

Quality Improvement for Dynamic Ordered Categorical Response Using Grey Relational Analysis

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Product design is increasingly complicated and some quality characteristics cannot be directly measured. These quality characteristics are generally classified into several categories by visually examining the externals of products and forming an ordered system response. Most studies of quality improvement focus primarily on the quantitative form. Studies of ordered categorical response in a state system have been proposed over recent years. However, cases of an ordered categorical response in a dynamic system have rarely been seen. This study utilises grey relational analysis, from grey system theory, to develop a procedure for improving the ordered categorical response in a dynamic system, based on Taguchi's parameter design. The proposed procedure can determine effectively the optimal factor level combination for an ordered categorical response in a dynamic system. A case study for improving the uniformity of plating in the lead frame process involved in semiconductor packaging is provided to demonstrate the effectiveness of the proposed procedure.

Keywords: Dynamic system; Grey relational analysis; Grey system theory; Ordered categorical quality characteristics; Taguchi's method

1. Introduction

Quality characteristics can be measured for most products or manufacturing processes. However, product design is now becoming increasingly complicated and, some quality characteristics cannot be directly measured or can only be measured at a cost so great as to be unbearable for a manufacturer. In such cases, these quality characteristics are typically classified into several categories by visually inspecting the externals of products and forming an ordered categorical response, to improve the quality of the manufacturing process. The ordered categorical data is qualitative response. Most studies of quality

improvement focus mainly on quantitative cases. Studies of the ordered categorical response in a state system have been proposed in recent years [1,2]. However, cases that involve signal factors for a dynamic, ordered categorical response have rarely been seen. The difference between a dynamic system and a state system is that signal factors are included in the dynamic system to achieve target performance or express the intended output. A quality characteristic is valued according to the level of the signal factor. Signal factors may be the steering angle in the steering mechanism of an automobile or the speed control setting of a fan. Hence, improving the quality of a dynamic, ordered categorical response has today become a relevant issue in industry.

The typical design of experiment is an effective tool for continuous improvement of the quality of industrial process in modern-day manufacturing. In industrial experiments, Taguchi's method has proven useful for improving product/process quality for a wide range of industrial applications. Taguchi's method seeks to resolve a quality problem based on the viewpoint of engineering and physics. Taguchi employs an orthogonal array to conduct experiments, which is equivalent to a fractional factorial experiment and which reduces the number of experiments required. The data used in Taguchi's optimisation process is therefore incomplete. Furthermore, improving a dynamic ordered categorical response requires that the variation in quality with control and signal factor levels be considered, along with the influence of the multivariable input information on the system. Thus, the improvement of the ordered categorical response in a dynamic system can be considered as a grey system with uncertainty.

Grey system theory [3] focuses mainly on resolving problems with uncertainty or systems with incomplete information, and can effectively resolve multivariable problems using system relational analysis, model construction, forecasting, or decision analysis. The calculations involved in grey system theory are rather simple. This study utilises grey relational analysis to develop a procedure to improve the ordered categorical response in a dynamic system based on Taguchi's parameter design. The proposed procedure can determine effectively the optimal factor level combination for an ordered categorical response in a dynamic system. Finally, a case study of improv-

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ing the uniformity of plating in a lead frame process for semiconductor packaging is provided to demonstrate the effectiveness of the proposed procedure.

2. Literature Review

2.1 Quality Improvement Techniques for Ordered Categorical Response

Fisher pioneered the analysis of ordered categorical data in 1925. Cochran [4] transformed the classified data into a percentage form, thereby simplifying problems that involve classified data. McCullagh [5] developed a logistic regression model using the maximum likelihood method to assess location and dispersion effects of ordered categorical data. These methods have significantly contributed to resolving problems related to ordered categorical data. However, the use of conventional statistics requires sufficient samples and the assumption that the data follows a specific distribution. The calculation is rather complicated for engineers without a strong statistical background. Furthermore, collecting many data samples in experiments is impractical, considering the cost and time requirements of manufacturers.

Taguchi developed the accumulation analysis (AA) method [6] to optimise the parameter settings for an ordered categorical response. The AA method includes the following four steps.

1. Define the corresponding cumulative categories.
2. Calculate the accumulated probabilities for each category and the factor level.
3. Plot the effect diagram of the factor level according to the corresponding accumulated probabilities.
4. Predict the accumulated probabilities for each category under optimal conditions.

Taguchi also recommended that the accumulated probability of the factor level be transferred to a corresponding omega (Ω) value using the Ω transformation, thus obtaining an additive response. The optimal factor level combination can be determined by screening the factor level effect diagram. However, the analysis of Taguchi's AA method involving location effects only cannot explain dispersion effects. The parameter optimisation using the AA method may result only in improving the response, without reducing its variation. Nair [2] proposed a scoring scheme (SS) to identify dispersion and location effects. The main calculation of SS initially divides observations of each category for each experimental run by the total number of observations of the corresponding category, to obtain the percentage of each category associated with each experimental run. The percentage data is then substituted into the developed mathematical model to yield the location and dispersion scores. Additionally, the significance of the location and dispersion effects on product quality is also obtained through analysis of variance. Finally, the optimal factor level combination is determined according to the proposed pseudo scores Eq., using an orthogonal array table. The SS method can indeed overcome the weakness of the AA method, which can account only for the location effects. However, the SS method involves compli-

cated mathematical theory and computation and thus cannot be used practically by engineers without a strong mathematical background.

Jean and Guo [1] proposed a weighted probability scoring scheme (WPSS) to simplify the complicated calculation of Nair's SS method. A performance index is determined by trading off location and dispersion effects. This method is simpler and more straightforward than Nair's SS method. Initially, a weighting is directly assigned to each ordered categorical response. A better category corresponds to a higher weighting. The location effect is then determined by the weighted probabilities of each category:

$$W_n = \sum_{i=1}^k w_i p_i \quad (1)$$

where k indicates the categorical response; n represents the number of the experimental run; w_i and $p_i = o_i/o_T$ represent the weight and probability of each category, respectively; o_T denotes the number of experiments under each run, and o_i is the number of each category. Based on the Euclidean norm, the distance between the location effect and the target value is determined to yield the dispersion effect as follows:

$$d_n^2 = \sum_{i=1}^k (w_i p_i - \text{Target}_i)^2 \quad (2)$$

where d_n represents the distance between the location effect and the target value under the n th experimental run, and Target_i is a target value of the dispersion effects for the i th category. Finally, the dispersion and location effects are incorporated into a single mean squared deviation (MSD) as in Eq. (3) to yield an overall performance index for ordered categorical responses. The optimal factor level combination is accordingly determined:

$$E(\text{MSD})_n \cong \frac{1}{W_n^2} \left(1 + \frac{3d_n^2}{W_n^2} \right) \quad (3)$$

where $E(\text{MSD})_n$ represents the overall performance index for the n th experimental run. The WPSS method can simply and directly determine location and dispersion effects. However, Eq. (3) is based on an assumption that a larger response is better. This performance index for the ordered categorical response is therefore inappropriate. The above optimisation methods are either too complex for industrial use or give an inappropriate index. The cases considered above are all for static systems; cases involving an ordered categorical response in a dynamic system have rarely been seen.

2.2 Grey Relational Analysis (GRA)

Deng [3] pioneered grey system theory in 1982. Grey system theory is concerned with solving problems that involve uncertainty or systems with incomplete information. Using system relational analysis, model construction, forecasting, or decision analysis, grey system theory can effectively resolve various problems that involve uncertainty, multiple variables or discrete data.

GRA has no requirements regarding sample size or specific probability assumptions, and involves simple calculations, since it is based on developmental data trends. GRA primarily uses discrete measurements to evaluate the distance between two sequences and to explore the extent of their relationship. The original numerals are transformed (or normalised) into numerals between zero and one. That is, the transformed numerals are scale-invariant. The sequence of the transformed numerals is a comparative sequence.

The calculation process for GRA is as follows [3].

Let X be a factor set of grey relations, $x = \{x_0, \mathbf{x}_1, \dots, \mathbf{x}_m\}$, where $\mathbf{x}_0 \in X$ denotes the referential sequence; $\mathbf{x}_i \in X$ represents the comparative sequence, and $i = 1, \dots, m$. Both \mathbf{x}_0 and \mathbf{x}_i include n elements and can be expressed as follows:

$$\mathbf{x}_0 = (x_0(1), x_0(2), \dots, x_0(k), \dots, x_0(n)) \quad (4)$$

$$\mathbf{x}_i = (x_i(1), x_i(2), \dots, x_i(k), \dots, x_i(n)) \quad (5)$$

where $i = 1, \dots, m$; $k = 1, \dots, n$; $n \in N$, and $x_0(k)$ and $x_i(k)$ are the numbers of referential sequences and comparative sequences at point k , respectively. In practical applications, the referential sequence can be an ideal objective and the comparative sequences are alternatives. The best alternative corresponds to the largest degree of the grey relation. If the grey relational coefficient (GRC) of the referential sequences and comparative sequences at point k is $\gamma(x_0(k), x_i(k))$, then the degree of grey relation for \mathbf{x}_0 and \mathbf{x}_i will be $\gamma(\mathbf{x}_0, \mathbf{x}_i)$ when the following four prerequisites are satisfied:

1. Normal interval:

$$\begin{aligned} 0 < \gamma(\mathbf{x}_0, \mathbf{x}_i) &\leq 1 \\ \gamma(\mathbf{x}_0, \mathbf{x}_i) = 1 &\Leftrightarrow \mathbf{x}_0 = \mathbf{x}_i \\ \gamma(\mathbf{x}_0, \mathbf{x}_i) = 0 &\Leftrightarrow \mathbf{x}_0, \mathbf{x}_i \in \phi \end{aligned}$$

2. Dual symmetry:

$$\begin{aligned} \mathbf{x}, \mathbf{y} &\in \mathbf{X} \\ r(\mathbf{x}, \mathbf{y}) = r(\mathbf{y}, \mathbf{x}) &\Leftrightarrow \mathbf{X} = \{\mathbf{x}, \mathbf{y}\} \end{aligned}$$

3. Wholeness:

$$\begin{aligned} \mathbf{x}_i, \mathbf{x}_j &\in \mathbf{X} \\ \gamma(\mathbf{x}_i, \mathbf{x}_j) &= \overset{\text{often}}{\gamma(\mathbf{x}_j, \mathbf{x}_i)} \end{aligned}$$

4. Approachability:

As $|x_0(k) - x_i(k)|$ increases, $\gamma(x_0(k), x_i(k))$ decreases.

The essential conditions and quantitative model for the grey relation are produced based on the above four prerequisites. The GRC of the referential sequences and comparative sequences at point k is expressed as follows:

$$\begin{aligned} \gamma(x_0(k), x_i(k)) & \quad (6) \\ &= \frac{\min_{i \in I} \cdot \min_k \cdot |x_0(k) - x_i(k)| + \zeta \max_{i \in I} \cdot \max_k \cdot |x_0(k) - x_i(k)|}{|x_0(k) - x_i(k)| + \zeta \max_{i \in I} \cdot \max_k \cdot |x_0(k) - x_i(k)|} \end{aligned}$$

where ζ is a determined coefficient with a value between zero and one. The ζ can be adjusted to suit practical requirements and it is normally set at 0.5.

The grey relational grade (GRG) stands for the degree of grey relation between the referential sequences and comparative

sequences and is defined as a GRC mean and can be expressed as follows:

$$\gamma(\mathbf{x}_0, \mathbf{x}_i) = \frac{1}{n} \sum_{k=1}^n \gamma(x_0(k), x_i(k)) \quad (7)$$

A larger GRG corresponds to a stronger degree of grey relation between the comparative and referential sequences.

3. Proposed Procedure

The ordered categorical response with signal factors forms a dynamic, ordered categorical response system. This study presents an optimisation process for an ordered categorical response in a dynamic system that includes the following five steps.

Step 1. Calculate the accumulated probability of each category for each signal factor level.

The number of observations of each category for each signal factor level is transformed into a corresponding accumulated probability, since the direction of optimisation of the observations for each category is uncertain. The response indicated by a higher accumulated probability represents a better characteristic. The accumulated probability of each category is calculated as follows:

$$A_{ij}(k) = \sum_{z=1}^k O_{ij}(z) \quad (8)$$

$$P_{ij}(k) = \frac{A_{ij}(k)}{A_{ij}(n)} \quad (9)$$

where $A_{ij}(k)$ denotes the number of accumulated observations of the k th category, for the j th signal factor level, under the i th experimental run, for $k = 1, \dots, n$, $j = 1, \dots, a$. $O_{ij}(z)$ is the number of observations of the z th category, for the j th signal factor level, for the i th experimental run. $P_{ij}(k)$ denotes the accumulated probability of the k th category, for the j th signal factor level, for the i th experimental run. $A_{ij}(n)$ is the number of accumulated observations of the n th category (that is, the final category), for the i th experimental run.

Step 2. Conduct grey relational generation according to the accumulated probability of each category for each signal factor level.

Conducting grey relational generation can yield a comparable series of data. Suppose the ordered categorical response is classified into n ordered categories. The grey relational analysis is based on the accumulated probability of the first $n - 1$ categories because the accumulated probability of the final (n th) category is unity. A higher accumulated probability corresponds to a better characteristic, and so the grey relational generation is as follows:

$$P_{ij}^*(k) = \frac{P_{ij}(k) - P_j^{\min}(k)}{P_j^{\max}(k) - P_j^{\min}(k)} \quad (10)$$

where P_{ij}^* denotes the grey relational generation of the k th category, for the j th signal factor level, for the i th experimental

run. $P_j^{\max}(k)$ and $P_j^{\min}(k)$ are the maximum and minimum accumulated probabilities of the k th category, for the j th signal factor level, respectively.

Step 3. Calculate the grey relational coefficient for each experimental run for each signal factor level.

The grey relational coefficient, $\gamma_j(x_0^*(P), x_i^*(P))$, can be determined after substituting the value for $P_{ij}^*(k)$ obtained in step 2 into Eq. (11), as follows:

$$\gamma_j(P_0^*(k), P_i^*(k)) = \frac{\min_i \cdot \min_k |P_{0j}^*(k) - P_{ij}^*(k)| + \zeta \max_i \cdot \max_k |P_{0j}^*(k) - P_{ij}^*(k)|}{|P_{0j}^*(k) - P_{ij}^*(k)| + \zeta \max_i \cdot \max_k |P_{0j}^*(k) - P_{ij}^*(k)|} \quad (11)$$

where $\gamma_j(P_0^*(k), P_i^*(k))$ is the grey relational coefficient of the j th signal factor level, under the i th experimental run, and $P_{0j}^*(k)$ represents the value of the referential series of the k th category, for the j th signal factor level, and is the maximum P_{ij}^* value of one.

Step 4. Calculate the grey relational grade for each signal factor level.

According to Eq. (7), the grey relational grade is calculated as follows:

$$r_j(P_0^*, P_i^*) = \frac{1}{n-1} \sum_{k=1}^{n-1} r_j(P_0^*(k), P_i^*(k)) \quad (12)$$

where $\gamma_j(P_0^*, P_i^*)$ represents the grey relational grade of the i th experimental run, for the j th signal factor level.

Step 5. Conduct grey relational analysis again, based on the $\gamma_j(P_0^*, P_i^*)$ value.

The $\gamma_j(P_0^*, P_i^*)$ value, obtained in step 4, is substituted into Eqs (10)–(12) and the grey relational analysis is performed again. Thereafter, the overall grey relational coefficient and the overall grey relational grade (OGRG) can be determined. The OGRG value lies between zero and one. A higher OGRG value produces a better quality for an ordered categorical response in a dynamic system.

Step 6. Establish the response table and diagram for OGRG to determine the optimal factor level combination.

The OGRG values for each factor level combination can be obtained according to the OGRG values obtained in step 5. The corresponding response table and response diagram can therefore also be established. The maximum OGRG value is accordingly determined as the optimal factor level combination, since a higher OGRG value corresponds to a better quality.

Step 7. Conduct the confirmation experiments.

The confirmation experiments are conducted under an optimal factor level combination to verify whether the quality performance is enhanced. The accumulated probabilities of each category, for each signal factor level obtained from the experiment are compared with the corresponding accumulated probabilities for the current factor level. Suitable control factors, or signal factors, must be selected once more to return

to step 1, if the actual accumulated probability improves only slightly compared with the current accumulated probability.

The procedure proposed above can be presented as a flow-chart, as in Fig. 1.

4. Examples

A lead frame process for semiconductor packaging is used to demonstrate the effectiveness of the proposed procedure. This type of lead frame is a quad flat package. The uniformity of the plating process is a crucial factor in obtaining a superior electric conductivity for the product. The lead frame process

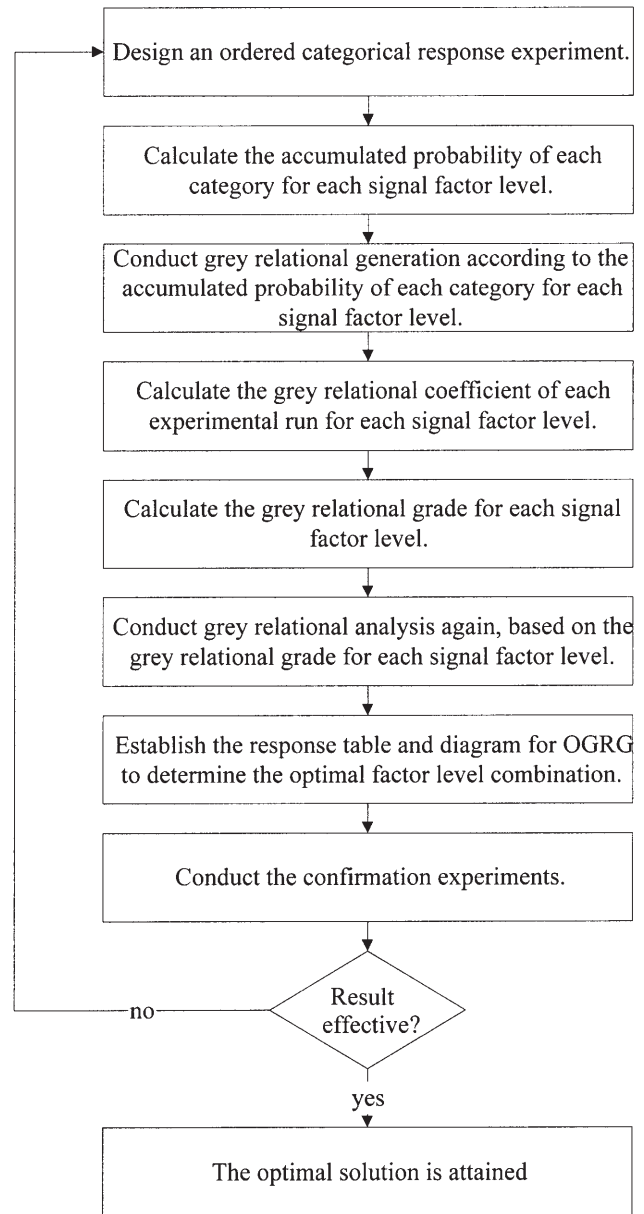


Fig. 1. Optimisation procedure for ordered categorical response in a dynamic system.

includes four primary processes: stamping, plating, downset and taping. Throughout a brainstorming analysis and pretest experiments, six control factors were chosen: electricity (A), reaction temperature (B), turning speed of the plating liquor (C), catalyst quantity (D), location of plating tank (E) and reaction time (F). Factor A is set to one of two levels, and the other factors are each set to one of three levels. The catalyst concentration was designed as a signal factor with three levels, 0.7, 1.0 and 1.3. The uniformity of plating must be inspected visually using an optical microscope, due to constraints of the measurement tool. Engineers utilise five measurement patterns, shown in Fig. 2, to inspect the external plating. Quality engineers accordingly assess the quality of the plated products. An L_{18} orthogonal array is employed in this experiment. Six control factors are allocated from the first to the sixth columns. The experimental data is therefore collected for each signal factor level.

The experimental data is analysed as follows, according to the proposed procedure.

The accumulated probability, $P_{ij}(k)$, for $k = 1, \dots, 5$, $i = 1, \dots, 18$, $j = 1, \dots, 3$ can be obtained by substituting the number of observations of each category, for each signal factor level, into Eqs (8) and (9). The value of the grey relational generation, $P_{ij}^*(k)$, can be obtained by substituting the $P_{ij}(k)$ value into Eq. (10). The grey relational grade of the j th signal factor level, under the i th experimental run, $r_j(\mathbf{P}_0^*, \mathbf{P}_i^*)$, can be determined using Eqs (11) and (12). Table 1 displays the $r_j(\mathbf{P}_0^*, \mathbf{P}_i^*)$ values for all the experimental runs.

The overall grey relational coefficient and OGRG value can be obtained by grey relational analysis, again by substituting $r_j(\mathbf{P}_0^*, \mathbf{P}_i^*)$ into Eqs (10)–(12). Table 2 presents the overall grey relational coefficient and the corresponding OGRG values for all the experimental runs, while Table 3 shows the main effects on OGRG. Figure 3 plots the corresponding factor effects. The factors that control the quality performance of the ordered categorical response in a dynamic system are, in order of significance: B, D, E, C, A and F. The optimal factor level combination is $A_1B_3C_1D_3E_1F_1$, because higher OGRG values yield better quality.

The confirmation experiment is performed with the optimal factor level combination, to confirm if the quality can be enhanced. The confirmation involved ten replications. The

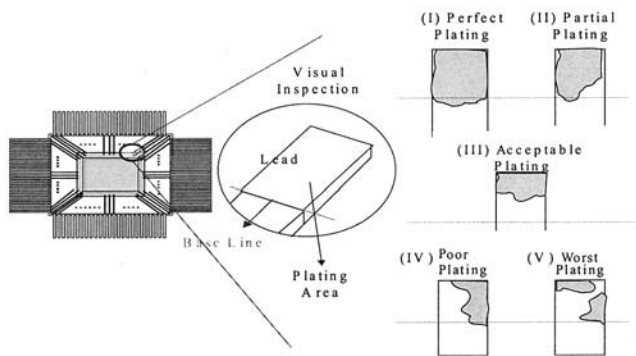


Fig. 2. Uniformity of plating in ordered categorical forms.

Table 1. Grey relational grade for each signal factor level.

Exp. no.	Control factor						$r_j(\mathbf{P}_0^*, \mathbf{P}_i^*)$		
	A	B	C	D	E	F	Level 1 ($j = 1$)	Level 2 ($j = 2$)	Level 3 ($j = 3$)
1	1	1	1	1	1	1	0.411	0.351	0.688
2	1	1	2	2	2	2	0.484	0.516	0.679
3	1	1	3	3	3	3	0.534	0.491	0.561
4	1	2	1	1	2	2	0.757	0.689	0.658
5	1	2	2	2	3	3	0.523	0.397	0.643
6	1	2	3	3	1	1	0.933	0.791	0.623
7	1	3	1	2	1	3	0.684	0.834	0.767
8	1	3	2	3	2	1	0.720	0.947	0.718
9	1	3	3	1	3	2	0.393	0.402	0.573
10	2	1	1	3	3	2	0.485	0.456	0.674
11	2	1	2	1	1	3	0.479	0.482	0.375
12	2	1	3	2	2	1	0.503	0.428	0.577
13	2	2	1	2	3	1	0.609	0.660	0.638
14	2	2	2	3	1	2	0.427	0.631	0.768
15	2	2	3	1	2	3	0.444	0.369	0.405
16	2	3	1	3	2	3	0.582	0.587	0.835
17	2	3	2	1	3	1	0.583	0.696	0.656
18	2	3	3	2	1	2	0.519	0.709	0.668

Table 2. Overall grey relational coefficient and OGRG value.

Exp. no.	Grey relational coefficient			OGRG
	Level 1 ($j = 1$)	Level 2 ($j = 2$)	Level 3 ($j = 3$)	
1	0.341	0.333	0.609	0.428
2	0.375	0.409	0.595	0.460
3	0.404	0.395	0.457	0.418
4	0.604	0.535	0.565	0.568
5	0.397	0.351	0.545	0.431
6	1.000	0.656	0.520	0.725
7	0.520	0.724	0.773	0.672
8	0.559	1.000	0.663	0.741
9	0.333	0.353	0.468	0.385
10	0.376	0.378	0.588	0.447
11	0.373	0.390	0.333	0.365
12	0.386	0.365	0.472	0.407
13	0.454	0.509	0.539	0.501
14	0.348	0.485	0.776	0.536
15	0.356	0.340	0.348	0.348
16	0.434	0.453	1.000	0.629
17	0.436	0.542	0.563	0.514
18	0.395	0.556	0.579	0.510

Table 3. Main effects on OGRG value.

Control factor	Level 1	Level 2	Level 3	Range
A	0.537	0.473	—	0.063
B	0.421	0.518	0.575	0.154
C	0.541	0.508	0.466	0.075
D	0.435	0.497	0.583	0.148
E	0.540	0.526	0.449	0.090
F	0.540	0.484	0.477	0.062

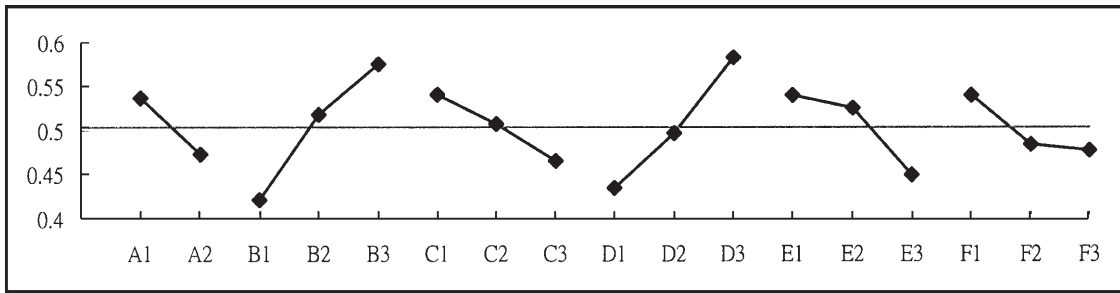


Fig. 3. Factors that affect OGRG.

Table 4. Comparison of the current and optimal factor levels.

Signal factor level	Level 1 (u = 0.7)					Level 2 (u = 1.0)					Level 3 (u = 1.3)				
	I	II	III	IV	V	I	II	III	IV	V	I	II	III	IV	V
Current factor level	0.15	0.37	0.61	0.93	1	0.15	0.40	0.70	0.85	1	0.16	0.54	0.76	0.88	1
Optimal factor level	0.40	0.73	0.92	0.95	1	0.47	0.79	0.92	0.99	1	0.59	0.79	0.93	0.98	1

average of the accumulated probabilities of each category for the signal factor level were determined according to the confirmation data, and compared to the current factor level combination, $A_2B_3C_2D_3E_1F_1$. Table 4 summarises the comparisons. Figure 4 plots the same comparisons. The accumulated probability of the first two categories for each signal factor level for the optimal factor level combination is 30% more than that of the current factor level combination. This result verifies that the optimal factor level combination can improve product/process quality for ordered categorical response in a dynamic system efficiently.

5. Conclusion

Product design is increasingly complex in modern manufacturing processes, due to the varied requirements of customers. Consequently, quality characteristics that cannot be measured directly must be improved in contemporary manufacturing processes. This study presents a novel approach based on grey relational analysis to enhance the quality for the ordered categorical response in a dynamic system. Grey relational analysis involves rather simple calculations. Engineers can easily adopt the proposed procedure. A case study for improving the uniformity of plating in the

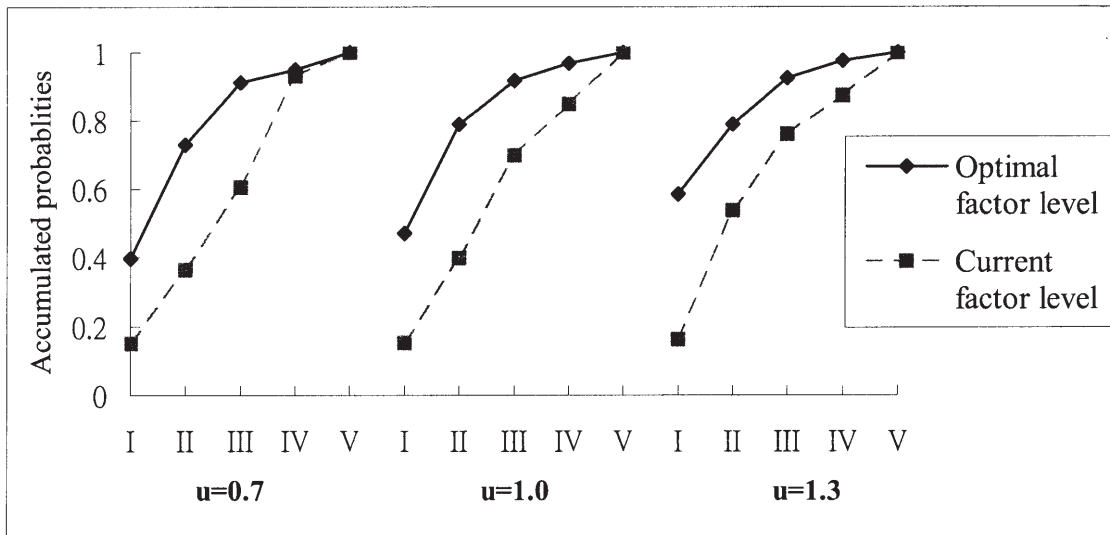


Fig. 4. Accumulated probability at each signal factor level.

lead frame process for semiconductor packaging demonstrates the effectiveness of the proposed procedure.

The proposed procedure has the following merits:

1. It can easily obtain an optimal factor level combination according to Taguchi's parameter design, since grey relational analysis can effectively resolve problems that involve incomplete information.
2. It does not involve complicated mathematical theory or computation and thus can be easily employed by engineers without a strong statistical background.
3. With some modification, the proposed procedure can also be utilised to optimise the ordered categorical response in a state system.

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