H_∞ Tracking-Based Sliding Mode Control for Uncertain Nonlinear Systems via an Adaptive Fuzzy-Neural Approach

Wei-Yen Wang, *Member, IEEE*, Mei-Lang Chan, Chen-Chien James Hsu, and Tsu-Tian Lee, *Fellow, IEEE*

***Abstract*—In this paper, a novel adaptive fuzzy-neural sliding mode controller with H_∞ tracking performance for uncertain nonlinear systems is proposed to attenuate the effects caused by unmodeled dynamics, disturbances and approximate errors. Because of the advantages of fuzzy-neural systems, which can uniformly approximate nonlinear continuous functions to arbitrary accuracy, adaptive fuzzy-neural control theory is then employed to derive the update laws for approximating the uncertain nonlinear functions of the dynamical system. Furthermore, the H_∞ tracking design technique and the sliding mode control method are incorporated into the adaptive fuzzy-neural control scheme so that the derived controller is robust with respect to unmodeled dynamics, disturbances and approximate errors. Compared with conventional methods, the proposed approach not only assures closed-loop stability, but also guarantees an H_∞ tracking performance for the overall system based on a much relaxed assumption without prior knowledge on the upper bound of the lumped uncertainties. Simulation results have demonstrated that the effect of the lumped uncertainties on tracking error is efficiently attenuated, and chattering of the control input is significantly reduced by using the proposed approach.**

***Index Terms*—Adaptive control, fuzzy-neural approximator, H_∞ tracking performance, sliding mode control, uncertain nonlinear systems.**

I. INTRODUCTION

OVER the past decade, fuzzy logic has been successfully applied to many control problems [1]–[3]. Parallel to the development of the fuzzy logic control, neural networks are also applied to several control problems [4]–[7] with satisfactory results. Because both the neural network and fuzzy logic system are universal approximators [8], [9], researches [10]–[12], [29] have been conducted to derive various fuzzy-neural controllers to obtain better control performance. Examples include an adaptive tracking control method with a radial basis function neural network (RBFNN) [13] proposed for nonlinear systems to adaptively compensate the nonlinearities of the systems, a direct and indirect adaptive control schemes using fuzzy systems and neural networks for nonlinear systems [14] to provide design algorithms for stable controllers, etc. In addition, control systems based on a fuzzy-neural control scheme are augmented with sliding mode control (SMC) [15], [16] to ensure global stability and robustness to disturbances. With the use of the adaptive fuzzy-neural control [10]–[12], [29] and the sliding mode control [17], [19], [31], two objectives can be achieved. First, modeling impression and bounded disturbance are effectively compensated. Secondly, the stability and robustness of the system can be verified.

Variable structure control with a sliding mode [36]–[39] has attracted great interest because of the essential property of the nonlinear feedback control, which has a discontinuity on one or more manifolds in the state space. It is particularly suited to the deterministic control of uncertain and nonlinear systems [36], [38]. Although sliding mode control has long being known for its capabilities in achieving robust control, however, it also suffers from large control chattering that may excite the unmodeled high frequency response of the systems due to the discontinuous switching and imperfect implementations. In general, there is a trade-off between chattering and robustness. Various controllers incorporating the sliding mode control and fuzzy control have been proposed [18]–[26] to reduce the chattering in sliding mode control. Most of the proposed methods, however, require that nonlinear functions of the dynamical system are known, which is impractical in real applications. Furthermore, sliding mode control rejects uncertainties and disturbances provided matching conditions are satisfied. The key assumption is that the matching uncertainties or disturbances are bounded, and bounds on norm of the uncertainties are available for design. However, due to the complexity of the structure of uncertainties, uncertainty bounds may not be easily obtained. Based on estimated upper bounds of the matching uncertainties, sliding mode controllers [33], [34] are proposed to guarantee asymptotic stability. However, in practical applications, the exact upper bounds of the uncertainties cannot be obtained in general. Though being solved to some extent through the abovementioned approaches, design problems for the uncertain nonlinear dynamical system are not well addressed.

To relax the assumption, we adopt the H_∞ tracking design technique [29], [32] in this paper because the lumped uncertainty is bounded rather than explicitly known. The fuzzy-neural appromximator is first used to approximate the unknown nonlinear functions of the dynamical systems through tuning by

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the derived update laws. Subsequently, the H_∞ tracking design technique and the sliding mode control method are incorporated into the adaptive fuzzy-neural control scheme to derive the control law. As a result, the overall system by using the H_∞ tracking-based adaptive fuzzy-neural sliding mode controller is robust with respect to unmodeled dynamics, disturbances, and approximate errors. Compared with conventional fuzzy sliding mode control approaches which generally require prior knowledge on the upper bound of the uncertainties, the proposed approach not only assures closed-loop stability, but also guarantees a desired H_∞ tracking performance for the overall system based on a much relaxed assumption. Moreover, control chattering inherent in conventional sliding mode control is significantly reduced by using the proposed approach.

This paper is organized as follows. Section II gives a brief description of the sliding mode control method and fuzzy-neural approximator, which form the basis to derive the H_∞ tracking-based adaptive fuzzy-neural sliding mode controller with H_∞ tracking performance in Section III. Examples are illustrated in Section IV. Conclusions are drawn in Section V.

II. Preliminaries

Consider the nth-order nonlinear dynamical system of the form

$$\dot{x}_n = f(\mathbf{x}) + g(\mathbf{x})u + d, \quad y = x_1 \tag{1}$$

where $\mathbf{x} = [x, \dot{x}, \ldots, x^{(n-1)}]^T = [x_1, x_2, \ldots, x_n]^T \in R^n$ is a vector of states which are assumed to be measurable, $u \in R$ and $y \in R$ are the control input and system output, respectively, d is the bounded external disturbance, i.e., $|d| \leq d^u, f(\mathbf{x})$ and $g(\mathbf{x})$ are smooth uncertain nonlinear functions, $g(\mathbf{x})$ is assumed strictly positive, i.e., $g(\mathbf{x}) \geq g^l > 0$. It is assumed that there exists a solution for (1) and the order of the nonlinear system (1) is known.

A. Sliding Mode Control

Sliding mode control generally assumes that $\mathbf{x}$ is measurable and that $f(\mathbf{x}), g(\mathbf{x})$ are given. Define a switching surface as

$$s = a_1 x_1 + a_2 x_2 + \cdots + a_n x_n, \quad a_n = 1 \tag{2}$$

where a_i are chosen such that $\sum_{i=1}^{n} a_i \lambda^{i-1}$ is a Hurwitz polynomial. Equation (2) implies

$$x_n = -a_1 x_1 - a_2 x_2 - \cdots - a_{n-1} x_{n-1} + s. \tag{3}$$

If $\tilde{\mathbf{x}} = [x_1 \ldots x_{n-1}]^T$, the dynamics-reduced $(n-1)$th-order system of (1) becomes

$$\dot{\tilde{\mathbf{x}}} = \mathbf{A}_1 \tilde{\mathbf{x}} + [0 \ldots 0 s]^T \tag{4}$$

where

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_1 & -a_2 & -a_3 & \cdots & -a_{n-1} \end{bmatrix}.$$

Following similar derivations in [35], we can obtain a control law for (1) by using the sliding mode control method shown in Lemma 1.

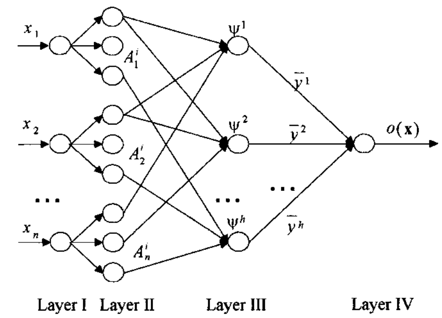

Fig. 1. Configuration of a fuzzy-neural approximator.

Lemma 1: Consider the nonlinear system (1) with given nonlinear functions $f(\mathbf{x})$ and $g(\mathbf{x})$. Suppose that control input is chosen as

$$u = \frac{-f(\mathbf{x}) - \sum_{i=1}^{n-1} a_i x_{i+1} - \sum_{i=1}^{n-1} p_{(n-1)i} x_i - k\,\mathrm{sgn}(s)}{g(\mathbf{x})} \tag{5}$$

and that $\mathbf{P} > 0, \mathbf{P} \in \mathbf{R}^{(n-1)\times(n-1)}$ satisfies the Lyapunov matrix equation

$$\mathbf{A}_1^T \mathbf{P} + \mathbf{P}\mathbf{A}_1 = -\mathbf{Q} \tag{6}$$

where s is the sliding surface defined in (2), $p_{(n-1)i}$ are elements of $\mathbf{P}, k = k_1 + d^u, k_1 > 0$, and $\mathbf{Q} > 0$ is given. Then $s \to 0$ and $\mathbf{x} \to 0$ as $t \to \infty$.

Proof: Given in Appendix A. □

In practical applications, however, $f(\mathbf{x})$ and $g(\mathbf{x})$ are generally uncertain rather than given. The controller of (5) derived in Lemma 1 is not always obtainable. Therefore, a new controller needs to be designed taking account the unknown nonlinear functions, which will be adequately approximated by a fuzzy-neural approximator.

B. Fuzzy-Neural Approximator

As shown in Fig. 1, the fuzzy-neural network [11], [12] consisting of fuzzy IF-THEN rules and a fuzzy inference engine is used as a function approximator. The fuzzy inference engine uses the fuzzy IF-THEN rules to perform a mapping from an input linguistic vector $\mathbf{x} = [x_1 x_2 \ldots x_n]^T \in \Re^n$ to an output linguistic variable $o(\mathbf{x}) \in \Re$. The lth fuzzy IF-THEN rule is written as

$$R^{(l)}\text{: if } x_1 \text{ is } A_1^l \text{ and } \cdots \text{ and } x_n \text{ is } A_n^l, \text{then } y \text{ is } B^l$$

where A_i^l and B^l are fuzzy sets with membership functions $\mu_{A_i^l}(x_i)$ and $\mu_{B^l}(y)$, respectively. By using product inference, center-average, and singleton fuzzifier, output $o(\mathbf{x})$ from the fuzzy-neural approximator can be expressed as

$$o(\mathbf{x}) = \frac{\sum_{i=1}^{h} \bar{y}^i \left(\prod_{j=1}^{n} \mu_{A_j^i}(x_j)\right)}{\sum_{i=1}^{h} \left(\prod_{j=1}^{n} \mu_{A_j^i}(x_j)\right)} = \boldsymbol{\theta}^T \boldsymbol{\psi}(\mathbf{x}) \tag{7}$$

where $\mu_{A_j^i}(x_j)$ is the membership function of the fuzzy variable x_j, h is the number of the total IF-THEN rules, and $\bar{y}^i$ is the point at which $\mu_{B^i}(\bar{y}^i) = 1$. $\boldsymbol{\theta} = [\bar{y}^1 \bar{y}^2 \ldots \bar{y}^h]^T$ is an adjustable parameter vector, and $\boldsymbol{\psi} = [\psi^1 \psi^2 \ldots \psi^h]^T$ is a fuzzy basis vector, where ψ^i is defined as

$$\psi^i(\mathbf{x}) = \frac{\left(\prod_{j=1}^n \mu_{A_j^i}(x_j)\right)}{\sum_{i=1}^h \left(\prod_{j=1}^n \mu_{A_j^i}(x_j)\right)}. \tag{8}$$

To approximate the uncertain nonlinear functions $f(\mathbf{x})$ and $g(\mathbf{x})$ in (1), adaptive update laws to adjust the parameter vector $\boldsymbol{\theta}$ in (7) of the fuzzy-neural approximator need to be developed. Let $\hat{f}(\mathbf{x})$ and $\hat{g}(\mathbf{x})$ be the estimation functions for the uncertain nonlinear functions $f(\mathbf{x})$ and $g(\mathbf{x})$, respectively. By using the fuzzy-neural approximator in (7), the estimation functions $\hat{f}(\mathbf{x})$ and $\hat{g}(\mathbf{x})$ can be obtained from the outputs of the fuzzy-neural approximator, which are defined as follows:

$$\hat{f}(\mathbf{x} \,|\, \boldsymbol{\theta}_f) = \boldsymbol{\theta}_f^T \boldsymbol{\psi}(\mathbf{x}) \tag{9}$$

and

$$\hat{g}(\mathbf{x} \,|\, \boldsymbol{\theta}_g) = \boldsymbol{\theta}_g^T \boldsymbol{\psi}(\mathbf{x}) \tag{10}$$

where $\boldsymbol{\theta}_f$ and $\boldsymbol{\theta}_g$ are adjustable parameter vectors. The fuzzy-neural approximator is valid under the following assumptions.

Assumption 1 [27]: Let $\mathbf{x}$ belongs to a compact set $\mathbf{U}_x = \{\mathbf{x} \in R^n : \|\mathbf{x}\| \le m_x < \infty\}$, and m_x is a designed parameter. It is known that optimal parameter vectors $\boldsymbol{\theta}_f^*$ and $\boldsymbol{\theta}_g^*$ lie in some convex regions

$$M_{\boldsymbol{\theta}_f} = \left\{\boldsymbol{\theta}_f \in R^h : \|\boldsymbol{\theta}_f\| \le m_{\boldsymbol{\theta}_f}\right\} \tag{11}$$

and

$$M_{\boldsymbol{\theta}_g} = \left\{\boldsymbol{\theta}_g \in R^h : \|\boldsymbol{\theta}_g\| \le m_{\boldsymbol{\theta}_g}\right\} \tag{12}$$

where the radii $m_{\boldsymbol{\theta}_g}$ and $m_{\boldsymbol{\theta}_f}$ are constants

$$\boldsymbol{\theta}_f^* = \arg \min_{\boldsymbol{\theta}_f \in M_{\boldsymbol{\theta}_f}} \left[\sup_{\mathbf{x} \in \mathbf{U}_\mathbf{x}} |f(\mathbf{x}) - \hat{f}(\mathbf{x} \,|\, \boldsymbol{\theta}_f)|\right] \tag{13}$$

and

$$\boldsymbol{\theta}_g^* = \arg \min_{\boldsymbol{\theta}_g \in M_{\boldsymbol{\theta}_g}} \left[\sup_{\mathbf{x} \in \mathbf{U}_\mathbf{x}} |g(\mathbf{x}) - \hat{g}(\mathbf{x} \,|\, \boldsymbol{\theta}_g)|\right]. \tag{14}$$

□

Assumption 2 [28]: The parameter vector $\boldsymbol{\theta}_g$ is chosen such that $\hat{g}(\mathbf{x} \,|\, \boldsymbol{\theta}_g)$ is bounded away from zero. □

Therefore, the fuzzy-neural approximator in the form of (7) can be used as a linearly parameterized approximator to approximate the uncertain nonlinear functions $f(\mathbf{x})$ and $g(\mathbf{x})$ to arbitrary accuracy [9] as Lemma 2.

Lemma 2 [9]: For any given real continuous function χ on a compact set $U \subset R^n$ and arbitrary $\varepsilon > 0$, there exists a fuzzy-neural approximator f in the form of (9) such that

$$\sup_{x \in U} |f(\mathbf{x}) - \chi(\mathbf{x})| < \varepsilon.$$

□

III. H_∞ TRACKING-BASED ADAPTIVE FUZZY-NEURAL SLIDING MODE CONTROLLER

As mentioned earlier, the controller of (5) can not be obtained by Lemma 1 if $f(\mathbf{x})$ and $g(\mathbf{x})$ are uncertain nonlinear functions. To solve this problem, the fuzzy-neural approximator is used to approximate the uncertain nonlinear functions by using the update laws derived to tune the adjustable parameter vector $\boldsymbol{\theta}$. In what follows, the H_∞ tracking design technique and the sliding mode control method are incorporated into the adaptive fuzzy-neural control scheme so as to attenuate the adverse effects caused by the unmodeled dynamics, disturbance, and approximate errors.

To approximate the uncertain nonlinear functions $f(\mathbf{x})$ and $g(\mathbf{x})$, (1) becomes

$$\begin{aligned} \dot{x}_n &= \boldsymbol{\theta}_g^{*T} \boldsymbol{\psi}_g(\mathbf{x}) u + \boldsymbol{\theta}_f^{*T} \boldsymbol{\psi}_f(\mathbf{x}) + w \\ y &= x_1 \end{aligned} \tag{15}$$

where $w = f - \boldsymbol{\theta}_f^{*T} \boldsymbol{\psi}_f + d + gu - \boldsymbol{\theta}_g^{*T} \boldsymbol{\psi}_g u$ is the lumped uncertainty. It is assumed that there exist optimal parameter estimates $\boldsymbol{\theta}_f^*, \boldsymbol{\theta}_g^*$ defined as (13) and (14), such that the approximation error is minimal. To facilitate the design process of the controller, the lumped uncertainty is generally assumed to have an upper bound.

Assumption 3: There exists a positive constant w^u, such that $|w| \le w^u$.

Based on Assumption 3, a controller which assures asymptotic stability for the uncertain nonlinear system can be obtained from Lemma 3 below.

Lemma 3: Consider the nonlinear system (1) with uncertain nonlinear functions $f(\mathbf{x})$ and $g(\mathbf{x})$, which is approximated as (15). Suppose *Assumptions* 1–3 are satisfied and control input is chosen as

$$u = \frac{-\hat{\boldsymbol{\theta}}_f^T \boldsymbol{\psi}_f - \sum_{i=1}^{n-1} a_i x_{i+1} - \sum_{i=1}^{n-1} p_{(n-1)i} x_i - k\,\mathrm{sgn}(s)}{\hat{\boldsymbol{\theta}}_g^T \boldsymbol{\psi}_g} \tag{16}$$

where $\hat{\boldsymbol{\theta}}_f$ and $\hat{\boldsymbol{\theta}}_g$ are the estimate of $\hat{\boldsymbol{\theta}}_f^*$ and $\hat{\boldsymbol{\theta}}_g^*$, respectively, and $p_{(n-1)i}$ are elements of $\mathbf{P}$ in (6), and the update laws are chosen as

$$\begin{aligned} \dot{\hat{\boldsymbol{\theta}}}_f &= \boldsymbol{\Gamma} s \boldsymbol{\psi}_f, \\ \dot{\hat{\boldsymbol{\theta}}}_g &= \boldsymbol{\Gamma} s \boldsymbol{\psi}_g u \end{aligned} \tag{17}$$

where $\boldsymbol{\Gamma} > 0$ is the adaptation gain matrix, $k = k_2 + w^u, k_2 > 0$, and s is the sliding surface defined in (2). Then $s \to 0$ and $\mathbf{x} \to 0$ as $t \to \infty$.

Proof: Given in Appendix B.

As shown in Lemma 3, $k = k_2 + w^u$ needs to be determined in advance to construct the control input u. In practical applications, however, the exact upper bound w^u cannot be obtained in general. Given that the upper bound w^u can be chosen so as to attenuate the uncertainties, large control chattering nevertheless occurs. Case 1 of the illustrative examples in this paper will show this effect for different k selected. To relax the impractical constraint, a new control law is designed by using the H_∞ tracking design technique based on a much relaxed assumption below. □

Assumption 4 [29], [32]: The lumped uncertainty is assumed such that $w \in L_2[0, T], \forall T \in [0, \infty)$.

To this end, we can proceed to introduce the main theorem to derive a control law, which guarantees an H_∞ tracking performance for the overall system without prior knowledge on the upper bound of the lumped uncertainties of the uncertain nonlinear system.

Theorem 1: Consider the nonlinear system (1) with uncertain nonlinear functions $f(\mathbf{x})$ and $g(\mathbf{x})$, which is approximated as (15). Suppose *Assumptions* 1, 2, and 4 are satisfied and control input is chosen as

$$u = \frac{-\hat{\boldsymbol{\theta}}_f^T \boldsymbol{\psi}_f - \sum_{i=1}^{n-1} a_i x_{i+1} - \sum_{i=1}^{n-1} p_{(n-1)i} x_i - \frac{s}{2\rho^2}}{\hat{\boldsymbol{\theta}}_g^T \boldsymbol{\psi}_g} \tag{18}$$

and the update laws as (17), where $\rho > 0$ is the design constant serving as an attenuation level, s is the sliding surface defined in (2), and $p_{(n-1)i}$ are elements of $\mathbf{P}$ in (6). Then the H_∞ tracking performance [29], [32] for the overall system satisfies the following relationship:

$$\begin{aligned} \frac{1}{2}\int_0^T \tilde{\mathbf{x}}^T(\tau)\mathbf{Q}\tilde{\mathbf{x}}(\tau)\,d\tau &\le \frac{1}{2}s^2(0) + \frac{1}{2}\tilde{\mathbf{x}}^T(0)\mathbf{P}\tilde{\mathbf{x}}(0) + \frac{1}{2}\tilde{\boldsymbol{\theta}}_f^T(0)\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}_f(0) \\ &\quad + \frac{1}{2}\tilde{\boldsymbol{\theta}}_{\mathrm{g}}^T(0)\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}_g(0) + \frac{1}{2}\rho^2\int_0^T w^2(\tau)\,d\tau \end{aligned} \tag{19}$$

where $\tilde{\boldsymbol{\theta}}_f = \boldsymbol{\theta}_f^* - \hat{\boldsymbol{\theta}}_f$, and $\tilde{\boldsymbol{\theta}}_g = \boldsymbol{\theta}_g^* - \hat{\boldsymbol{\theta}}_g$.

Proof: Given in Appendix C.

As shown in Theorem 1, the design constant ρ serving as an attenuation level is specified by designers during the design process. The constraint to specify an upper bound w^u of the unknown lumped uncertainties required in Lemma 3 is therefore removed. Furthermore, chattering effect of the control input is substantially reduced by using this approach, as will be demonstrated in Case 2 of the illustrative examples in this paper. The desired effect comes at no surprise because the term $k * \mathrm{sgn}(s)$ accounting for the control chattering in the control law of (16) is replaced by a much smoother term $s/(2\rho^2)$ in the derived control law of (18). □

Remark 1: If a set of initial conditions $\tilde{\mathbf{x}}(0) = \mathbf{0}, s(0) = 0, \hat{\boldsymbol{\theta}}_f(0) = \boldsymbol{\theta}_f^*(0)$ and $\hat{\boldsymbol{\theta}}_g(0) = \boldsymbol{\theta}_g^*(0)$ can be obtained, and $\mathbf{Q} = \mathbf{I}$, then control performance of the overall system satisfies

$$\frac{\|\tilde{\mathbf{x}}\|_2}{\|w\|_2} \le \rho \tag{20}$$

where $\|\tilde{\mathbf{x}}\|_2^2 = \int_0^T \tilde{\mathbf{x}}^T(\tau)\tilde{\mathbf{x}}(\tau)\,d\tau, \|w\|_2^2 = \int_0^T w^2(\tau)\,d\tau$. That is, an arbitrary attenuation level can be obtained, if ρ is adequately chosen.

Design Algorithm:

Step 1) Select control parameters $a_1, a_2, \ldots, a_{n-1}$ such that matrix $\mathbf{A}_1$ is a Hurwitz matrix. Determine $m_{\boldsymbol{\theta}_f}$ and $m_{\boldsymbol{\theta}_g}$.

Step 2) Choose an appropriate $\mathbf{Q}$ to solve the Lyapunov matrix equation (6).

Step 3) Construct membership functions of the fuzzy sets to approximate the uncertain nonlinear functions $f(\mathbf{x})$ and $g(\mathbf{x})$.

Step 4) Choose an appropriate adaptation gain matrix $\boldsymbol{\Gamma}$ to establish the Lyapunov function.

Step 5) Obtain the update laws from (17), and control laws from (16) or (18), respectively, depending on different assumptions on the lumped uncertainties.

Remark 2: This paper investigates mainly on SISO systems. However, it can be easily extended to MIMO systems via an input-output linearizaton technique [36]. A brief description on the derivations toward a similar design approach for MIMO systems is given in Appendix D.

IV. Illustrative Examples

Example 1: Consider the following nonlinear dynamical system [35]:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= f(\mathbf{x}) + u + d \end{aligned} \tag{21}$$

where $f(\mathbf{x}) = x_1 x_2$ denotes *the uncertain nonlinear function*, and $d = 0.5\sin(t)$ is the disturbance. Let the sliding surface be defined as $s = x_1 + 2x_2 + x_3$, and $\mathbf{Q} = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$. We obtain $\mathbf{P} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 1.5 \end{bmatrix}$ by solving the Lyapunov matrix equation. A set of membership functions are constructed for $f(\mathbf{x}) = x_1 x_2$ as

$$\begin{aligned} &\mu_{F_i^1}(x_i) = \exp[-(x_i+1)^2]; \quad \mu_{F_i^2}(x_i) = \exp[-(x_i+0.5)^2]; \\ &\mu_{F_i^3}(x_i) = \exp\left(-x_i^2\right); \quad \mu_{F_i^4}(x_i) = \exp[-(x_i-0.5)^2]; \\ &\mu_{F_i^5}(x_i) = \exp[-(x_i-1)^2]; \quad \text{for } i = 1, 2, 3. \end{aligned}$$

Let $\Gamma = 0.1$ I, and initial states $\mathbf{x}(0) = [1\ 0.5\ 0]$. Control laws will be derived by using Lemma 3 (Case 1) and Theorem 1 (Case 2), respectively, depending on different assumptions on the lumped uncertainties.

Case 1) Assume that the upper bound w^u of the lumped uncertainty is known, i.e., $|w| \le w^u$, and $k = k_2 + w^u$ is chosen as 0.3 and 0.5, respectively. According to Lemma 3, the control input can be obtained as $u = -\hat{\boldsymbol{\theta}}^T \boldsymbol{\psi} - x_2 - 2x_3 - (0.5x_1 + 1.5x_2) - k\,\mathrm{sgn}(s)$, with update law $\dot{\hat{\boldsymbol{\theta}}} = \Gamma s \boldsymbol{\psi}$.

Case 2) Assume that the upper bound w^u of the lumped uncertainty is unknown. The design constant ρ, which serves as an attenuation level, is chosen as 0.1 and 0.2, respectively. According to Theorem 1, the control input can be obtained as $u = -\hat{\boldsymbol{\theta}}^T \boldsymbol{\psi} - x_2 - 2x_3 - (0.5x_1 + 1.5x_2) - (s/2\rho^2)$, with update law $\dot{\hat{\boldsymbol{\theta}}} = \Gamma s \boldsymbol{\psi}$.

Figs. 2 and 3 show the time responses of the states x_i and control input u with $k = 0.3$ of Case 1 in Example 1 by using Lemma 3, assuming that the upper bound of the lumped uncertainties is available for design. As clearly demonstrated in Fig. 2, the time responses of the states are oscillatory due to the disturbance $[d = 0.5\sin(t)]$. This comes at no surprise because the improper selection of k cannot effectively suppress the disturbance. If $k = 0.5$ is selected, the time responses of the states x_i are satisfactory as shown in Fig. 4. Although the impact of the disturbance is alleviated as shown in Fig. 4 with the selection of a better $k = 0.5$, the problem of control chattering, however, becomes much serious as clearly demonstrated in Fig. 5, in comparison to that of Fig. 3. In general, the control law obtained by

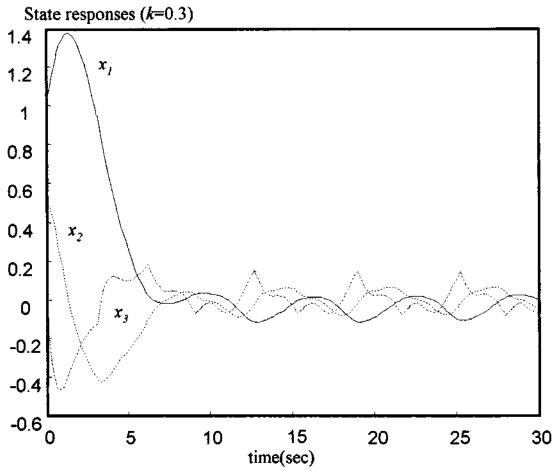

Fig. 2. State responses in Case 1 with $k = 0.3$ in Example 1.

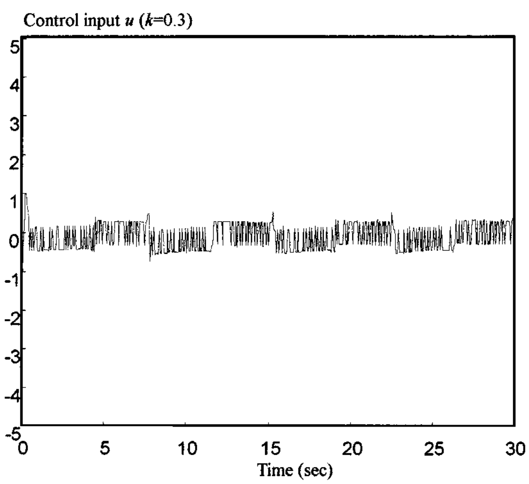

Fig. 3. Time response of the control input in Case 1 with $k = 0.3$ in Example 1.

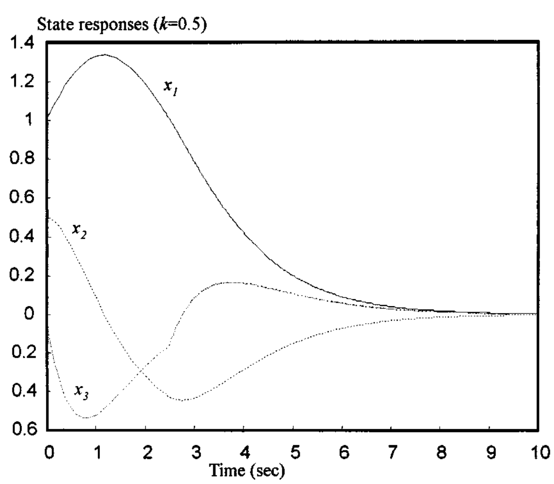

Fig. 4. State responses in Case 1 with $k = 0.5$ in Example 1.

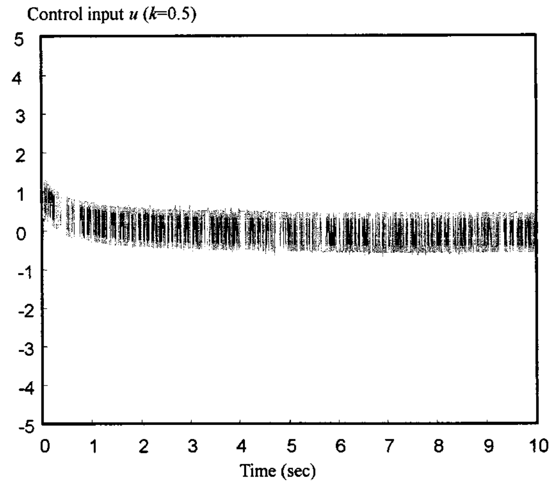

Fig. 5. Time response of the control input in Case 1 with $k = 0.5$ in Example 1.

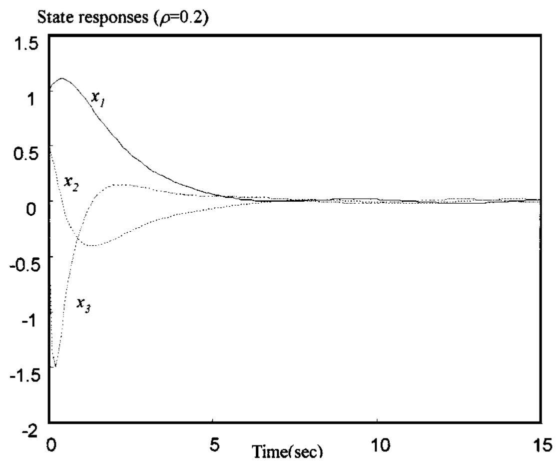

Fig. 6. State responses in Case 2 (the proposed method) with $\rho = 0.2$ in Example 1.

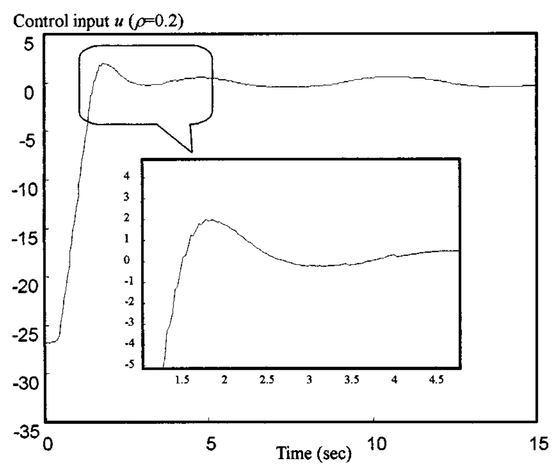

Fig. 7. Time response of the control input in Case 2 (the proposed method) with $\rho = 0.2$ in Example 1.

Lemma 3 to compensate the lumped uncertainties results in serious control chattering. In general, there is a trade-off between chattering and robustness.

On the other hand, Figs. 6 and 7 show the time responses of the states x_i and control input u with $\rho = 0.2$ of Case 2 in Example 1 by using the proposed method (Theorem 1), under Assumption 4 without prior knowledge on the upper bound of the lumped uncertainties. As demonstrated in Fig. 7, the control chattering is significantly reduced while maintaining satisfactory state responses, compared with those shown in Figs. 3 and

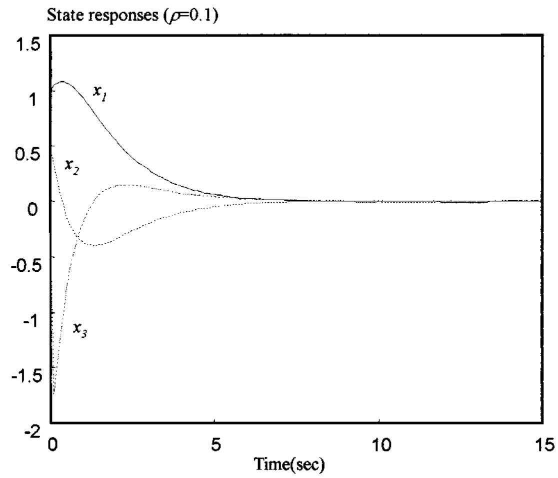

Fig. 8. State responses in Case 2 (the proposed method) with $\rho = 0.1$ in Example 1.

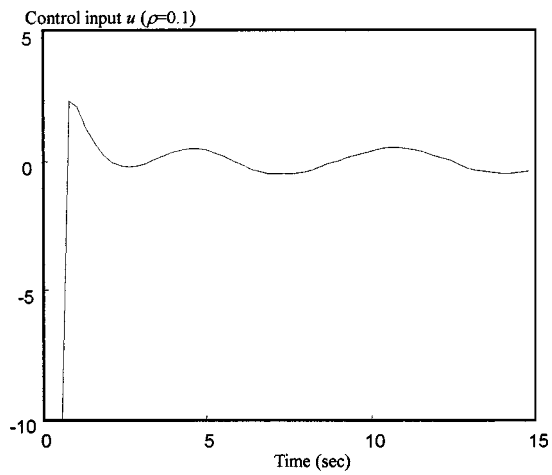

Fig. 9. Time response of the control input in Case 2 (the proposed method) with $\rho = 0.1$ in Example 1.

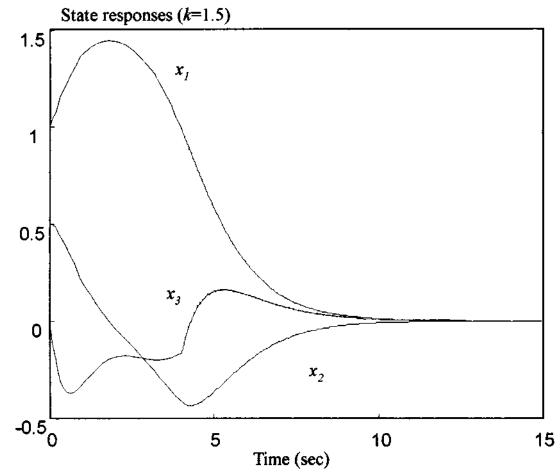

Fig. 10. State responses in Case 1 with $k = 1.5$ in Example 2.

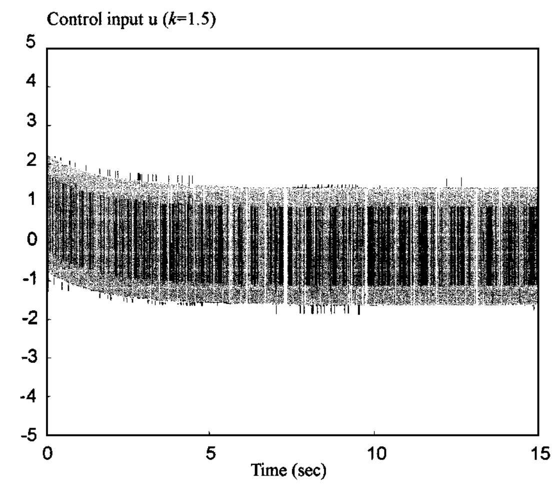

Fig. 11. Time response of the control input in Case 1 with $k = 1.5$ in Example 2.

5 with various k by Lemma 3. If the design parameter ρ, which serves as an attenuation level, is further reduced to $\rho = 0.1$, better state responses can be obtained as shown in Fig. 8. Note that the problem of control chattering does not occur as demonstrated in Fig. 9.

Example 2: Consider the uncertain nonlinear system having the same model as (21), where the uncertain nonlinear function and external disturbance are $f(\mathbf{x}) = x_1^2 + x_2^2 + x_3^2$ and $d = 0.5\sin(t)$, respectively. If the same design parameters ($k = 0.3$ or 0.5) as Case 1 in Example 1 are taken for computer simulation by Lemma 3 for this example. We find that the disturbance cannot be effectively suppressed because the upper bound of the lumped uncertainties has changed due to the change of the uncertain nonlinear function. Therefore, the previously selected parameter k can not be applied to different systems in general. It is therefore a typical trial-and-error process to determine a suitable k, which satisfies Assumption 3 that the upper bound of the lumped uncertainties is available for design as required by Lemma 3. Figs. 10 and 11 show the time responses of the states x_i and control input u with $k = 1.5$ of Case 1 in Example 2 by using Lemma 3 via a trial-and-error process. As demonstrated in Fig. 11, the problem of control chattering becomes serious in order to obtain the acceptable state responses shown in Fig. 10.

On the contrary, the proposed method (Theorem 1) uses the design parameter ρ as an attenuation level under Assumption 4 without prior knowledge on the upper bound of the lumped uncertainties. Figs. 12 and 13 show the time responses of the states x_i and control input u with $\rho = 0.1$ of Case 2 in Example 2 by using Theorem 1. As demonstrated in Fig. 13, the control chattering is significantly reduced, compared with that shown in Fig. 11, in which $k = 1.5$ is used by Lemma 3.

In summary, a design parameter ρ serving as an attenuation level, rather than an estimated upper bound, for the lumped uncertainties can be specified by the designer, so that a desired system performance can be obtained via the proposed H_∞ tracking-based adaptive fuzzy-neural sliding mode controller.

V. Conclusion

In this paper, an adaptive fuzzy-neural control scheme incorporating both the H_∞ tracking design technique and the sliding mode control method for uncertain nonlinear systems has been developed, in which a fuzzy-neural model is used to approximate the uncertain nonlinear functions of the dynamical

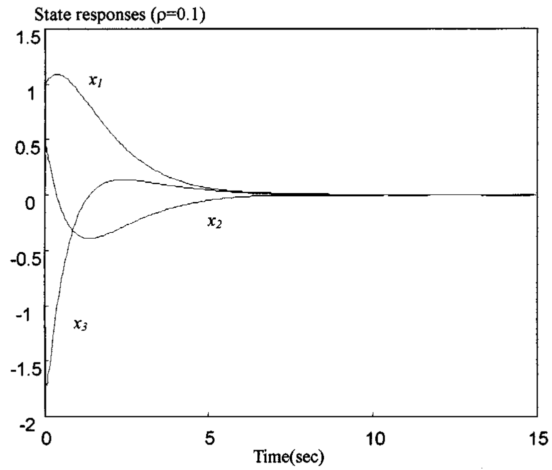

Fig. 12. State responses in Case 2 (the proposed method) with $\rho = 0.1$ in Example 2.

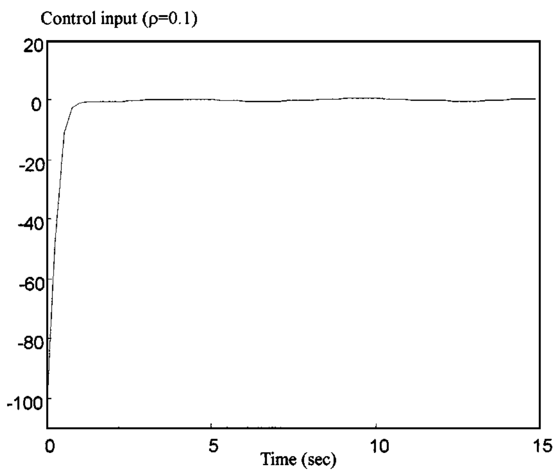

Fig. 13. Time response of the control input in Case 2 (the proposed method) with $\rho = 0.1$ in Example 2.

system. To facilitate the design process, an design algorithm, which can be computerized to derive the H_∞ tracking-based adaptive fuzzy-neural sliding mode controller for the uncertain nonlinear system, is also presented. As shown in this paper, the proposed H_∞ tracking-based adaptive fuzzy-neural sliding mode controller not only attenuates the lumped uncertainties caused by the unmodeled dynamics, disturbances, and approximate errors associated with the uncertain nonlinear system, but also significantly reduces the control chattering inherent in conventional sliding mode control. Furthermore, the constraint demanding prior knowledge on upper bounds of the lumped uncertainties is removed through the design algorithm of the proposed approach. As demonstrated in the illustrated examples, the H_∞ tracking-based adaptive fuzzy-neural sliding mode controller proposed in this paper can achieve a better control performance over the conventional methods.

APPENDIX A

Proof of Lemma 1: Consider the Lyapunov function

$$v = \frac{1}{2}s^2 + \frac{1}{2}\tilde{\mathbf{x}}^T\mathbf{P}\tilde{\mathbf{x}}. \tag{22}$$

The time derivative of (22) is

$$\begin{aligned}\dot{v} = s[a_1x_2 + a_2x_3 + \cdots + a_{n-1}x_n + f(\mathbf{x}) + g(\mathbf{x})u + d]\\ + \frac{1}{2}\tilde{\mathbf{x}}^T\left[\mathbf{PA}_1 + \mathbf{A}_1^T\mathbf{P}\right]\tilde{\mathbf{x}} + [0\dots 0s]\mathbf{P}\tilde{\mathbf{x}}.\end{aligned} \tag{23}$$

Apply (5) to (23) and let $k = k_1 + d^u, k_1 > 0$. We have the following relationship:

$$\dot{v} = -k|s| - \frac{1}{2}\tilde{\mathbf{x}}^T\mathbf{Q}\tilde{\mathbf{x}} + sd \le -k_1|s| - \frac{1}{2}\tilde{\mathbf{x}}^T\mathbf{Q}\tilde{\mathbf{x}} \le 0. \tag{24}$$

We conclude $s \to 0$ and $\tilde{\mathbf{x}} \to 0$ $(\mathbf{x} \to 0)$ as $t \to \infty$. This completes the proof. □

APPENDIX B

Proof of Lemma 3: Consider the Lyapunov function

$$v = \frac{1}{2}s^2 + \frac{1}{2}\tilde{\mathbf{x}}^T\mathbf{P}\tilde{\mathbf{x}} + \frac{1}{2}\tilde{\boldsymbol{\theta}}_f^T\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}_f + \frac{1}{2}\tilde{\boldsymbol{\theta}}_g^T\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}_g \tag{25}$$

where $\tilde{\boldsymbol{\theta}}_f = \boldsymbol{\theta}_f^* - \hat{\boldsymbol{\theta}}_f$, and $\tilde{\boldsymbol{\theta}}_g = \boldsymbol{\theta}_g^* - \hat{\boldsymbol{\theta}}_g$. The time derivative of (25) is

$$\begin{aligned}\dot{v} = s\left[\sum_{i=1}^{n-1} a_ix_{i+1} + \boldsymbol{\theta}_f^{*^T}\boldsymbol{\psi}_f + \boldsymbol{\theta}_g^{*^T}\boldsymbol{\psi}_g u + w\right]\\ + \frac{1}{2}\tilde{\boldsymbol{x}}^T\left[\mathbf{PA}_1 + \mathbf{A}_1^T\mathbf{P}\right]\tilde{\boldsymbol{x}}\\ + s\sum_{i=1}^{n-1} p_{(n-1)i}x_i - \tilde{\boldsymbol{\theta}}_f^T\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}}_f - \tilde{\boldsymbol{\theta}}_g^T\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}}_g.\end{aligned} \tag{26}$$

Apply (16) and (17) to (26) and let $k = k_2 + w^u, k_2 > 0$. We have the following relationship:

$$\begin{aligned}\dot{v} &\le -k_2|s| - \frac{1}{2}\tilde{\mathbf{x}}^T\mathbf{Q}\tilde{\mathbf{x}} - \tilde{\boldsymbol{\theta}}_f^T\boldsymbol{\Gamma}^{-1}(\dot{\hat{\boldsymbol{\theta}}}_f - \boldsymbol{\Gamma}s\boldsymbol{\psi}_f)\\ &\quad - \tilde{\boldsymbol{\theta}}_g^T\boldsymbol{\Gamma}^{-1}(\dot{\hat{\boldsymbol{\theta}}}_g - \boldsymbol{\Gamma}s\boldsymbol{\psi}_g u)\\ &\le -k_2|s| - \frac{1}{2}\tilde{\mathbf{x}}^T\mathbf{Q}\tilde{\mathbf{x}} \le 0.\end{aligned} \tag{27}$$

By using Barbalat's lemma in [30] and Theorem 2 in [11], (27) implies $s \to 0$ and $\tilde{\mathbf{x}} \to 0$ $(\mathbf{x} \to 0)$ as $t \to \infty$. This completes the proof. □

APPENDIX C

Proof of Theorem 1: Consider the Lyapunov function

$$v = \frac{1}{2}s^2 + \frac{1}{2}\tilde{\mathbf{x}}^T\mathbf{P}\tilde{\mathbf{x}} + \frac{1}{2}\tilde{\boldsymbol{\theta}}_f^T\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}_f + \frac{1}{2}\tilde{\boldsymbol{\theta}}_g^T\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}_g. \tag{28}$$

The time derivative of (28) is

$$\begin{aligned}\dot{v} = s\left[\sum_{i=1}^{n-1} a_ix_{i+1} + \boldsymbol{\theta}_f^{*^T}\boldsymbol{\psi}_f + \boldsymbol{\theta}_g^{*^T}\boldsymbol{\psi}_g u + w\right]\\ + \frac{1}{2}\tilde{\mathbf{x}}^T\left[\mathbf{PA}_1 + \mathbf{A}_1^T\mathbf{P}\right]\tilde{\mathbf{x}}\\ + s\sum_{i=1}^{n-1} p_{(n-1)i}x_i - \tilde{\boldsymbol{\theta}}_f^T\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}}_f - \tilde{\boldsymbol{\theta}}_g^T\boldsymbol{\Gamma}^{-1}\dot{\hat{\boldsymbol{\theta}}}_g.\end{aligned} \tag{29}$$

Substituting (17) and (18) into (29), we have

$$\dot{v}=\frac{-1}{2}\tilde{\mathbf{x}}^T\mathbf{Q}\tilde{\mathbf{x}}-\frac{1}{2}\left(\frac{s}{\rho}-\rho w\right)^2+\frac{1}{2}\rho^2 w^2\le\frac{-1}{2}\tilde{\mathbf{x}}^T\mathbf{Q}\tilde{\mathbf{x}}+\frac{1}{2}\rho^2 w^2. \tag{30}$$

By *Assumption 4*, we integrate (30) from $t=0$ to $t=T$, and obtain

$$\frac{1}{2}\int_0^T \tilde{\mathbf{x}}^T(\tau)\mathbf{Q}\tilde{\mathbf{x}}(\tau)\,d\tau \le v(0)+\frac{1}{2}\rho^2\int_0^T w^2(\tau)\,d\tau. \tag{31}$$

Substituting (28) into (31), we have the H_∞ tracking performance, satisfying

$$\begin{aligned}\frac{1}{2}\int_0^T &\tilde{\mathbf{x}}^T(\tau)\mathbf{Q}\tilde{\mathbf{x}}(\tau)\,d\tau\\ &\le \frac{1}{2}s^2(0)+\frac{1}{2}\tilde{\mathbf{x}}^T(0)\mathbf{P}\tilde{\mathbf{x}}(0)+\frac{1}{2}\tilde{\boldsymbol{\theta}}_f^T(0)\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}_f(0)\\ &\quad+\frac{1}{2}\tilde{\boldsymbol{\theta}}_g^T(0)\boldsymbol{\Gamma}^{-1}\tilde{\boldsymbol{\theta}}_g(0)+\frac{1}{2}\rho^2\int_0^T w^2(\tau)\,d\tau.\end{aligned} \tag{32}$$

This completes the proof. □

APPENDIX D

A. Derivations of a Similar Design Approach for MIMO Systems

According to [36], the input-output linearization of MIMO systems can be obtained by differentiating the outputs of MIMO systems, until at least one of the inputs appears. Consider the MIMO nonlinear dynamical system

$$\begin{aligned}\dot{\mathbf{z}}&=\mathbf{f}'(\mathbf{z})+\sum_{i=1}^{p}\mathbf{g}_i'(\mathbf{z})u_i+\mathbf{d}'\\ \mathbf{y}&=\mathbf{h}(\mathbf{z})\end{aligned} \tag{33}$$

where $\mathbf{z}\in\Re^n$ is a vector of states, $\mathbf{d}'\in\Re^p$ represents a vector of the external bounded disturbances, $\mathbf{u}=[u_1\ u_2\ \cdots\ u_p]^T\in\Re^p$ and $\mathbf{y}=[y_1\ y_2\ \cdots\ y_p]^T\in\Re^p$ are the control inputs and system outputs, respectively, and $\mathbf{f}'(\mathbf{z}),\mathbf{g}_i'(\mathbf{z}),i=1,2,\ldots p,$ and $\mathbf{h}(\mathbf{z})$ are unknown and smooth vector functions.

Input-output linearization of MIMO systems is obtained by differentiating the outputs $y_i, i=1,2,\ldots,p,$ until at least one of the inputs appears. We have

$$\begin{aligned}y_i^{(r_i)}=&L_{\mathbf{f}'}^{r_i}h_i+\sum_{j=1}^{p}L_{\mathbf{g}_j'}L_{\mathbf{f}'}^{r_i-1}h_iu_j+L_{\mathbf{f}'+\sum_{k=1}^p\mathbf{g}_k'u_k+\mathbf{d}'}^{r_i-1}L_{\mathbf{d}'}h_i\\ &+\sum_{j=1}^{r_i-1}L_{\mathbf{f}'+\sum_{k=1}^p\mathbf{g}_k'u_k+\mathbf{d}'}^{j-1}L_{\mathbf{d}'}L_{\mathbf{f}'}^{r_i-j}h_i,\\ &\qquad i=1,2,\ldots,p\end{aligned} \tag{34}$$

where r_i is the smallest integer such that at least one of the inputs appears in $y_i^{(r_i)}$, and the operator $L_{\mathbf{f}}(\cdot)$ denotes the Lie derivatives with respective to $\mathbf{f}$. We define $y_1=x_1, y_2=x_{r_1+1},\ldots,y_p=x_{r_1+r_2+\cdots+r_{p-1}+1}$. Then the input-output form of (33) can be described as

$$\begin{bmatrix}y_1^{(r_1)}\\ y_2^{(r_2)}\\ \vdots\\ y_p^{(r_p)}\end{bmatrix}=\mathbf{f}(\mathbf{x})+\mathbf{G}(\mathbf{x})\mathbf{u}+\mathbf{d} \tag{35}$$

where we get the equation shown at the top of the next page. Specifically, (35) can be also rewritten as

$$\begin{aligned}\dot{x}_1&=x_2\\ \dot{x}_2&=x_3\\ &\vdots\\ \dot{x}_{r_1}&=f_1(\mathbf{x})+\sum_{i=1}^{p}g_{1i}(\mathbf{x})u_i+d_1\\ \dot{x}_{r_1+1}&=x_{r_1+2}\\ \dot{x}_{r_1+2}&=x_{r_1+3}\\ &\vdots\\ \dot{x}_{r_1+r_2}&=f_2(\mathbf{x})+\sum_{i=1}^{p}g_{2i}(\mathbf{x})u_i+d_2\\ &\vdots\\ \dot{x}_{r_1+r_2+\cdots r_{p-1}+1}&=x_{r_1+r_2+\cdots r_{p-1}+2}\\ \dot{x}_{r_1+r_2+\cdots r_{p-1}+2}&=x_{r_1+r_2+\cdots r_{p-1}+3}\\ &\vdots\\ \dot{x}_{r_1+r_2+\cdots+r_p}&=f_p(\mathbf{x})+\sum_{i=1}^{p}g_{pi}(\mathbf{x})u_i+d_p\\ y_1&=x_1\\ y_2&=x_{r_1+1}\\ &\vdots\\ y_p&=x_{r_1+r_2+\cdots+r_{p-1}+1}.\end{aligned} \tag{36}$$

Equation (36) is basically a set of p SISO nonlinear dynamical systems with different order similar to (1) in the paper. Design methodology developed in the paper for SISO systems can then be applied to the MIMO system.

In order to derive a control law for the MIMO system by using the input-output linearizaton technique, the following assumptions are required.

Assumption (5): $y_i^{(k)}$ can be realizable for $i=1,2,\ldots,p,$ $k=0,1,\ldots,r_i-1$, where r_i represents the relative degree of y_i, and is finite and known. Moreover, $r_1+r_2+\cdots r_p=n$.

Assumption (6): $\mathbf{G}(\mathbf{x})$ is bounded away from singularity over a compact set $U_x\subset\Re^n$. Moreover, $\mathbf{d}$ is bounded.

Based on the above-mentioned assumptions and the design procedures proposed in the paper, similar results extended for the MIMO systems can be easily obtained. □

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$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \\ \vdots \\ f_p(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} L_{\mathbf{f}'}^{r_1} x_1 \\ L_{\mathbf{f}'}^{r_2} x_{r_1+1} \\ \vdots \\ L_{\mathbf{f}'}^{r_p} x_{r_1+r_2+\cdots+r_p+1} \end{bmatrix}$$

$$\mathbf{G}(\mathbf{x}) = \begin{bmatrix} g_{11}(\mathbf{x}) & g_{12}(\mathbf{x}) & \cdots & g_{1p}(\mathbf{x}) \\ g_{21}(\mathbf{x}) & g_{22}(\mathbf{x}) & \cdots & g_{2p}(\mathbf{x}) \\ \vdots & \vdots & \ddots & \vdots \\ g_{p1}(\mathbf{x}) & g_{p2}(\mathbf{x}) & \cdots & g_{pp}(\mathbf{x}) \end{bmatrix}$$

$$= \begin{bmatrix} L_{\mathbf{g}_1'} L_{\mathbf{f}'}^{r_1-1} x_1 & L_{\mathbf{g}_2'} L_{\mathbf{f}'}^{r_1-1} x_1 & \cdots & L_{\mathbf{g}_p'} L_{\mathbf{f}'}^{r_1-1} x_1 \\ L_{\mathbf{g}_1'} L_{\mathbf{f}'}^{r_2-1} x_2 & L_{\mathbf{g}_2'} L_{\mathbf{f}'}^{r_2-1} x_2 & \cdots & L_{\mathbf{g}_p'} L_{\mathbf{f}'}^{r_2-1} x_2 \\ \vdots & \vdots & \ddots & \vdots \\ L_{\mathbf{g}_1'} L_{\mathbf{f}'}^{r_p-1} x_p & L_{\mathbf{g}_2'} L_{\mathbf{f}'}^{r_p-1} x_p & \cdots & L_{\mathbf{g}_p'} L_{\mathbf{f}'}^{r_p-1} x_p \end{bmatrix}$$

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_p \end{bmatrix} = \begin{bmatrix} L_{\mathbf{f}'+\sum_{k=1}^{p} \mathbf{g}_k' u_k + \mathbf{d}'}^{r_1-1} L_{\mathbf{d}'} x_1 + \sum_{j=1}^{r_1-1} L_{\mathbf{f}'+\sum_{k=1}^{p} \mathbf{g}_k' u_k + \mathbf{d}'}^{j-1} L_{\mathbf{d}'} L_{\mathbf{f}'}^{r_1-j} x_1 \\ L_{\mathbf{f}'+\sum_{k=1}^{p} \mathbf{g}_k' u_k + \mathbf{d}'}^{r_2-1} L_{\mathbf{d}'} x_2 + \sum_{j=1}^{r_2-1} L_{\mathbf{f}'+\sum_{k=1}^{p} \mathbf{g}_k' u_k + \mathbf{d}'}^{j-1} L_{\mathbf{d}'} L_{\mathbf{f}'}^{r_i-j} x_2 \\ \vdots \\ L_{\mathbf{f}'+\sum_{k=1}^{p} \mathbf{g}_k' u_k + \mathbf{d}'}^{r_p-1} L_{\mathbf{d}'} x_p + \sum_{j=1}^{r_p-1} L_{\mathbf{f}'+\sum_{k=1}^{p} \mathbf{g}_k' u_k + \mathbf{d}'}^{j-1} L_{\mathbf{d}'} L_{\mathbf{f}'}^{r_p-j} x_p \end{bmatrix}.$$

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