Fast IMDCT and MDCT Algorithms—A Matrix Approach

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***Abstract*—This paper presents a systematic investigation of the modified discrete cosine transform/inverse modified discrete cosine transform (MDCT/IMDCT) algorithm using a matrix representation. This approach results in new understanding of the MDCT/IMDCT, enables the development of new algorithms, and makes clear the connection between the algorithms. We represent in a matrix form the IMDCT as the product of the type-IV DCT with simple scaling, sign-changing, and permutation operations such that fast algorithms for the type-IV DCT can be simply modified for the IMDCT, and vice versa. Then, the simple symmetry and inversion properties of the type-IV DCT are used to develop new algorithms and establish the connection between existing fast IMDCT algorithms. This approach also enables us to show that MDCT and IMDCT share common core operation and present an efficient architecture for implementing both the MDCT and the IMDCT in one hardware.**

***Index Terms*—MDCT/IMDCT, type-IV DCT.**

I. Introduction

THE modified discrete cosine transform/inverse modified discrete cosine transform (MDCT/IMDCT) is used to realize the analysis/synthesis filterbank of time-domain aliasing cancellation scheme [1], [2] for subband coding. This transform, because of its efficiency and flexibility in window-size switching, has been adopted in several international standards and commercial products, such as MPEG-1 [3], MPEG-2 [4], and AC-3 [5] for audio coding. Thus, fast algorithms for realizing the MDCT/IMDCT are important for real-time audio applications.

Several fast algorithms for the MDCT/IMDCT have been proposed. In [6], the independent variables of the MDCT are formulated as a Fourier transform such that the fast Fourier transform (FFT) can be employed for realization. Later, more computationally efficient IMDCT algorithms using the type-II DCT [7] transform have been presented in [8]–[13]. It is also noted that MDCT is equivalent to the modulated lapped transform introduced by Malvar [14], and several fast algorithms have been developed in his books and papers [15]–[17]. Note that these algorithms are developed mainly by using trigonometric identities; the development process requires ingenious work, and it is hard to grasp its fundamental idea. Naturally, the relationship between algorithms is obscure and hard to understand.

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In this paper, we present a systematic approach for investigating the MDCT/IMDCT using a matrix representation. This approach results in new understanding of the MDCT/IMDCT, enables the development of new algorithms, and makes clear the connection between algorithms. We first formulate the IMDCT in a matrix form and represent it as the product of the type-IV DCT transformation matrix, a scaling factor, a simple sign-changing matrix, and a simple permutation matrix. Hence, fast algorithms for the type-IV DCT can be simply modified for the IMDCT, and vice versa. Then, the simple symmetry and inversion properties of the type-IV DCT are used to develop an efficient architecture for realizing both the MDCT and the IMDCT. These properties further enable us to develop several fast IMDCT/MDCT algorithms. These algorithms encompass most existing IMDCT algorithms in the literature. Since all derivations use simple matrix manipulations, it is much easier to grasp the basic idea of the development of new algorithms and the relationship between them.

This paper is organized as follows. In Section II, the IMDCT is first represented in a matrix form as the product of the type-IV DCT and some simple operations. Then, simple but useful symmetry and inversion properties of the type-IV DCT transformation matrix are disclosed. In Section III, the symmetry property of the type-IV DCT is used to develop one MDCT realization, which requires the same core function as that of the IMDCT. Therefore, an efficient architecture is obtained for applications requiring the realization of both the MDCT and IMDCT in one integrated circuit. In Section IV, we apply the fast algorithm proposed in [18]–[20] for the type-IV DCT to develop the first IMDCT algorithm; this algorithm, via the matrix approach, yields other three algorithms by using the symmetry and inversion property of the type-IV DCT transformation matrix. These four algorithms encompass new and most existing IMDCT algorithms. Thus, the matrix approach makes clear the connection between existing algorithms and simplifies the development of new algorithms.

II. IMDCT and MDCT

The concerned MDCT and IMDCT in this paper are, respectively, defined as

$$\text{MDCT}: \quad X(k) = \sum_{n=0}^{N-1} x(n) \cos\left[\frac{\pi}{2N}\left(2n+1+\frac{N}{2}\right)(2k+1)\right]$$
$$k = 0, 1, \ldots, \frac{N}{2}-1 \qquad (1)$$

$$\text{IMDCT}: \quad \hat{x}(n) = \sum_{k=0}^{N/2-1} X(k)\cos\left[\frac{\pi}{2N}\left(2n+1+\frac{N}{2}\right)(2k+1)\right] \quad n=0,1,\ldots,N-1. \tag{2}$$

Define the following vectors:

$$\boldsymbol{X} = \left[X(0), X(1), \ldots, X\left(\frac{N}{2}-1\right)\right]^T \tag{3}$$

$$\boldsymbol{x} = [x(0), x(1), \ldots, x(N-1)]^T \tag{4}$$

$$\hat{\boldsymbol{x}} = [\hat{x}(0), \hat{x}(1), \ldots, \hat{x}(N-1)]^T \tag{5}$$

where the superscript T denotes the transpose operation. The MDCT in (1), in a matrix form, can be written as

$$\boldsymbol{X} = M\boldsymbol{x} \tag{6}$$

where M is an $(N/2) \times N$ matrix with its elements

$$m(i,j) = \cos\left[\frac{\pi}{2N}\left(2j+1+\frac{N}{2}\right)(2i+1)\right] \quad i=0,1,\ldots,\frac{N}{2}-1; j=0,1,\ldots,N-1. \tag{7}$$

Similarly, the IMDCT in (2) is expressed as

$$\hat{\boldsymbol{x}} = M^T\boldsymbol{X}. \tag{8}$$

Hence, if a realization of the IMDCT is developed, one realization for the MDCT is obtained by transposing the signal flow graph for realizing the IMDCT. It is also known that the MDCT and the IMDCT defined above are related as

$$MM^T = \frac{N}{2} I_{N/2} \tag{9}$$

where $I_{N/2}$ denotes the $N/2 \times N/2$ identity matrix.

Most works in the literature assume that the MDCT window size N is a power of 2; the following development only assumes that N is a multiple of 4. If N is not a multiple of 4, then it can be shown that the IMDCT after simple sign changing and permutation operations can be reduced to a type-II DCT. We do not further elaborate on this point because in most real-world applications, N is a multiple of 4. It is known from (2) that $\hat{x}(n)$ has only $N/2$ independent variables and has the following symmetries:

$$\hat{x}\left(\frac{3N}{4}+n\right) = \hat{x}\left(\frac{3N}{4}-1-n\right) \quad n=0,1,\ldots,\frac{N}{4}-1 \tag{10}$$

$$\hat{x}\left(\frac{N}{2}-1-n\right) = -\hat{x}(n) \quad n=0,1,\ldots,\frac{N}{4}-1. \tag{11}$$

Therefore, if $\hat{x}(n), n = 0,1,\ldots,(N/4)-1$, and $\hat{x}(n), n = (3N/4)-1, (3N/4)-2, \ldots, N/2$, are given, then another $\hat{x}(n)$ can be directly obtained using (10) and (11). Define the following vector:

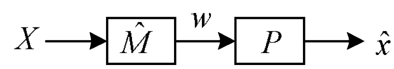

Fig. 1. Block diagram for the IMDCT realization.

$$\boldsymbol{w} = \left[\hat{x}\left(\frac{3N}{4}-1\right), \ldots, \hat{x}\left(\frac{N}{2}+1\right), \hat{x}\left(\frac{N}{2}\right), \hat{x}(0), \hat{x}(1), \ldots, \hat{x}\left(\frac{N}{4}-1\right)\right]^T \tag{12}$$

which generates the vector $\hat{\boldsymbol{x}}$ using (10) and (11), yielding

$$\hat{\boldsymbol{x}} = P\boldsymbol{w} \tag{13}$$

where

$$P = \begin{bmatrix} 0 & I_{N/4} \\ 0 & -J_{N/4} \\ J_{N/4} & 0 \\ I_{N/4} & 0 \end{bmatrix} \tag{14}$$

and $J_{N/4}$ is an $N/4 \times N/4$ matrix given by

$$J = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 1 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}. \tag{15}$$

Note that the matrix P in (13) consists of simple permutations and sign-changing operations. Thus, the main effort for realizing the IMDCT is to obtain the vector $\boldsymbol{w}$. Using (12) and the IMDCT definition (2), we can derive

$$\boldsymbol{w} = \hat{M}\mathbf{X} \tag{16}$$

where $\hat{M}$ is an $N/2 \times N/2$ matrix. Therefore, the block diagram shown in Fig. 1, for the IMDCT realization, first realizes the $\hat{M}$ matrix and then performs the permutations and sign-changing operations of the matrix P. The fast algorithm is developed, naturally, to realize the operation of the $\hat{M}$ matrix.

The matrix $\hat{M}$ can be written in the following form:

$$\hat{M} = \begin{bmatrix} \hat{M}_{11} & \hat{M}_{12} \\ \hat{M}_{21} & \hat{M}_{22} \end{bmatrix} \tag{17}$$

where $\hat{M}_{11}$, $\hat{M}_{12}$, $\hat{M}_{21}$, and $\hat{M}_{22}$ all are $N/4 \times N/4$ matrices, and the components of each are obtained by evaluating (2), yielding

$$\hat{M}_{11}(i,j) = -\cos\left[\frac{\pi}{2N}(2i+1)(2j+1)\right] \quad i,j=0,1,\ldots,\frac{N}{4}-1 \tag{18}$$

$$\hat{M}_{12}(i,j) = -\cos\left[\frac{\pi}{2N}(2i+1)\left(2j+1+\frac{N}{2}\right)\right] \quad i,j=0,1,\ldots,\frac{N}{4}-1 \tag{19}$$

$$\hat{M}_{21}(i,j) = \cos\left[\frac{\pi}{2N}\left(2i+1+\frac{N}{2}\right)(2j+1)\right] \quad i,j=0,1,\ldots,\frac{N}{4}-1 \tag{20}$$

$$\hat{M}_{22}(i,j) = \cos\left[\frac{\pi}{2N}\left(2i+1+\frac{N}{2}\right)\left(2j+1+\frac{N}{2}\right)\right] \quad i,j=0,1,\ldots,\frac{N}{4}-1. \tag{21}$$

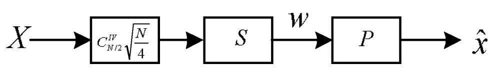

Fig. 2. Type-IV DCT in the block diagram for the IMDCT realization.

Closely investigating the matrix $\hat{M}$, we observe that the matrix $\hat{M}$ and the $N/2$-point type-IV DCT transformation matrix $C_{N/2}^{IV}$ are related by

$$\hat{M} = \sqrt{\frac{N}{4}} S C_{N/2}^{IV} \tag{22}$$

where S is an $N/2 \times N/2$ matrix given by

$$S = \begin{bmatrix} -I_{N/4} & 0 \\ 0 & I_{N/4} \end{bmatrix}. \tag{23}$$

The above result illustrates the relation between the type-IV DCT and the IMDCT transform. Note that they are different only by a scaling factor and one simple sign-changing operation. Hence, fast algorithms for the type-IV DCT can be simply modified for the IMDCT, and vice versa. This relation can be directly shown because the components of an $N/2$-point type-IV DCT transformation matrix [7] are

$$c^{IV}(i,j) = \sqrt{\frac{4}{N}} \cos\left[\frac{\pi}{2N}(2i+1)(2j+1)\right] \qquad i,j = 0,1,\ldots,\frac{N}{2}-1. \tag{24}$$

Note that the relation between IMDCT and the type-IV DCT has been known in the literature [9], [11], [12]. However, they do not further exploit the properties of the type-IV DCT. Here, we present this relation in matrix form and use the simple matrix properties of the type-IV DCT to develop IMDCT and MDCT algorithms and build up the relationship between existing algorithms. In terms of (22), the block diagram for realizing the IMDCT using the type-IV DCT is depicted in Fig. 2. Since S and P are merely sign-changing and permutation operations, the most computation-demanding operation is therefore the type-IV DCT.

The following simple properties of the type-IV DCT transformation matrix will be used to develop new algorithms and establish the connection between existing fast IMDCT algorithms.

A. *Property 1 (Symmetry)*

The DCT type-IV matrix is symmetric. That is

$$C_{N/2}^{IV} = \left(C_{N/2}^{IV}\right)^T. \tag{25}$$

B. *Property 2 (Inversion)*

The DCT type-IV matrix is involutory, that is

$$\left(C_{N/2}^{IV}\right)^2 = I_{N/2}. \tag{26}$$

Hence

$$C_{N/2}^{IV} = \left(C_{N/2}^{IV}\right)^{-1}. \tag{27}$$

In the following section, we will use these properties to develop new IMDCT algorithms, build up the connection between existing IMDCT algorithms, and design an efficient architecture for realizing both the MDCT and IMDCT by one hardware.

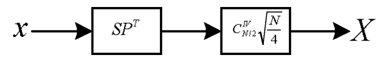

Fig. 3. Block diagram for realizing the MDCT.

III. MDCT Realization

The MDCT is commonly realized by transposing the operations of the IMDCT. Such realization often requires a separate hardware. In this section, we show that because of the symmetry property of the type-IV DCT, the main core operation for realizing the MDCT and the IMDCT can be identical. Hence, we can design a more efficient hardware for realizing both the MDCT and IMDCT.

Using the results of (8), (13), (16), and (22), we obtain the IMDCT, which is given by

$$M^T = P\hat{M} \tag{28}$$

$$= \sqrt{\frac{N}{4}} PSC_{N/2}^{IV}. \tag{29}$$

Hence, the MDCT transformation matrix can be derived as

$$M = \sqrt{\frac{N}{4}} \left(C_{N/2}^{IV}\right)^T S^T P^T \tag{30}$$

$$= \sqrt{\frac{N}{4}} C_{N/2}^{IV} SP^T. \tag{31}$$

Therefore, the MDCT can be realized in the following way:

$$\boldsymbol{X} = M\boldsymbol{x} \tag{32}$$

$$= \sqrt{\frac{N}{4}} C_{N/2}^{IV} SP^T \boldsymbol{x}. \tag{33}$$

Note that

$$SP^T\boldsymbol{x} = \begin{bmatrix} -\left[x\left(\frac{3N}{4}-1\right) + x\left(\frac{3N}{4}\right)\right] \\ -\left[x\left(\frac{3N}{4}-2\right) + x\left(\frac{3N}{4}+1\right)\right] \\ \vdots \\ -\left[x(\frac{N}{2}) + x(N-1)\right] \\ x(0) - x\left(\frac{N}{2}-1\right) \\ x(1) - x\left(\frac{N}{2}-2\right) \\ \vdots \\ x(\frac{N}{4}-1) - x\left(\frac{N}{4}\right) \end{bmatrix}. \tag{34}$$

Fig. 3 illustrates the block diagram for realizing the MDCT. Compared with the IMDCT realization block diagram shown in Fig. 2, we observe that the MDCT and the IMDCT have the identical main core operation, which is the realization of $\sqrt{(N/4)}C_{N/2}^{IV}$. Therefore, an efficient architecture for realizing both the MDCT and IMDCT is derived.

This result is useful for audio applications because in designing an application-specific integrated circuit (ASIC) for performing the audio encoding as well as decoding, the MDCT and the IMDCT can share the same hardware for realizing the transform $\sqrt{(N/4)}C_{N/2}^{IV}$ such that the circuit complexity is significantly reduced.

IV. Fast IMDCT Algorithms

The type-IV DCT, as discussed above, can be used to realize the IMDCT. This section develops four fast IMDCT algorithms and illustrates their relationships using the symmetry and inversion properties of the type-IV DCT. We first use the fast algorithm for computing the type-IV DCT transform in [18]–[20]

to develop the first fast IMDCT algorithm. Then, the symmetry property is applied to the first algorithm to obtain the second fast IMDCT algorithm in Section IV-B. In Section IV-C, the inversion property is used for the first algorithm, resulting in the third algorithm; several existing IMDCT algorithms [8]–[12] can be categorized in this class. Similarly, the third algorithm will yield the fourth algorithm using the symmetry property, as discussed in Section IV-D. Section IV-E contains a short discussion and summary.

A. First Algorithm

As discussed above, the main core operation of the IMDCT is to realize the type-IV DCT. Therefore, a fast computing method for $C^{IV}_{N/2}$ is easily adapted to develop a fast IMDCT algorithm. Here, we use this result to develop the first algorithm.

In [18]–[20], the fast type-II DCT algorithm is used to derive a fast algorithm to realize the type-IV DCT transform; its realization for the $N/2$-point type-IV DCT, in matrix form, can be expressed as

$$C^{IV}_{N/2} = \sqrt{\frac{4}{N}} L C^{II}_{N/2} D \tag{35}$$

where D is a diagonal matrix of diagonal elements

$$d(i,i) = 2\cos\left[\frac{\pi}{2N}(2i+1)\right], \quad i = 0, 1, \ldots, \frac{N}{2}-1. \tag{36}$$

$C^{II}_{N/2}$ is the transformation matrix of the $N/2$-point type-II DCT with its elements

$$c^{II}(i,j) = \cos\left[\frac{\pi}{2(\frac{N}{2})}(2i+1)j\right], \; i, j = 0, 1, \ldots, \frac{N}{2}-1 \tag{37}$$

and L is a lower triangular matrix given by

$$L = \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 & \cdots & 0 \\ -\frac{1}{2} & 1 & 0 & 0 & \cdots & 0 \\ \frac{1}{2} & -1 & 1 & 0 & \cdots & 0 \\ -\frac{1}{2} & 1 & -1 & 1 & & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -\frac{1}{2} & 1 & -1 & 1 & \cdots & 1 \end{bmatrix}. \tag{38}$$

Note that the L matrix can be realized by using simple addition/subtraction recursively. For the matrix L of size $N/2 \times N/2$, its realization only requires $N/2$ addition/subtraction operations.

Hence

$$\begin{aligned} \sqrt{\frac{N}{4}} C^{IV}_{N/2} &= \sqrt{\frac{N}{4}} \left(\sqrt{\frac{4}{N}} L C^{II}_{N/2} D \right) \\ &= L C^{II}_{N/2} D. \end{aligned} \tag{39}$$

Therefore, the first fast IMDCT, with its configuration shown in Fig. 4, is obtained. Note that this algorithm has been presented in [17]. The algorithm first multiplies the input data by the cosine coefficients and then performs the type-II DCT transform; the final results are obtained after recursive addition/subtraction. The main operation of the algorithm is the type-II DCT transform for which several fast algorithms have been proposed [7], [21]–[25].

B. Second Algorithm

We develop the second algorithm using the symmetry property of the type-IV DCT. Using (35) and the result $C^{IV}_{N/2} = (C^{IV}_{N/2})^T$, we have

$$\begin{aligned} \sqrt{\frac{N}{4}} C^{IV}_{N/2} &= \sqrt{\frac{N}{4}} \left(C^{IV}_{N/2} \right)^T \\ &= \sqrt{\frac{N}{4}} \left(\sqrt{\frac{4}{N}} L C^{II}_{N/2} D \right)^T \\ &= D \left(C^{II}_{N/2} \right)^T L^T. \end{aligned} \tag{40}$$

Hence, we obtain the second fast IMDCT algorithm, with its configuration depicted in Fig. 5. The algorithm first performs the recursive addition/subtraction and then executes the type-II IDCT transform. The final results are obtained after multiplication by the cosine coefficients. Similarly, abundant fast algorithms for the type-II IDCT transform have been reported [7], [21]–[25].

The second algorithm is new, although the differences between the first and the second algorithms are only the operation sequence and the DCT for the first but the IDCT for the second. The computational complexities of these two algorithms are identical if the post-windowing in subband synthesis is not considered. If the post-windowing is involved, however, the second algorithm requires less computational complexity than the first because the windowing operation for the second algorithm can be absorbed in the multiplication of the cosine coefficients such that no extra computational burden is required to realize the post-windowing. Hence, $N/2$ multiplications are saved.

C. Third Algorithm

This section applies the inversion property to obtain the third fast IMDCT algorithm. Using (35) and the inversion property, we obtain

$$\sqrt{\frac{N}{4}} C^{IV}_{N/2} = \sqrt{\frac{N}{4}} \left(C^{IV}_{N/2} \right)^{-1} \tag{41}$$

$$= \frac{N}{4} \left(L C^{II}_{N/2} D \right)^{-1} \tag{42}$$

$$= \frac{N}{4} D^{-1} \left(C^{II}_{N/2} \right)^{-1} L^{-1}. \tag{43}$$

Note that the type-II DCT transformation matrix $C^{II}_{N/2}$ defined in (37) has its inverse as in [7]

$$\left(C^{II}_{N/2} \right)^{-1} = \frac{4}{N} \left(C^{II}_{N/2} \right)^T D^{II}_{N/2} \tag{44}$$

where

$$D^{II}_{N/2} = \begin{bmatrix} \frac{1}{2} & 0 & \cdots & 0 & 0 \\ 0 & 1 & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & 1 & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}. \tag{45}$$

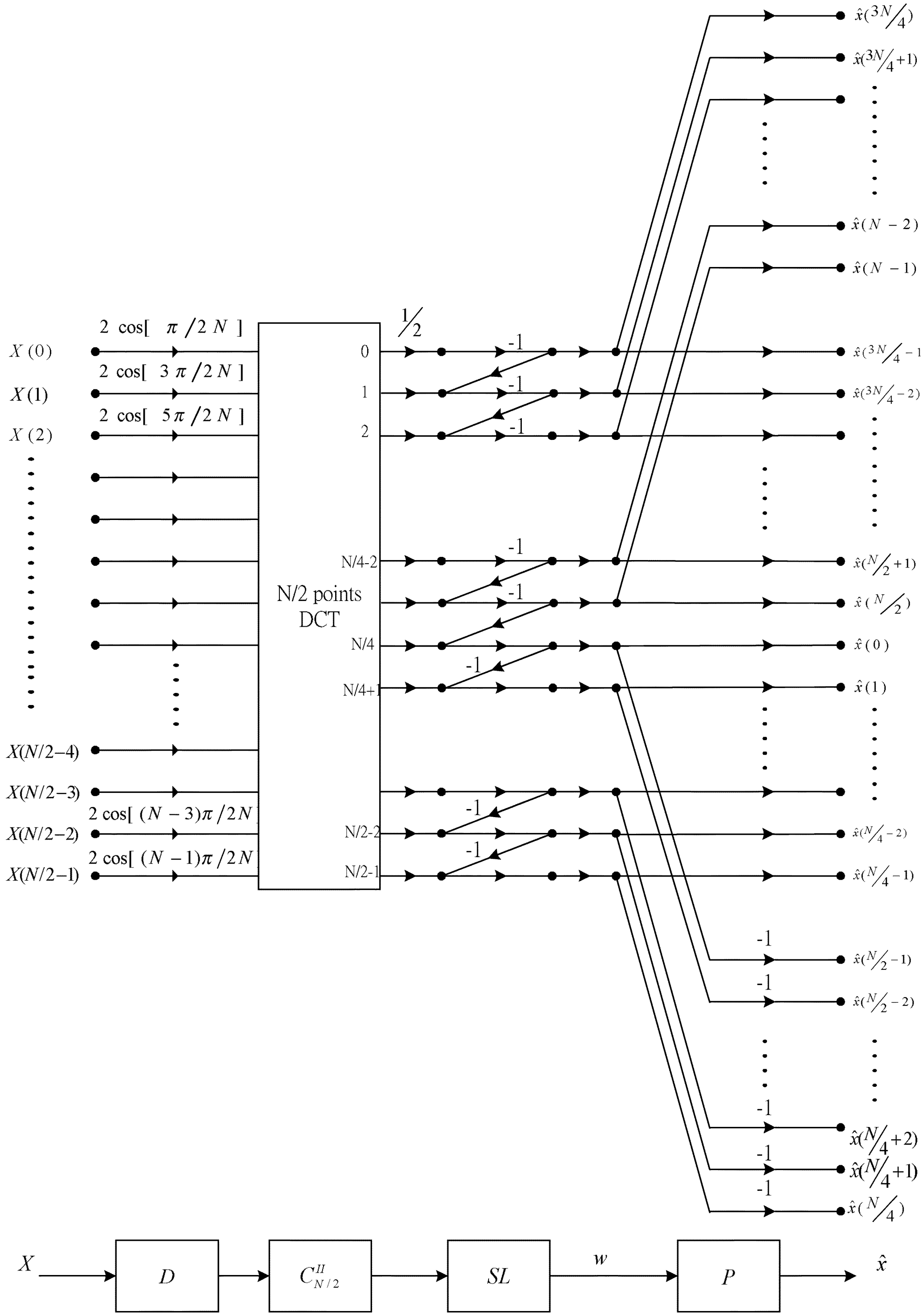

Fig. 4. Configuration of the first IMDCT algorithm.

Using (44) in (43) results in

$$\sqrt{\frac{N}{4}} C_{N/2}^{IV} = D^{-1} \left(C_{N/2}^{II} \right)^T D_{N/2}^{II} L^{-1}. \qquad (46)$$

Observe that the matrix L given by (38) can be decomposed into two terms

$$L = L_1 D_{N/2}^{II} \qquad (47)$$

where

$$L_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ -1 & 1 & 0 & 0 & \cdots & 0 \\ 1 & -1 & 1 & 0 & \cdots & 0 \\ -1 & 1 & -1 & 1 & & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -1 & 1 & -1 & 1 & \cdots & 1 \end{bmatrix}. \qquad (48)$$

Substituting (47) into (46), we have the expression for the third

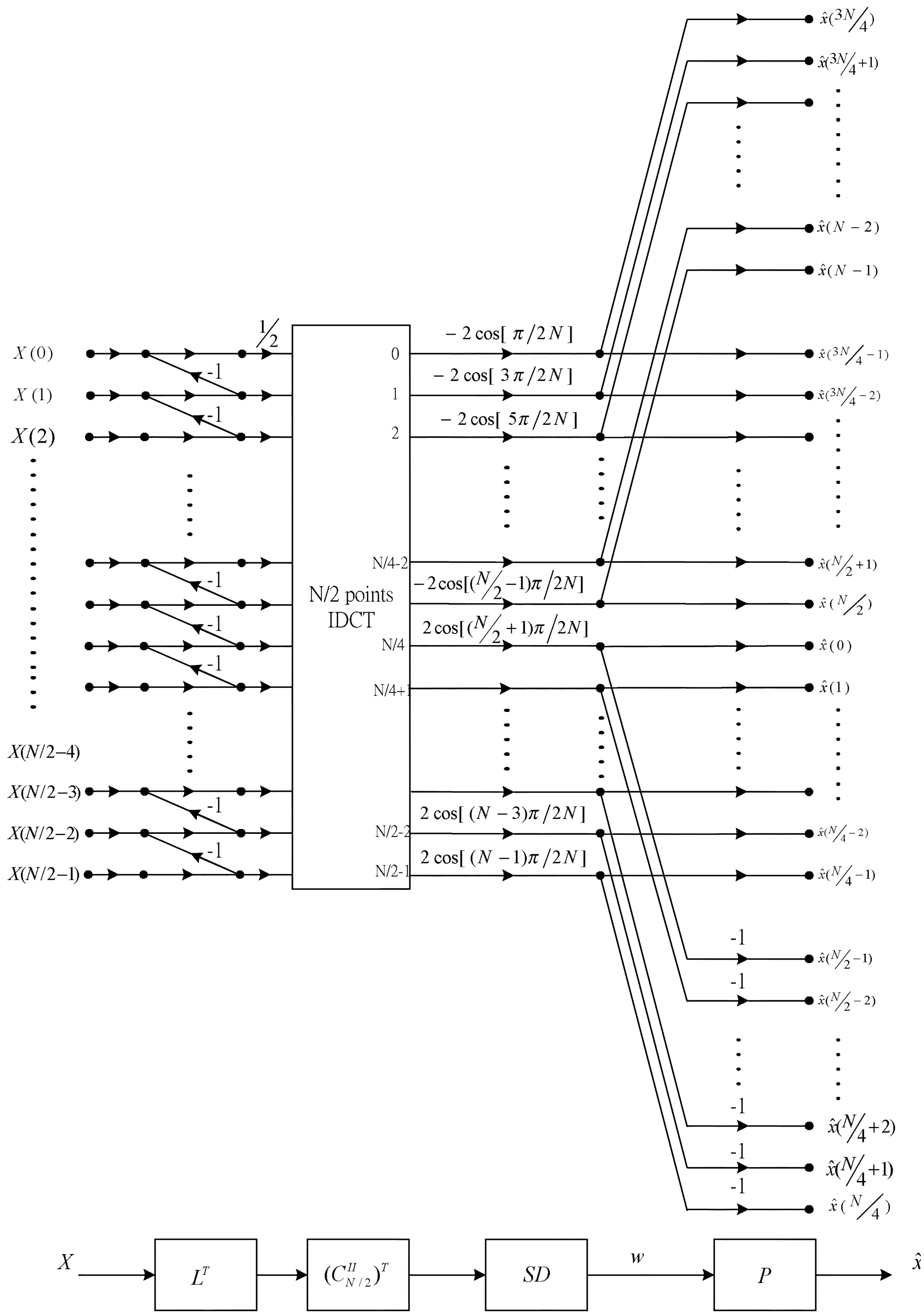

Fig. 5. Configuration of the second IMDCT algorithm.

algorithm

$$\sqrt{\frac{N}{4}} C_{N/2}^{IV} = D^{-1} \left(C_{N/2}^{II} \right)^T L_1^{-1} \tag{49}$$

where L_1^{-1} has the following simple form:

$$L_1^{-1} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 0 & & 0 & 0 \\ 0 & 1 & 1 & & 0 & 0 \\ \vdots & & & \ddots & & \vdots \\ 0 & 0 & 0 & & 1 & 0 \\ 0 & 0 & 0 & \cdots & 1 & 1 \end{bmatrix}. \tag{50}$$

The third algorithm, with its configuration shown in Fig. 6, first performs the simple addition and then realizes the type-II IDCT. The final results are obtained after the multiplication by the inverse of the cosine coefficients. This algorithm has the same computational complexity as the above two algorithms but has poorer numerical accuracy because of the inverse of the cosine sequence.

Note that four existing IMDCT algorithms [9]–[12] have the same configurations as the third algorithm. They are different only in the way of realizing the type-II DCT. Each of these al-

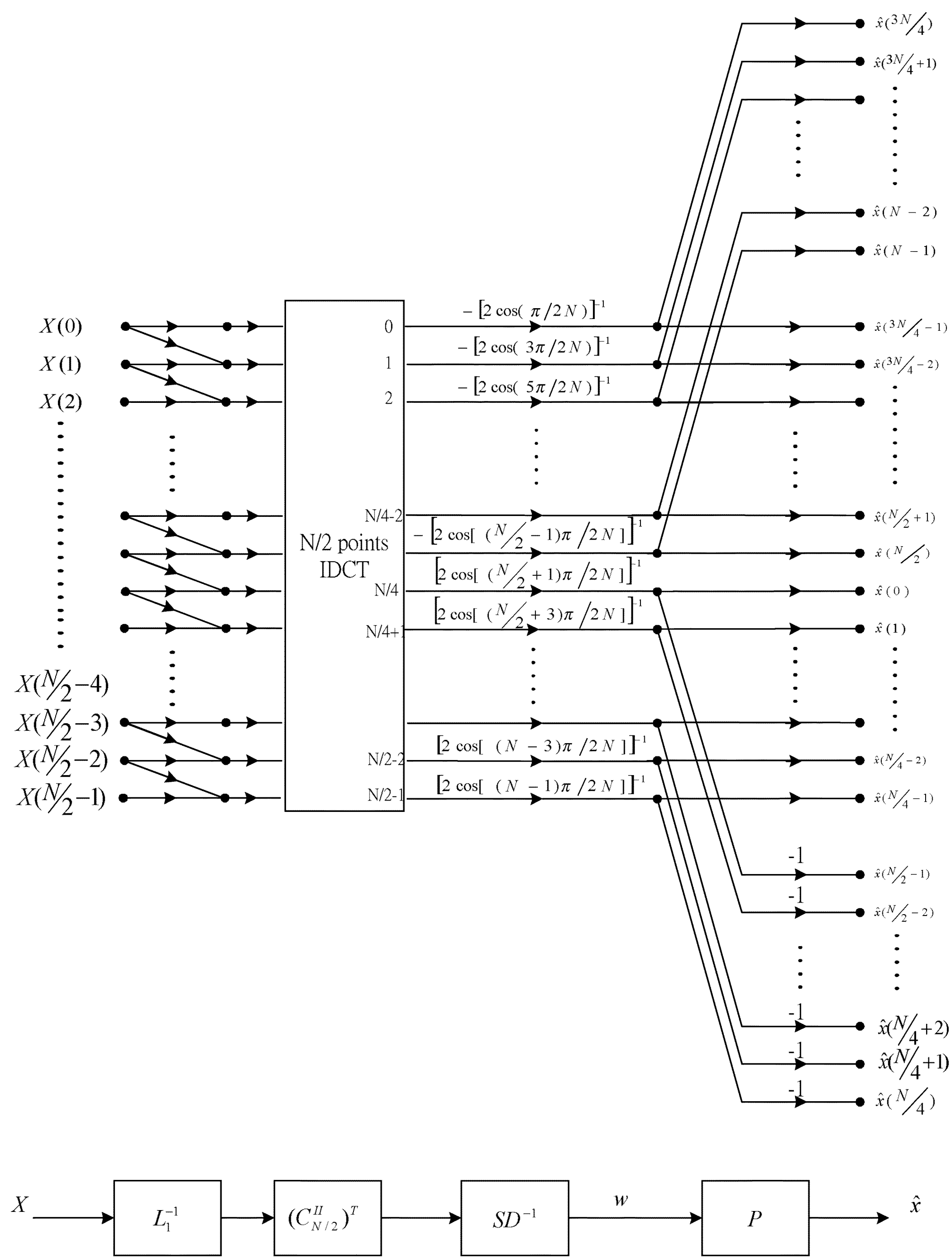

Fig. 6. Configuration for the third fast IMDCT algorithm.

gorithms is developed independently using trigonometric identities. Our approach establishes the connection between these algorithms and makes them easy to understand.

D. Fourth Algorithm

Similarly, applying the symmetry property to the third algorithm results in the fourth fast IMDCT algorithm. Using (49) and the symmetry property, we obtain the expression for the fourth algorithm

$$\sqrt{\frac{N}{4}}C^{IV}_{N/2} = \left(D^{-1}(C^{II}_{N/2})^T L_1^{-1}\right)^T \tag{51}$$

$$= L_1^{-T} C^{II}_{N/2} D^{-1}. \tag{52}$$

The configuration of the fourth algorithm is shown in Fig. 7. This algorithm has been derived using trigonometric identities in [8]. The algorithm first performs the multiplication by the inverse of the cosine coefficients and then realizes the type-II DCT operation. The final results are obtained after the simple addition.

E. Discussion and Summary

The third and the fourth algorithms have poorer numerical performance because of the inverse of the cosine sequence. If the latency in hardware realization is considered, however, the third and the fourth algorithms have a critical path of length 1, whereas the first and the second algorithms have a critical

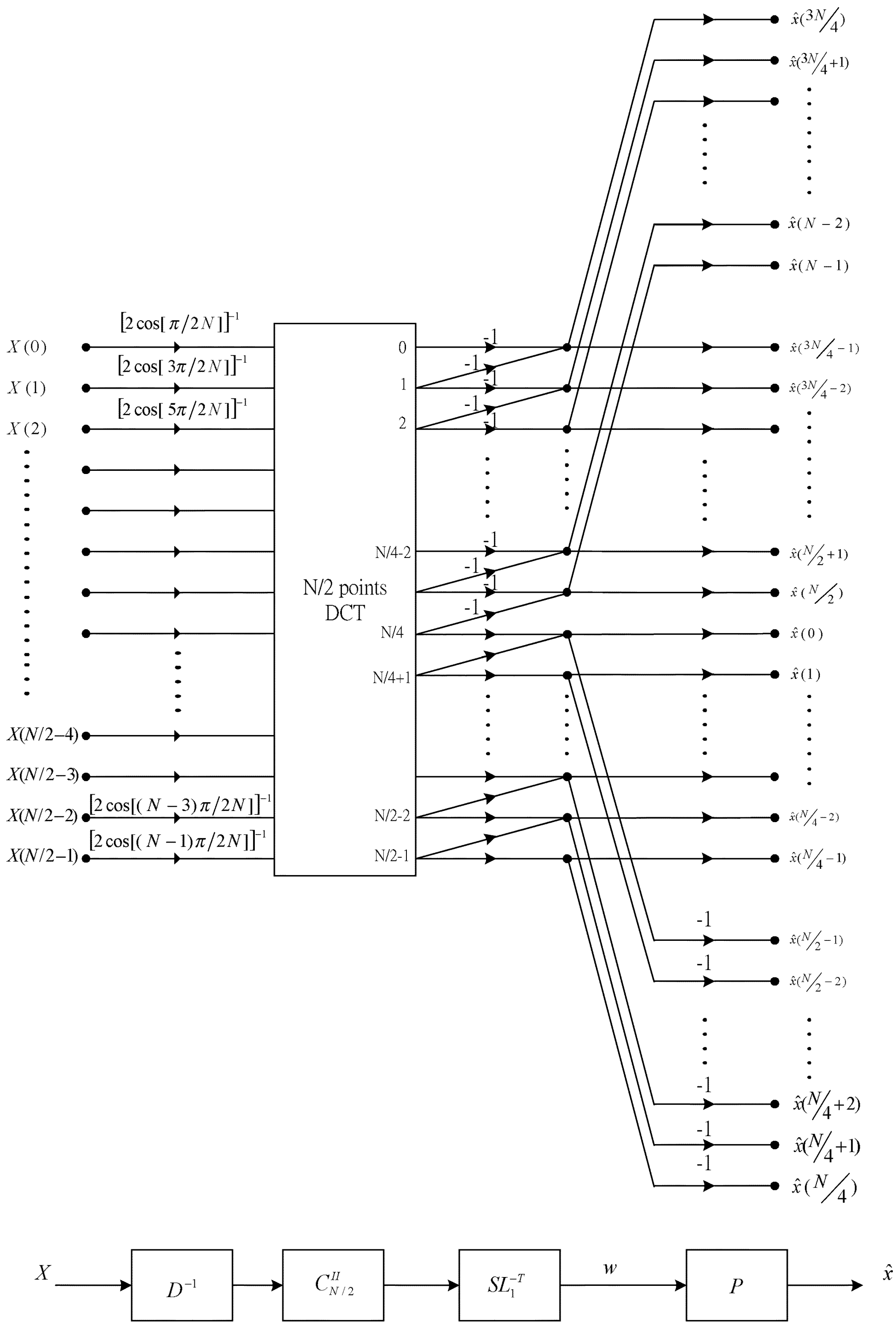

Fig. 7. Fourth fast IMDCT algorithm.

path of length $N/2$ because of the recursive computation for the triangular matrix given by L (38). Therefore, if both the latency in hardware and the numerical stability are considered, then one alternative is to realize the DCT-IV directly using the algorithm in [25] at the price of increased computational complexity.

Four different IMDCT algorithms have been developed by applying the simple symmetry and inversion properties of the type-IV DCT transformation matrix. These algorithms can be derived from each other in terms of the simple matrix manipulations. Therefore, the matrix approach makes clear the relationship between the algorithms and simplifies the development of new algorithms.

V. Conclusions

In this paper, we use the matrix approach to investigate the fast MDCT/IMDCT algorithms. We represent the relation between the IMDCT and the type-IV DCT in matrix form such that the fast algorithm for the type-IV DCT can be applied to realize the IMDCT by simple modification, and vice versa. The simple symmetry and inversion properties of the type-IV DCT are further exploited to develop new algorithms and establish the connection between existing fast IMDCT algorithms. We also propose, by using the symmetry property, an efficient architecture for realizing both the MDCT and the IMDCT in one hardware. Note that this matrix approach builds up the link between the IMDCT, the type-II DCT, and the type-IV DCT. Thus, any new type-IV DCT algorithm can be applied to develop several IMDCT algorithms by using the symmetry and inversion properties. Similarly, new IMDCT algorithms can also be simply modified to realize the type-II or type-IV DCT.

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