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Sliding-Mode Controller Design for Spacecraft Attitude Tracking Maneuvers

This paper presents a robust sliding-mode control law to deal with the spacecraft attitude tracking problems. Two important natural properties related to the spacecraft model of motion are shown and then by using these properties and the second method of Lyapunov theory, the system stability in the sliding mode can be easily achieved. The success of the sliding-mode controller and the robustness to uncertainties are illustrated by an example of multi-axial attitude tracking maneuvers.

Manuscript received November 18, 1992; revised January 13, 1993.

IEEE Log No. T-AES/29/4/10997.

Research was supported by National Science Council, Taiwan, R.O.C., under Contract NSC 80-E-sp-009-02D.

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I. INTRODUCTION

Recently, some investigators [2, 3, 8] have applied the sliding-mode control (also called the variable-structure control) to the spacecraft attitude maneuvers, for example the detumbling and reorientation. A new spacecraft sliding-mode controller is introduced here to deal with the attitude tracking problems.

The sliding-mode control [7] is known as a nonlinear technique which possesses two important features. First, it can reject the external disturbances and second, it is robust to the system parameter variations. Many linear or nonlinear systems, such as the dc motor and robot [5, 1], have applied the sliding-mode theory to the controller design and obtained good results.

In general, the spacecraft motion is governed by the so-called kinematic equations and dynamic equations [3]. Actually, these mathematical descriptions are highly nonlinear and thus, the conventional linear control techniques are not suitable for the controller design, especially when large-angle spacecraft maneuvers are required. Besides, since the spacecraft system is always ideally modeled, these mathematical descriptions cannot completely describe the spacecraft motion. Hence, the robust sliding-mode control has been considered as a useful scheme for the spacecraft attitude maneuvers. By Dywer and Sira-Ramirez [3], the sliding-mode control has been successfully developed for the spacecraft attitude reorientation and detumbling maneuvers. However, with the sliding vector they introduced, complicated algorithms were resulted in their sliding-mode controller. Here, we present a new sliding vector to attain simpler control laws which, significantly, can be applied to not only the attitude reorientation but also the tracking maneuvers.

In the controller design, we choose the sliding vector based on two important natural properties related to the system model. The first one is that the inertia matrix J is symmetric and positive definite. Second, the matrix $T(\rho)$ in the kinematic equation satisfies $T(\rho) \geq \frac{1}{2}I$ where ρ is the Gibbs vector of Rodrigues attitude parameter and I is the identity matrix. Based on these two natural properties, the control law can be easily derived to guarantee the reaching and sliding conditions and once the system is controlled to be restricted in the sliding mode the attitude tracking can be achieved. Besides, the direct method of Lyapunov stability theory is adopted to both the development of sliding-mode control law and the analysis of system stability in the sliding mode. As for the chattering caused by the practical implementation, it can be alleviated by the use of sliding layers [6].

This paper is organized as follows. In the next section, the spacecraft model with two natural properties is presented. In Section III, with these properties, a robust sliding-mode control law is

designed for multiaxial attitude tracking maneuvers and the stability analysis of the system in the sliding mode is shown. A numerical example of multiaxial attitude tracking maneuvers is illustrated in Section IV to verify the usefulness of the control laws. Finally, Section V gives the concluding remarks.

II. SPACECRAFT MODEL DESCRIPTION

The mathematical model of a spacecraft, treated as a rigid body, is commonly composed of two sets of equations called the kinematic equations and dynamic equations which can be described, respectively, by [3, 4]

$$\dot{\rho} = T(\rho)\omega \quad (1)$$

$$J\dot{\omega} = H\omega + \tau \quad (2)$$

where $\rho \in \mathcal{R}^3$ is the Gibbs vector of Rodrigues attitude parameters, $\omega \in \mathcal{R}^3$ represents the angular velocity, and $\tau \in \mathcal{R}^3$ is the control input applied to the system. As for the matrices J , $T(\rho)$, and H , they can be found as

$$J = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{12} & J_{22} & J_{23} \\ J_{13} & J_{23} & J_{33} \end{bmatrix}$$

$$T(\rho) = \frac{1}{2}[I + \rho\rho^T + \rho \times]$$

$$= \frac{1}{2} \begin{bmatrix} 1 + \rho_1^2 & \rho_1\rho_2 - \rho_3 & \rho_1\rho_3 + \rho_2 \\ \rho_1\rho_2 + \rho_3 & 1 + \rho_2^2 & \rho_2\rho_3 - \rho_1 \\ \rho_1\rho_3 - \rho_2 & \rho_2\rho_3 + \rho_1 & 1 + \rho_3^2 \end{bmatrix}$$

$$H = [h \times] = \begin{bmatrix} 0 & -h_3 & h_2 \\ h_3 & 0 & -h_1 \\ -h_2 & h_1 & 0 \end{bmatrix}$$

where $h \in \mathcal{R}^3$ denotes the angular momentum in the body coordinates. Here, the matrix J is known as the inertia matrix which is inherently symmetric and positive definite by choosing the body-fixed frame appropriately. Besides, by directly calculating

$$x^T [T(\rho) - \frac{1}{2}I]x = \frac{1}{2}(\rho^T x)^2 \geq 0 \quad \forall x \in \mathcal{R}^3$$

we can conclude that $T(\rho) \geq \frac{1}{2}I$. In the next section, we employ these two natural properties, i.e., 1) the inertia matrix J is symmetric and positive definite, and 2) $T(\rho) \geq \frac{1}{2}I$, in the controller design. Both of them are shown useful in the stability analysis by the second method of Lyapunov theory.

III. SLIDING-MODE CONTROLLER DESIGN

The sliding-mode controller introduced here is to deal with the multiaxial attitude tracking maneuvers of the spacecraft modeled by (1) and (2). In the controller design, the required feedback signals ω and ρ are

assumed to be measurable. Besides, to demonstrate the robustness of the controller, we allow the dynamic equations (2) to possess bounded input disturbances d and parameter variations ΔJ and ΔH ; thus, the dynamic equations are rewritten as

$$J\dot{\omega} = H\omega + \tau + d \quad (3)$$

where

$$J = J_0 + \Delta J$$

$$H = H_0 + \Delta H$$

and J_0 and H_0 represent the nominal parts of J and H , respectively. Note that the inertia matrix J is assumed to be time invariant and its variation ΔJ , for example, may result from the change of payloads of the spacecraft.

The objective of the tracking control is to drive the spacecraft such that $\rho(t) = \rho_d(t)$, i.e., the Gibbs vector $\rho(t)$ is controlled to follow the given reference vector $\rho_d(t)$. Note that if the vector $\rho_d(t)$ is set as constant, it refers to the attitude orientation problem. The design procedure of sliding-mode control generally contains two fundamental steps. The first step is to choose the sliding vector such that in the sliding mode the goal of control is achieved. In our case, significantly, in addition to the successful tracking of $\rho(t)$, the choice of sliding vector is also related to the convergent rate of $[\rho(t) - \rho_d(t)]$. The second step is to design the control laws such that the reaching and sliding conditions are satisfied, and thus the system is strictly constrained in the sliding mode. Next, we show the sliding-mode controller design as follows.

Step 1: Choose the sliding vector such that in the sliding mode the goal of control is achieved.

Let us first define the error signal as

$$\varepsilon(t) = \rho(t) - \rho_d(t) \quad (4)$$

and then choose the sliding vector as

$$s(t) = [\omega(t) - \hat{\omega}(t)] + K_p \varepsilon(t) \quad (5)$$

where

$$\hat{\omega}(t) = T^{-1}(\rho(t))\dot{\rho}_d(t)$$

$$T^{-1}(\rho) = \frac{2}{1 + \rho^T \rho} [I - \rho \times]$$

$$= \frac{2}{(1 + \rho^T \rho)} \begin{bmatrix} 1 & \rho_3 & -\rho_2 \\ -\rho_3 & 1 & \rho_1 \\ \rho_2 & -\rho_1 & 1 \end{bmatrix}$$

and K_p is a symmetric and positive definite constant matrix whose eigenvalues are shown later related to the rate of convergence of the error signal $\varepsilon(t)$. From the sliding-mode theory [7], once the reaching and sliding conditions are satisfied, the system is finally forced to

stay in the sliding mode

$$s(t) = [\omega(t) - \hat{\omega}(t)] + K_p \varepsilon(t) = 0. \quad (6)$$

By using the kinematic equations (1) and premultiplying $T(\rho)$ into (6), we have

$$(\dot{\rho} - \dot{\rho}_d) + T(\rho)K_p \varepsilon = 0$$

or

$$\dot{\varepsilon} + T(\rho)K_p \varepsilon = 0. \quad (7)$$

Let the candidate of Lyapunov function be

$$V_\varepsilon = \frac{1}{2} \varepsilon^T K_p \varepsilon. \quad (8)$$

Then, the derivative of V_ε is

$$\begin{aligned} \dot{V}_\varepsilon &= \varepsilon^T K_p \dot{\varepsilon} \\ &= -\varepsilon^T K_p T(\rho) K_p \varepsilon \\ &= -(K_p \varepsilon)^T T(\rho) (K_p \varepsilon). \end{aligned} \quad (9)$$

From property 2, we can further obtain

$$\dot{V}_\varepsilon \leq -\frac{1}{2} (K_p \varepsilon)^T (K_p \varepsilon) \leq 0 \quad (10)$$

and $\dot{V}_\varepsilon = 0$ only if $\varepsilon = 0$. Thus, V_ε is really a Lyapunov function so that the error signal ε will converge to zero. That means the Gibbs vector $\rho(t)$ can track $\rho_d(t)$ successfully. As for the angular velocity vector ω , we can easily derive $[\omega(t) - \hat{\omega}(t)] \rightarrow 0$ from (6). Therefore, by introducing the sliding vector (5) the stability problem of the system in the sliding mode is guaranteed.

Next, let us further discuss about the choice of K_p . Since K_p is symmetric and positive definite, it can be decomposed as

$$K_p = U^T \Sigma U$$

where U is the unitary matrix and $\Sigma = \text{diag}[\lambda_1 \ \lambda_2 \ \lambda_3]$ with λ_i s being the positive eigenvalues of K_p . Then the Lyapunov function becomes

$$\begin{aligned} V_\varepsilon &= \frac{1}{2} \varepsilon^T K_p \varepsilon = \frac{1}{2} \varepsilon^T U^T \Sigma U \varepsilon = \frac{1}{2} \bar{\varepsilon}^T \Sigma \bar{\varepsilon} \\ &= \frac{1}{2} [\lambda_1 \bar{\varepsilon}_1^2 + \lambda_2 \bar{\varepsilon}_2^2 + \lambda_3 \bar{\varepsilon}_3^2] \end{aligned} \quad (11)$$

where $\bar{\varepsilon} = U \varepsilon$ and the derivative of V_ε in (10) can be rewritten as

$$\begin{aligned} \dot{V}_\varepsilon &\leq -\frac{1}{2} \varepsilon^T K_p^2 \varepsilon = -\frac{1}{2} \varepsilon^T U^T \Sigma^2 U \varepsilon = -\frac{1}{2} \bar{\varepsilon}^T \Sigma^2 \bar{\varepsilon} \\ &\leq -\frac{1}{2} [\lambda_1^2 \bar{\varepsilon}_1^2 + \lambda_2^2 \bar{\varepsilon}_2^2 + \lambda_3^2 \bar{\varepsilon}_3^2]. \end{aligned} \quad (12)$$

From (11) and (12), we have

$$\begin{aligned} \dot{V}_\varepsilon &= \frac{\dot{V}_\varepsilon}{V_\varepsilon} V_\varepsilon \\ &\leq -\frac{[\lambda_1^2 \bar{\varepsilon}_1^2 + \lambda_2^2 \bar{\varepsilon}_2^2 + \lambda_3^2 \bar{\varepsilon}_3^2]}{[\lambda_1 \bar{\varepsilon}_1^2 + \lambda_2 \bar{\varepsilon}_2^2 + \lambda_3 \bar{\varepsilon}_3^2]} V_\varepsilon \\ &\leq -\lambda_{\min} V_\varepsilon \end{aligned} \quad (13)$$

where $\lambda_{\min} = \min\{\lambda_1, \lambda_2, \lambda_3\}$. As a result,

$$V_\varepsilon(\varepsilon(t)) \leq V_\varepsilon(\varepsilon(t_h)) e^{-\lambda_{\min}(t-t_h)} \quad (14)$$

where t_h denotes the time reaching the sliding mode.

Since $\|\bar{\varepsilon}\| = \|U\varepsilon\| = \|\varepsilon\|$, from (11), we can obtain

$$\frac{1}{2} \lambda_{\min} \|\varepsilon\|^2 \leq V_\varepsilon \leq \frac{1}{2} \lambda_{\max} \|\varepsilon\|^2 \quad (15)$$

where $\lambda_{\max} = \max\{\lambda_1, \lambda_2, \lambda_3\}$, and then (14) can be further extended to

$$\begin{aligned} \frac{1}{2} \lambda_{\min} \|\varepsilon(t)\|^2 &\leq V_\varepsilon(\varepsilon(t)) \leq V_\varepsilon(\varepsilon(t_h)) e^{-\lambda_{\min}(t-t_h)} \\ &\leq \frac{1}{2} \lambda_{\max} \|\varepsilon(t_h)\|^2 e^{-\lambda_{\min}(t-t_h)} \end{aligned}$$

or

$$\|\varepsilon(t)\| \leq \sqrt{\frac{\lambda_{\max}}{\lambda_{\min}}} \|\varepsilon(t_h)\| e^{-(1/2)\lambda_{\min}(t-t_h)}. \quad (16)$$

This inequality implies the norm $\|\varepsilon\|$ of the tracking errors converge to zero related to an exponential rate of $\frac{1}{2}\lambda_{\min}$. Note that if $\lambda_1 = \lambda_2 = \lambda_3 = \lambda$, i.e., $K_p = \lambda \cdot I$, (16) becomes

$$\|\varepsilon(t)\| \leq \|\varepsilon(t_h)\| e^{-(\lambda/2)(t-t_h)}$$

and from (6),

$$(\omega - \hat{\omega}) = -\lambda(\rho - \rho_d) = -\lambda \varepsilon$$

or equivalently,

$$\|\omega - \hat{\omega}\| = \lambda \|\varepsilon\|.$$

That means $\|\omega - \hat{\omega}\|$ will converge to zero with the same rate $-\lambda/2$ as $\|\varepsilon\|$. Hence, in the next step, for simplicity we let $K_p = \lambda I$ and thus the sliding vector (5) becomes

$$s(t) = [\omega(t) - \hat{\omega}(t)] + \lambda[\rho(t) - \rho_d(t)]. \quad (17)$$

Step 2: Design the control laws such that the reaching and sliding conditions are satisfied.

From the truth that J is symmetric and positive definite, the candidate of Lyapunov function is set as

$$V_s = \frac{1}{2} s^T J s \geq 0 \quad (18)$$

and $V_s = 0$ only when $s = 0$. Taking the first derivative of V_s and adopting (1), (3), and (17), we have

$$\begin{aligned} \dot{V}_s &= s^T J \dot{s} \\ &= s^T [J\dot{\omega} - J\dot{\hat{\omega}} + \lambda J(\dot{\rho} - \dot{\rho}_d)] \\ &= s^T [H\omega + \tau + d - J\dot{\hat{\omega}} + \lambda J T(\rho)\omega - \lambda J \dot{\rho}_d]. \end{aligned} \quad (19)$$

Let the control law be

$$\tau = -H_0\dot{\omega} + J_0\dot{\omega} - \lambda J_0[T(\rho)\omega - \dot{\rho}_d] + \tau' \quad (20)$$

where

$$\begin{aligned} \dot{\omega} &= \frac{d}{dt}(T^{-1}(\rho)\dot{\rho}_d) \\ &= \frac{d}{dt}(T^{-1}(\rho))\dot{\rho}_d + T^{-1}(\rho)\ddot{\rho}_d \\ \frac{d}{dt}(T^{-1}(\rho)) &= -\frac{2}{1+\rho^T\rho}[(T(\rho)\omega) \times] \\ &\quad - \frac{4\rho^T(T(\rho)\omega)}{(1+\rho^T\rho)^2}[I - \rho \times] \\ \tau' &= [\tau'_1 \quad \tau'_2 \quad \tau'_3]^T \\ \tau'_i &= -k_i \cdot \text{sgn}(s_i) \\ \text{sgn}(s_i) &= \begin{cases} 1 & s_i > 0 \\ -1 & s_i < 0 \end{cases} \quad i = 1, 2, 3 \end{aligned} \quad (21)$$

then (19) becomes

$$\begin{aligned} \dot{V}_s &= s^T[\delta + \tau'] \\ &= \sum_{i=1}^3 s_i(\delta_i + \tau_i) \\ &= \sum_{i=1}^3 -k_i |s_i| \left[1 - \frac{\delta_i}{k_i} \text{sgn}(s_i) \right] \end{aligned} \quad (22)$$

where

$$\begin{aligned} \delta &= [\delta_1 \quad \delta_2 \quad \delta_3]^T \\ &= \Delta H\omega + \Delta J\dot{\omega} + \lambda\Delta J[T(\rho)\omega - \dot{\rho}_d] + d. \end{aligned}$$

Since the external disturbances d and uncertain parameters ΔJ and ΔH are all bounded, the upper bound of $|\delta_i|$ can be found and denoted as $\delta_i^{\max}(\omega, \rho, \rho_d, \dot{\rho}_d, \ddot{\rho}_d, t)$. It is evident that if we choose

$$k_i = \delta_i^{\max}(\omega, \rho, \rho_d, \dot{\rho}_d, \ddot{\rho}_d, t), \quad \text{for } i = 1, 2, 3 \quad (23)$$

then (22) becomes

$$\dot{V}_s = -\sum_{i=1}^3 \delta_i^{\max} |s_i| \left[1 - \frac{\delta_i}{\delta_i^{\max}} \text{sgn}(s_i) \right] < 0$$

for $s \neq 0$. This implies V_s is really a Lyapunov function. Therefore, the reaching and sliding of the sliding mode $s = 0$ is guaranteed.

However, due to the existence of nonideality in the practical implementation of the sign function $\text{sgn}(s_i)$, the control law τ in (20) always suffers from the chattering problem. To alleviate such undesirable performance, the sign function can be simply replaced

by the saturation function [5]

$$\text{sat}(s_i, \epsilon) = \begin{cases} 1 & s_i > \epsilon \\ s_i/\epsilon & |s_i| < \epsilon \\ -1 & s_i < -\epsilon \end{cases} \quad (24)$$

The system is now no longer forced to stay in the sliding mode but is constrained within the sliding layer $|s_i| \leq \epsilon$. The cost of such substitution is a reduction in the accuracy of the desired performance.

In the next section, simulation results are given to verify the success of the sliding-mode control laws.

IV. MULTIAXIAL ATTITUDE TRACKING MANEUVERS

The mathematical model of a spacecraft driven by pairs of opposing thrusters is shown as

$$\dot{\rho} = T(\rho)\omega \quad (25)$$

$$J\dot{\omega} = H\omega + \tau + d \quad (26)$$

where the components h_i s in H is determined from $h = J\omega$ and the numerical data are

$$\begin{aligned} J &= J_0 + \Delta J (\text{kg} \cdot \text{m}^2) \\ J_0 &= \begin{bmatrix} 87.212 & 0 & 0 \\ 0 & 86.067 & 0 \\ 0 & 0 & 114.562 \end{bmatrix} \\ \Delta J &= \begin{bmatrix} \Delta J_{11} & 0 & 0 \\ 0 & \Delta J_{22} & 0 \\ 0 & 0 & \Delta J_{33} \end{bmatrix} \end{aligned} \quad (27)$$

$$|\Delta J_{11}| < 8.7212 (10\% \text{ of } J_{011})$$

$$|\Delta J_{22}| < 4.3034 (5\% \text{ of } J_{022})$$

$$|\Delta J_{33}| < 17.1843 (15\% \text{ of } J_{033})$$

$$d = \begin{bmatrix} -0.005 \cdot \sin(t) \\ +0.005 \cdot \sin(t) \\ -0.005 \cdot \sin(t) \end{bmatrix} (N \cdot m).$$

Based on this model, the multiaxial attitude tracking maneuvers are illustrated with the following desired reference vector function

$$\rho_d(t) = \begin{bmatrix} \sin\left(\frac{\pi}{50}t\right) \\ -\sin\left(\frac{\pi}{50}t\right) \\ 0.5\cos\left(\frac{\pi}{50}t\right) \end{bmatrix}.$$

The initial conditions are set as $\omega = [0.001 \quad -0.005 \quad 0.001]^T$ and $\rho = [1 \quad 1 \quad -1]^T$. Following the design procedure, in the first step we choose the sliding vector

$$s(t) = [\omega(t) - \dot{\omega}(t)] + \lambda[\rho(t) - \rho_d(t)]$$

as shown in (17). With this sliding vector, it has been proved that the system in the sliding mode is stable and the convergent rate of the error signal $\varepsilon(t)$ is $\lambda/2$. Here, we let $\lambda = 0.5$. In the second step, from (20), we have the control laws

$$\tau = -H_0\omega + J_0\dot{\omega} - \lambda J_0[T(\rho)\omega - \dot{\rho}_d] - K \cdot \text{sgn}(s) \quad (28)$$

where

$$H_0\omega = h_0 \times \omega$$

$$h_0 = J_0\omega$$

$$K \cdot \text{sgn}(s) = \begin{bmatrix} k_1 \cdot \text{sgn}(s_1) \\ k_2 \cdot \text{sgn}(s_2) \\ k_3 \cdot \text{sgn}(s_3) \end{bmatrix}.$$

Here, from (23), the elements k_i , $i = 1, 2, 3$, are determined by the upper bounds of δ and can be found as

$$k_1 = |(\Delta J_{22} - \Delta J_{33})\omega_2\omega_3| + \Delta J_{11}|\dot{\omega}_1| + \lambda\Delta J_{11}|T_{\omega_1} - \dot{\rho}_{d1}| + |d_1|_{\max} + 1$$

$$k_2 = |(\Delta J_{33} - \Delta J_{11})\omega_3\omega_1| + \Delta J_{22}|\dot{\omega}_2| + \lambda\Delta J_{22}|T_{\omega_2} - \dot{\rho}_{d2}| + |d_2|_{\max} + 1$$

$$k_3 = |(\Delta J_{11} - \Delta J_{22})\omega_1\omega_2| + \Delta J_{33}|\dot{\omega}_3| + \lambda\Delta J_{33}|T_{\omega_3} - \dot{\rho}_{d3}| + |d_3|_{\max} + 1$$

where T_{ω_i} is the i th element of $T(\rho)\omega$, $i = 1, 2, 3$, and a constant 1 is added in order to shorten the reaching time t_h . Note that the terms $\dot{\omega}_i$ s can be evaluated from (21). Furthermore, to avoid the chattering problem of the control torques, we replace the sign function with the saturation function (24) to obtain the control law

$$\tau = -H_0\omega + J_0\dot{\omega} - \lambda J_0[T(\rho)\omega - \dot{\rho}_d] - K \cdot \text{sat}(s, \epsilon)$$

with $\epsilon = 0.05$. Therefore, the system is no longer restricted in the sliding mode but constrained in the sliding layers $|s_i| \leq \epsilon_i$, $i = 1, 2, 3$.

The simulation results are shown in Fig. 1 through Fig. 3 with the inertia matrix given by

$$J = \begin{bmatrix} 95.933 & 0 & 0 \\ 0 & 81.763 & 0 \\ 0 & 0 & 131.746 \end{bmatrix}$$

which satisfies the variation ΔJ listed in (27). Fig. 1 shows that the attitude vector traces the reference vector $\rho_d(t)$ successfully. In Fig. 2, the angular velocity vector $\omega(t)$ approach $\dot{\omega}(t)$ as expected. Finally, Fig. 3 shows the input torques which do not possess any undesirable chattering. Therefore, the robustness of the sliding-mode control to parameter variation ΔJ and external disturbance $d(t)$ is verified.

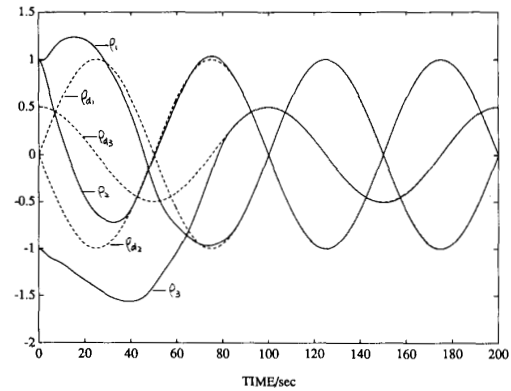


Fig. 1. Attitude components ρ_i , $i = 1, 2, 3$.

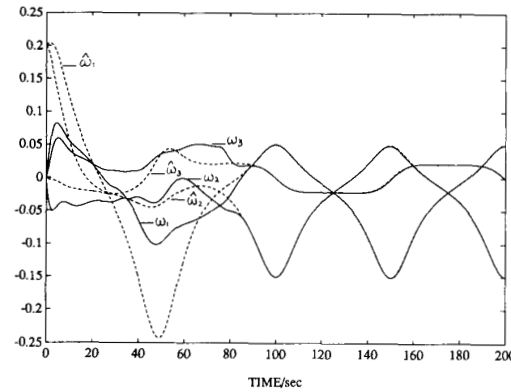


Fig. 2. Angular velocities ω_i , $i = 1, 2, 3$.

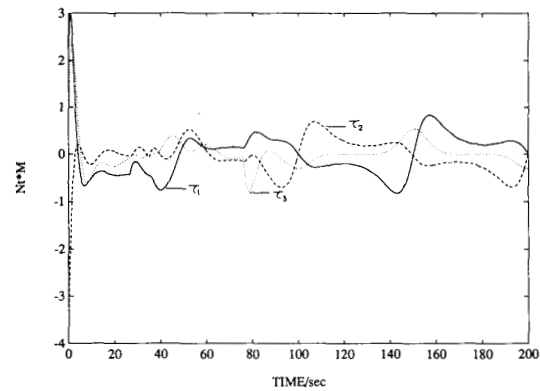


Fig. 3. Control torques τ_i , $i = 1, 2, 3$.

V. CONCLUSION

In this paper, sliding-mode theory is applied to the controller design for spacecraft attitude tracking maneuvers. There exist two important natural properties of the spacecraft model. First, the inertia matrix J is symmetric positive definite and second, the matrix $T(\rho)$ in the kinematic equation satisfies

$T(\rho) \geq \frac{1}{2}I$. With these properties and based on the direct method of the Lyapunov stability theory, a new sliding vector and two significant Lyapunov functions are introduced in the controller design and system stability analysis. Besides, the convergent rate of the error signal can be determined by suitably choosing the sliding vector. As for the chattering problem, the saturation functions have been suggested to replace the sign functions in the control laws. Finally, the example of spacecraft driven by pairs of opposing thrusters verifies the success and robustness of the sliding-mode controller.

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Removal of Alignment Errors in an Integrated System of Two 3-D Sensors

An algorithm is presented to relatively align two 3-D sensors using targets that are tracked by both sensors. The algorithm estimates and removes sensor biases and sensor frame orientation errors. For illustrative purposes, the alignment algorithm is applied to simulated track data from two sensors.

I. INTRODUCTION

Interest in integrating stand-alone sensors into multisensor systems for command, control, and communications (C^3) has been increasing in recent years. Rather than develop new sensors to achieve more accurate tracking and improved surveillance in C^3 systems, it is less costly to integrate existing stand-alone sensors into a single system to obtain performance improvements and enhanced capabilities for tracking and surveillance. However, before the benefits of multisensor integration can be realized, the sensor registration (or alignment) problem must be addressed. Registration refers to the process of expressing the multisensor data in a common reference frame, where the data is free from errors due to improper alignment of the sensors, orientation errors in the reference frames of the sensors, and sensor location errors [1]. That is, the data from each sensor must be transformed to a common reference frame that is free from errors in the transformation process. Unfortunately, attempts to integrate multiple sensors into a single system for C^3 have had limited success, due largely to a failure to solve the registration problem [1, 2].

One source of registration errors is sensor calibration errors (i.e., offsets). Although the sensors are usually calibrated in an initial calibration procedure, the calibration may deteriorate over time. Another source of registration errors is attitude (or orientation) errors in the reference frames of the sensors. Attitude errors can be caused by bias errors

Manuscript received January 26, 1993.

IEEE Log No. T-AES/29/4/10994.

This work was supported by the Independent Research Program at the Naval Surface Warfare Center Dahlgren Division (NSWCDD), Dahlgren, VA.

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