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Note

A note on the ultimate categorical matching in a graph ☆

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Abstract

Let $m(G)$ denote the number of vertices covered by a maximum matching in a graph G. The ultimate categorical matching $m^*(G)$ is defined as $m^*(G)=\lim_{n\to\infty} m(G^n)^{1/n}$ where the categorical graph product is used. In (Discrete Math. 232 (2001) 1), Albert et al. ask that "Is there a graph G, with at least one edge, such that for all graphs H, $m^*(G\times H)=m^*(G)m^*(H)$?". Actually, $m^*(G\times H)=m^*(G)m^*(H)$ holds for any graphs G and H with the previous result of Hsu et al. (Discrete Math. 65 (1987) 53). © 2002 Elsevier Science B.V. All rights reserved.

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For the graph definition and notation we follow [2]. An (*undirected simple*) *graph* $G=(V,E)$ consists of a finite set (*vertices*) V and a subset (*edges*) E of $\{[u,v] \mid u\neq v$, $[u,v]$ is an unordered pair of elements of $V\}$. Let $m(G)$ denote the number of vertices covered by a maximum matching in G.

Let $G=(X,E)$ and $H=(Y,F)$ be two graphs. the *categorical product* of G and H is defined as the graph $G\times H=(Z,K)$, where $Z=X\times Y$, the Cartesian product of X and Y, and edge set $K=\{[(x_1,y_1),(x_2,y_2)] \mid [x_1,x_2]\in E$ and $[y_1,y_2]\in F\}$. In [1], the *ultimate categorical matching*, $m^*(G)$, is defined as $\lim_{n\to\infty} m(G^n)^{1/n}$. In [1] Albert et al. ask that "Is there a graph G, with at least one edge, such that for all graphs H, $m^*(G\times H)=m^*(G)m^*(H)$?".

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Given a graph G, the G-matching function, γ_G, assigns to any graph H to the maximum integer k such that there are k disjoint copies of G as a subgraph of H. The *graph capacity function* for G, $P_G : \mathscr{G} \to \mathscr{R}$, is defined as $P_G(H) = \lim_{n\to\infty}[\gamma_G(H^n)]^{1/n}$. Some properties of graph capacity functions are studied [3,4,5,6,7].

It is obvious that $m(G) = 2 \times \gamma_{K_2}(G)$ for any graph G. Hence,

$$m^*(G) = \lim_{n\to\infty} [2 \times \gamma_{K_2}(G^n)]^{1/n} = \lim_{n\to\infty} [\gamma_{K_2}(G^n)]^{1/n} = P_{K_2}(G).$$

In [5], it is proved that $P_{K_2}(G \times H) = P_{K_2}(G)P_{K_2}(H)$ for any graphs G and H. Hence, $m^*(G \times H) = m^*(G)m^*(H)$ holds for any graphs G and H.

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