge impurities in the finite quantum well, with different V_0 corresponding to $x=0.36$ (curve 1), 0.30 (curve 2), and 0.14 (curve 3).

ous works,[12,13] but no calculation was given since they only considered the case of the infinite quantum well. Our work is an explicit calculation and presents the results for different potential barrier heights. For example, for the on-center impurity with $L = a^*$, the percentage of polaronic correction is 10% for the barrier height $V_0(x = 0.36)$, and decreases to about 7% for $V_0(x = 0.14)$. It is clear to see that, in general, for a lower quantum-well height where the binding energy is smaller, the polaron effect becomes weaker. It is also worth noting that the tendency of decreasing polaron effect as the barrier is lowered is quite fast for smaller well thicknesses. As the well width becomes larger, the polaronic correction is almost the same for different barrier heights, and approaches the limiting bulk value.

IV. CONCLUSION

We have studied the polaron effect on the hydrogenic impurity in the GaAs-$Ga_{1-x}Al_xAs$ quantum-well system. It is found that the shifts of binding energy due to electron-phonon couplings are quite important for both the on-center and on-edge impurities. The polaronic correction can be as large as 23% for the small well thickness. In this work both the interactions of the electron with the confined bulk phonon and interface phonon are taken into account. Therefore, our result seems to be more reliable than those of previous calculations where the phonon confinement effect and the interface phonon mode are neglected. The competition between the interface and confined bulk phonons is also investigated. Our results show that the dominant contribution comes from the interface mode in the case of thin layers, and the bulk phonon is more important as the well thickness becomes larger than $10a^*$. We have extended the work for the case of the finite quantum well with different potential barrier heights. It is found that the higher the quantum-well barrier where the binding energy is larger, the larger the polaronic correction due to the increasing importance of the electron-phonon interaction. We also find that the polaron effect decreases very quickly as the barrier height is lowered, for a small well width and approaches the limiting bulk result for the large well size.

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