Monotone iterative simulation of the multilane traffic dispersion model

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Abstract

In this study, the multilane traffic flow is modeled as a nonlinear Poisson equation in a two-dimensional space. The model is derived from the interaction among vehicles and the assumption that vehicles will tend toward the equilibrium state under a given traffic condition. A monotone iterative scheme for the nonlinear model, which is a finite difference approximation of the model, is presented. The convergency is also discussed herein. At last, a numerical example is employed to explain the model. © 2002 Elsevier Science B.V. All rights reserved.

1. Introduction

Traffic congestion generates the interest in traffic flow researches. Since most roads are multilane in each direction in the real world, traffic flow theory evolves the multilane theory from the single lane models. Therefore, the four main methodologies: car-following models, kinetic models, Boltzmann-like models and cellular automation (CA) toward modeling traffic flow phenomena are extended from single lane models to multilane models. Wiedemann [1] incorporated the lane-changing behavior and the overtaking behavior with the car-following theory. His job becomes the foundation of a number of microscopic mul-

tilane models [2]. The kinetic model [3] is extended to multilane freeway by Munjal et al. [4,5], Michalopoulos, Beskos, and Yamauchi [6] and others discussed the lane-changing behavior directly by mathematical model instead of considering lane changing as a perturbation of equilibrium. The higher-order kinetic model is also formulated to describe the multilane traffic [7,8]. Furthermore, Hoogendoorn and Bovy [9–11] have developed a multilane multiclass traffic flow model based on mesoscopic principles. The model inherits some underlying properties of the gas-kinetic equations (e.g., a description using platoons, finite-space requirements). The modeling approach of Helbing [7,8] is similar to Hoogendoorn and Bovy [9–11]. They started with the Boltzmann-like model and the result is a gas-kinetic model. Boltzmann-like models are also developed directly to describe the characteristics of traffic on multilane highways [12–14]. Rickert

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et al. [15], Nagel et al. [16,17] and Hattori [18] extended CA to two-lane traffic in different conditions such as signalized intersection and etc.

Multilane traffic flow is a difficult problem because of modeling the lane-changing behavior and model computation. Therefore, a multilane traffic dispersion model, which describes how traffic density distributes on a multilane road, is suggested in this study. The dispersion model is derived from the interaction among vehicles and results in a linear Poisson equation. Poisson equation is a stationary diffusion or wave equation, which is employed to describe steady fluid or heat flow, electrostatics and so on. In addition, it is assumed that the traffic will tend toward the equilibrium state under a given situation. An equilibrium distribution of density is obtained. Coupling the linear Poisson equation and the equilibrium distribution eventually results in the nonlinear dispersion model. To solve the model, a monotone iterative scheme is presented because of its convergence [19]. The monotone iterative scheme is firstly introduced into the numerical simulation in nonlinear traffic phenomena.

The paper is organized in the following way. In Section 2, the description of the traffic dispersion model is presented. Section 3 illustrates the monotone iterative scheme and its convergency. After that, the numerical result is discussed in Section 4. The paper ends with some conclusions and perspectives in Section 5.

2. Description of the model

In order to understand the multilane traffic behavior, a multilane dispersion model is proposed in this section. Firstly, the interaction between a single vehicle and other vehicles is concerned by the car-following theory [2,20–23]. The car-following theory discusses the acceleration of a single vehicle by its spacing and velocity. The original idea is based on Newton's law [21], which implies that acceleration is induced by external force (which is denoted by $\mathbf{F}$). Therefore, we can consider the acceleration is influenced by the interaction among vehicles, which is in terms of traffic force. The concept of traffic field ($\widetilde{\mathbf{E}}$) is introduced to replace the force, since the interaction is not really a contact force. Thus, it is assumed that an external field applies to a road and vehicles on the road will move along the direction of the exter-

nal field. For the sake of safety, one vehicle on a road adjusts its velocity and spacing according to the relative position among others in order to avoid accidents. Therefore, it is assumed that each vehicle has its own field and the vehicles interact with each other by their field. As mentioned above, the traffic force produced by the interaction is a resistance against the external field. Here are the further assumptions of the derivation of the traffic model. The first one is that the influence of vehicles in the same lane is M times larger than that in the adjacent lanes, where M is a scalar. It is reasonable to assume that the influence of vehicles in the same lane is larger than that in the adjacent lanes. Owing to the assumption, the traffic field can be transformed to a conservative field, which will be described in detail in the following context. The second one is that the traffic field produced by the interaction is determined by the spacing among vehicles. The interaction is larger when the spacing is smaller. Therefore, it is assumed that the traffic field produced by the interaction satisfies the inverse-square law (the gravity model). The traffic field acting on a specific vehicle is represented as

$$
\begin{aligned}
\widetilde{\mathbf{E}} &= \sum_{i=1}^{N} \frac{e_i}{\hat{\varepsilon}_i} \left(\frac{\tilde{x}_0 - \tilde{x}_i}{|\tilde{x}_0 - \tilde{x}_i|^3} \mathbf{i} + \frac{\tilde{y}_0 - \tilde{y}_i}{M^2 |\tilde{y}_0 - \tilde{y}_i|^3} \mathbf{j} \right) \\
&= \sum_{i=1}^{N} \left(\frac{e_i \widetilde{\mathbf{X}}_i}{\hat{\varepsilon}_i \|\widetilde{\mathbf{X}}_i\|^3} \right),
\end{aligned} \tag{1}
$$

where N is the number of vehicles that may interact with the specific vehicle, $\widetilde{\mathbf{X}}_i$ denotes the spacing from vehicle i to vehicle 0. e_i is the passenger car equivalent of vehicle i and $\hat{\varepsilon}_i$ is the interacting parameter of vehicle i. We transform the traffic field ($\widetilde{\mathbf{E}}$) into a conservative field ($\mathbf{E}$) by letting $x = \tilde{x}$, $y = M \tilde{y}$ for convenience. In the continuous space, Eq. (1) can be represented as

$$
\mathbf{E} = \frac{e}{\hat{\varepsilon}} \int_{\Omega} \left((k - k_s) / \|\mathbf{X}\|^2 \right) d\Omega, \tag{2}
$$

where Ω is the multilane section, which is a two-dimensional space. $\mathbf{X}$ is the spacing after transformation. It is assumed that all vehicles have the same characteristics and driving behavior, i.e. the passenger car equivalent and the interacting parameter are constants, which are denoted by e and $\hat{\varepsilon}$, respectively. k is the density and k_s is the unstrained density, which is the

density that vehicles do not disturb each other, under a given condition. If the actual density is equal to the unstrained density, vehicles will spread out immediately. Since the field is conservative, a potential function ϕ exists by the potential theory. The potential function ϕ satisfies $\mathbf{E} = -\nabla_{\mathbf{x}}\phi$. Thus, the magnitude of traffic field is illustrated as

$$\operatorname{div}\mathbf{E} = -\Delta\phi = e(k - k_s)/\varepsilon + K_a, \tag{3}$$

where $\operatorname{div}\mathbf{E}$ denotes the magnitude of traffic field, $\varepsilon = \hat{\varepsilon}/4\pi$, $K_a = K_a(\mathbf{x})$, which depends on the the position $\mathbf{x}$, is the adjust term of the road condition if the road condition is ideal $K_a = 0$. Eq. (3) is a linear Poisson equation, which describes the diffusion effect under the steady state. The analytical solution of the linear model can be solved directly by separation of variables. However, the linear model implies that density is independent of potential. Since the traffic field affects the movement of vehicles, density should be distributed by the field (or potential). The relation between density and potential is obtained from the assumption that density will tend toward its equilibrium state under a given traffic situation. The equilibrium state defined in this study is the most possible microscopic state of a given macroscopic condition. The macroscopic condition includes the conservation of vehicle numbers and the conservation of total velocity variance. An equilibrium distribution, which is in the form of Maxwell–Boltzmann distribution, is finally obtained as shown in Eq. (4).

$$k = K_0 \exp\bigl((e\psi - e\phi)/\Theta_e\bigr), \tag{4}$$

where K_0 is the essential density, Θ_e is the equilibrium velocity variance, ψ is the potential equivalent of the velocity variance threshold. ψ is named as the potential barrier here. Velocity variance is defined as $\Theta = \int \|\mathbf{v} - \mathbf{u}\|^2 f(\mathbf{x}, \mathbf{v}, t)\,d\mathbf{v}$ where $\mathbf{v}$ is individual velocity, $\mathbf{u}$ is average velocity and $f(\mathbf{x}, \mathbf{v}, t)$ is the phase plane distribution [12]. Θ_e is the velocity variance under the equilibrium state. Eq. (4) is the equilibrium distribution of density. Several points are induced from the equation. First, density decreases as traffic potential increases. Second, when the equilibrium velocity variance increases, the variation of density increases, which means the traffic, is sensitive. The third one is when the potential barrier is low, the density is small; that is, drivers are aggressive. By coupling Eqs. (3) and (4), the multilane traffic dispersion model is obtained. In the following paragraphs, the application of

the monotone iterative scheme in the dispersion model is derived.

3. The monotone iterative scheme

Monotone iterative techniques are widely applied in solving nonlinear differential equation. Pao [19] mentioned that the point method is not efficient computation in two or higher space dimension and proposed a block monotone iterative scheme for nonlinear elliptic equations. Since the dispersion model is a nonlinear elliptic equation, the block monotone iterative scheme is employed. In order to demonstrate the fundamental of monotone iterative method Eqs. (3), (4) and the boundary condition are shown in a simplified form as follows:

$$\begin{aligned} \Delta\phi &= f(x, y, \phi), &&\text{in } \Omega, \\ a\,\partial\phi/\partial\mathbf{n} + b\phi &= g(x, y) &&\text{on } \partial\Omega, \end{aligned} \tag{5}$$

where Ω is a two-dimensional rectangular domain $\Omega = (0, L_x) \times (0, L_y)$ with boundary $\partial\Omega$, where $\partial\phi/\partial\mathbf{n}$ is the outward normal derivative of ϕ on $\partial\Omega$, $a \equiv a(x, y)$ and $b \equiv b(x, y)$ are nonnegative functions on $\partial\Omega$ with $a + b > 0$, and f and g are prescribed nonlinear function in its domain. Let $h_x = L_x/M_x$, $h_y = L_y/M_y$ and $x_i = ih$, $y_j = jh$ for $i = 0, 1, \ldots, M_x$, $j = 0, 1, \ldots, M_y$, where M_x and M_y are the total number of intervals along the x and y directions, respectively. The set of points (x_i, y_j) in Ω and $\overline{\Omega} = \Omega \cup \partial\Omega$ are denoted, respectively, by Λ and $\overline{\Lambda}$. Define $\phi_{i,j} = \phi(x_i, y_j)$, $f_{i,j}(\phi_{i,j}) = f(x_i, y_j, \phi(x_i, y_j))$ and $g_{i,j} = g(x_i, y_j)$, then the standard finite difference approximation for the equations in (5) leads to a finite difference system in the form

$$\begin{aligned} &\tilde{a}_{ij}\phi_{ij} - \tilde{b}_{ij}\phi_{i-1,j} - \tilde{c}_{ij}\phi_{i+1,j} - \tilde{d}_{ij}\phi_{i,j-1} - \tilde{e}_{ij}\phi_{i,j+1} \\ &\quad = hkf_{ij}(\phi_{ij}) + g_{i,j}^*, \quad\text{for all } (i, j) \in \overline{\Lambda}, \end{aligned} \tag{6}$$

where the coefficients $\tilde{a}_{ij}$, $\tilde{b}_{ij}$, etc. are associated with the coefficient of the discretized Laplace operator as well as the boundary coefficients $a \equiv a(x, y)$ and $b \equiv b(x, y)$, $g_{i,j}^*$ is associated with the boundary functions of $g_{i,j}$. In a more compact form

$$\mathbf{A}\Phi = -\mathbf{F}(\Phi) - \mathbf{G}, \tag{7}$$

where $\mathbf{A}$ is a $\mathbf{M} \times \mathbf{M}$ matrix, where

$$\mathbf{M} = (M_x + 1)(M_y + 1),$$

$$\mathbf{F}_j(\Phi_j) = \left(-h_x h_y f_{0,j}(\phi_{0,j}), \dots, \right.$$
$$\left. -h_x h_y f_{M_x,j}(\phi_{M_x,j})\right)^{\mathrm{T}},$$

$$\mathbf{G}_j = (-h_x h_y g_{0,j}, \dots, -h_x h_y g_{M_x,j})^{\mathrm{T}} \quad \text{and}$$

$$\Phi = (\phi_{0,j}, \dots, \phi_{M_x,j})^{\mathrm{T}} \quad \text{for } j = 0, 1, 2, \dots, M_y.$$

Denoting $\mathbf{F}$ and Φ by $\mathbf{M}$, then

$$\mathbf{F}(\Phi) \equiv \left(-F_0(\Phi_0), \dots, -F_{\mathbf{M}}(\Phi_{\mathbf{M}})\right)^{\mathrm{T}},$$

$$\mathbf{G} \equiv (-G_0, \dots, -G_{\mathbf{M}})^{\mathrm{T}} \quad \text{and}$$

$$\Phi \equiv (\phi_{0,j}, \dots, \phi_{M_x,j})^{\mathrm{T}}.$$

Typical choice of the coefficients in Eq. (6) can be easily derived if a finite difference approximation is specified. For the boundary points (x_i, y_j) where $i = 0, M_x$ and $j = 0, M_y$. If the coefficients $\tilde{a}_{ij}$, $\tilde{b}_{ij}$, $\tilde{c}_{ij}$, $\tilde{d}_{ij}$, $\tilde{e}_{ij}$ are associated with the boundary coefficients a_{ij} and b_{ij}, and possess the following property for the case $a_{ij} > 0$.

$$\tilde{a}_{ij} \geqslant \tilde{b}_{ij} + \tilde{c}_{ij} + \tilde{d}_{ij} + \tilde{e}_{ij}, \tag{8}$$

for $i = 0, M_x$ and $j = 0, M_y$. If the mixed boundary condition holds, then strict inequality in the above relation holds for at least one (i, j). In either case, these coefficients satisfy the condition:

$$\tilde{a}_{ij} \geqslant 0, \quad \tilde{b}_{ij} \geqslant 0, \quad \tilde{c}_{ij} \geqslant 0, \quad \tilde{d}_{ij} \geqslant 0, \quad \tilde{e}_{ij} \geqslant 0,$$
$$\tilde{a}_{ij} \geqslant \tilde{b}_{ij} + \tilde{c}_{ij} + \tilde{d}_{ij} + \tilde{e}_{ij}, \quad \text{for all } (i, j) \in \overline{\Lambda}. \tag{9}$$

Condition (9) is the fundamental hypothesis for our discussion of the monotone iterative method for the nonlinear algebraic system (7). It is clear from the matrix $\mathbf{A}$ in Eq. (7) and condition (9) that $\mathbf{A}$ is a M-matrix, i.e. $\mathbf{A}^{-1} \geqslant 0$ is a strictly diagonal dominant and invertible matrix provided that strict inequality in the relation $\tilde{a}_{ij} \geqslant \tilde{b}_{ij} + \tilde{c}_{ij} + \tilde{d}_{ij} + \tilde{e}_{ij}$ in condition (9) holds for at least one (i, j) and the domain $\overline{\Lambda}$ is connected. The monotone iterative method is based on the system (7) with $\mathbf{A}$ in the split form $\mathbf{A} = \mathbf{D} - \mathbf{L} - \mathbf{U}$, where $\mathbf{D}$, $\mathbf{L}$ and $\mathbf{U}$ are block diagonal, lower triangular and upper triangular matrices of $\mathbf{A}$, respectively. In addition, we observe that D is a M-matrix, $\mathbf{L} \geqslant 0$, and $\mathbf{U} \geqslant 0$. A direct fact that Eq. (7) has at most a solution holds, if $\mathbf{F}_j(\Phi_j) \in C^1$, for $0 \leqslant j \leqslant \mathbf{M}$ and $\partial \mathbf{F}_j(\Phi_j)/\partial \phi \geqslant 0$, $-\infty < \phi < \infty$.

It is assumed that there is a function $S_0(c)$ such that

$$\left|\mathbf{F}_j(\Phi_1) - \mathbf{F}_j(\Phi_2)\right| \leqslant S_0(c)|\Phi_1 - \Phi_2|, \tag{10}$$

for $0 \leqslant j \leqslant \mathbf{M}$ provided that $|\Phi_1|, |\Phi_2| \leqslant c$. $\mathbf{F}_j(\Phi_j)$ is bounded uniformly if there exists a constant $Z > 0$ such that $|\mathbf{F}_j(\Phi_j)| \leqslant Z$, $|\Phi_j| < \infty$, $j = 1, \dots, \mathbf{M}$. Let $\lambda = S_0(c)$, where $c_0 = \|\widehat{\Phi}^{(0)}\|$ and $\widehat{\Phi}^{(0)}$ is the solution of $\mathbf{A}\widehat{\Phi}^{(0)} = (Z + 1)\mathbf{E} - \mathbf{G}$, where $\mathbf{E} = (e_j)$ for $0 \leqslant j \leqslant \mathbf{M}$. The following result is the well-known theorem and has been reported by Pao [19].

Theorem 1. *Let the functions $\mathbf{F}_j(\Phi)$ be bounded uniformly, and $\widehat{\Phi}^{(n+1)}$, $n = 0, 1, 2, \dots$, be the solution of*

$$(\mathbf{D} + \lambda\mathbf{I})\widehat{\Phi}^{(n+1)}$$
$$= (\mathbf{L} + \mathbf{U})\widehat{\Phi}^{(n)} - \mathbf{F}(\widehat{\Phi}^{(n)}) - \mathbf{G} + \lambda\widehat{\Phi}^{(n)}, \tag{11}$$

then, there exists a solution of Eq. (7).

As discussion above, let the coefficient $\xi_{i,j}$ associated with the Laplace operator in Eq. (3) satisfies the condition (9), i.e. $\xi_{i,j} \geqslant 0$ for all $(i, j) \in \overline{\Lambda}$ and $\xi_{i,j} \geqslant \xi_{i+1,j} + \xi_{i-1,j} + \xi_{i,j+1} + \xi_{i,j-1}$. As last, when using the corresponding finite difference and monotone iterative methods for the nonlinear Poisson equation, we obtain the following finite difference system, i.e.

$$\phi_{i,j}\xi_{i,j} = (e/\varepsilon)\big\{\xi_{i+1,j}\phi_{i+1,j} + \xi_{i-1,j}\phi_{i-1,j}$$
$$+ \xi_{i,j+1}\phi_{1,j+1} + \xi_{i,j-1}\phi_{i,j-1}$$
$$+ K_0 \exp\big(e(\psi_{i,j} - \phi_{i,j})/\Theta_e\big) - k_{si,j}\big\}$$
$$+ K_{ai,j} \tag{12}$$

is the finite difference system corresponding to the nonlinear Poisson equation, and

$$\phi_{i,j}^{(n)} = \big(e/\varepsilon(\lambda + \xi_{i,j})\big)\big\{\xi_{i+1,j}\phi_{i+1,j}^{(n)} + \xi_{i-1,j}\phi_{i-1,j}^{(n)}$$
$$+ \xi_{i,j+1}\phi_{1,j+1}^{(n)} + \xi_{i,j-1}\phi_{i,j-1}^{(n)}$$
$$+ K_0 \exp\big(e(\psi_{i,j} - \phi_{i,j}^{(n)})/\Theta_e\big) - k_{si,j} + \lambda\phi_{i,j}^{(n)}\big\}$$
$$+ K_{ai,j}/(\lambda + \xi_{i,j}) \tag{13}$$

is the associated monotone iterative equations where $\lambda = \partial f/\partial \phi$ for all $(x, y, \phi) \in \overline{\Omega} \times [\inf \underline{\phi}, \sup \overline{\phi}]$, where $\underline{\phi}$ and $\overline{\phi}$ is so-called a subsolution and supersolution, respectively.

Theorem 2. *If a monotone iterative number λ is chosen, and the coefficients $\xi_{i,j}$ satisfy the condition (9), then sequences $\{\phi_{i,j}^{(n)}\}_{n=0}^{\infty}$ which is generated by Eq. (13) converge to a solution $\phi_{i,j}$ of Eq. (12) monotonically.*
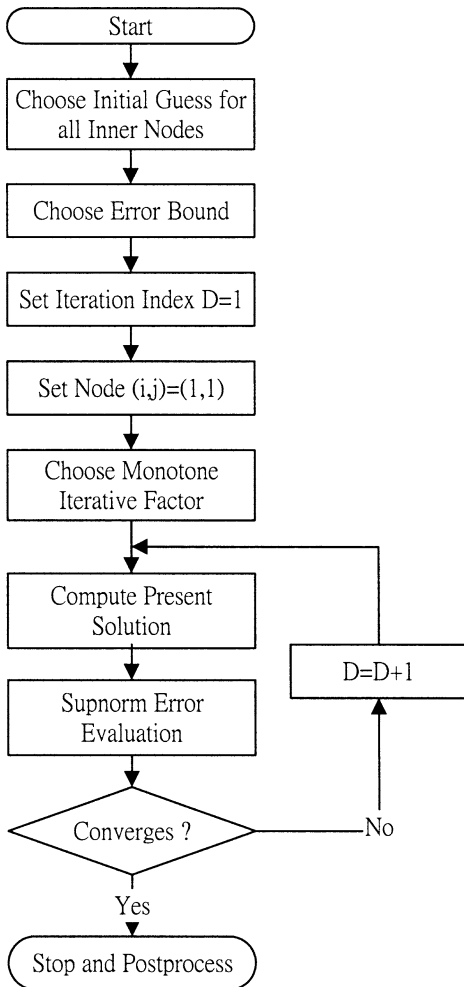
Fig. 1. Flowchart of monotone iterative method.

4. Numerical results and discussions

In order to describe the traffic situation, the model and the boundary conditions should be considered together. Since the boundary conditions of potential can't be obtained directly, we have to transform the boundary conditions of density into the boundary conditions of potential by Eqs. (3) and (4). Therefore, the boundary conditions of density in traffic flow problems have to be discussed first. If the traffic condition on a boundary can be described by a deterministic function, a Dirichlet condition is chosen, such as deterministic inflow or outflow ($k(x, y) = f(y)$) and no entrance and exit on the roadsides ($k(x, y) = 0$). If we only have the changing rate of density on a boundary, a Neumann condition is chosen. The situation takes place behind the on-ramp, the signal intersection and the toll collection station where the service rate is easily obtained. Mostly, a Robin condition mixed by both Dirichlet and Neumann condition is employed. A traffic flow problem often involves two or three types of boundary conditions, which is a mixed boundary condition problem. A boundary condition of density can be corresponded to a boundary condition of potential when it is formulated.

The numerical example in this study considers a basic section of freeway, which is behind an on-ramp. The research domain is a three-lane section with 1 km in length. Under the uncongested traffic, the boundary conditions are given as follows:

$$
\begin{cases}
\partial\phi(x,0)/\partial\mathbf{n} = \partial\phi(x,60)/\partial\mathbf{n} = 0, & x \in \partial\Omega, \\
\phi(0,y) - \partial\phi(0,y)/\partial\mathbf{n} \\
\quad = 4 - 2 \times (\Theta_e/e) \times \ln(k_s(0,y)/K_0), & y \in \partial\Omega, \\
\phi(100,y) \\
\quad = -(\Theta_e/e) \times \ln(k_s(100,y)/K_0), & y \in \partial\Omega,
\end{cases}
$$
(14)

where the passenger car equivalent $e = 1$, the equilibrium velocity variance $\Theta_e = 25$ km^2/h^2 and the $K_0 = 3$, which is about 108 passenger car unit per kilometer per lane (pcu/(km · lane)) in the original problem (before transforming to the conservative field). Assume the interacting parameter $\varepsilon = 10$ and the domain is ideal in geometry, then $K_a = 0$. The unstrained density k_s and the research domain are given as Fig. 2. The
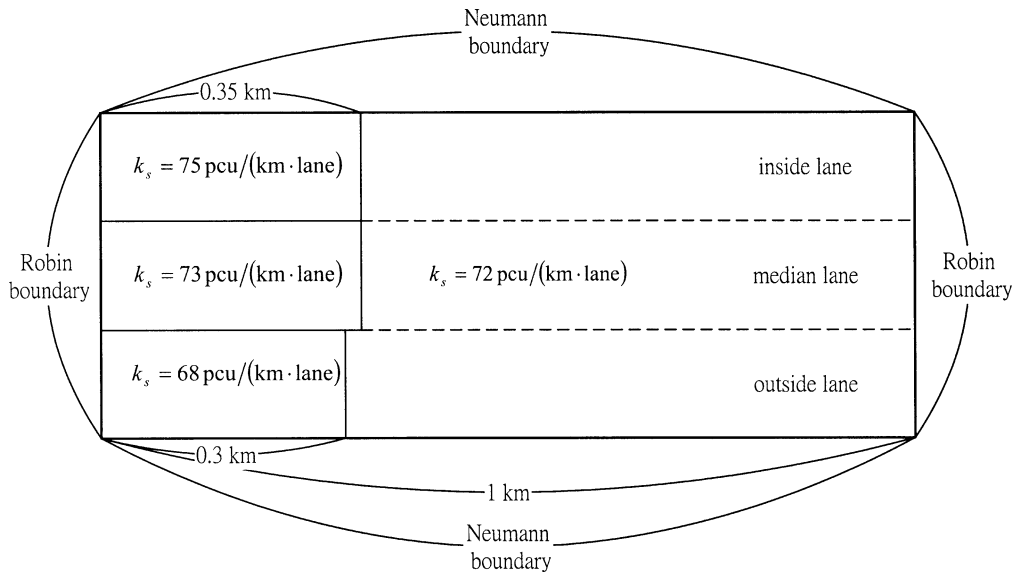
Theorems 1 and 2 give us the existence and the uniqueness of solution of the monotone iterative scheme for our nonlinear dispersion model. The implementation of the monotone iterative method is illustrated in Fig. 1. Furthermore, the initial guess of the iteration is determined by the physical meaning of the model and the boundary conditions in general. Fortunately, the nonlinear function (4) is a monotonic function, and then any arbitrarily initial guess will converge to the unique solution. A numerical example is employed to explain the nonlinear dispersion model and to show the convergence of the scheme in the following section.

Fig. 2. The research domain and the unstrained density (k_s).

first condition in Eq. (14) means that there is no entrance and exit on both roadsides; that is, $k = 0$, which implies $\nabla\phi = 0$. The second condition is obtained from the Robin condition of density, which means the inflow depends on the existent density and the inflow density. The third condition means that the outflow is determined by the passable density. Through the boundary conditions, traffic flow is forced to move forward and the mobility of the vehicles is blocked by the density (platoon of vehicles). The passable density is assumed to describe that the inside lane makes more vehicles pass through than outside lanes, since the outside lanes are disturbed by the inflow of the on-ramp.

The numerical results are illustrated in Figs. 3–5. The traffic field is derived from the traffic potential when it is computed. The traffic field is illustrated in Fig. 4, which implies the following behavior. Vehicles tend toward the inside lane and the trend of the vehicles in the outside lane is stronger than that of the vehicles in the median lane. That is, there are more vehicles trying to change lanes in the front part of the outside lane. However, the lane-changing trend keeps a longer distance in the median lane. At last, the traffic flow will become uniform. This case is an uncongested flow, which means that if the traffic is disturbed, the density will decrease. Fig. 5 shows the density distributed on the road. Since the front part
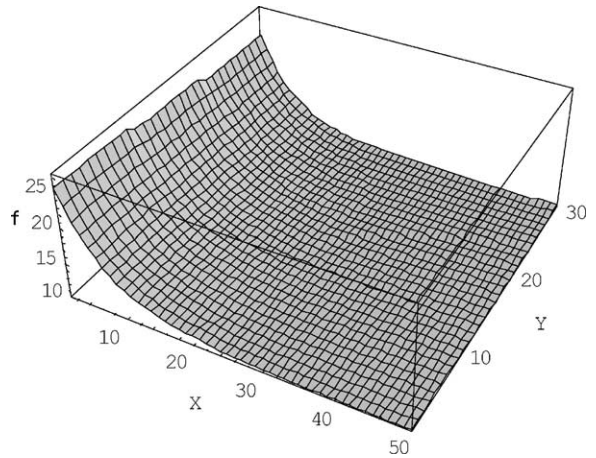

Fig. 3. The potential of the numerical example. Each interval is 20 m in x direction and 0.4 m in y direction. $0 \leqslant y \leqslant 10$ is the outside lane; $10 \leqslant y \leqslant 20$ is the median lane; $20 \leqslant y \leqslant 30$ is the inside lane.

(right-hand side) of the domain is disturbed by the inflow of the on-ramp, the density there is lower than the rear part (left-hand side) of the domain. Behind the merging area, the traffic flow becomes uniform. This information can help to design the on-ramp area (the length of the auxiliary lane). Sup norm error is chosen to check the convergence. The numerical result supports the conclusions of Theorems 1 and 2 that the
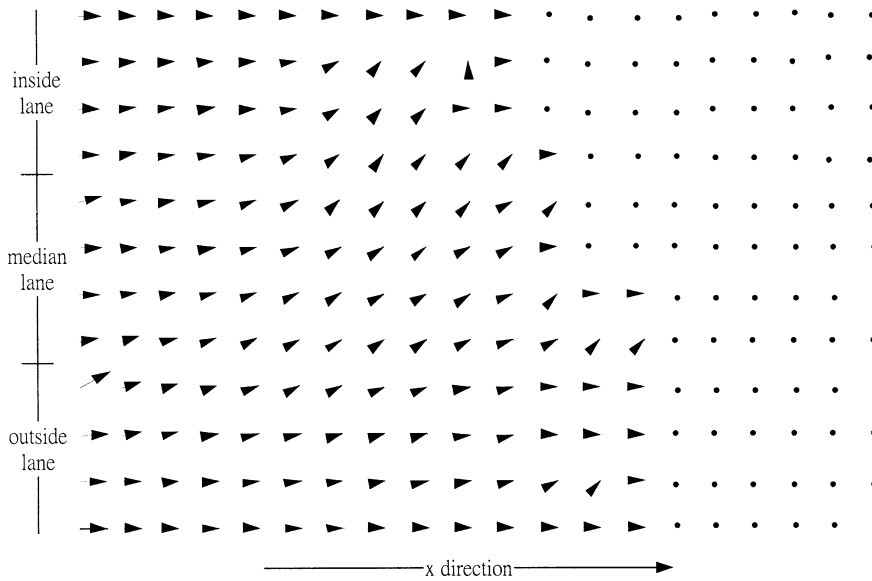
Fig. 4. The traffic field of the numerical example, which shows that vehicles tend to move to the inside lane and the traffic flow will become uniform (the upper side of the figure is the inside lane and the lower side is the outside lane).
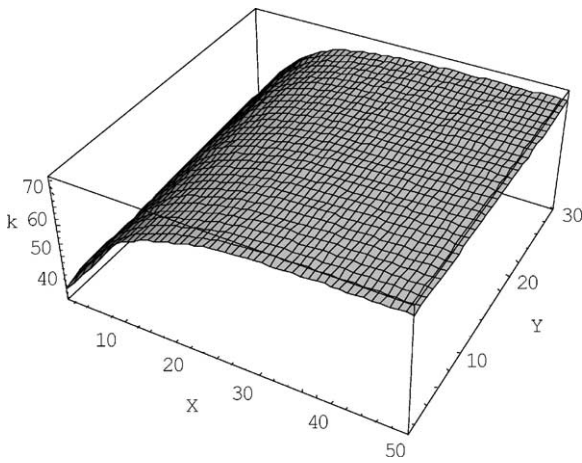

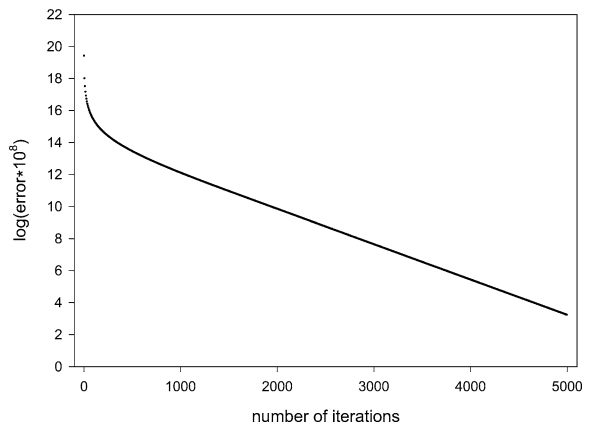Fig. 5. The density of the numerical example.

Fig. 6. Plot of the convergence (log(error $\times 10^8$) v.s. number of iterations).

monotone iterative scheme converges to the unique solution. This fact is inferred in Fig. 6, which also illustrates the speed of convergence is very fast.

5. Conclusions and perspectives

In this study, a multilane traffic dispersion model is used to describe the multilane traffic flow. The model, which is a nonlinear Poisson equation, pro-

vides the multilane traffic information (density) without microscopic analyses. The multilane dispersion model is a nonlinear Poisson equation. Poisson equation is widely used in physical researches, such as heat, fluid, electricity and so on, to discuss the distribution of density or volume. We introduce the application of Poisson equation into the traffic flow research in this study. Also, the monotone scheme is firstly employed to solve the traffic problem. A numerical example is employed to explain the multilane traffic by the model,

and it also shows the convergence of the monotone iterative scheme is fast. Scenarios of boundary conditions should be formulated according to different traffic situations. The model developed herein is a stationary dispersion model. The time dependent dispersion model is left for further researches.

Acknowledgements

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