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Femtosecond soliton propagating in slow group-velocity fiber

Boren Luo[1], Sien Chi[1], Chi-Feng Chen[2]

[1] Institute of Electro-Optical Engineering, National Chiao Tung University, 1001 Ta Hsueh Road, Hsin-Chu 300, Taiwan R. O. C.
[2] Industrial Technology Research Institute, Hsin-Chu 300, Taiwan, R. O. C.

Abstract: The slow group-velocity pulse in fiber described by nonlinear Schrödinger equation were demonstrated and investigated extensively. We derive a more generalized nonlinear Schrödinger equation as the superposition of monochromatic waves and numerically study the propagations of 2.5-fs fundamental and 5-fs second-order solitons. It is found that, for a slow-group velocity fiber, the magnitude of time shift is related with the group velocity and the more generalized NLSE is more suitable than the conventional generalized NLSE. When the pulse is slow down to 50% of normal group velocity (c/n_0), the effect of the higher nonlinear terms is significant.

Keywords: Self-steepening effect – slow group-velocity – propagation equation – nonlinear Schrödinger equation – nonlinear optics – soliton

1. Introduction

The versatile nonlinear phenomena described by the nonlinear Schrödinger equation (NLSE) were widely investigated. The optical solitons maintain its shape unchanged after propagating long distance under the balance of group velocity dispersion (GVD) and self-phase modulation (SPM) [1, 2]. For higher order nonlinear terms, the pulse edge arises because resulting from the term associated with an intensity-dependent group velocity. It could be realized as self-steeping phenomena [3–6], which processes a temporal pulse distortion and asymmetry in pulse spectrum. It can develop optical shock, understood as an extremely sharp in tailing edge.

Novel phenomenons of pulses slowing down are extensively investigated in various topics of physics, such as resonant active medium, Bose-Einstein condensation medium and periodic structure medium etc.. They would be roughly cataloged as the nonlinear processes of interaction between the pulse and medium. Therefore, we investigate the slow light based on the NLSE and study the propagation characteristic of nonlinear pulse in slow-group medium.

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Correspondence to: B. Luo
Fax: ++886-3-5716631
E-mail: boren.eo86g@nctu.edu.tw

An accurate wave equation beyond the slowly varying envelope approximation for femtosecond soliton propagation in an optical fiber [7] was suggested. The derived equation contains higher nonlinear terms than the generalized nonlinear Schrödinger equation obtained previously. The optical shock terms are corrected and associated with group velocity. For a slow-group velocity fiber, the validity of the conventional generalized NLSE becomes questionable.

In this paper, we investigate the propagation of ultrashort pulses along the slow-group velocity fiber. We derive a more generalized nonlinear Schrödinger equation as the superposition of monochromatic waves [8] and numerically study the propagations of 2.5-fs fundamental and 5-fs second-order solitons. For a slow-group velocity fiber, the phenomenon of the propagation pulse is different for different group velocity and the more generalized NLSE is more suitable than the conventional generalized NLSE. When the pulse is slow down to 50% of normal group velocity (c/n_0), the effect of the higher nonlinear terms is significant.

2. Derivation of propagation equation

We derive the propagation of the ultrashort light propagation with third-order nonlinearity based on scalar electric filed $E(t, z)$ in one dimension:

$$\frac{\partial^2 E(t,z)}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 D(t,z)}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 P^{(3)}(t,z)}{\partial t^2}\,, \qquad (1)$$

where c is the light velocity in vacuum. The linear electric induction $D(t, z)$ and the nonlinear third-order polarization $P^{(3)}(t, z)$ are

$$D(t,z) = \int \varepsilon(t', z)\, E(t - t', z)\, \mathrm{d}t'\,, \qquad (2)$$

$$P^{(3)}(t,z) = E(t,z) \int R(t') |E(t - t', z)|^2\, \mathrm{d}t'\,, \qquad (3)$$

where $\varepsilon(t, z)$ and $R(t)$ are linear and nonlinear response function of the medium, respectively, and $R(t)$ consists of the instantaneous electronic and delayed Raman responses. The electronic field $E(t, z)$ can be separated as a superposition of monochromatic waves,

$$E(t,z) = \int E(\omega, z)\, \exp\,(\mathrm{i}\beta(\omega)\, z - \mathrm{i}\omega t)\, \mathrm{d}\omega\,. \qquad (4)$$

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Substituting eqs. (2)–(4) into eq. (1), one can obtain

$$\left[\frac{\partial^2}{\partial z^2} + i2\beta(\omega)\,\frac{\partial}{\partial z}\right] E(\omega, z) = \frac{4\pi\omega^2}{c^2} \iint E(\omega', z)\, E(\omega'', z)\, E(\omega' + \omega'' - \omega, z) \times \chi^{(3)}(\omega - \omega') \exp(i\Delta\beta z)\, d\omega'\, d\omega'' , \tag{5}$$

here $\Delta\beta = \beta(\omega') - \beta(\omega'') - \beta(\omega' + \omega'' - \omega)$, and $\chi^{(3)}(\omega) = \int R(t) \exp(i\omega t)\, dt$ is the third-order susceptibility. $\chi^{(3)}(\omega)$ is a complex value. The real part of which is responsible for parametric and self-phase modulation effects, and the imaginary part of which is responsible for the Raman effect. Considering the propagation of ultrashort pulse, we retain the term of $\partial^2 E(t, z)/\partial z^2$. By substituting $\tilde{E}(\Delta\omega, z) = E(\omega, z) \exp(i[\beta(\omega) - \beta_0]\, z)$ into eq. (5), one can obtain

$$\frac{\partial \tilde{E}(\Delta\omega, z)}{\partial z} - i[\beta(\omega) - \beta_0]\, \tilde{E}(\Delta\omega, z) = \frac{i\pi\omega^2}{2c^2\beta(\omega)} \int d\omega' \int d\omega''\, \tilde{E}(\Delta\omega', z)\, \tilde{E}(\Delta\omega'', z) \times \tilde{E}(\Delta\omega' + \Delta\omega'' - \Delta\omega)\, \chi^{(3)}(\Delta\omega - \Delta\omega') + \frac{1}{2\beta(\omega)} \frac{\partial^2 \tilde{E}(\Delta\omega, z)}{\partial z^2} . \tag{6}$$

Expanding $\beta(\omega)$ about ω_0 and correcting to the mth order,

$$\beta(\omega) = \beta_0 + \beta_1\, \Delta\omega + \beta_2\, \Delta\omega^2/2 + \beta_3\, \Delta\omega^3/6 + \ldots + \beta_m \Delta\omega^m/m! , \tag{7}$$

$$\Delta\omega = \omega - \omega_0 , \qquad \beta_m = \left.\frac{\partial^m \beta(\omega)}{\partial\omega^m}\right|_{\omega=\omega_0} . \tag{8}$$

Expanding and simplifying eq. (6) about $\Delta\omega$,

$$\frac{\partial \tilde{E}(\Delta\omega, z)}{\partial z} - i[\beta_1\, \Delta\omega + \beta_2\, \Delta\omega^2/2 + \beta_3\, \Delta\omega^3/6]\, \tilde{E}(\Delta\omega, z) = \frac{i\kappa\omega_0^2}{2c^2\beta_0} \left(1 + \left(\frac{2}{\omega_0} - \frac{\beta_1}{\beta_0}\right) \Delta\omega - \left(\frac{1}{\omega_0^2} - \frac{2\beta_1}{\omega_0\beta_0} + \frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{2\beta_0}\right) \Delta\omega^2\right) \times \int d\omega' \int d\omega''\, \tilde{E}(\Delta\omega', z) \cdot \tilde{E}(\Delta\omega'', z) \times \tilde{E}(\Delta\omega' + \Delta\omega'' - \Delta\omega)\, \chi^{(3)}(\Delta\omega - \Delta\omega') + \frac{1}{2\beta(\omega)} \frac{\partial^2 \tilde{E}(\Delta\omega, z)}{\partial z^2} . \tag{9}$$

We use the iteration method to derive the wave equation beyond the slowly varying envelope approximation (SEVA) [9]. The iteration method only modified the nonlinear terms, which higher than the last terms in eq. (9). It is verified that this terms could be negligible for 2.5-fs fundamental soliton by using finite difference time domain method [7]. Normalizing eq. (9), we obtain

$$\frac{\partial}{\partial\zeta}\, \psi(\tau, \zeta) - \frac{i}{2} \frac{\partial^2}{\partial\tau^2}\, \psi(\tau, \zeta) - \frac{\beta_3}{6|\beta_2|\, T_0} \frac{\partial^3}{\partial\tau^3}\, \psi(\tau, \zeta) = i\psi(\tau, \zeta) \int |\psi(\tau - \tau')|^2\, r(\tau')\, d\tau' - \alpha_1 \frac{\partial}{\partial\tau} \left[\psi(\tau, \zeta) \int |\psi(\tau - \tau')|^2\, r(\tau')\, d\tau'\right] - i\alpha_2 \frac{\partial^2}{\partial\tau^2} \left[\psi(\tau, \zeta) \int |\psi(\tau - \tau')|^2\, r(\tau')\, d\tau'\right] , \tag{10}$$

where

$\zeta = \dfrac{z}{L_D}$, $\tau = \dfrac{t - \beta_1 z}{T_0}$, $L_D = \dfrac{T_0^2}{|\beta_2|}$, T_0 is pulse duration.

$u = \tilde{E}(\Delta\omega, z)$, $r(\tau) = \dfrac{R(\tau)}{\int R(\tau')\, d\tau'}$,

$\alpha_1 = \left(\dfrac{2}{\omega_0} - \dfrac{\beta_1}{\beta_0}\right) \dfrac{1}{T_0}$, $\alpha_2 = \left(\dfrac{1}{\omega_0^2} - \dfrac{2\beta_1}{\omega_0\beta_0} + \dfrac{\beta_1^2}{\beta_0^2} - \dfrac{\beta_2}{2\beta_0}\right) \dfrac{1}{T_0^2}$,

$\psi(\tau, \zeta) = \dfrac{\tilde{E}(\tau, \zeta)}{E_0}$, $E_0 = \dfrac{2c|\beta_2|}{\omega_0 n_2 T_0^2}$, n_2 is the nonlinear refractive index.

The relation of α_1 and α_2 to β_1 is shown in fig. 1.

We obtain the more generalized nonlinear Schrödinger equation with more nonlinear terms. The coefficient α_1 and α_2 is corrected and different with the NLSE, we conventional used. The factor of the steepening effect α_1 also describes the Stokes losses associated with the material excitation during the Raman self-scattering process. In the conventional condition, carry frequency of the propagation waves far away resonant frequency of the dipole. It makes the constraint

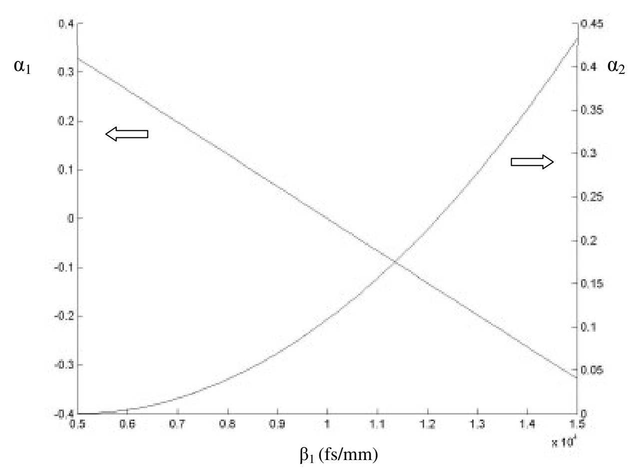

Fig. 1. The relation of α_1 and α_2 to β_1 is shown.

$\frac{\beta_1}{\beta_0} \sim \frac{1}{\omega_0}$ and obtains the α_1 approaching to $\frac{1}{\omega_0 T_0}$, just the same as the conventional nonlinear Schrödinger equation.

$$\frac{\partial}{\partial \zeta} \psi(\tau, \zeta) - \frac{\mathrm{i}}{2} \frac{\partial^2}{\partial \tau^2} \psi(\tau, \zeta) - \frac{\beta_3}{6|\beta_2| T_0} \frac{\partial^3}{\partial \tau^3} \psi(\tau, \zeta) - \frac{\mathrm{i}\beta_4}{24|\beta_2| T_0^2} \frac{\partial^4}{\partial \tau^4} \psi(\tau, \zeta) = \mathrm{i}\psi(\tau, \zeta) \, |\psi(\tau, \zeta)|^2 - \frac{1}{\omega_0 T_0} \frac{\partial}{\partial \tau} \left(\psi(\tau, \zeta) \, |\psi(\tau - \tau')|^2\right). \tag{11}$$

3. Numerical result and discussion

The fiber parameters used to numerically solve eqs. (10) and (11) are: soliton wavelength $\lambda = 1.55$ μm, $\beta_2 = -20$ fs^2/mm and $n_2 = 3.2 \times 10^{-20}$ m^2/W. Eqs. (10) and (11) are solved by using split-step Fourier method with the initial condition $u(\zeta = 0, \tau) = \sec h(\tau)$.

To show the validity of the propagating phenomenon described by the more generalized NLSE for the slow-group velocity fiber, we simulate the propagations of 2.5-fs fundamental and 5-fs second-order solitons without the third-order dispersion and without Raman effect. The power evolution of pulse shapes for the different group velocity $v_g = 1/\beta_1$ over $10Ld$ along the slow-group velocity fiber simulated by using the more generalized NLSE (eq. (10)) and conventional generalized NLSE (eq. (11)) are shown in fig. 2. The thick solid line, thin dotted line, and thin dashed line

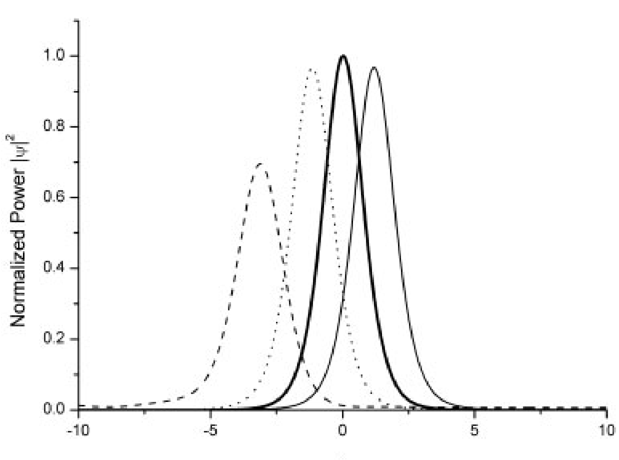

Fig. 2. Pulse shapes of 2.5-fs fundamental soliton propagate over $10L_d$. The thick solid line, thin dotted line, and thin dashed line are simulated by using the more generalized nonlinear Schrödinger equation for $\beta_1 = 2\frac{n_0}{c}$, $3\frac{n_0}{c}$, and $4\frac{n_0}{c}$, respectively. The thin solid line is simulated by conventional nonlinear Schrödinger equation for $\beta_1 = 2\frac{n_0}{c}$.

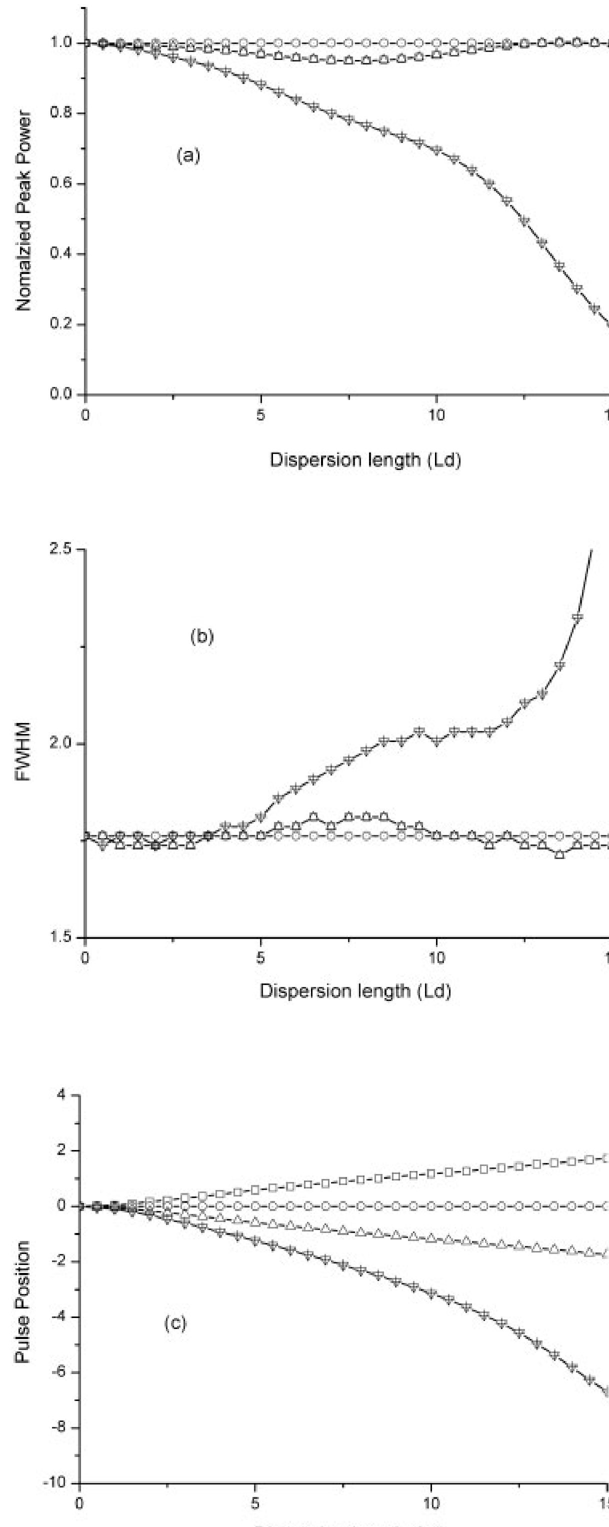

Fig. 3. a) The peak power, b) FWHM, and c) time shift evolution of 2.5-fs fundamental soliton versus the propagation distance. Square, circle, up-triangle, and down-triangle present the cases simulated by using the more generalized NLSE for $\beta_1 = \frac{n_0}{c}$ (normal velocity), $2\frac{n_0}{c}$, $3\frac{n_0}{c}$, and $4\frac{n_0}{c}$, respectively.

are simulated by using the more generalized NLSE for $\beta_1 = 2\,\frac{n_0}{c}$, $3\,\frac{n_0}{c}$ and $4\,\frac{n_0}{c}$, respectively. The thin solid line is simulated by conventional generalized NLSE. It is seen that, for the thin solid line in fig. 2, the pulses have shift for the tailing side. Obviously, it makes no difference that the propagation of ultrashort pulses simulated by using conventional generalized NLSE in any group velocity. The propagating phenomenon simulated by using the more generalized NLSE is difference for the different group velocity. Comparing these results, it is found that the validity of the conventional generalized NLSE becomes questionable when the propagations of ultrashort pulses along the slow-group velocity fiber. The more generalized NLSE is more suitable to describe these cases. Figs. 3 shows a) the peak power, b) FWHM, and c) time shift evolution of 2.5-fs fundamental soliton versus the propagation distance. Square, circle, up-triangle, and down- triangle present

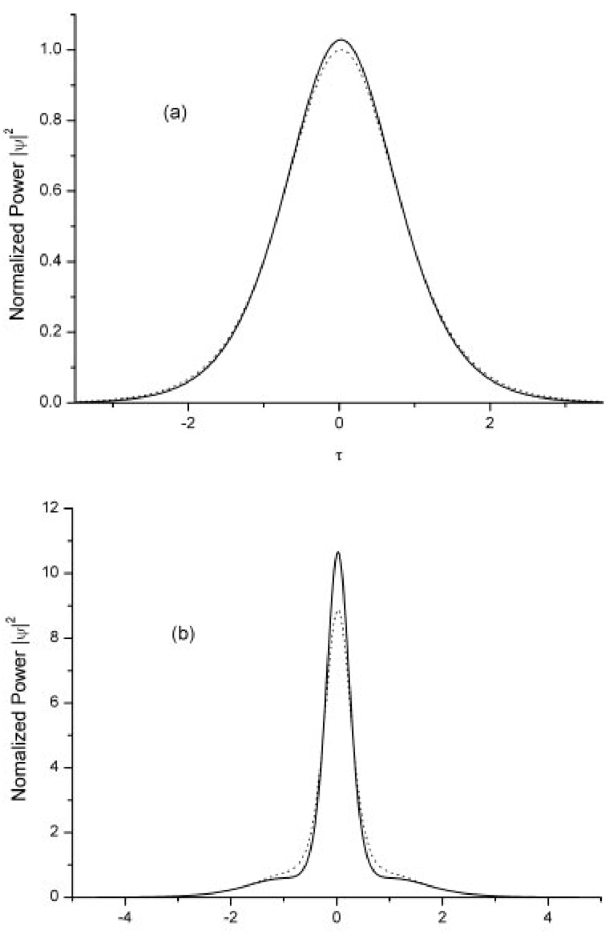

Fig. 4. Pulse shapes of a) 2.5-fs fundamental soliton and b) 5-fs second order soliton simulated by using the more generalized NLSE propagates over $10L_d$ with (solid line) and without (dot line) α_2 for $\beta_1 = 2\,\frac{n_0}{c}$.

the cases simulated by using the more generalized NLSE for $\beta_1 = \frac{n_0}{c}$ (normal velocity), $2\,\frac{n_0}{c}$, $3\,\frac{n_0}{c}$, and $4\,\frac{n_0}{c}$, respectively. Comparing the cases of $\beta_1 = \frac{n_0}{c}$ and $3\,\frac{n_0}{c}$, the changes of the peak power, FWHM, and time shift are same except for the direction of time shift. The $\beta_1 = \frac{n_0}{c}$ case is shift for tailing side and the $\beta_1 = 3\,\frac{n_0}{c}$ case is shift for leading side. In $\beta_1 = 2\,\frac{n_0}{c}$ case, the shock terms are canceled. The phenomenon of time shift is not produced and the changes of the peak power and FWHM are very small. One can see that, for $\beta_1 = 4\,\frac{n_0}{c}$ case, the pulse broaden larger than the other cases and the pulse shift rapidly for the leading side. The time shift is related with the group velocity. For the group velocity of $0.5\,\frac{c}{n_0}$, the shock terms are just canceled. To understand the effect of the higher nonlinear terms, we show pulse shapes of a) 2.5-fs fundamental soliton and b) 5-fs second order soliton simulated by using the more generalized NLSE propagates over $10L_d$ with (solid line) and without (dot line) α_2 in figs. 4. One can see that the effect of the higher nonlinear terms symmetrically compresses pulse shape. For the 5-fs second soliton case, the propagation pulse is much narrower than the initial one, i.e., the effect of the higher nonlinear terms must be consider.

4. Conclusion

The propagation of ultrashort pulses along the slow-group velocity fiber is investigated. We derive a more generalized nonlinear Schrödinger equation as the superposition of monochromatic waves and numerically study the propagations of 2.5-fs fundamental and 5-fs second-order solitons. It is found that, for a slow-group velocity fiber, the magnitude of time shift is related with the group velocity and the more generalized NLSE is more suitable than the conventional generalized NLSE. When the pulse is slow down to 50% of normal group velocity (c/n_0), the effect of the higher nonlinear terms is significance.

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