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# Design optimization of a current mirror amplifier integrated circuit using a computational statistics technique

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#### Abstract

In this work, we implement a computational statistics technique for design optimization of integrated circuits (ICs). Integration of a well-known circuit simulation software and central composite design method enables us to construct a second-order response surface model (RSM) for each concerned constraint. After construction of RSMs, we verify the adequacy and accuracy using the normal residual plots and their residual of squares. The constructed models are further employed for design optimization of current mirror amplifier ICs with 0.18  $\mu$ m CMOS devices. By considering the voltage gain, cut-off frequency, phase margin, common-mode rejection ratio and slew-rate, six designing parameters including the width and length of different transistors are selected and optimized to fit the targets.

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# 1. Introduction

It is known that integrated circuits (ICs) design nowadays plays a crucial role for microelectronics industry; in particular, for highly competitive consumer products [8,9,11-13,15,16,18,19,30]. In modern ICs design flow and chip implementation, IC designers perform a series of functional examination and analysis of the characteristics by circuit simulation tools to match specifications. To meet specified electrical characteristics and performance of designed product, designers in general have to tune parameters of the passive and active devices ranging from resistors, capacitors, inductors, line width, line length, to transistor size, etc. [11-16,18]. It thus requires experienced designers to accomplish such complicated works. Diverse approaches have been proposed to reduce this designing cycle, which includes numerical optimization techniques and evolutionary algorithms; and have demonstrated their merit and validity [1,2,11-16,18,20,21,24-26,29,31]. Furthermore, integration of circuit simulation tool, design of experiment [4,27], and response surface methodology may also provide a cost-effective way to advanced IC design optimization and sensitivity analysis of performance.

In this paper, a computational statistics technique for the design optimization of ICs is developed. Based on HSPICE circuit simulator [11,12,18,28], a central composite design (CCD), and a second-order response surface model (RSM),

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the circuit performances can be systematically optimized with respect to different specified constraints. The investigated current mirror amplifier IC with 0.18  $\mu$ m CMOS devices has the specifications that includes the voltage gain within 50–100 db, the cut-off frequency (FT) within 20–70 MHz, the common-mode rejection ratio (CMRR) within 60–85 db, the slew-rate (SR)+ within 20–80 V/  $\mu$ s, and the range of SR – is 20–70 V/  $\mu$ s. We firstly use the circuit simulator and the central composite design to construct the second-order response surface models. Seventy-seven experimental runs of circuit simulations are completed to generate the necessary data for construction of the quadratic response models. We notice that, for validating the constructed model, the model adequacy checking and the accuracy verification are necessary [3,5–7,20,23]. With the second-order RSMs [3,5,14,20,23], we apply optimization approaches, such as the least squares method and desirability function approach, to extract the optimal parameters to fit the specifications. In the examined current mirror amplifier IC, six parameters including the width and length of different transistors are selected and optimized to satisfy the circuit specifications. The examined results verify the usability and efficiency of the proposed method. We believe that this approach benefits the design optimization for more diverse ICs.

The rest of this paper is organized as follows. Section 2 introduces the proposed computational statistics approach. Section 3 discusses the achieved results. Conclusions are drawn in Section 4, along with recommendations for future research.

## 2. The computational statistics technique

The procedure of the proposed methodology is given by

The computational statistics approach {

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Step 1. Variables selection by screening design or empirical check
Step 2. Central composite design
Step 3. Model construction
Step 4. Accuracy and adequacy verifications
Step 5. If not satisfies the verifications in Step 4 perform transformation of the parameters, then go to Step 1
Step 6. Optimization of characteristics
Step 7. Check the results with target
If results not achieve the target, repeat Steps 1–6
Step 8. Output the optimized results
}
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The first step in our approach is variables selection by using a screening design or empirical check so that we can determine the significant designing parameters. After determining the important factors, we will execute the central composite design by running the HSPICE circuit simulator. A second-order response surface model is then established between circuit performance (i.e., responses) and circuit parameters (i.e., factors), and the optimization for the circuit design then can be followed. In the following subsections, we state the details of some steps in the proposed approach.

#### 2.1. Variables selection

Variables selection is a step to find the few significant factors from a list of many potential ones. Conventionally, we can use a screening design or empirical check to identify significant main effects, rather than interaction effects, the latter being assumed an order of magnitude less important. To determine factor's significance, two-level fractional factorial design or Plackett–Burman design is ideally suited for screening design [25]. Two-level fractional factorial design can reasonably assume that high-order interactions are negligible. We can run only a fraction of the complete factorial experiment to obtain information on the main effects and low-order interactions. For example, in one-half fraction of the  $2^3$  design ( $2^{3-1}$  design), A and BC are aliases, B and AC are aliases, C and AB are aliases, where A, B, and C are factors. When designs with resolution III, main effects are aliases with two-factor interactions and two-factor interactions may be aliased with each other. Sometimes designs with resolution IV are also used for screening designs. In this design main effects are aliased with, at worst, three-factor interactions. This is better from the confounding viewpoint, but the designs require more runs than a resolution III design.

Plackett–Burman design is two-level fractional designs [31] for studying up to k = N - 1 variables in N runs, where N is a multiple of 4. In a Plackett–Burman design, main effects are heavily confounded with two-factor interactions

in general. For example, N = 12, every main effect is partially aliased with every two-factor interaction. Each main effect is partially aliased with 45 two-factor interactions. And the plus and minus signs are

$$K = 11, \qquad N = 12 + + - + + - - - + -. \tag{1}$$

When we analyze data from screening designs, the use of an error mean square obtained by pooling high-order interactions is inappropriate occasionally. To overcome this problem a half-normal probability plot of the estimates of the effects is suggested. The half-normal plot consists of the point:

$$\left(\Phi^{-1}\left(0.5 + \frac{0.5[i - 0.5]}{I}\right), |\theta|_{(i)}|\right),\tag{2}$$

for i = 1, ..., I. The  $\Phi$  is the cumulated density function of the standard normal distribution. If factors are unimportant, the effects with mean zero and variance  $\sigma^2$  will tend to fall along a straight line on this plot, whereas important factors will not lie along the straight line [5,20,23].

In our current mirror amplifier IC, six variables are selected from eight parameters according to the empirical check. The selected parameters include the width or length of different transistors.

## 2.2. Central composite design

Response models are constructed relating the characteristics of circuits using data generated from statistical experimentation. Mathematically, response surface models may be represented as second-order polynomials:

$$Y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \beta_{ii} x_i^2 + \sum_{i=1}^k \sum_{i\neq j}^k \beta_{ij} x_i x_j + \varepsilon,$$
(3)

where k is the number of input factors,  $x_i$  is the *i*th input factor,  $\beta_i$  is the *i*th regression coefficient, and  $\varepsilon$  represents model error.

Many applications of response surface models involve constructing and checking the adequacy of a second-order model. The central composite design (CCD) is perhaps the most common experimental design used to generate second-order response models. These designs combine a two-level full factorial or fractional factorial design of  $n_f$  runs with 2k axial runs and  $n_c$  center runs, where k represents the number of control factors [5,20,23]. The axial points represent new extreme values for each factor in the design. There are three varieties of CCD which include central composite circumscribed (CCC), central composite inscribed (CCI), and face-centered cube (CCF) design [25].

Central composite designs include five input levels for each control factor  $(0; \pm 1; \pm \alpha)$ . Level 0, the nominal factor level, represents the base processing conditions. The cube levels  $(\pm 1)$  are selected to reflect the design space of interest. These values are typically set to a multiple of the factor's standard deviation or a percentage of its nominal value. The precise value of  $\alpha$  depends on certain properties desired for the design and on the number of factors involved. To maintain rotatability, the value of  $\alpha$  depends on the number of experimental runs in the factorial portion of the central composite design:

$$\alpha = [n_c]^{1/4}.\tag{4}$$

#### 2.3. Response surface model construction

It is necessary to develop an approximate model for the true response surface. If n observations are collected in an experiment, the model for them takes the form:

$$y = X\beta + \varepsilon, \tag{5}$$

where

$$y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}, \qquad X = \begin{pmatrix} 1 & x_{11} & x_{12} \cdots & x_{1k} \\ 1 & x_{21} & x_{22} \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{n1} & x_{n2} \cdots & x_{nk} \end{pmatrix}, \qquad \beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{pmatrix}, \quad \text{and} \quad \varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix}.$$

In general, y is an  $n \times 1$  vector of the observations, X is an  $n \times k$  matrix of the levels of the independent variables,  $\beta$  is a  $k \times 1$  vector of the regression coefficients, and  $\varepsilon$  is an  $n \times 1$  vector of random errors.

We want to find the least squares estimators,  $\hat{\beta}$ , that minimizes

$$L = \sum_{i=1}^{n} \varepsilon_i^2 = \varepsilon^{\mathrm{T}} \varepsilon = (y - X\beta)^{\mathrm{T}} (y - X\beta).$$
(6)

As the result of our calculation, the least squares estimator of  $\beta$  is

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{X}^{\mathrm{T}}\boldsymbol{X})^{-1}\boldsymbol{X}^{\mathrm{T}}\boldsymbol{y}.$$
(7)

The fitted regression model is

$$\hat{y} = X\hat{\beta}.$$
(8)

The difference between the responses  $y_i$  and the fitted value  $\hat{y}_i$  is a residual, say  $e_i = y_i - \hat{y}_i$ . The vector of residual is denoted by

$$\mathbf{e} = \mathbf{y} - \hat{\mathbf{y}}.\tag{9}$$

To check the normality assumption, we prepare a normal probability plot of the residual values. If the assumption holds, this plot will resemble a straight line. If the assumption is violated, a non-linear data transformation (e.g., y' = log(y)) may be applied and new models are generated in an attempt to improve model adequacy [23]. A second plot showing the residual values versus the predicted response values is used to verify if the variance of the original observation is constant. A random scattering of the residual values indicates that no correlation exists between the observed variance and the mean level of the response [5,14,20].

To develop an estimator of this parameter, we consider the sum of squares of the residuals, say

$$SS_E = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{n} e_i^2 = e^T e.$$
 (10)

Eq. (10) is called the error or residual of squares, and it has n - k degrees of freedom associated with it. It can be shown that

$$E(SS_E) = \sigma^2(n-k), \tag{11}$$

so an unbiased estimator of  $\sigma^2$  is given by

$$\hat{\sigma}^2 = \frac{\mathrm{SS}_\mathrm{E}}{n-k}.$$

To determine if there is a linear relationship between the response variable *y* and a subset of the regressor variables  $x_1, x_2, \ldots, x_k$  we test for significance of regression. The appropriate hypotheses are [23]:

$$H_0: \quad \beta_1 = \beta_2 = \dots = \beta_k = 0, \qquad H_1: \quad \beta_j \neq 0 \text{ for at least one } j. \tag{13}$$

If we reject  $H_0$ , it implies that at least one of the regressor variables  $x_1, x_2, \ldots, x_k$  contributes significantly to the model. The test procedure involves partitioning the total sum of squares due to residual:

$$SS_{T} = SS_{R} + SS_{E}.$$
(14)

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where  $SS_R$  is the sum of squares due to regression. A relatively simple procedure is performed to check for model significance in relation to random error. This test involves calculating the test statistic:

$$F_{0} = \frac{\mathrm{MS}_{\mathrm{R}}}{\mathrm{MS}_{\mathrm{E}}} = \frac{\mathrm{SS}_{\mathrm{R}}/k}{\mathrm{SS}_{\mathrm{E}}/(n-k-1)} = \frac{(1/k)\sum_{j=1}^{n} (\hat{y}_{i} - \bar{y})^{2}}{(1/(n-k-1))\sum_{j=1}^{n} (y_{i} - \hat{y}_{i})^{2}},$$
(15)

where  $\bar{y}$  is the average of measured response values.  $y_i$ ,  $\hat{y}_i$ , and *n* are the *i*th measured response, the *i*th predicted response, and the number of simulated runs, respectively [23]. If this statistic exceeds the corresponding value of the *F* distribution value ( $F_{\alpha,k,n-k-1}$ ), the response model is considered significant in relation to random error.

A second statistic, the coefficient of multiple determination  $R^2$  is defined as

$$R^{2} = \frac{SS_{R}}{SS_{T}} = 1 - \frac{SS_{E}}{SS_{T}} = 1 - \frac{\sum_{i=1}^{n} (y_{i} - \hat{y}_{i})^{2}}{\sum_{i=1}^{n} (y_{i} - \bar{y})^{2}}.$$
(16)

 $R^2$  measures the amount of reduction in variability of the response *y* achieved, using the input factors  $x_1, x_2, \ldots, x_k$ . From Eq. (14) we see that  $R^2$  varies from zero to one [5,14,20,23]. However, a large value of  $R^2$  does not necessarily imply that the regression model is good one. Adding a variable to the model will always increase  $R^2$ , regardless of whether the additional variable is statistically significant or not. About this problem, some regression model builders prefer to use an adjusted  $R^2$  statistic defined as

$$R_{\rm adj}^2 = 1 - \frac{\mathrm{SS}_{\mathrm{E}}/(n-k-1)}{\mathrm{SS}_{\mathrm{T}}/(n-1)} = 1 - \frac{n-1}{n-k-1}(1-R^2). \tag{17}$$

In general, the adjusted  $R^2$  statistic will not always increase as variables are added to the model. In fact, if unnecessary terms are added, the value of  $R^2_{adi}$  will often decrease.

#### 2.4. Optimization of characteristics

After the construction of models, we can use several techniques, such as the normality assumption and plot of residuals versus predicted value, to verify the adequacy of the response surface model. We further apply some numerical methods to perform the design optimization of our circuits. A useful approach to optimization of multiple responses is to use the simultaneous optimization technique popularized by Derringer and Suich [32]. Their procedure makes use of desirability functions. The general approach is to first convert each response  $y_i$  into an individual desirability function  $d_i$  that varies over the range  $0 \le d_i \le 1$ , where if the response are transformed into individual desirabilities, the individual desirabilities are then combined using geometric mean to maximize the overall desirability D:

$$D = (d_1 \times d_2 \times \dots \times d_m)^{1/m},$$
(18)

where *m* is the number of responses [22]. By the equation, if any  $d_i$  is equal to zero, then the overall desirability is zero.

According to the specification for the responses, response is to be maximized, minimized, or achieved a target value. For the *i*th response  $y_i$  is a maximum value:

$$d_{i} = \begin{cases} 0, & \hat{y}_{i} < L_{i} \\ \left(\frac{\hat{y}_{i} - L_{i}}{T_{i} - L_{i}}\right)^{s}, & L_{i} \leq \hat{y}_{i} \leq T_{i} \\ 1, & \hat{y}_{i} > T_{i} \end{cases}$$
(19)

For the response  $y_i$  is a minimum value:

$$d_{i} = \begin{cases} 1, & \hat{y}_{i} < T_{i} \\ \left(\frac{U_{i} - \hat{y}_{i}}{U_{i} - T_{i}}\right)^{s}, & T_{i} \le \hat{y}_{i} \le U_{i} \\ 0, & \hat{y}_{i} > U_{i} \end{cases}$$
(20)

For the response is achieved a target value:

$$d_{i} = \begin{cases} 0, & \hat{y}_{i} < L_{i} \\ \left(\frac{\hat{y}_{i} - L_{i}}{T_{i} - L_{i}}\right)^{s}, & L_{i} \leq \hat{y}_{i} \leq T_{i} \\ \left(\frac{U_{i} - \hat{y}_{i}}{U_{i} - T_{i}}\right)^{t}, & T_{i} \leq \hat{y}_{i} \leq U_{i} \\ 0, & \hat{y}_{i} > U_{i} \end{cases}$$
(21)

where the weight *s* and *t* determine how important it is close to the target value. When the weight s = 1, t = 1 the desirability function is linear. Choosing s > 1, t > 1 means more important to close the target value with the function that is concave, and choosing 0 < s < 1, 0 < t < 1 means this less important with the function that is convex.  $L_i$ ,  $U_i$ , and  $T_i$  are the lower, upper, and target value, respectively.

## 3. Numerical results and discussion

Operational amplifier plays an indispensable role of analog integrated circuits. We explore the design optimization of a current mirror amplifier [8–10,19] with 0.18  $\mu$ m CMOS devices to examine the validity of the proposed method. Fig. 1 shows the studied current mirror amplifier circuit. In this experiment, the specifications are including the ranges of voltage gain, cut-off frequency, phase margin, common-mode rejection ratio and slew-rate; and the adjustable parameters are related to the size of transistors used in the designed IC. The gain is a measure of the circuit's ability, and it is defined as the mean ratio of the signal output to the signal input of a system. The term cut-off frequency represents a boundary in the system response. CMRR measures the tendency of the device to reject input signals. The slew-rate represents the maximum rate of change of signal at any point in a circuit. The phase margin is the additional phase required to bring the phase of the loop gain to  $-180^\circ$ . These are important factors to measure the ability and performance of the current mirror amplifier IC. According to the results of the variable selection, we selected six factors which includes width of transistor M1 (W1), width and length of M3 (W3 and L3), width and length of M7 (W7 and L7) and width of MS (Ws) for our next design, and the face-centered cube design (CCF) is also used [5,14].



Fig. 1. The explored current mirror amplifier circuit. The transistors MB and MS provide the current source for the differential amplifier. The transistors M1 and M2 form the differential amplifier, and transistors M3–M6 make up the current mirror.

Table 1 List of the residual of squares in the six response surface models

Response	Residual of squares
GAIN	0.9993
1.0/Sqrt(FT)	0.9999
PM <sup>1.57</sup>	0.9999
CMRR <sup>-2.34</sup>	0.9997
Sqrt(SR+)	0.9999
1.0/Sqrt(SR-)	0.9984

To generate the necessary data for construction of the quadratic response models, 77 experimental runs (1 center point, 12 axial points, and 26 cube points) are completed by running the HSPICE circuit simulations. We choose the CCF design from the three type of the CCD (CCC, CCI, and CCF), because the CCF design is more suitable for designing the response surface models. Following equations are the responses, and the variables, *A*, *B*, *C*, *D*, *E* and *F* are the factors of W1, W3, L3, W7, L7 and Ws, respectively:

$$GAIN = 61.3078 - 0.33473 \times E^{2} + 1.415906 \times A + 0.285844 \times A \times F + 0.013121 \times B + 0.069125$$
$$\times B \times F + 0.839219 \times C - 0.04759 \times C \times D - 0.16788 \times D + 0.240156 \times C \times E + 1.906719$$
$$\times E + 0.176656 \times C \times F - 0.87466 \times F + 0.054844 \times D \times E - 1.09336 \times A^{2} + 0.098156$$
$$\times D \times F - 0.14331 \times C^{2} - 0.23459 \times E \times F.$$
(22)

$$\frac{1.0}{\text{Sqrt(FT)}} = 0.0001651 + 8.46 \times 10^{-8} \times E^2 - 1.25 \times 10^{-5} \times A + 3.07 \times 10^{-8} \times A \times B + 5.3 \times 10^{-7} \times B + 6.96 \times 10^{-7} \times A \times C + 4.632 \times 10^{-6} \times C + 3.86 \times 10^{-8} \times A \times E + 2.344 \times 10^{-8} \times D - 5.7 \times 10^{-7} \times A \times F + 3.123 \times 10^{-7} \times E + 3.28 \times 10^{-7} \times B \times C - 2.625 \times 10^{-5} \times F - 6.8 \times 10^{-8} \times B \times F + 1.421 \times 10^{-5} \times A^2 - 5 \times 10^{-8} \times C \times E + 6.348 \times 10^{-7} \times C^2 - 2.2 \times 10^{-7} \times C \times F.$$
(23)



Fig. 2. (a) The residual normal probability plot and (b) the residuals vs. predicted plot for the response GAIN.



Fig. 3. (a) The residual normal probability plot and (b) the residuals vs. predicted plot for the response 1.0/Sqrt(FT).

- $PM^{1.57} = 721.3923 0.80638 \times E^2 55.5958 \times A 6.95119 \times A \times C 15.3572 \times B 1.8323 \times A \times E 132.616 \times C 11.4127 \times A \times F 1.42565 \times D 3.27457 \times B \times C 18.9915 \times E2.38953 \times C \times E 27.2108 \times F 4.90147 \times C \times F + 25.36517 \times A^2 0.91534 \times D \times E + 0.880197 \times B^2 0.82643 \times E \times F + 9.737562 \times C^2.$ (24)
- $\begin{aligned} \text{CMRR}^{-2.34} &= 6.66626 \times 10^{-5} + 3.713 \times 10^{-7} \times A \times D 1.0922 \times 10^{-5} \times A 8.542 \times 10^{-7} \times A \times E \\ &- 9.3073 \times 10^{-7} \times B 5.454 \times 10^{-7} \times A \times F + 2.13336 \times 10^{-6} \times C 3.928 \times 10^{-7} \\ &\times B \times C 5.7487 \times 10^{-7} \times D 1.065 \times 10^{-7} \times B \times E + 1.36801 \times 10^{-6} \times E \\ &- 3.767 \times 10^{-7} \times B \times F + 1.61471 \times 10^{-5} \times F 1.027 \times 10^{-7} \times C \times D + 1.08167 \\ &\times 10^{-7} \times B^2 + 2.357 \times 10^{-7} \times C \times E 1.1113 \times 10^{-7} \times C^2 + 7.426 \times 10^{-7} \times C \times F \end{aligned}$



Fig. 4. (a) The residual normal probability plot and (b) the residuals vs. predicted plot for the response PM<sup>1.57</sup>.



Fig. 5. (a) The residual normal probability plot and (b) the residuals vs. predicted plot for the response  $CMRR^{-2.34}$ .

$$-2.2466 \times 10^{-7} \times E^{2} - 4.531 \times 10^{-7} \times D \times F - 1.4178 \times 10^{-7} \times A \times B$$
  
+ 1.025 \times 10^{-6} \times E \times F + 3.63478 \times 10^{-7} \times A \times C. (25)

$$Sqrt(SR+) = 6498.221 + 22.16759 \times C^{2} - 15.1833 \times A - 6.28594 \times A \times C + 4.293962 \times B$$
$$- 3.11049 \times A \times F - 90.0353 \times C + 7.492849 \times B \times C + 1805.042 \times F$$
$$+ 8.133772 \times B \times F + 59.15891 \times A^{2} - 83.5258 \times C \times F.$$
(26)

 $\frac{1.0}{\text{Sqrt(SR})} = 0.000163577 - 4.3 \times 10^{-7} \times A \times B + 9.28055 \times 10^{-7} \times A + 4.58 \times 10^{-7} \times A \times C$ 

$$-6.0608 \times 10^{-7} \times B + 3.17 \times 10^{-7} \times A \times F + 2.31514 \times 10^{-6} \times C - 9.3 \times 10^{-7} \times B \times C - 3.5869 \times 10^{-5} \times F - 1 \times 10^{-6} \times B \times F + 1.44356 \times 10^{-5} \times A^{2} + 9.77 \times 10^{-7} \times C \times F.$$
(27)



Fig. 6. (a) The residual normal probability plot and (b) the residuals vs. predicted plot for the response Sqrt(SR+).



Fig. 7. (a) The residual normal probability plot and (b) the residuals vs. predicted plot for the response 1.0/Sqrt(SR-).



Fig. 8. Scatter plot calculated from the response surface model vs. values obtained from circuit simulator for (a) GAIN and (b) 1.0/Sqrt(FT).

The SR+ and SR- in Eqs. (26) and (27) are the positive and negative slew-rate. After we construct the response models, we need to verify the adequacy of the models. We notice that the transformation of FT, PM, CMRR and SR- by BOX-COX transformation or experience [22] can improve the model adequacy. Table 1 shows the residual of squares of each response surface model. Fig. 2 shows the (a) residual normal probability plot and (b) the residuals versus predicted plot for the response GAIN, and Figs. 3–7 show the model adequacy checking for the 1.0/Sqrt(FT),

Table 2

List of the range of designing parameters and the optimized parameters for the current mirror amplifier circuit

Parameter	Range	Extracted result
A: W1 (μm)	40–100	100
<i>B</i> : W3 (μm)	60-80	80
<i>C</i> : L3 (µm)	1.2–2.5	1.21
D: W7 (µm)	14.4–20	18.24
<i>E</i> : L7 (µm)	1.2–2.5	2.47
<i>F</i> : Ws (µm)	60–160	66.16



Fig. 9. Scatter plot calculated from the response surface model vs. values obtained from circuit simulator for (a) PM<sup>1.57</sup> and (b) CMRR<sup>-2.34</sup>.



 $Fig. \ 10. \ Scatter \ plot \ calculated \ from \ the \ response \ surface \ model \ vs. \ values \ obtained \ from \ circuit \ simulator \ for \ (a) \ Sqrt(SR+) \ and \ (b) \ 1.0/Sqrt(SR-).$ 

 $PM^{1.57}$ ,  $CMRR^{-2.34}$ , Sqrt(SR+), and 1.0/Sqrt(SR-). Figs. 8–10 show the scatter plots of values calculated from the response surface models versus values that obtained from the HSPICE circuit simulator for the models of all responses. The results show that there is a high linearity between actual and predicted values and thus confirm the accuracy of the constructed models. Based on the response models, we extract the factors to satisfy the targets. Table 2 shows the list of the range of designing parameters and the optimized parameters. Table 3 shows the list of targets and the extracted models.

Table 3

List of the specified targets and the extracted results for the current mirror amplifier circuit

Specification	Goal	Range	Result
Gain (db)	Maximize	50-100	61.54
FT (MHz)	Maximize	20–70	43.87
PM (°)	In the range	55-70	70
CMRR (db)	Maximize	60-85	64.01
$SR + (V/\mu s)$	Maximize	20-80	56.27
$SR-(V/\mu s)$	Maximize	20–70	39.40

results for the current mirror amplifier circuit. It shows that the extracted parameters can satisfy all the specified targets, and the results confirm the validity of the proposed method. The Design-Expert software is employed in this work, and the proposed method is now developed in our unified optimization framework.

# 4. Conclusions

In this work, we are proposed an computational statistics technique for design optimization of ICs. Based on circuit simulation and central composite design method, second-order response surface models have been constructed for IC design optimization. The current mirror amplifier IC examined in our experiment has verified the efficiency of the proposed method. The integration of circuit simulation tools, design of experiment and response surface models indicate a cost-effective way to IC design. This statistics computational approach is now embed in our unified optimization framework (UOF) [17] for more applications, and we believe that it benefits the field of ICs design optimization and engineering automation.

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