Lapped Unimodular Transform and Its Factorization

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***Abstract*—Two types of lapped transforms have been studied in detail in the literature, namely, the lapped orthogonal transform (LOT) and its extension, the biorthogonal lapped transform (BOLT). In this paper, we will study the lapped unimodular transform (LUT). All three transforms are first-order matrices with finite impulse response (FIR) inverses. We will show that like LOT and BOLT, all LUTs can be factorized into degree-one unimodular matrices. The factorization is both *minimal and complete*. We will also show that all first-order systems with FIR inverses can be minimally factorized as a cascade of degree-one LOT, BOLT, and LUT building blocks. Two examples will be given to demonstrate that despite having a very small system delay, the LUTs have a satisfactory performance in comparison with LOT and BOLT.**

***Index Terms*—Filterbank, polynomial matrix, transform, unimodular matrix.**

I. Introduction

FILTER banks (FBs) and transforms have found many applications in signal processing [1]–[3]. When the polyphase matrix has order one, such an FB is also known as a lapped transform. Two classes of lapped transforms [the lapped orthogonal transform (LOT) [1], [4], [6] and the biorthogonal lapped transform (BOLT) [5]] have been studied in detail. The LOTs [6] and its generalization (GenLOT [3], [7]) have been widely applied in various applications. Many properties of LOTs and GenLOTs, such as the factorization and phase linearity, have been developed. In [5], Vaidyanathan and Chen relax the orthogonality condition and introduce a more general class of transforms called the BOLTs. BOLT is the class of lapped transforms that have anticausal FIR inverses. It includes the LOT as a useful special case. Like LOT, it was shown [5] that BOLT can also be factorized into degree-one matrices. A design example showed that BOLT has more design freedom and that its filters have better frequency responses than those of LOT with the same degree.

In this paper, we will study a class of lapped transform called the lapped unimodular transform (LUT). LUTs are first-order unimodular matrices. When the polyphase matrix of an FB is unimodular, we say it is a unimodular FB. Like LOTs and BOLTs, the LUTs and unimodular matrices have the advantage that both their inverses and themselves are FIR matrices. If they are used as the polyphase matrices, the FBs have FIR analysis and synthesis filters and achieve perfect reconstruction (PR). In additional to having causal FIR inverses, unimodular FBs also enjoy the advantage of having the smallest system delay among all FBs. The system delay of M-channel unimodular FBs is always $(M-1)$, no matter how long the analysis and synthesis filters are. System delay is of particular importance in applications such as speech coding and adaptive subband filtering. In speech coding, excessive delay can be very annoying [8]. In adaptive subband filtering, long system delay can degrade the performance [9]. Although there are efficient design methods for low delay FBs [10]–[12], there are relatively few results on unimodular FBs.

The earliest paper that studied the relationship between unimodular matrices and FIR PR FB is [13]. Using system-theoretic concepts, the authors derived a number of properties for causal FIR unimodular matrices. In particular, the authors showed that there are examples of second-order unimodular matrices that cannot be factorized into degree-one unimodular matrices. Moreover, it was shown that any causal FIR matrix $\mathbf{H}(z)$ with $[\det \mathbf{H}(z)] = cz^{-L}$ can always be decomposed into a product of a unimodular matrix and a paraunitary matrix. Even though such a decomposition is not necessarily minimal, it proved that all FIR PR FBs can be captured by a paraunitary matrix and a unimodular matrix. In [5], the authors showed that all BOLTs can be decomposed into degree-one building blocks. In addition, the authors showed that the lapped transform $\mathbf{A}_0 + \mathbf{A}_1 z^{-1}$ is a LUT if and only if the matrix $\mathbf{A}_1$ has all the eigenvalues equal to zero. The most general degree-one unimodular matrix was also given [5]. However, the factorization of LUTs into these degree-one unimodular matrices was not established. Another type of factorization of unimodular matrices has been studied before. It is also shown in a corollary in [14, Sect. II, ch. 6] that unimodular matrices can be expressed as a product of elementary matrices containing delay elements. Elementary matrices can be realized by using the lifting schemes [15], [16]. Lifting schemes enjoy the advantages of having low complexity and being structurally PR, that is, the FB continues to have PR even when the lifting coefficients are quantized. However, such a representation is not minimal and not unique. It would not be useful for the parameterization of filter banks as it does not give a structure with a fixed number of multipliers.

The following results are the main contributions of this paper.

1) All LUTs can be factorized into degree-one unimodular matrices. The factorization is both minimal and complete.
2) All lapped transforms with FIR inverses, which include LOTs, BOLTs and LUTs as special cases, can be minimally factorized as a cascade of degree-one building blocks.

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3) There do not exist any set of finite-degree unimodular matrices that form the building blocks of all unimodular matrices.
4) All factorable unimodular matrices can be captured using orthogonal matrices and unit-delay lifting matrices.
5) There do not exist linear-phase unimodular FBs with equal filter length.
6) Two examples demonstrate that despite having a very small system delay, we are able to i) design LUT with stopband attenuation comparable with that of LOTs and BOLTs ii) obtain a satisfactory coding performance.

A. Paper Outline

In Section II, we will first review some results from [5] and [13]. Then, we will show that we cannot have linear-phase unimodular FBs with equal filter length. The factorization theorem of LUTs is presented in Section III. In Section IV, we consider unimodular matrices of higher order. We first give a class of undecomposable unimodular matrices. Then, we show that factorable unimodular matrices, not restricting to first order, can be completely parameterized in terms of orthogonal matrices and unit-delay lifting matrices. In Section V, we will show that all lapped transforms with FIR inverse can be minimally factorized into degree-one building blocks. Section VI gives two examples to demonstrate the potential applications of LUTs. A conclusion is given in Section VII. Parts of the results in this paper have been presented in [17].

B. Notations and Definitions

Boldfaced upper and lower case letters are used to denote matrices and vectors, respectively. All matrices and vectors are $M \times M$ and $M \times 1$, respectively. For a causal polynomial $\mathbf{A}(z) = \mathbf{A}_0 + \mathbf{A}_1 z^{-1} + \cdots + \mathbf{A}_N z^{-N}$ with $\mathbf{A}_N \neq \mathbf{0}$, its *order* is equal to N, whereas its *degree* is the minimum number of delay required to realize the matrix. For example, the matrix $\mathbf{A}(z) = \mathbf{A}_0 + \mathbf{A}_1 z^{-1}$ has order one, whereas its degree is equal to the rank of $\mathbf{A}_1$. An implementation of a polynomial matrix is said to be *minimal* if it uses the minimum number of delays needed to implement the matrix. A representation or structure is said to be *complete* for a certain class of matrices if every matrix in that class can be expressed in such a representation.

The first-order matrix $\mathbf{A}(z) = \mathbf{A}_0 + \mathbf{A}_1 z^{-1}$, will be called the *lapped transform*. It is well known that such a matrix has an FIR inverse if and only if its determinant is $\det[\mathbf{A}(z)] = cz^J$ for some nonzero constant c and integer J. The lapped transform $\mathbf{A}(z)$ is a BOLT if it has an anticausal inverse [5]. Moreover, if the coefficients of BOLT satisfy $\mathbf{A}_0^T \mathbf{A}_1 = \mathbf{0}$ and $\mathbf{A}_0^T \mathbf{A}_0 + \mathbf{A}_1^T \mathbf{A}_1 = \mathbf{I}$, then it is a LOT [4], [6]. Both the BOLTs and LOTs have been studied in detail [1], [5], [6], [19].

II. Properties of Unimodular Matrices and LUT

A causal matrix $\mathbf{A}(z)$ is unimodular if its determinant $\det[\mathbf{A}(z)] = c$ for some nonzero constant. The inverse of a causal unimodular matrix is also causal unimodular. When the first-order matrix

$$\mathbf{A}(z) = \mathbf{A}_0 + \mathbf{A}_1 z^{-1} \tag{1}$$

is unimodular, it is called the LUT. Many useful properties of unimodular matrices can be found in [1], [5], and [13]. In this section, we will first review some results from [5] and [13] that are useful for later discussions. Then some new results will be stated.

A. Some Known Results From [5] and [13]

Parts of the results on unimodular matrices from [5] and [13] will be given later. For proofs and more results see [5] and [13].

Theorem 1: Let $\mathbf{A}(z) = \mathbf{A}_0 + \mathbf{A}_1 z^{-1} + \cdots + \mathbf{A}_N z^{-N}$ with $\mathbf{A}_N \neq \mathbf{0}$ be unimodular. Then, we have i) that $\mathbf{A}_N$ is singular and ii) that $\mathbf{A}_0$ is nonsingular.

Theorem 2: The degree-one matrix

$$\mathbf{I} - \mathbf{u}\mathbf{v}^\dagger + z^{-1}\mathbf{u}\mathbf{v}^\dagger$$

is unimodular if and only if $\mathbf{v}^\dagger \mathbf{u} = 0$, and more generally, the order-one matrix

$$\mathbf{I} - \mathcal{U}\mathcal{V}^\dagger + z^{-1}\mathcal{U}\mathcal{V}^\dagger \tag{2}$$

where $\mathcal{U}$ and $\mathcal{V}$ are $M \times \rho$ matrices with rank ρ, is a degree-ρ matrix having FIR inverse if and only if $\mathcal{V}^\dagger\mathcal{U}$ has all eigenvalues equal to zero or one. Moreover, the degree-ρ matrix in (2) is unimodular if and only if $\mathcal{V}^\dagger\mathcal{U}$ has all eigenvalues equal to zero.

As a result of Theorem 1, we can also express the unimodular matrix in (2) as

$$(\mathbf{I} - \mathcal{U}\mathcal{V}^\dagger)(\mathbf{I} + z^{-1}\mathcal{U}\mathcal{V}^\dagger).$$

Thus, it is also true (see [5, Ex. 5.4]) that $(\mathbf{I} + z^{-1}\mathcal{U}\mathcal{V}^\dagger)$ is unimodular if and only if $\mathcal{V}^\dagger\mathcal{U}$ has all eigenvalues equal to zero. In particular, the degree-one matrix $\mathbf{I} + \mathbf{u}\mathbf{v}^\dagger z^{-1}$ is unimodular if and only if $\mathbf{v}^\dagger \mathbf{u} = 0$. In this paper, we will use $\mathbf{I} + \mathbf{u}\mathbf{v}^\dagger z^{-1}$ as the building block because it gives a neater expression. All the derivations in Section III can also be done using $\mathbf{I} - \mathbf{u}\mathbf{v}^\dagger + z^{-1}\mathbf{u}\mathbf{v}^\dagger$ as the building block. Though all LUTs are factorable, as we will show in Section III, there are higher order unimodular matrices that cannot be decomposed into degree-one building block. One such example was given in [5]

$$\begin{pmatrix} 1 & 0 \\ z^{-2} & 1 \end{pmatrix}. \tag{3}$$

It was shown [5] that the matrix in (3) is unimodular and that it cannot be decomposed into degree-one building blocks.

B. Existence of Linear Phase Unimodular FBs With Equal Filter Length

In [20], it was shown that there exist nontrivial linear-phase paraunitary FBs when the number of channels $M > 2$. Let $\mathbf{E}(z) = \sum_{i=0}^{N} \mathbf{E}_i z^{-i}$ be the polyphase matrix. The class of linear phase paraunitary FBs derived in [20] satisfy the symmetry constraint

$$\mathbf{E}_{N-i} = \mathbf{D}\mathbf{E}_i\mathbf{J} \tag{4}$$

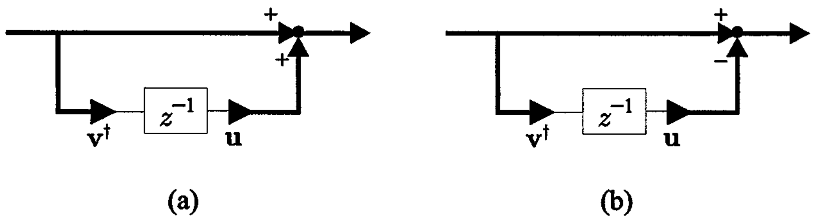

Fig. 1. Implementation of (a) a degree-one unimodular matrix $\mathbf{D}(z)$ and the inverse $\mathbf{D}^{-1}(z)$. Here, the vectors satisfy $\mathbf{v}^\dagger\mathbf{u} = 0$.

where $\mathbf{D}$ is a diagonal matrix with ± 1 diagonal entries, and $\mathbf{J}$ is the $M \times M$ reversal matrix

$$\mathbf{J} = \begin{pmatrix} 0 & \dots & 0 & 1 \\ 0 & \dots & 1 & 0 \\ \vdots & \dots & \vdots & \vdots \\ 1 & \dots & 0 & 0 \end{pmatrix}.$$

Note that all linear-phase FBs with filter length equal to kM (k integer) satisfy this constraint. When M is even, it was shown [20] that minimal factorization theorem exists for such matrices. We may ask if there are unimodular FBs satisfying the linear-phase constraint (4). The answer is, unfortunately, no. To see this, note that (4) implies that the ranks of $\mathbf{E}_N$ and $\mathbf{E}_0$ are the same. This contradicts the fact that for a unimodular FB, $\mathbf{E}_0$ is nonsingular, and $\mathbf{E}_N$ is singular. Therefore, we conclude that *there does not exist any unimodular FB satisfying the linear-phase constraint* (4).

III. Factorization of LUT

We will first derive the most general degree-one unimodular matrix and then show that all LUTs can be factorized into these building blocks. From Theorem 1, we know that the unimodular matrix $\mathbf{A}(z) = \mathbf{A}_0 + \mathbf{A}_1 z^{-1}$ can always be written as $\mathbf{A}_0[\mathbf{I} + \mathbf{P}z^{-1}]$. The matrix $\mathbf{A}(z)$ has degree one if and only if $\mathbf{P}$ has rank one. Since $\mathbf{P}$ has rank one, $\mathbf{P} = \mathbf{u}\mathbf{v}^\dagger$ for some nonzero $M \times 1$ vectors $\mathbf{u}$ and $\mathbf{v}$. From Section II-A, we know that $[\mathbf{I} + \mathbf{u}\mathbf{v}^\dagger z^{-1}]$ is unimodular if and only if $\mathbf{v}^\dagger\mathbf{u} = \mathbf{0}$. Hence, the most general degree-one unimodular matrix is a cascade of a nonsingular matrix $\mathbf{A}_0$ and a building block $\mathbf{D}(z)$ of the form

$$\mathbf{D}(z) = \mathbf{I} + \mathbf{u}\mathbf{v}^\dagger z^{-1}, \quad \mathbf{v}^\dagger\mathbf{u} = 0. \tag{5}$$

Its inverse is given by $\mathbf{D}^{-1}(z) = \mathbf{D}(-z) = \mathbf{I} - \mathbf{u}\mathbf{v}^\dagger z^{-1}$, which is also a degree-one unimodular system. The implementations of $\mathbf{D}(z)$ and its inverse using one delay are shown in Fig. 1. Using $\mathbf{D}(z)$ as a building block, we are now ready to show the factorization of LUTs.

Theorem 3: The $M \times M$ matrix $\mathbf{A}(z) = \mathbf{A}_0 + \mathbf{A}_1 z^{-1}$ is a degree-ρ LUT if and only if it can be expressed as

$$\mathbf{A}(z) = \mathbf{A}_0\mathbf{D}_0(z)\mathbf{D}_1(z)\dots\mathbf{D}_{\rho-1}(z) \tag{6}$$

where $\mathbf{D}_i(z) = \mathbf{I} + \mathbf{u}_i\mathbf{v}_i^\dagger z^{-1}$, and $\mathbf{A}_0$ is nonsingular. The vectors $\mathbf{u}_i$ and $\mathbf{v}_i$ are such that i) both $\mathbf{U} = [\mathbf{u}_0 \quad \mathbf{u}_1 \quad \dots \quad \mathbf{u}_{\rho-1}]$ and $\mathbf{V} = [\mathbf{v}_0 \quad \mathbf{v}_1 \quad \dots \quad \mathbf{v}_{\rho-1}]$ have full rank, and ii) their product satisfies (here "$\times$" denotes the don't-care term)

$$\mathbf{V}^\dagger\mathbf{U} = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 \\ \times & 0 & 0 & \dots & 0 \\ \times & \times & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \times & \times & \times & \dots & 0 \end{pmatrix}. \tag{7}$$

Proof: If $\mathbf{A}(z)$ can be expressed as the product in (6) and (7), it is not difficult to verify that it is a degree-ρ LUT. The LUT matrix will have the form of $\mathbf{A}_0(\mathbf{I} + \mathbf{U}\mathbf{V}^\dagger z^{-1})$. Suppose that $\mathbf{A}(z)$ is an LUT; then, it can be rewritten as $\mathbf{A}(z) = \mathbf{A}_0[\mathbf{I} + \mathbf{P}z^{-1}]$. As $\mathbf{A}(z)$ has degree ρ, the rank of $\mathbf{P}$ is also ρ. Therefore, there exist $M \times \rho$ full rank matrices $\widetilde{\mathbf{U}}$ and $\widetilde{\mathbf{V}}$ such that $\mathbf{P} = \widetilde{\mathbf{U}}\widetilde{\mathbf{V}}^\dagger$. From Section II-A, we know that the $\rho \times \rho$ matrix $\widetilde{\mathbf{V}}^\dagger\widetilde{\mathbf{U}}$ has all the eigenvalues equal to zero. Using Schur's unitary triangularization theorem, we can find a $\rho \times \rho$ unitary matrix $\mathbf{T}$ such that

$$\mathbf{T}^\dagger\widetilde{\mathbf{V}}^\dagger\widetilde{\mathbf{U}}\mathbf{T} = \triangle$$

for some lower triangular matrix $\triangle$ with all the diagonal elements equal to zero. Letting $\mathbf{U} = \widetilde{\mathbf{U}}\mathbf{T}$ and $\mathbf{V} = \widetilde{\mathbf{V}}\mathbf{T}$, one can verify that

$$\begin{aligned} \mathbf{I} + \mathbf{P}z^{-1} &= \mathbf{I} + \widetilde{\mathbf{U}}\widetilde{\mathbf{V}}^\dagger z^{-1} \\ &= \mathbf{I} + \mathbf{U}\mathbf{V}^\dagger z^{-1} \\ &= \left[\mathbf{I} + \mathbf{u}_0\mathbf{v}_0^\dagger z^{-1}\right] \dots \left[\mathbf{I} + \mathbf{u}_{\rho-1}\mathbf{v}_{\rho-1}^\dagger z^{-1}\right] \end{aligned}$$

where the vectors $\mathbf{u}_i$ and $\mathbf{v}_i$ are obtained from $\mathbf{U} = [\mathbf{u}_0 \quad \mathbf{u}_1 \quad \dots \quad \mathbf{u}_{\rho-1}]$ and $\mathbf{V} = [\mathbf{v}_0 \quad \mathbf{v}_1 \quad \dots \quad \mathbf{v}_{\rho-1}]$. Note that the number of degree-one building blocks in the factorization (6) is equal to ρ, which is the degree of the LUT. Hence, such a factorization is minimal. $\triangle\triangle\triangle$

From (6), the inverse of LUT can be expressed as

$$\mathbf{A}^{-1}(z) = \mathbf{D}_{\rho-1}(-z)\dots\mathbf{D}_1(-z)\mathbf{D}_0(-z)\mathbf{A}_0^{-1}.$$

Although the inverse $\mathbf{A}^{-1}(z)$ is also unimodular, its order is, in general, higher than one when $\rho > 1$. It is not difficult to show that $\mathbf{A}^{-1}(z)$ also has order one if and only if the vectors $\mathbf{u}_i$ and $\mathbf{v}_i$ satisfy

$$\mathbf{V}^\dagger\mathbf{U} = \begin{pmatrix} 0 & \times & \times & \dots & \times \\ 0 & 0 & \times & \dots & \times \\ 0 & 0 & 0 & \dots & \times \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}.$$

Combining the above result and (7), we conclude that both $\mathbf{A}(z)$ and $\mathbf{A}^{-1}(z)$ are LUTs if and only if the vectors $\mathbf{u}_i$ and $\mathbf{v}_j$ are orthogonal, that is, $\mathbf{V}^\dagger\mathbf{U} = \mathbf{0}$.

A note on the degree of cascade of unimodular systems: It is well known if we cascade two causal paraunitary matrices of degree ρ_1 and ρ_2, the resulting system is a causal paraunitary system with degree $(\rho_1 + \rho_2)$. The same is true for the class of causal FIR matrix with anticausal FIR inverse (CAFACAFI) [5]. The LOT is a member of the paraunitary family, and the BOLT belongs to the CAFACAFI class. Therefore, cascading degree-one LOT and BOLT building blocks always results in systems with higher degree. For unimodular matrices, this is no longer true. For example, if we cascade two degree-one unimodular system, namely, $\mathbf{D}(z)$ as in (5) and $\mathbf{D}(-z)$, the resulting system is the identity matrix, which has a degree of zero. Therefore, cascading more unimodular systems does not always result in an unimodular system with a higher degree. However, in the LUT case, the degree-one system $\mathbf{D}_i(z)$ in (6) cannot cancel itself since the vector sets $\{\mathbf{u}_i\}$ and $\{\mathbf{v}_i\}$ are both linearly independent sets.

Degrees of freedom: Any $M \times M$ degree-ρ LUT system is characterized by (6). The constant matrix $\mathbf{A}_0$ has M^2 elements, and the 2ρ vectors $\mathbf{u}_i$ and $\mathbf{v}_i$ have $2M\rho$ elements, but there are $0.5\rho(\rho+1)$ constraints in (7). Therefore, the degrees of freedom are given by $M^2 + 2M\rho - 0.5\rho(\rho+1)$.

Factorization using a different degree-one building block: In [5], a different degree-one unimodular system is introduced. It has the form

$$\widehat{\mathbf{D}}(z) = \mathbf{I} - \mathbf{u}\mathbf{v}^\dagger + \mathbf{u}\mathbf{v}^\dagger z^{-1}, \quad \mathbf{v}^\dagger\mathbf{u} = 0.$$

Comparing $\widehat{\mathbf{D}}(z)$ with $\mathbf{D}(z)$ in (5), one can verify that $\widehat{\mathbf{D}}(z) = (\mathbf{I} - \mathbf{u}\mathbf{v}^\dagger)\mathbf{D}(z)$. Using an approach similar to the proof of Theorem 3, one can also factorize LUTs in terms of $\widehat{\mathbf{D}}(z)$. It is not difficult to show that $\mathbf{A}(z) = \mathbf{A}_0 + \mathbf{A}_1 z^{-1}$ is a degree-ρ LUT if and only if

$$\mathbf{A}(z) = \mathbf{A}(1)\widehat{\mathbf{D}}_0(z)\widehat{\mathbf{D}}_1(z)\ldots\widehat{\mathbf{D}}_{\rho-1}(z)$$

where $\widehat{\mathbf{D}}_k(z) = \mathbf{I} - \mathbf{u}_k\mathbf{v}_k^\dagger + \mathbf{u}_k\mathbf{v}_k^\dagger z^{-1}$, and the vectors $\mathbf{u}_k$ and $\mathbf{v}_k$ satisfy the relation given in (7).

IV. Higher Order Unimodular Matrices

From Section III, we know that all first-order unimodular matrices (LUTs) are factorable. For higher order unimodular matrices, there are examples that are not factorable. For example, see the unimodular matrix in (3). In the following, we will first show that in fact, we can never capture all unimodular matrices with any set of finite-degree unimodular matrices. Then, we will show that we can capture the class of factorable unimodular matrices, not restricting to first-order, with orthogonal matrices and unit-delay lifting matrices.

A. Class of Undecomposable Unimodular Matrices

Consider the following matrix:

$$\mathbf{G}(z) = \mathbf{I} + \mathbf{a}\mathbf{b}^\dagger z^{-L}$$

where $\mathbf{b}^\dagger\mathbf{a} = 0$, and $L \geq 2$. One can verify that $\mathbf{G}(z)$ is unimodular and that its inverse is given by $\mathbf{I} - \mathbf{a}\mathbf{b}^\dagger z^{-L}$. Its degree is equal to L. We will show that $\mathbf{G}(z)$ cannot be minimally decomposed into any unimodular matrices of degree $< L$. Suppose that

$$\mathbf{G}(z) = \mathbf{G}_0(z)\mathbf{G}_1(z) \tag{8}$$

where $\mathbf{G}_0(z)$ and $\mathbf{G}_1(z)$ are unimodular matrices with degree equal to L_0 and L_1, respectively. Using minimality, we have $L_0 + L_1 = L$. To avoid triviality, we assume that $1 \leq L_0$, $L_1 \leq L - 1$. Let the inverse of $\mathbf{G}_0^{-1}(z) = \mathbf{C}_0 + \mathbf{C}_1 z^{-1} + \cdots + \mathbf{C}_k z^{-k}$ with $\mathbf{C}_k \neq \mathbf{0}$. Then, $\mathbf{G}_0^{-1}(z)$ is unimodular, and hence, its first coefficient $\mathbf{C}_0$ is nonsingular. Moreover, its order satisfies $k \leq$ degree of $\mathbf{G}_0^{-1}(z) = L_0 \leq L-1$,, where we have used the fact that for any invertible square matrix, its inverse has the same degree [13]. From (8), we have

$$\left(\mathbf{C}_0 + \mathbf{C}_1 z^{-1} + \cdots + \mathbf{C}_k z^{-k}\right)\left(\mathbf{I} + \mathbf{a}\mathbf{b}^\dagger z^{-L}\right) = \mathbf{G}_1(z).$$

As the degree of $\mathbf{G}_1(z)$ is $L_1 \leq L-1$, the order of $\mathbf{G}_1(z) \leq L-1$. From the previous equation, if we compare the coefficients of the term z^{-L}, we get

$$\mathbf{C}_0\mathbf{a} = \mathbf{0}.$$

As $\mathbf{a}$ is a nonzero vector, this contradicts the fact that $\mathbf{C}_0$ is nonsingular. Therefore, $\mathbf{G}(z)$ cannot be decomposed into any unimodular matrix of smaller degree. As L can be any integer, the matrix $\mathbf{G}(z)$ can have an arbitrary degree. We conclude that there do not exist any set of finite-order unimodular matrices that forms a building block for all unimodular matrices.

B. Parameterization of Factorable Unimodular Matrices Using Orthogonal and Unit-Delay Lifting Matrices

It is well known [1] that all paraunitary matrices can be decomposed as degree-one building blocks. They can also be parameterized in terms of orthogonal matrices and diagonal matrices with a single delay. Define the unit-delay matrix

$$\mathbf{\Lambda}(z) = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & z^{-1} \end{pmatrix}.$$

Then, all paraunitary matrices $\mathbf{E}(z)$ can be expressed as

$$\mathbf{E}(z) = \mathbf{T}_0\mathbf{\Lambda}(z)\mathbf{T}_1\mathbf{\Lambda}(z)\mathbf{T}_2\ldots\mathbf{\Lambda}(z)\mathbf{T}_J$$

for some orthogonal matrices $\mathbf{T}_i$. Similarly, it was shown ([5, Fig. 6 and (27)])[1] that all factorable CAFACAFI matrices can be expressed in the previous form with $\mathbf{T}_i$ being nonsingular matrices. In the following, we will derive a similar parameterization of factorable unimodular matrices.

Consider the degree-one building block $\mathbf{D}(z)$ in (5). By simple normalization, we can express the building block using unit-norm vectors

$$\mathbf{D}_i(z) = \mathbf{I} + \lambda_i\mathbf{u}_i\mathbf{v}_i^\dagger z^{-1} \tag{9}$$

[1]Although it was derived only for BOLT in [5], it can easily be generalized to the class of factorable CAFACAFI matrices.

where λ_i are nonzero scalars, and $\mathbf{u}_i$ and $\mathbf{v}_i$ are unit-norm vectors such that $\mathbf{v}_i^\dagger \mathbf{u}_i = 0$. As $\mathbf{u}_i$ and $\mathbf{v}_i$ are orthonormal vectors, we can always form the unitary matrix $\tilde{\mathbf{T}}_i = [\mathbf{t}_0 \quad \ldots \quad \mathbf{t}_{M-3} \quad \mathbf{v}_i \quad \mathbf{u}_i]$. Using $\tilde{\mathbf{T}}_i$, we can rewrite $\mathbf{D}_i(z)$ as

$$\begin{aligned}\mathbf{D}_i(z) =& \mathbf{I} + \lambda_i z^{-1} \tilde{\mathbf{T}}_i \\ & \times \begin{pmatrix} \mathbf{0}_{M-1\times M-2} & \mathbf{0}_{M-1\times 1} & \mathbf{0}_{M-1\times 1} \\ \mathbf{0}_{1\times M-2} & 1 & 0 \end{pmatrix} \tilde{\mathbf{T}}_i^\dagger \\ =& \tilde{\mathbf{T}}_i \theta_i(z) \tilde{\mathbf{T}}_i^\dagger\end{aligned}$$

where the matrix $\theta_i(z)$ is given by

$$\theta_i(z) = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & \cdots & \lambda_i z^{-1} & 1 \end{pmatrix}.$$

Note that $\theta_i(z)$ is an elementary row operation matrix with a delay element. Its inverse is $\theta_i(-z)$. The matrix $\theta_i(z)$ is therefore also unimodular. As $\theta_i(z)$ can be implemented using a lifting with a single delay, it will be called a unit-delay lifting matrix. Suppose that $\mathbf{A}(z)$ is a unimodular matrix that can be factorized in terms of degree-one building blocks, that is

$$\mathbf{A}(z) = \mathbf{A}_0 \mathbf{D}_0(z) \ldots \mathbf{D}_{J-1}(z). \tag{10}$$

Then, using the above derivation in (10), $\mathbf{A}(z)$ can be rewritten as the following cascade form:

$$\mathbf{A}(z) = \mathbf{T}_0 \theta_0(z) \mathbf{T}_1 \theta_1(z) \ldots \mathbf{T}_{J-1} \theta_{J-1}(z) \mathbf{T}_J \tag{11}$$

where $\mathbf{T}_0$ is a nonsingular matrix, and $\mathbf{T}_i$ for $i \geq 1$ are unitary matrices. This cascade form is shown in Fig. 2. On the other hand, if a matrix can be implemented using Fig. 2, it is clearly a unimodular matrix. Moreover, each section can be rewritten as $\tilde{\mathbf{T}}_i \theta_i(z) \tilde{\mathbf{T}}_i^\dagger$ for some unitary $\tilde{\mathbf{T}}_i$. Therefore, any unimodular matrix of the form (11) can be factorized as in (10). In other words, Fig. 2 captures all degree-J factorable unimodular matrices.

V. Lapped Transforms With FIR Inverses

In this section, we will first compare the three classes of first-order systems, namely, LOTs [1], [4], [6], BOLTs [5], and LUTs. All of these transforms are first-order matrices with FIR inverses. Then, we will show that using three different degree-one matrices as building blocks, we are able to factorize any lapped transforms having FIR inverse. A summary on factorization theorems will be given at the end of the section.

The LOTs, BOLTs and LUTs can, respectively, be factorized into the following three different degree-one building blocks:

$$\begin{aligned}\mathbf{B}_k(z) =& \mathbf{I} - \mathbf{v}_k \mathbf{v}_k^\dagger + \mathbf{v}_k \mathbf{v}_k^\dagger z^{-1}, \quad \mathbf{v}_k^\dagger \mathbf{v}_k = 1; \\ \mathbf{C}_k(z) =& \mathbf{I} - \mathbf{u}_k \mathbf{v}_k^\dagger + \mathbf{u}_k \mathbf{v}_k^\dagger z^{-1}, \quad \mathbf{u}_k^\dagger \mathbf{v}_k = 1; \\ \mathbf{D}_k(z) =& \mathbf{I} - \mathbf{u}_k \mathbf{v}_k^\dagger + \mathbf{u}_k \mathbf{v}_k^\dagger z^{-1}, \quad \mathbf{u}_k^\dagger \mathbf{v}_k = 0.\end{aligned}$$

Note that $\mathbf{B}_k(z)$, $\mathbf{C}_k(z)$, and $\mathbf{D}_k(z)$ are, respectively, degree-one LOT, BOLT, and LUT matrices. Combining our earlier results and those in [1], [4], and [5], we can conclude that the first-order degree-ρ system $\mathbf{A}(z) = \mathbf{A}_0 + \mathbf{A}_1 z^{-1}$ is

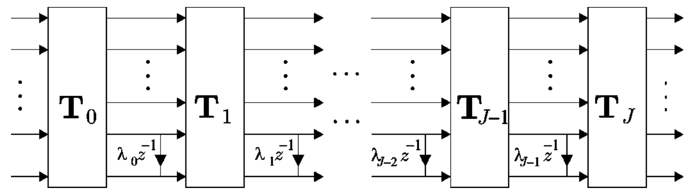

Fig. 2. Characterization of all degree-J unimodular matrices using a nonsingular matrix $\mathbf{T}_0$ and unitary matrices $\mathbf{T}_i$ $(i \geq 1)$.

1) an LOT if and only if $\mathbf{A}(z) = \mathbf{A}(1)\mathbf{B}_0(z)\mathbf{B}_1(z) \ldots \mathbf{B}_{\rho-1}(z)$, where the vectors $\mathbf{v}_k$ are such that $[\mathbf{v}_0 \quad \mathbf{v}_1 \quad \ldots \quad \mathbf{v}_{\rho-1}]^\dagger [\mathbf{v}_0 \quad \mathbf{v}_1 \quad \ldots \quad \mathbf{v}_{\rho-1}] = \mathbf{I}_\rho$ [1];
2) a BOLT if and only if $\mathbf{A}(z) = \mathbf{A}(1)\mathbf{C}_0(z)\mathbf{C}_1(z) \ldots \mathbf{C}_{\rho-1}(z)$, where the vectors $\mathbf{u}_k$ and $\mathbf{v}_k$ are such that $[\mathbf{v}_0 \quad \mathbf{v}_1 \quad \ldots \quad \mathbf{v}_{\rho-1}]^\dagger [\mathbf{u}_0 \quad \mathbf{u}_1 \quad \ldots \quad \mathbf{u}_{\rho-1}] = \triangle$ for some lower triangular matrix with all diagonal elements equal to one [5];
3) an LUT if and only if $\mathbf{A}(z) = \mathbf{A}(1)\mathbf{D}_0(z)\mathbf{D}_1(z) \ldots \mathbf{D}_{\rho-1}(z)$, where the vectors $\mathbf{u}_k$ (linearly independent) and $\mathbf{v}_k$ (linearly independent) are such that $[\mathbf{v}_0 \quad \mathbf{v}_1 \quad \ldots \quad \mathbf{v}_{\rho-1}]^\dagger [\mathbf{u}_0 \quad \mathbf{u}_1 \quad \ldots \quad \mathbf{u}_{\rho-1}] = \triangle$ for some lower triangular matrix with all diagonal elements equal to zero.

In fact, not only LOTs, BOLTs and LUTs are factorable, but all lapped transforms with FIR inverses are factorable. Consider a degree-ρ system $\mathbf{A}(z) = \mathbf{A}_0 + \mathbf{A}_1 z^{-1}$. It is well known [1] that a polynomial matrix $\mathbf{A}(z)$ has an FIR inverse if and only if $[\det \mathbf{A}(z)] = z^{-L}$ for some integer L. A complete characterization of first-order matrices having an FIR inverse was given in [5]. The authors showed that the degree-ρ lapped transform $\mathbf{A}(z) = \mathbf{A}_0 + \mathbf{A}_1 z^{-1}$ has an FIR inverse if and only if it can be expressed as

$$\mathbf{A}(z) = \mathbf{A}(1) \left[\mathbf{I} - \widetilde{\mathbf{U}} \widetilde{\mathbf{V}}^\dagger + \widetilde{\mathbf{U}} \widetilde{\mathbf{V}}^\dagger z^{-1} \right]$$

where $\widetilde{\mathbf{U}}$ and $\widetilde{\mathbf{V}}$ are $M \times \rho$ matrices such that the eigenvalues of their product $\widetilde{\mathbf{V}}^\dagger \widetilde{\mathbf{U}}$ are either one or zero. Using this result and Schur's triangularization theorem, we can find a unitary matrix $\mathbf{T}$ such that $\mathbf{T}^\dagger \widetilde{\mathbf{V}}^\dagger \widetilde{\mathbf{U}} \mathbf{T} = \triangle$, where $\triangle$ is a lower triangular matrix with its diagonal elements equal to either one or zero. Using a procedure similar to the proof of Theorem 3, one can show the following theorem.

Theorem 4: Let $\mathbf{A}(z)$ be a degree-ρ lapped transform with an FIR inverse. Then, $\mathbf{A}(z) = \mathbf{A}(1)(\mathbf{I} - \mathbf{U}\mathbf{V}^\dagger + z^{-1}\mathbf{U}\mathbf{V}^\dagger$, where $\mathbf{V} = [\mathbf{v}_0 \quad \mathbf{v}_1 \quad \ldots \quad \mathbf{v}_{\rho-1}]$ and $\mathbf{U} = [\mathbf{u}_0 \quad \mathbf{u}_1 \quad \ldots \quad \mathbf{u}_{\rho-1}]$ are such that $\mathbf{V}^\dagger \mathbf{U}$ is a lower triangular matrix with diagonal entries equal to zero or one. Moreover, $\mathbf{A}(z)$ can be decomposed as

$$\mathbf{A}(z) = \mathbf{A}(1)\mathbf{E}_0(z)\mathbf{E}_1(z) \ldots \mathbf{E}_{\rho-1}(z)$$

where

$$\mathbf{E}_k(z) = \begin{cases} \mathbf{B}_k(z), & \text{if } \mathbf{u}_k = \mathbf{v}_k \\ \mathbf{C}_k(z), & \text{if } \mathbf{v}_k^\dagger \mathbf{u}_k = 1 \\ \mathbf{D}_k(z), & \text{if } \mathbf{v}_k^\dagger \mathbf{u}_k = 0. \end{cases}$$

A summary of factorization theorems for FIR PR FBs: For the past 15 years, there has been a lot of interest in the study of the

TABLE I
SUMMARY OF FACTORIZATION THEOREM OF FBS

Class of FBs	Always Factorable?
Paraunitary FB [19]	YES
CAFACAFI FB [5]	NO
Unimodular FB [13]	NO
Time-varying Lossless FB [21]	NO
Paraunitary FB over GF(2) [22]	NO
Order-One Paraunitary FB (LOT) [19]	YES
Order-One CAFACAFI FB (BOLT) [5]	YES
LOT over GF(2) [22]	YES
Order-One Unimodular FB (LUT)	YES
Order-One FIR FB with FIR Inverse	YES

factorization theorem for FIR PR FBs. The advantage of a factorized form is that it efficiently (i.e., minimally) captures all FBs within the specific class of FB by using simple building blocks. Using the cascade structure from the factorization theorem, PR is guaranteed, and the free parameters can be designed to achieve the desired goal. The earliest minimal factorization results for FIR PR FBs was shown by Vaidyanathan. In [19], Vaidyanathan showed that paraunitary FBs can always be minimally factorized into degree-one building blocks. Since then, the factorization of various classes of FBs has been investigated by a number of researchers. In Table I, we summarize some of the results. Note that all the FBs in Table I have FIR analysis and FIR synthesis filters. Unless stated explicitly, the FBs listed in Table I are assumed to be LTI and over the real or complex field. From the table, we can conclude the following.

1) Except for the paraunitary case, FBs with arbitrary order are, in general, unfactorable. These FBs include the CAFACAFI FB [5], the unimodular FB [5], [13], the time-varying lossless FB [21], and the paraunitary FB over the finite field GF(2) [22].
2) All the listed order-one FBs (i.e., lapped transforms) are factorable. These FBs include the LOT [1], [19], the BOLT [5], the LUT, the order-one FB with FIR inverse, and the LOT over GF(2) [22].

VI. TWO EXAMPLES

In this section, we provide two examples to demonstrate potential applications of LUTs. In both examples, we use the factorization in Theorem 3 to design the vectors $\mathbf{u}_i$ and $\mathbf{v}_i$ using nonlinear optimization packages [24]. Hence, the results are not optimal, but they show that despite having a very small system delay, their performance is comparable with that of LOTs and BOLTs.

Example 1—LUT with Good Frequency Response: In this example, we apply the factorization theorem in Section III to the design of LUTs with good filters. The number of channel is $M = 8$, and the degree is $\rho = 3$. The system delay of the LUT is therefore 7. The free parameters are optimized so that the total stopband energy of the eight filters is minimized. The result is shown in Fig. 3. The filters have a stopband attenuation of at least 21.7 dB. Comparing the result with those in [5], the LOT and BOLT ($M = 8$ and $\rho = 3$) have a stopband attenuation of around 20 and 26 dB, respectively. The system delays of LOT and BOLT are 15 and 31, respectively. We see that the LUT, which has a much smaller system delay, has a better stopband attenuation than the LOT. Although BOLT is better than LUT, its system delay is significantly larger.

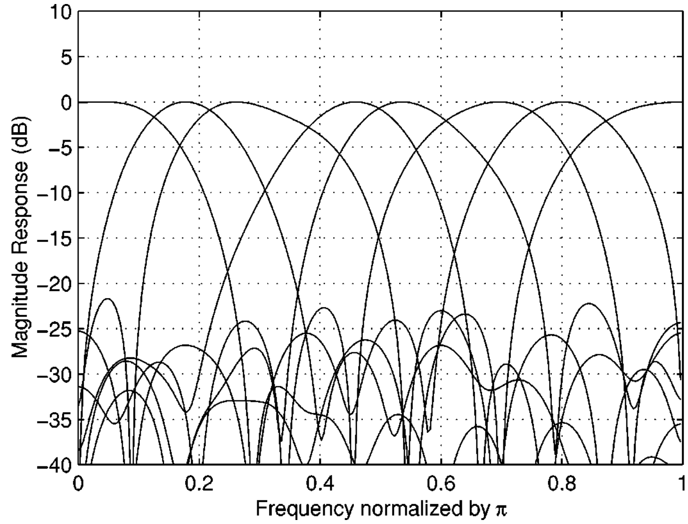

Fig. 3. Magnitude response of the analysis filters for an eight-channel LUT with degree $\rho = 3$.

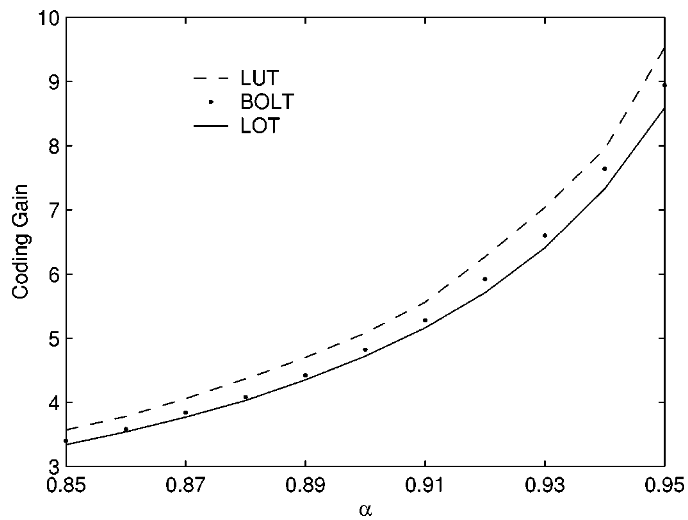

Fig. 4. Comparison of coding gain of eight-channel degree-two LUT, BOLT, and LOT for AR(1) input with correlation α.

Example 2—Coding Gain for AR(1) Processes: In this example, we compare the coding performances of LOTs, BOLTs, and LUTs for AR(1) process. The input signal $\mathbf{x}(n)$ (which is a vector of dimension M) is taken as the blocked version of a scalar AR(1) process with correlation coefficients α. The number of channels is $M = 8$, and the number of degrees is $\rho = 2$. For $0.85 \le \alpha \le 0.95$, we optimize the coding gain of LOTs, BOLTs, and LUTs.[2] The results are plotted in Fig. 4. From Fig. 4, we see that the LUTs always outperform the BOLTs, whereas the BOLTs always outperform the LOTs.

[2]Like other nonparaunitary matrices, the direct application of LUTs in subband coding will suffer from noise amplification at the synthesis end. To avoid this problem, we employ the minimum noise structure in [23], which is closely related to the closed-loop vector DPCM.

The system delays of the LUTs, BOLTs, and LOTs are, respectively, 7, 23, and 15. We see that the LUTs have the highest coding gain and the smallest system delay when the input is an AR(1) process. The gain can be substantial when the correlation coefficient α is close to 1.

VII. Conclusions

In this paper, we have shown that like LOTs and BOLTs, all LUTs can be minimally factorized. Moreover, all first-order matrices with FIR inverse can be decomposed into a cascade of degree-one LOT, degree-one BOLT, and degree-one LUT building blocks. For higher order unimodular matrices, we show that there exists a class of unimodular matrices that cannot be decomposed into any unimodular matrices with a smaller degree. This shows that there are no finite degree building blocks that can capture all unimodular matrices. Design examples show that the LUTs, which have a much smaller system delay, can achieve comparable or better performance than LOTs and BOLTs. Thus, the LUT is an attractive candidate for applications where low delay is a desired feature.

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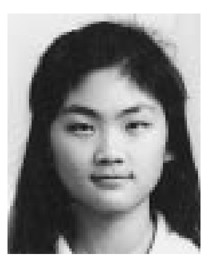

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