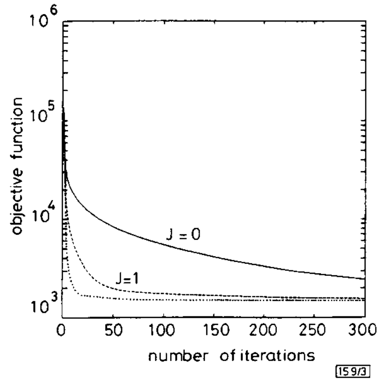

Fig. 3 *Convergence curves of energy function using different resolution levels*

16 August 1993

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Bandwidth of crossbars for general reference model

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Indexing term: Multiprocessor interconnection networks

The bandwidth of crossbar multiprocessor systems is analysed for the general memory reference model. Previous solutions are restricted to several specified models: uniform memory reference, favorite memory reference and hot-spot; the presented analysis includes these as special cases.

Introduction: In a tightly coupled multiprocessor system, processors are connected via an interconnection network (IN) to memory modules so that the memory modules are shared by all processors. The readers are referred to [5] for a survey of INs.

Fig. 1 shows an $M \times N$ crossbar connecting M processors and N memory modules. A crossbar provides the capacity for all memory modules to be accessed simultaneously provided the requested memory modules are distinct. A memory conflict occurs when two or more processors attempt to access the same memory module. The bandwidth, which is defined as the expected number of requests accepted per unit time [5], is an important metric with which to estimate the performance of an IN.

Analyses of the bandwidth of crossbars for three specified reference models have appeared in the literature. They are the uniform memory reference [6], favourite memory reference [3] and hot-spot [1,2,8]. In [4], a survey was reported. In this Letter we analyse the bandwidth of crossbars for the general reference model.

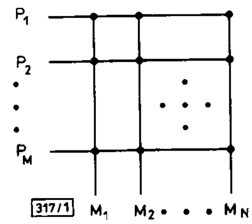

Fig. 1 *M × N crossbar*

Assumptions and notation: The analysis of this Letter is based on the following assumptions:

(i) The crossbar operates in a synchronous mode, i.e. a process can generate a request, if any, at the beginning of a memory cycle.

(ii) Requests from different processors are mutually independent.

(iii) When two or more requests are intended for the same memory modules, only one of the requests is accepted, and the others are rejected.

(iv) The requests which are rejected are discarded, i.e. the requests generated at successive cycles are independent.

Note that this Letter does not make any assumption of the memory reference model. Let P_i and M_j denote processor i and memory module j, respectively, $1 \le i \le M$, $1 \le j \le N$. The memory reference model is defined by a matrix $Q = \{q_{ij}\}_{M \times N}$, where q_{ij} is the probability that a request from P_i is intended for M_j and

$$\sum_{j=1}^{N} q_{ij} = 1$$

The probability that P_i makes a request at the beginning of each memory cycle is r_i, $0 \le r_i \le 1$. Thus the probability that P_i makes a request to M_j at the beginning of any memory cycle is $r_i q_{ij}$.

Bandwidth analysis: For $1 \le j \le N$, let X_j be a binary random variable such that $X_j = 1$ denotes the event that M_j receives one or more requests from the processors; then $X_j = 0$ denotes the event that no request from any processor is sent to M_j.

The expected value of X_j, $E\{X_j\}$, is

$$\begin{aligned} E\{X_j\} &= \Pr\{X_j = 1\} \\ &= 1 - \Pr\{X_j = 0\} \\ &= 1 - \prod_{i=1}^{M}(1 - r_i q_{ij}) \end{aligned} \qquad (1)$$

The bandwidth of the crossbar is the expected value of $X_1 + \ldots + X_N$. According to assumption (ii), $X_1, \ldots, X_N$ are independent random variables. Thus, the bandwidth of the crossbar, BW, is

$$\begin{aligned} BW &= E\left\{\sum_{j=1}^{N} X_j\right\} \\ &= \sum_{j=1}^{N} E\{X_j\} \\ &= \sum_{J=1}^{N}\left(1 - \prod_{i=1}^{M}(1 - r_i q_{ij})\right) \end{aligned} \qquad (2)$$

Comparisons: Eqn. 2 is compared with previous works including: the uniform memory reference model [6], favourite memory [3], and hot memory [2,8].

(*a*) *Uniform reference model*: In this model, for an $M \times N$ crossbar, $r_i = m$, $q_{ij} = 1/N$ for all $1 \le i \le M$, $1 \le j \le N$; then eqn. 2 becomes

$$BW = N\left(1 - \left(1 - \frac{m}{N}\right)^M\right) \qquad (3)$$

which is consistent with eqn. 3 in [6].

(*b*) *Favourite memory*: For an $N \times N$ crossbar, Bhuyan [3] proposed a model that P_i communicates more often with M_i. Formally, in this model, $r_i = p_0$, $q_{ii} = m$, $1/N \le m \le 1$ for all $1 \le i \le N$ and $q_{ij} = (1 - m)/(N - 1)$, for all $1 \le i, j \le N$, $i \ne j$; then

$$BW = N\left(1 - (1 - p_0 m)\left(1 - p_0\frac{1-m}{N-1}\right)^{N-1}\right) \qquad (4)$$

which is identical to eqn. 4 in [3].

(*c*) *Hot memory*: Hot memory (hot spot contention) was first introduced by Pfister and Norton [7]. Without loss of generality, for an $N \times N$ crossbar, we assume M_1 is the hot memory. It was assumed in [2,8] that a fraction h ($0 \le h \le 1$) of all references is aimed at M_1 (hot memory) and the remaining fraction $1 - h$ of references is distributed uniformly over all N memory modules. In this model, $r_i = r$, $q_{i1} = h + (1 - h)/N$, $0 \le h \le 1$, for all $1 \le i \le N$ and $q_{ij} = (1 - h)/N$, for all $1 \le i \le N$, $2 \le j \le N$; then

$$BW = 1 - \left(1 - rh - \frac{r(1-h)}{N}\right)^N + (N-1)\left(1 - \left(1 - \frac{r(1-h)}{N}\right)^N\right)$$

which is consistent with the comment in [2].

In summary, this Letter analyses the bandwidth of crossbars for the general reference model. Previous analyses are special cases of our solution.

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Transmission of planar, cylindrical and spherical multiple dielectric layer systems

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Indexing terms: Electromagnetic waves, Wave transmission

The transmission of planar, cylindrical and spherical waves has been studied for single dielectric layer, resonant tunnelling and multiple period Bragg reflector systems. Interesting features originating from the geometry are found in the cylindrical and spherical systems and their applications are discussed.

A classical example of electromagnetic wave propagation is the transmission and the reflection of waves through single and multiple dielectric layer systems with planar geometry. Applications of these systems include the well known Fabry-Perot interferometer, Bragg reflectors, antireflection coating and various filters. A quantum analogue of this wave propagation phenomenon, resonant tunnelling, was discovered in semiconductor materials [1] and has lead to many applications in high speed electronics [2,3]. The systems that have been studied so far consist mostly of planar layers because they are easy to fabricate. Systems with curved surfaces have been studied very recently by Ping *et al.* [4] for the semiconductor double-barrier quantum well resonant tunnelling system with cylindrical and spherical geometries. Interesting features originating from the different geometries have been revealed. They also demonstrated that to see the new features, the structure ought to be very small, imposing difficulties in the physical realisation. We present an analogous study for the propagation of the electromagnetic waves in single and multiple layer systems with planar, cylindrical and spherical geometry.

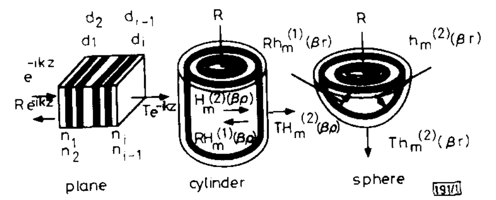

Fig. 1 *Schematic diagram for multiple dielectric layer system with planar, cylindrical and spherical geometries and corresponding travelling waves in these structures*

Refractive index and thickness are n_1 to n_i and d_1 to d_i, respectively; radius of inner cylinder and sphere is R

The structure to be studied is depicted in Fig. 1, in which the geometrical parameters are also shown. The sources of the waves are the remote source for the plane wave, an electric line source for the cylindrical wave and a point source for the spherical wave, respectively. The quantities necessary for determining the nonzero electric and magnetic fields can be found in [5]. Applying the boundary conditions of the electric and magnetic fields at the interfaces, the transmission coefficient could be obtained by the standard transfer matrix approach

$$\begin{pmatrix}1\\R\end{pmatrix} = M(1)M(2)\cdots M(i)\begin{pmatrix}T\\0\end{pmatrix} = \begin{pmatrix}M_{11} & M_{12}\\M_{21} & M_{22}\end{pmatrix}\begin{pmatrix}T\\0\end{pmatrix} \qquad (1)$$

with $M(n)$ being the transfer matrix for the nth layer, which relates to e^{ikz}, $H_1^{(1,2)}(\beta\rho)$ and $h_1^{(1,2)}(\beta r)$, the plane, cylindrical and spherical waves, respectively. In the following, we assume that the layers are isotropic and the dimensions of the layers are measured by a unit wavelength λ_0. Other than specified, the medium is assumed to be air ($n = 1$).

Fig. 2 is the transmission spectra of a single layer ($n = 2$) embedded in air as a function of the wave vector with planar, cylindrical and spherical geometries. The thickness of the layer is $\lambda_0/(4n)$ and the inner radius of the cylinder and sphere is λ_0. As seen from Fig. 2, the transmission spectra for these geometries are very similar to each other for large wave vectors (high frequency), which is expected because the cylindrical and spherical waves