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1 August 2002

Optics Communications 209 (2002) 167–172

OPTICS COMMUNICATIONS

www.elsevier.com/locate/optcom

Measurement of wavelength shift by using surface plasmon resonance heterodyne interferometry

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Received 8 March 2002; received in revised form 13 May 2002; accepted 30 May 2002

Abstract

A linearly polarized light is incident on a surface plasmon resonance (SPR) apparatus at the resonant angle, the surface plasmons are excited. Small wavelength shifts will introduce phase difference variations between s- and p-polarizations of the reflected light. These phase difference variations can be measured accurately by using heterodyne interferometry. Based on these facts, a novel method for measuring small wavelength shifts is proposed. It has the advantages of both common-path interferometry and heterodyne interferometry. © 2002 Elsevier Science B.V. All rights reserved.

1. Introduction

A monochromatic interferometer is used to differentiate the test wave with a reference wave in the unit of the light wavelength. It is necessary to measure the wavelength variations of a light source to insure the measurement resolution. In addition, an optical interferometric sensor technique for measuring small wavelength shift is becoming important [1–4]. A common spectrometer and an unequal-path interferometer [4] are always used to measure the wavelength shifts. Although the latter has a better resolution than the former, it needs an additional stabilized light source and a feedback control system. So it becomes more complicated and is difficult to operate.

In this paper, a novel method for measuring small wavelength shifts is presented. A linearly polarized light enters a surface plasmon resonance (SPR) [5,6] apparatus. If the incident angle is just equivalent with the resonant angle, the surface plasmons are excited. At this time, phase differences between s- and p-polarizations of the reflected light is changed with the variation of the refractive index of the thin metal film of SPR apparatus. The refractive index is related to the wavelength. And the phase difference variations can be accurately measured by using heterodyne interferometry. Based on these effects, a novel method for measuring small wavelength shifts is presented by using the specified dispersive equation of the thin metal film. It has the advantages of both common-path interferometry and heterodyne interferometry.

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PII: S0030-4018(02)01641-3

2. Principle

2.1. Phase difference resulting from reflection of SPR apparatus

A ray of light in the air is incident at θ on one side surface of the SPR [5,6] apparatus of a Krestschmann configuration [7] as shown in Fig. 1. This apparatus is an isosceles right-prism with a thin metal film of thickness d_2 deposited on the hypotenuse surface. The refractive indices of the prism and the thin metal film are n_1 and n_2, respectively. As θ equals the resonant angle θ_{sp}, surface plasmons are excited. Then the reflection coefficients of p- and s-polarization components can be expressed as [5]

$$r_q = \frac{r_{12}^q + r_{23}^q e^{i2k_{z2}d_2}}{1 + r_{12}^t r_{23}^t e^{i2k_{z2}d_2}} \quad q = \text{p}, \text{s}, \tag{1}$$

where r_{ij}^q is the Fresnel reflection coefficient between the ith and jth media and is given as

$$r_{ij}^q = \frac{X_i^q - X_j^q}{X_i^q + X_j^q}, \tag{2a}$$

and

$$X_i^q = \begin{cases} n_i^2/k_{zi} & q = \text{p}, \\ k_{zi} & q = \text{s}, \end{cases} \tag{2b}$$

where k_{zi} is the component of the wave vector in medium i in the z direction and is given as

$$k_{zi} = k_0(n_i^2 - n_1^2 \sin^2\theta)^{1/2}, \tag{2c}$$

and k_0 is the free-space wave vector. The amplitude reflection coefficients r_p and r_s can be written as

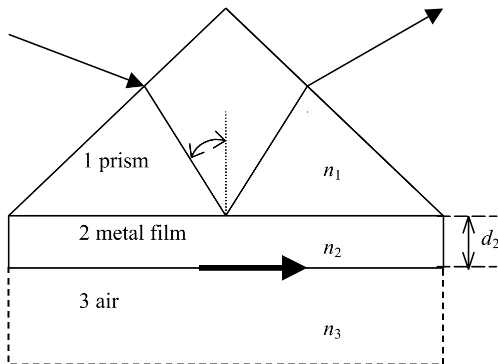

Fig. 1. Reflection in a surface plasmon resonance apparatus.

$$r_p = |r_p| e^{i\phi_p}, \quad r_s = |r_s| e^{i\phi_s}, \tag{3}$$

then the phase difference variations ϕ between p and s polarization components is

$$\phi = \phi_p - \phi_s. \tag{4}$$

It is obvious from Eqs. (1)–(4) that the phase difference is strongly dependent on n_1 and n_2. In general the dispersion equations of an absorption material are given as [8]

$$n(\lambda) = a_0 + a_1\lambda + a_2\lambda^2 + a_3\lambda^3 + a_4\lambda^4 + \cdots, \tag{5a}$$

and

$$k(\lambda) = b_0 + b_1\lambda + b_2\lambda^2 + b_3\lambda^3 + b_4\lambda^4 + \cdots, \tag{5b}$$

where n and k are the real and imaginary indices; $a_0, a_1, a_2, a_3, \ldots$, and $b_0, b_1, b_2, b_3, \ldots$ are the coefficients and λ is the wavelength. If the wavelength has small variation $\Delta\lambda$, then the variation in phase difference is

$$\Delta\phi = \left(\frac{\partial\phi(n(\lambda), k(\lambda))}{\partial\lambda} \right) \Delta\lambda. \tag{6}$$

Eq. (6) can be rewritten as

$$\Delta\lambda = \left(\frac{\partial\lambda}{\partial\phi(n(\lambda), k(\lambda))} \right) \Delta\phi. \tag{7}$$

If the thin film with specified dispersion equation is used, then it is seen that from Eqs. (1), (4), (5a), (5b) and (7) that the small wavelength variation $\Delta\lambda$ can be calculated with the measurement of the phase difference variation $\Delta\phi$.

2.2. Phase-difference measurements with heterodyne interferometry

Chiu et al. [9] proposed a method for measuring the refractive index of a transparent material by using total-internal-reflection heterodyne interferometry. A schematic diagram of the optical arrangement of our method, which is based on similar considerations, was designed and is shown in Fig. 2. A linearly polarized light passes through a half-wave plate H and its polarization plane is at α with respective to the horizontal axis. Then it passes through an electro-optic modulator (EO), and is incident at θ_{sp} upon a surface plasma resonance apparatus. The reflected light passes an

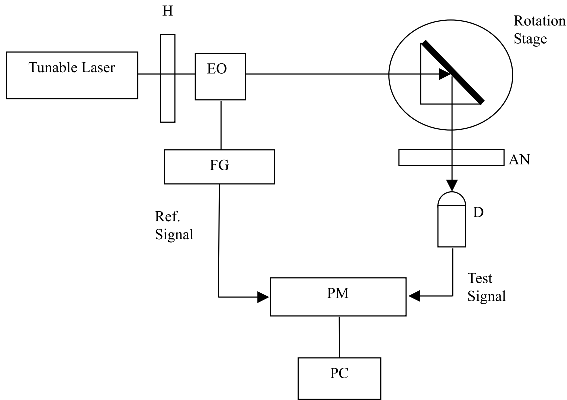

Fig. 2. Schematic diagram of measurement of the phase difference of the reflected light: H, half-wave plate; EO, electro-optic modulator; FG, function generator; PM, phase meter; AN, analyzer; D, detector; PC: personal computer.

analyzer AN with the transmission axis at 45° to the horizontal axis and is detected by a photodetector D. If the fast axis of EO under an applied electric field is in the horizontal direction and a sawtooth signal of angular frequency ω and amplitude $V_{\lambda/2}$, the half-wave voltage of the EO, is applied to the EO, the intensity measured by D can be derived as [9]

$$I_{t1} = |E_t|^2 = \frac{1}{4}\Big[r_p^2\cos^2\alpha + r_s^2\sin^2\alpha + 2r_pr_s\cos\alpha\sin\alpha\cos(\omega t + \phi_1)\Big]. \quad (8)$$

Here I_{t1} is the test signal. On the other hand, the electrical signal generated by the function generator FG is filtered and becomes the reference signal. It has the form as

$$I_r = \frac{1}{2}[1 + \cos(\omega t)]. \quad (9)$$

Both of these two sinusoidal signals are sent to the phase meter PM, then ϕ_1 can be obtained. In the second measurement let wavelength be changed to $\lambda + \Delta\lambda$, then the test signal has the form

$$I_{t2} = |E_t|^2 = \frac{1}{4}\Big[r_p^2\cos^2\alpha + r_s^2\sin^2\alpha + 2r_pr_s\cos\alpha\sin\alpha\cos(\omega t + \phi_2)\Big]. \quad (10)$$

ϕ_2 can be obtained in a similar way. Finally, by substituting the value of $\Delta\phi = \phi_2 - \phi_1$ into Eq. (7), we can evaluate the small wavelength shift $\Delta\lambda$.

3. Experiments and results

In order to show the feasibility of this method, we used an SPR apparatus with thin gold film of thickness 35 nm to measure the wavelength variation in the range 632.6 and 633.9 nm, with 633.3 nm as the initial wavelength. This apparatus consists of a BK7 glass prism and the thin gold film deposited by the commercial sputtering system (Model BA510, Balzers) with the ±1 nm thickness accuracy. To insure its quality, an ellipsometer (Model eta, Steag) was used to measure the refractive indices and the thickness of the thin gold film and the refractive index of the prism in situ. The refractive index of prism is 1.5151 and it does not change in this wavelength range. And the measured results of the thin gold film are shown in Fig. 3. If these data are substituted into Eqs. (5a) and (5b), we can obtain the coefficients of the dispersion equations by using polynomial fitting technique that is calculated by the software "Origin". They are $a_0 = 65.00949$, $a_1 = -0.36354$, $a_2 = 7.70989 \times 10^{-4}$, $a_3 = -7.3235 \times 10^{-7}$, $a_4 = 2.6242 \times 10^{-10}$, $b_0 = -14.32305$, $b_1 = -0.05643$, $b_2 = -6.13036 \times 10^{-5}$, $b_3 = 2.65412 \times 10^{-8}$, respectively. Then $\theta_{sp} = 43.9°$ can be calculated [10]. It could be achieved by using a high-resolution rotation stage with an angular resolution 0.005° (Model PS-θ-90, Japan Chuo Precision Industrial Company), and was made sure by measuring the critical minimum reflectance [5]. A phase meter with an angular resolution of 0.01° was used to measure the phase difference. A velocity laser (Model 6304, New Focus) and an EO (Mode 4002, New Focus) with a half-voltage of 125 V were used in this test. The frequency of the sawtooth signal that was applied to the EO was 1 kHz. In addition, we used a personal computer to record and analyze the data. The experimental results of the phase difference variation $\Delta\phi$ versus the wavelength shift $\Delta\lambda$ calculated with Eqs. (1), (4) and (7), are shown in Fig. 4. For comparison

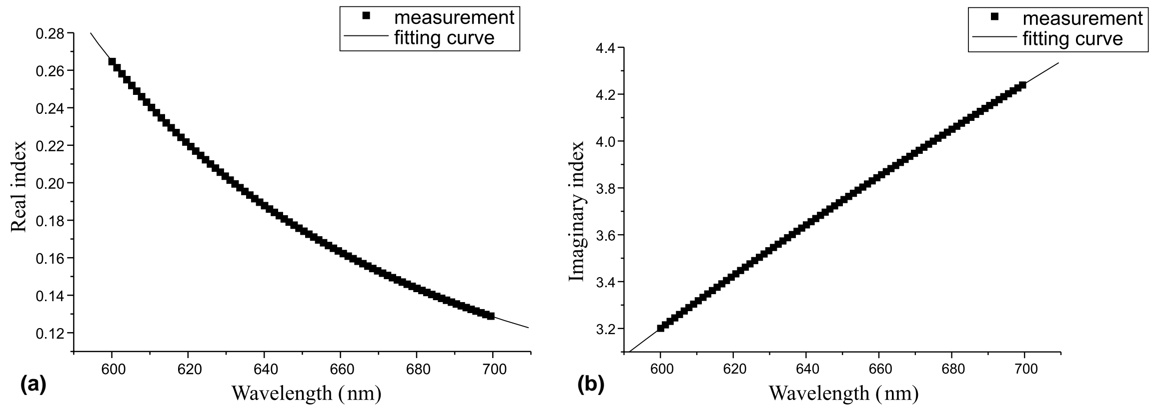

Fig. 3. Measurement results for (a) n and (b) k of gold film in the wavelength range 600–700 nm.

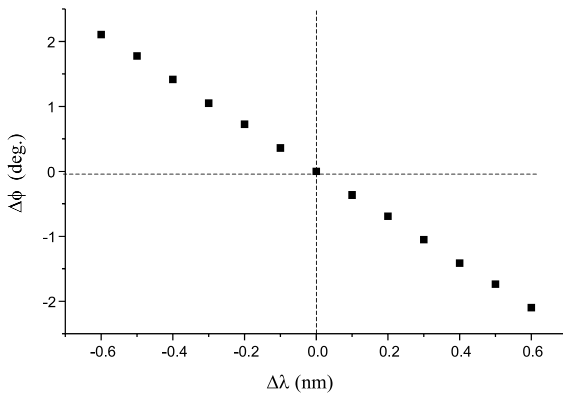

Fig. 4. Experimental results of $\Delta\phi$ versus $\Delta\lambda$.

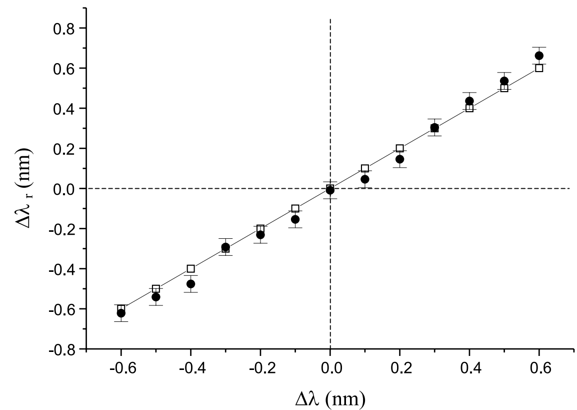

Fig. 5. Relation curve of $\Delta\lambda_r$ versus $\Delta\lambda$.

with the readout data of the wavelength shift on the controller of the velocity laser, the relation curve between the readout data $\Delta\lambda_r$ and the measured data $\Delta\lambda$ was depicted and shown in Fig. 5. In this figure, the symbols □ and • represent the readout data and the measured data, respectively. Here the quantity of the error bar is 0.04215 nm which is the standard deviation of the difference between the measured data and readout data. Because the relation curve is nearly a straight line, it can be seen that the measured data show good correspondence with the readout data. Hence it can be realized that if the SPR apparatus consisting of the thin metal film with known dispersion equations is used, as described in this method, then $\Delta\lambda$ can be estimated from the measured $\Delta\phi$.

4. Discussion

The thin gold film was deposited on BK7 prism by sputtering processes. Owing to the associated adhesion, its dispersion equation will change slightly. Although the dispersion equation of gold material could be obtained from reference book [8], it is necessary to measure its dispersion equation in situ to enhance its accuracy.

As θ equals the resonance angle θ_{sp}, the reflection coefficient r_p is very small. To enhance the contrast of the test signal, the component of p-polarization of the incident linearly polarized light should be increased. So a half-wave plate is located before the EO to rotate the azimuth angle of the polarization plane of the incident light. In our

experiments, the angle between its fast axis and the horizontal axis is set to 10°, the contrast of the test signal is about 0.88.

Angular resolution of the phase meter, second harmonic error, and polarization-mixing errors are factors that may influence the accuracy in the phase difference in this method. So the total phase difference errors $\Delta\phi_{err}$ can be decrease to 0.03° in our experiments [11]. Assuming the measured wavelength shift range $\Delta\lambda$ is 0.1 nm, then the phase difference variation is about 0.37° according to Eqs. (1)–(5b) and (7). Substituting these data into the following equation

$$\Delta\lambda_{err} = \Delta\lambda\left|\frac{\Delta\phi_{err}}{\Delta\phi}\right|, \tag{11}$$

a resolution $\Delta\lambda_{err}$ of 0.00853 nm can be obtained. If the thickness d changes and the complex refractive index of the thin gold film remains unchanged, then the relation curve of the $\Delta\lambda_{err}$ versus d can be obtained as shown in Fig. 6. Although theoretically the resolution gets better as d increases, the reflection coefficient r_p will be too small to detect. Compromising these conditions, $d = 35$ nm is chosen in our experiments.

If there are damages, such as, aging, contamination scratching, and dirt, on the thin metal film, θ_{sp} will be changed. So the condition $\theta = \theta_{sp}$ is no more valid, and the measurement resolution decreases obviously. To avoid these damages, the SPR apparatus is located into a protection box as shown with dotted lines in Fig. 1. The condition $\theta = \theta_{sp}$ can be obtained by measuring the critical minimum reflectance, and every time it should be operated in advance. In general the cross-section of the laser beam is about 4 mm^2, and the area on the thin metal film illuminated by the laser beam is nearly 7 mm^2. Within this small area, the thin metal film has a good uniformity.

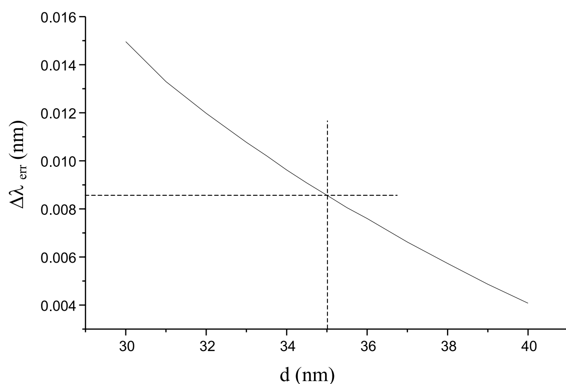

Fig. 6. Relation curve of d versus $\Delta\lambda_{err}$.

5. Conclusions

In this paper, a novel method for measuring a small wavelength shift is presented. A linearly polarized light enters a surface plasmon resonance apparatus of Krestschmann configuration. If the incident angle is exactly equivalent with the resonant angle, the surface plasmons are excited. At this time, small wavelength shifts will introduce phase difference variations between s- and p-polarizations of the reflected light. They can be measured accurately by using heterodyne interferometry. A surface plasmon resonance apparatus with thin gold thin film of thickness 35 nm was used to measure the wavelength shift in the range 632.6 and 633.9 nm at $\theta_{sp} = 43.9°$. The measured data show good correspondence with the readout data on the controller of the light source, its resolution is 0.00853 nm. It has the advantages of both common-path interferometry and heterodyne interferometry. And it may have some applications such as the determination of wavelength variations in a wavelength division multiplexing system or the measurement of the wavelength change induced in optical sensors used for monitoring gas exhausts, temperature fluctuations, and mechanical vibration [2,4].

Acknowledgement

This study was support in part by National Science Council, Taiwan, under contract NSC 89-2112-M-009-022.

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