Polarization and distribution function of the Λ_b baryon

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The polarization of the Λ_b baryon has been measured in ALEPH, OPAL, and DELPHI experiments. A significant loss on the transfer of the b quark polarization to the Λ_b baryon polarization has been noticed. This implies that the hadronization effects cannot be neglected. Therefore we may make use of the polarization measurements to look for a suitable model for the Λ_b distribution function. To investigate the Λ_b polarization, we construct four models based on a perturbative QCD factorization formula. The models are the quark model, the modified quark model, the parton model, and the modified parton model. The modified models mean the models having transverse degrees of freedom and the associated soft radiative corrections having been resummed. The quark and parton models cannot describe all experiments at the same time. On the other hand, the modified models can have the power to explain all data in the same formalism.

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I. INTRODUCTION

The polarization of bottom baryons Λ_b's has been measured by ALEPH [1], OPAL [2], and DELPHI [3]. The ALEPH data showed that the Λ_b polarization has a value $P=-0.23^{+0.24}_{-0.20}(\text{stat})\pm0.08(\text{syst})$. The OPAL data indicated the polarization $P=-0.56^{+0.20}_{-0.13}(\text{stat})\pm0.09(\text{syst})$. The DELPHI experiment gave $P=-0.49^{+0.32}_{-0.30}(\text{stat})\pm0.17(\text{syst})$. Although these three measurements are compatible with each other, the Λ_b polarization still has a wide range of value from $+0.01$ to -0.79. To improve the situation, it is better to find out a sensitive measurable quantity on the polarization. However, this is very difficult before we have a more qualitative understanding on the spin properties of Λ_b baryons. This paper is intended to understand the behind mechanisms by constructing physical models based on perturbative QCD formalism.

Measurement of a large longitudinal polarization of the Λ_b may indicate the polarization of a primary b quark produced from a Z^0 decay. The b quarks produced in the reaction $e^+e^-\to Z^0\to b\overline{b}$ are highly polarized with polarization $P=-0.94$ [4–6]. The corrections from hard gluon emissions and mass effects can change the polarization of the final state b quarks by only 3% [7,8]. The b quark can fragment into mesons and baryons. The decays of b mesons into spin zero pseudoscalar states do not retain any polarization information. The hadronization to b baryons might preserve a large fraction of the initial b quark polarization. In the heavy quark mass limit, the spin degrees of freedom of the b quark are decoupled from a spin-zero light diquark. The initial polarization of the b quark can therefore be preserved until the Λ_b decays. The higher mass b baryon states can decay into the Λ_b baryon but transfer little spin degrees of freedom. These effects have been estimated from different scenarios as about 30%. This leads to that the final Λ_b polarization could be $P=-0.6$–0.70 [9,10].

The ALEPH Collaboration measured the Λ_b polarization by employing the method suggested by Bovicini and Randall [11]. In the ratio $y(P)=\langle E_l(P)\rangle/\langle E_{\bar{\nu}}(P)\rangle$ with $\langle E_l\rangle$ and $\langle E_{\bar{\nu}}\rangle$ the average lepton and antineutrino energies in the laboratory reference frame, the fragmentation effects are largely cancelled out. Also, the spectra of the electrons and antineutrinos produced from the inclusive semileptonic decays of polarized Λ_b baryons are harder relative to the spectra of unpolarized decays. The ALEPH Collaboration proposed to measure the ratio $\mathrm{R}(P)=y(P)/y(0)$, which is a Lorentz invariant quantity. The Λ_b polarization is then extracted from a comparison between the measured ratio $\mathrm{R}^{\mathrm{ALEPH}}=1.12\pm0.10$ and a Monte Carlo simulation ratio $\mathrm{R}^{\mathrm{MC}}(P)$ with varying P. Because the Λ_b polarization is best defined in the rest frame of the Λ_b baryons, one can rewrite the ratio R in terms of the average variables in the rest frame to determine the polarization

$$P=\frac{\langle E_l^*\rangle\langle E_\nu^*\rangle(1-\mathrm{R})}{\langle E_l^*\rangle\langle P_\nu^*\rangle\mathrm{R}-\langle E_\nu^*\rangle\langle P_l^*\rangle}, \tag{1}$$

where the star variables denote the average quantities in the rest frame and are evaluated with $P=-1$. Theoretically, the values of average quantities in the above equation are model dependent. They are sensitive to the nonperturbative model employed for calculation. For example, if we apply the free quark model to calculate the star variables, we can obtain the polarization $P=-0.23$, which is closed to the ALEPH's result [1]. The situation will become interesting as we apply the same model for the DELPHI experiment. The DELPHI experiment measured the same ratio $\mathrm{R}^{\mathrm{DELPHI}}=1.21^{+0.16}_{-0.14}$ and obtained $P^{\mathrm{DELPHI}}=-0.49^{+0.32}_{-0.30}$. In the same way, it is easy to check that substituting DELPHI's ratio $\mathrm{R}^{\mathrm{DELPHI}}$ into Eq. (1) can derive a different value $P=-0.38$ in the free quark model. On the other hand, in the same model, if we employ the OPAL's polarization $P^{\mathrm{OPAL}}=-0.56$ into the ALEPH's and DELPHI's Monte Carlo simulation ratios R^{MC}, we can extract the central value of corresponding R as $\mathrm{R}_1=1.30$ and $\mathrm{R}_2=1.27$, respectively. This seems to imply that there requires more investigations to find a consistent picture for Λ_b polarization. That is we need to find a model which can explain the experiments self-consistently. The model dependence in the equation for P, such as Eq. (1), arises from the z variable $z_{l(\bar{\nu})} = \langle P^*_{l(\bar{\nu})} \rangle / \langle E^*_{l(\bar{\nu})} \rangle$. Using the z variables, Eq. (1) can be recast as

$$P = \frac{(1-\mathrm{R})}{z_{\bar{\nu}}\mathrm{R} - z_l}. \tag{2}$$

It will become clear in the following sections that different models would give different values of ratio z_l but almost the same $z_{\bar{\nu}}$ due to the characteristics of the lepton and antineutrino spectra.

In order to explore the mechanisms controlling the spin properties of polarized Λ_b baryons, we shall investigate four models. They are the (free) quark model (QM), the modified quark model (MQM), the parton model (PM), and the modified parton model (MPM). The parton model describes the probability of finding the b quark carrying a momentum fraction of the momentum of the Λ_b baryon by a parton distribution function. The quark model assumes that the Λ_b baryon contains only one b quark and two light quarks and the corresponding parton distribution function is just a delta function of the momentum fraction. This means that the b quark carries almost all the Λ_b baryon momentum. The modified quark model and the modified parton model mean that the quark model and the parton model contain an additional Sudakov form factor and transverse momentum. The Sudakov form factor arises from a resummation over radiative corrections of soft gluons and have the effects to enhance the perturbative QCD contributions.

We emphasize the importance of transverse degrees of freedom of partons inside a Λ_b baryon in our analysis. First, the transverse momenta regularize the divergences when the outgoing c quark in the process $b \to cW$ is approaching the end point. Second, the transverse momenta also enhance the contributions from the spin vector along the polarization direction. For completeness, we also introduce the intrinsic transverse momentum for the distribution function. We assume that the form of the intrinsic transverse momentum part of the parton distribution function can be parametrized as $\exp[-tM^2b^2]$ with an impact parameter b, which is the conjugate variable of the transverse momentum. The other factors are the Λ_b baryon mass M and a dimensionless parameter t. The impact parameter b will be integrated out in our perturbative QCD (PQCD) formalism. The z variables are functions of t. To determine the parameter t we rely on the experiments. The OPAL Collaboration determined the polarization by comparing the measured distribution of the ratio $E_{\bar{\nu}}/E_l$ against a simulation of this ratio using the JETSET Monte Carlo event generator. The polarization P^{OPAL} of the OPAL experiment and results of the ALEPH and DELPHI experiments will determine the range of parameter t.

The arrangement of our paper is as follows. In the next section, we shall demonstrate the factorization formula for the inclusive semileptonic decay of the Λ_b baryon. In this formula, the hard scattering amplitude, describing the short distance subprocess $b \to c l \bar{\nu}$, convolutes with a jet function and a universal soft function. For simplicity, we shall assume that the charm quark mass can be ignored. That is we shall neglect the corrections like m_c^2/M^2 with m_c the charm quark mass. This approximation is less than 10% and is safe as compared with the accuracy of the experiments. However, it requires one to consider the collinear divergences due to our ignorance of the charm quark mass. The jet function is then necessary for absorbing the collinear divergences. The universal soft function involves the b quark matrix element. The matrix element contains a large scale factor, the b quark mass M_b. To have a well established matrix element, we need to employ heavy quark effective theory (HQET) to scale out this large scale. We also need to separate the leading order matrix elements in $1/M_b$ expansion from the higher order ones. We shall develop a description for separating the leading order from the higher order mass corrections. This description is equivalent to the OPE approach. In Sec. III, we shall construct four models based on the factorization formula. Section IV gives the numerical result. The conclusion is given in Sec. V.

II. FACTORIZATION FORMULA

We shall investigate the quadruple differential decay rate for polarized Λ_b baryons, $\Lambda_b \to X_c l \bar{\nu}$,

$$\frac{d^4\Gamma}{dE_l dq^2 dq_0 d\cos\theta_l} = \frac{|V_{cb}|^2 G_F^2}{256\pi^4 M} L_{\mu\nu} W^{\mu\nu}, \tag{3}$$

where M denotes the mass of the Λ_b baryon, $L_{\mu\nu}$ represents the leptonic tensor

$$L_{\mu\nu} = \mathrm{Tr}[\not{P}_l \Gamma_\mu \not{P}_{\bar{\nu}} \Gamma_\mu], \tag{4}$$

and $W^{\mu\nu}$ means the hadronic tensor

$$W_{\mu\nu} = (2\pi)^3 \delta^{(4)}(P_{\Lambda_b} - q - P_{X_c}) \times \sum_X \langle \Lambda_b | \bar{c} \Gamma^\mu \dagger b | X_c \rangle \langle X_c | \bar{c} \Gamma^\nu b | \Lambda_b \rangle \frac{d^3\mathbf{P_{X_c}}}{(2\pi)^3 2E_{X_c}}, \tag{5}$$

where Γ^μ denotes the $V-A$ operator $\gamma^\mu(1-\gamma_5)$,$|X_c\rangle$ means the hadronic states containing a charm quark, and q is total momentum carried by the lepton and antineutrino. We choose the normalization for the Λ_b state $|\Lambda_b\rangle$ as $\langle \Lambda_b(P'_{\Lambda_b},S) | \Lambda_b(P_{\Lambda_b},S) \rangle = (2\pi)^3 (2P^0_{\Lambda_b}) \delta^3(\vec{S} - \vec{S}') \delta^3(\vec{P}_{\Lambda_b} - \vec{P}'_{\Lambda_b})$. The kinematical variables E_l, q, q_0, and $\cos\theta_l$ are expressed as follows. We choose the Λ_b baryon rest frame such that the initial Λ_b baryon momentum, P_{Λ_b}, and the final state lepton and antineutrino momenta, p_l and $p_{\bar{\nu}}$, can be defined as

$$P_{\Lambda_b} = \frac{M}{\sqrt{2}}(1,1,\mathbf{0}), p_l = (p_l^+, 0, \mathbf{0}), p_{\bar{\nu}} = (p_{\bar{\nu}}^+, p_{\bar{\nu}}^-, \mathbf{p}_{\bar{\nu}\perp}). \tag{6}$$

The variables E_l, q, and q_0 are related to $p_l^+, p_{\bar{\nu}}^+, p_{\bar{\nu}}^-$ as $E_l = p_l^+/\sqrt{2}, q^2 = 2p_l^+ p_{\bar{\nu}}^-$, and $q_0 = (p_l^+ + p_{\bar{\nu}}^+ + p_{\bar{\nu}}^-)/\sqrt{2}$. We let $P_b = P_{\Lambda_b} - l$ represent the b quark momentum whose square is set as $P_b^2 \approx M_b^2$ with M_b the b quark mass. The momentum l of the light degree of freedom of the Λ_b baryon can have a large plus component and small transverse components $\mathbf{l}_\perp$. The final state charm quark momentum is equal to $P_c = P_{\Lambda_b} - l - q$. The angle θ_l is defined as the angle between the third component of p_l and that of the b quark polarization vector, S_b. The differential decay rate can also be rewritten in terms of $E_{\bar{\nu}}, y, y_0$, and $\theta_{\bar{\nu}}$ with $E_{\bar{\nu}}$ the antineutrino energy and corresponding angle $\theta_{\bar{\nu}}$. Because the right-hand side of Eq. (3) is independent of which parametrization for the leptonic variables, we shall use both parametrizations for the differential decay rate.

It is convenient to use the scaled variables $x_{l(\bar{\nu})} = 2E_{l(\bar{\nu})}/M, y = q^2/M^2$, and $y_0 = 2q_0/M$. The integration regions for $x_{l(\bar{\nu})}, y$, and y_0 are

$$0 \leqslant x_{l(\bar{\nu})} \leqslant 1, \quad 0 \leqslant y \leqslant x_{l(\bar{\nu})},$$

$$\frac{y}{x_{l(\bar{\nu})}} + x_{l(\bar{\nu})} \leqslant y_0 \leqslant 1 + y. \tag{7}$$

Note that we have chosen M as the scale factor. For simplicity, we have chosen $m_c = 0$ and left $m_c \neq 0$ to other work. This approximation is safe as compared with the accuracy of available experiments. The leading corrections to this approximation are of order $O(m_c^2/M_b^2)$ being less than 10%.

By optical theorem, the hadronic tensor $W^{\mu\nu}$ can be related to the forward matrix element $T^{\mu\nu}$ through the formula

$$W^{\mu\nu} = -\frac{\text{Im}(T^{\mu\nu})}{\pi}. \tag{8}$$

The lowest order of $T^{\mu\nu}$ is defined as

$$T^{\mu\nu} = -i \int d^4 y e^{iq\cdot y} \langle \Lambda_b | \mathcal{T}[J^{\dagger\mu}(0), J^\nu(y)] | \Lambda_b \rangle \tag{9}$$

with $J^\mu = \bar{q}\gamma^\mu(1-\gamma_5)b$ the $V-A$ current. We have abbreviated the state vector $|\Lambda_b(P_{\Lambda_b}, q, S)\rangle$ as $|\Lambda_b\rangle$. The forward matrix element can be expressed in the momentum space

$$T^{\mu\nu} = -i \int \frac{d^4 P_b}{(2\pi)^4} \text{Tr}[S^{\mu\nu}(P_b - q) T(P_b)], \tag{10}$$

where the trace is taken over the fermion indices and color indices, the hard function $S^{\mu\nu}(P_b - q)$ describes the short distance decay subprocess, $b \to Wc$, and the soft function $T(P_b)$ denotes the long distance matrix element

$$T(P_b) = \int d^4 y e^{iP_b \cdot y} \langle \Lambda_b | \bar{b}(0) b(y) | \Lambda_b \rangle. \tag{11}$$

Because we are only interested in the leading contributions in this note, we are required to separate the leading contributions from subleading contributions. To specify the leading contributions, we also need to consider correction terms. The correction contributions may come from radiative correction terms like α_s^n and power correction terms like $1/M^m$ for $n, m \geqslant 1$. Among the radiative corrections, the contributions from soft gluons will become dominant at the end points, at which the final state quark is approaching on-shell. As discussed in the Introduction, the hard gluon emissions can only contribute about 3%. Therefore we shall retain the soft gluon contributions at the end points. To discuss the power correction terms, we need to be more careful. As investigated in the operator product expansion (OPE) and heavy quark effective theory (HQET) approach, the power corrections can have two sources; one from the short distance expansion for the forward matrix element and the other one from the heavy quark mass expansion for the expanded matrix elements. Here, we shall present a different approach in which the leading order matrix elements are in terms of nonlocal heavy quark currents composed of heavy quark effective fields in HQET.

We now demonstrate this description. To start up, we express the forward matrix element

$$\begin{aligned} iT^{\mu\nu} = & \int \frac{d^4 P_b}{(2\pi)^4} \text{Tr}[S^{\mu\nu}(P_b - q) T(P_b)] \\ & + \int \frac{d^4 P_b}{(2\pi)^4} \int \frac{d^4 P_b'}{(2\pi)^4} \text{Tr}[S_\alpha^{\mu\nu}(P_b - q, P_b' - q) \\ & \times T^\alpha(P_b, P_b')] + \cdots \end{aligned} \tag{12}$$

by including a higher order term from triple parton matrix elements containing gluon fields

$$\begin{aligned} T^\alpha(P_b, P_b') = & \int d^4 y \int d^4 z e^{iP_b \cdot y} e^{i(P_b - P_b')\cdot z} \\ & \times \langle \Lambda_b | \bar{b}(0)[-gA^\alpha(z)] b(y) | \Lambda_b \rangle \end{aligned} \tag{13}$$

with $A^\alpha(z)$ the gluon fields. The P_b and P_b' denote the outgoing and incoming b quark momenta. The hard functions $S^{\mu\nu}$ and $S_\alpha^{\mu\nu}$ have the following expressions:

$$S^{\mu\nu} = -\frac{\Gamma^\mu \not{P}_c \Gamma^\nu}{P_c^2 + i\epsilon}, \tag{14}$$

$$S_\alpha^{\mu\nu} = \frac{\Gamma^\mu \not{P}_c \gamma_\alpha \not{P}_c' \Gamma^\nu}{(P_c^2 + i\epsilon)(P_c'^2 + i\epsilon)}, \tag{15}$$

where $P_c(P_c') = P_b(P_b') - q$. We shall employ the light-cone gauge $A^+ = n \cdot A = 0$. To continue, it is useful to introduce the light-cone vectors p and n in the $+$ and $-$ directions, respectively. These two vectors satisfy properties $p^2 = n^2 = 0$ and $p \cdot n = P_{\Lambda_b} \cdot n$. The Λ_b baryon momentum P_{Λ_b} is then recast as

$$P_{\Lambda_b}^\mu = p^\mu + \frac{M^2}{2p\cdot n} n^\mu. \tag{16}$$

For the b quark inside the Λ_b baryon, we parametrize its momentum P_b as

$$P_b^\mu = zp^\mu + \frac{P_b^2 + P_{b\perp}^2}{2P_b\cdot n}n^\mu + P_{b\perp}^\mu \quad (17)$$

$$= \hat{P}_b^\mu + \frac{P_b^2 - M_b^2}{2P_b\cdot n}n^\mu, \quad (18)$$

where $\hat{P}_b^2 = M_b^2$ is the on-shell part of P_b and the momentum fraction z defined by $z = P_b^+/P_{\Lambda_b}^+ = 1 - l^+/P_{\Lambda_b}^+$. By the parametrization of P_b, the b quark propagator is then expressed as

$$\frac{i}{\not{P}_b - M_b + i\epsilon} = \frac{i(\hat{\not{P}}_b + M_b)}{P_b^2 - M_b^2 + i\epsilon} + \frac{i\not{n}}{2P_b\cdot n + i\epsilon}$$

$$\equiv F_b^L(P_b) + F_b^S(P_b). \quad (19)$$

There are two scenarios to be discussed. The first situation is for small z. That is we allow l^+ to be of order $P_{\Lambda_b}^+$ and $z \sim \Lambda/M$ with $\Lambda \sim \Lambda_{\rm QCD}$. The $F_b^S(P_b)$ term having power like $1/\Lambda$ is as important as the $F_b^L(P_b)$ term. We shall see later that the power correction completely comes from the short distance hard function. There is also the second situation in which the value of z is large of order $1 - \Lambda/M$. This corresponds to small l^+. In this case, the $F_b^S(P_b) \sim 1/M$ is power suppressed than $F_b^L(P_b) \sim 1/\Lambda$. In this configuration, the hard function is in the end points $x \to 1$ and $y \to 0, y_0 \to 1$. The power corrections come from the noncollinear momentum of the b quark. We now investigate the contributions from the above two scenarios. For simplicity, we shall ignore the transverse component of the neutrino momentum. This will not affect the following analysis.

For the large l^+ scenario, we need to separate the leading terms of the hard functions $S^{\mu\nu}, S_\alpha^{\mu\nu}$ from the higher order terms. The hard function $S^{\mu\nu}$ is a function of l and $S_\alpha^{\mu\nu}$ a function of l and l'. The l and l' are momenta of light degrees of freedom of the Λ_b and are defined as $l = P_{\Lambda_b} - P_b$ and $l' = P_{\Lambda_b} - P_b'$. We can make Taylor expansion for $S^{\mu\nu}$ and $S_\alpha^{\mu\nu}$ with respect to l^+ and $l^{+\prime}$. This is because the momentum $l(l')$ have a large plus component $l^+ = \xi p(l^{+\prime} = \xi' p)$ with $\xi = 1 - z(\xi' = 1 - z')$. By performing Taylor expansions for $S^{\mu\nu}(l)$ and $S_\alpha^{\mu\nu}(l,l')$ around $S^{\mu\nu}(\xi p)$ and $S_\alpha^{\mu\nu}(\xi p, \xi' p)$, we then obtain

$$S^{\mu\nu}(l) = S^{\mu\nu}(\xi p) + \left.\frac{\partial S^{\mu\nu}}{\partial l^\alpha}\right|_{l=\xi p}(l - \xi p)^\alpha + \cdots, \quad (20)$$

$$S_\alpha^{\mu\nu}(l,l') = S_\alpha^{\mu\nu}(\xi p, \xi' p) + \cdots. \quad (21)$$

The following low energy theorems are assumed to hold

$$\frac{\partial S^{\mu\nu}}{\partial l^\alpha}(\xi p) = -S_\alpha^{\mu\nu}(\xi p, \xi p). \quad (22)$$

The minus sign in the above equations is due to the direction of the momentum flow of l. The effects of the second terms in the right-hand side of Eq. (20) are to replace the gluonic field operators in the first term in the right-hand side of Eq. (21) by covariant derivative operators. Let us explain this. By substituting $S_\alpha^{\mu\nu}(\xi p, \xi p)$ and $S_\alpha^{\mu\nu}(\xi p, \xi' p)$ into Eq. (12), we can arrive at

$$-\int \frac{d^4 l}{(2\pi)^4} \mathrm{Tr}[S_\alpha^{\mu\nu}(\xi p, \xi p)(l - \xi p)^\alpha T(l)]$$

$$+\int \frac{d^4 l}{(2\pi)^4}\int \frac{d^4 l'}{(2\pi)^4} \mathrm{Tr}[S_\alpha^{\mu\nu}(\xi p, \xi' p) T^\alpha(l,l')]. \quad (23)$$

In light cone gauge $n\cdot A = 0$, we can rewrite the above equation as

$$\int \frac{d^4 l}{(2\pi)^4}\int \frac{d^4 l'}{(2\pi)^4} \mathrm{Tr}\Bigg[S_\alpha^{\mu\nu}(\xi p, \xi' p) w_{\alpha'}^\alpha\Bigg\{(-l^{\alpha'})T(l,l')$$

$$\times (2\pi)^4 \delta^{(4)}(l - l')\int \frac{d\xi'}{2\pi}\delta(\xi - \xi') + T^{\alpha'}(l,l')\Bigg\}\Bigg], \quad (24)$$

where the projection tensor $w_{\alpha'}^\alpha = g_{\alpha'}^\alpha - p^\alpha n_{\alpha'}$ has been employed. Since $S_\alpha^{\mu\nu}(\xi p, \xi' p)$ is independent of momenta l and l', we can move it out of the integrals and employ the identities

$$\int d\xi \int \frac{d\lambda}{2\pi} e^{-i\lambda(\xi - l\cdot n)} = 1,$$

$$\int \frac{d^4 l}{(2\pi)^4} e^{-il\cdot y} e^{i\lambda l\cdot n} = \delta^{(4)}(y - \lambda n), \quad (25)$$

we obtain

$$\int d\xi \int d\xi' \, \mathrm{Tr}[S_\alpha^{\mu\nu}(\xi p, \xi' p) w_{\alpha'}^\alpha \{T_1^{\alpha'}(\xi,\xi')$$

$$+ T_2^{\alpha'}(\xi,\xi')\}], \quad (26)$$

where

$$T_1^{\alpha'}(\xi,\xi') = \int \frac{d\lambda}{2\pi}\int \frac{d\eta}{2\pi} e^{i\lambda n\cdot P_{\Lambda_b}} e^{-i\lambda\xi} e^{-i\eta(\xi - \xi')}$$

$$\times \langle \Lambda_b | \overline{b}(0) i\partial^{\alpha'}(\eta n) b(\lambda n) | \Lambda_b\rangle, \quad (27)$$

$$T_2^{\alpha'}(\xi,\xi') = \int \frac{d\lambda}{2\pi}\int \frac{d\eta}{2\pi} e^{i\lambda n\cdot P_{\Lambda_b}} e^{-i\lambda\xi} e^{-i\eta(\xi - \xi')}$$

$$\times \langle \Lambda_b | \overline{b}(0)[-gA^{\alpha'}(\eta n)] b(\lambda n) | \Lambda_b\rangle. \quad (28)$$

Adding up $T_1^{\alpha'}$ and $T_2^{\alpha'}$ leads to

$$\int \frac{d\lambda}{2\pi}\int \frac{d\eta}{2\pi} e^{i\lambda n\cdot P_{\Lambda_b}} e^{-i\lambda\xi} e^{-i\eta(\xi-\xi')} \times\langle\Lambda_b|\overline{b}(0)D^{\alpha'}(\eta n)b(\lambda n)|\Lambda_b\rangle, \tag{29}$$

with $D^\alpha = i\partial^\alpha - gA^\alpha$. It is easy to find that the contributions from $S^{\mu\nu}_\alpha(\xi p,\xi' p)$ terms are power suppressed than $S^{\mu\nu}(\xi p)$ by at least $O(1/M)$ due to an additional charm quark propagator. This can be realized as follows. First we note that the charm quark propagator in $S^{\mu\nu}$ has terms proportional to $\not p$ and $\not n$ and, in this scenario, both terms are compatible. Because the light cone gauge and projection tensor $w^\alpha_{\alpha'}$, the subscript α in $S^{\mu\nu}_\alpha$ must be transverse. Also considering the property of light vector $p^2=n^2=0$ and the momentum $P_c(P'_c)$ with terms proportional to $\not p$ or $\not n$, we can conclude that the leading possible combination of the superscripts $\mu\nu$ can only be transverse. This implies that $S^{\mu\nu}_\alpha$ has terms proportional to γ_β with transverse index $\beta=\perp$. Thus we can safely ignore the contributions from the $S^{\mu\nu}_\alpha$ terms at leading order.

We now consider the second scenario, the large z or small l^+ case. To derive leading order contributions, we need to extract the leading contribution from the term

$$iT_0^{\mu\nu} = \int \frac{d^4P_b}{(2\pi)^4}\,\mathrm{Tr}[S^{\mu\nu}(P_b)T(P_b)] \tag{30}$$

which is just the first term of Eq. (12). The hard function has the structure

$$S^{\mu\nu}(P_b) = -\frac{\Gamma^\mu\left[(z-\alpha)\not p + \beta\not n\frac{M^2}{2}\right]\Gamma^\nu}{M^2\beta(z-z_B)+i\epsilon} \tag{31}$$

with

$$\alpha = y_0 - \frac{y}{x},\quad \beta=\left(1-\frac{y}{x}\right),\quad z_B = \frac{\alpha-y}{\beta}.$$

In the end point regime $x\to 1$ and $y\to 0, y_0\to 1$, the variables become $\alpha\to 1,\beta\to 1$ and $z_B\to 1$. The imaginary part of $T_0^{\mu\nu}$ is equivalent to taking a cut over the charm quark propagator. This implies that

$$\delta(P_c^2)\to\frac{1}{M^2\beta}\,\delta(z-z_B), \tag{32}$$

and the $S^{\mu\nu}(k)$ becomes

$$S^{\mu\nu}(k)\to i\pi\frac{\Gamma^\mu\left[(z_B-\alpha)\not p + \beta\not n\frac{M^2}{2}\right]\Gamma^\nu}{M^2\beta}. \tag{33}$$

It is clear that in this scenario, the large z, the charm quark propagator in $S^{\mu\nu}(P_b)$, after taking the cut, has a vanishing $\not p$ component and a large $\not n$ component. The μ,ν indices take one of the possible combinations $p^\mu p^\nu, p^\mu n^\nu, n^\mu p^\nu, n^\mu n^\nu, d^{\mu\nu} = p^\mu n^\nu + n^\mu p^\nu - g^{\mu\nu}$. Because $p^2=n^2=0$, the hard function is then proportional to $\not n$ with transverse μ,ν, or $\not p$ with $\mu=\nu=-$. The existence of the term proportional to $\not p$ implies that $iT_0^{\mu\nu}$ can have subleading power suppression terms. We now demonstrate this fact. The contraction of $T(P_b)$ with $\not p$ requires one to consider that the b quark propagator contacts with $\not p$. We first consider the effects from the out-going b quark propagator. Since in the second scenario, the b quark propagator has a on-shell part $F_b^L(P_b)$ and a power suppressed part $F_b^S(P_b)$. The situation as the on-shell part $F_b^L(P_b)$ contacts with $\not p$ can lead to

$$F_b^L(P_b)\not p = F_b(P_b)[(P_b - zp)^\alpha(i\gamma_\alpha) - iM_b]F_b^S(P_b)\not p. \tag{34}$$

The contribution from the power suppression part $F_b^S(P_b)$ contacting with $\not p$ has the form

$$F_b^S(P_b)\not p = F_b(P_b)[-gA^\alpha](i\gamma_\alpha)F_b^S(P_b)\not p. \tag{35}$$

Similar considerations can be applied for the incoming b quark propagator. The total effects from the b quark propagator contacting with $\not p$ leads to

$$iT_0^{\mu\nu} = \int\frac{d^4P_b}{(2\pi)^4}\,\mathrm{Tr}[S^{\mu\nu}(P_b)T(P_b)] + \int\frac{d^4P_b}{(2\pi)^4}\,\mathrm{Tr}[S_M^{\mu\nu}(P_b)T_M(P_b)] + \int\frac{d^4P_b}{(2\pi)^4}\int\frac{d^4P'_b}{(2\pi)^4}\,\mathrm{Tr}[S'^{\mu\nu}_\alpha(P_b,P'_b)w^\alpha_{\alpha'}T^{\alpha'} \times(P_b,P'_b)], \tag{36}$$

where the new hard functions $S_M^{\mu\nu}(P_b)$ and $S'^{\mu\nu}_\alpha(P_b,P'_b)$ take expressions

$$S_M^{\mu\nu}(P_b) = -iF_b^S(P_b)S^{\mu\nu}(P_b) + iS^{\mu\nu}(P_b)[F_b^S(P_b)]^\dagger, \tag{37}$$

$$S'^{\mu\nu}_\alpha(P_b,P'_b) = i\gamma_\alpha F_b^S(P_b)S^{\mu\nu}(P_b) - iS^{\mu\nu}(P_b)[F_b^S(P_b)]^\dagger\gamma_\alpha, \tag{38}$$

and the new matrix elements $T_M(P_b)$ and $T^{\alpha'}(P_b,P'_b)$ have the forms

$$T_M(P_b) = \int d^4y\, e^{iP_b\cdot y}\langle\Lambda_b|\overline{b}(0)M_b b(y)|\Lambda_b\rangle, \tag{39}$$

$$T^{\alpha'}(P_b,P'_b) = \int d^4z\int d^4y\, e^{iP_b\cdot y}e^{i(P'_b-P_b)\cdot z} \times\langle\Lambda_b|\overline{b}(0)D^\alpha(z)b(y)|\Lambda_b\rangle. \tag{40}$$

It is easy to see that the contributions from $S_M^{\mu\nu}$ and $S'^{\mu\nu}_\alpha$ are power suppressed than $S^{\mu\nu}$ by at least $O(1/M)$. To leading order, we can consider the first term of $iT_0^{\mu\nu}$.

The b quark field in the leading matrix element T in Eq. (36) contains a large phase factor $\exp(-iM_b v\cdot x)$ with $v \equiv P_{\Lambda_b}/M$ the Λ_b velocity. This is unsuitable to define a matrix element at low energies. To solve this, we can employ the HQET. In HQET, we can rescale the b quark field, $b(x)$, into $b_v(x)=\exp(iM_b v\cdot x)(1+\not{v}/2)b(x)$. The rescaled b_v field is a small fluctuation quantity of coordinate, since the remaining scale in its phase factor is only about Λ_{QCD} scale. In HQET, P_b is parametrized as $P_b=M_b v+k$, with k the residual momentum. The rescaled b quark field, $b_v(x)$, carries the residual momentum k and has a small effective mass $\bar{\Lambda}$, with $\bar{\Lambda}\equiv M-M_b$. Under the heavy quark mass expansion

$$b_v(x)=h_v(x)+O\left(\frac{1}{M}\right)+\cdots, \quad (41)$$

the matrix element T in terms of b_v can be expanded as

$$T=T_0+O\left(\frac{1}{M^2}\right)+\cdots. \quad (42)$$

The T_0 is in terms of the effective heavy quark field h_v, which is defined as the b_v field in the infinite mass limit $M_b\to\infty$. The missing of the $O(1/M)$ term is due to the equation of motion. The expression for T_0 is easily written down as

$$T_0=\int d^4y e^{ik\cdot y}\langle\Lambda_b(v,S)|\overline{h_v}(0)h_v(y)|\Lambda_b(v,S)\rangle. \quad (43)$$

Note that we have replaced the hadronic state vector $|\Lambda_b(P_{\Lambda_b},S)\rangle$ by its equivalent representation $|\Lambda_b(v,S)\rangle$. The final task is to extract the leading contributions of $iT_0^{\mu\nu}$. This can be achieved by means of Fierz identity. As a result, the leading order forward matrix element $T_0^{\mu\nu}$ takes the form

$$T_0^{\mu\nu}(P_{\Lambda_b},q,S)\approx -i\int\frac{d^4k}{(2\pi)^4}\left\{\mathrm{Tr}[S^{\mu\nu}(k=-\xi p,q)\not{P}_b]\mathrm{Tr}\left[T_0(P_{\Lambda_b},S,k=-\xi p)\frac{\not{n}}{4P_b\cdot n}\right]\right.$$
$$\left.-\mathrm{Tr}[S^{\mu\nu}(k=-\xi p,q)\not{S}_b\gamma_5]\mathrm{Tr}\left[T_0(P_{\Lambda_b},S,k=-\xi p)\frac{\not{n}\gamma_5}{4S_b\cdot n}\right]\right\}+O\left(\frac{1}{M^2}\right), \quad (44)$$

where we have inserted the Fierz identity

$$I_{ij}I_{mn}=\frac{1}{4}(\gamma^\mu)_{im}(\gamma_\mu)_{jn}+\frac{1}{4}(\gamma^\mu\gamma_5)_{im}(\gamma_5\gamma_\mu)_{jn}+\cdots, \quad (45)$$

where i,j,m,n denote the fermion indices and the dots represent the other gamma matrix that would result in higher order terms. In summary, the leading contributions are from large z configuration. This fact will be employed to make a model for the distribution function.

We now briefly describe how to derive the factorization formula for the inclusive semileptonic decay $\Lambda_b\to X_c l\nu$. The details about the derivation of the following factorization formula can be found in [12]. We shall only demonstrate the main ideas and try not to give a repeated proof. The formula for the quadruple differential decay rate can be expressed as a convolution integral over the soft function S, the jet function J, and a hard function H

$$\frac{1}{\Gamma^{(0)}}\frac{d^3\Gamma}{dxdydy_0 d\cos\theta}=\frac{M^2}{2}\int_{z_{\min}}^{z_{\max}}dz\int d^2\mathbf{l}_\perp S(z,\mathbf{l}_\perp,\mu)$$
$$\times J(z,P_c^-,\mathbf{l}_\perp,\mu)H(z,P_c^-,\mu), \quad (46)$$

where $x=x_l(x_{\bar{\nu}}),\theta=\theta_l(\theta_{\bar{\nu}})$, and $\Gamma^{(0)}=G_F^2/16\pi^3|V_{cb}|^2M^5$. The scale μ is introduced as a renormalization and factorization scale. The transverse momentum $\mathbf{l}_\perp$ has been reintroduced for regularization of the end point singularities [12]. The end point singularities arise from the end point region $x\to 1$ and $y\to 0,y_0\to 1$. The charm quark (assumed as massless) has a large minus component $P_c^-=(1-y/x)M/\sqrt{2}$ and a small plus component $P_c^+=(1-y_0-y/x)M/\sqrt{2}$. This implies that there is a very small invariant $P_c^2=M^2(1-y_0+y)$, which leads to an on-shell jet subprocess. The $\mathbf{l}_\perp$ integrals can be finished only when we know the exact dependence of the jet function on $\mathbf{l}_\perp$. But the jet function is nonperturbative and cannot be determined theoretically, so far. Fortunately, this difficulty for integration over $\mathbf{l}_\perp$ can be removed by means of a Fourier transformation for the jet function into its impact space representation as

$$J(z,P_c^-,\mathbf{l}_\perp,\mu)=\int\frac{d^2\mathbf{b}}{(2\pi)^2}\tilde{J}(z,P_c^-,\mathbf{b},\mu)e^{i\mathbf{l}_\perp\cdot\mathbf{b}}. \quad (47)$$

The $\mathbf{l}_\perp$ integrals then decouple from the jet function and the remaining factor $e^{i\mathbf{l}_\perp\cdot\mathbf{b}}$ is then associated with the soft function. The factorization formula Eq. (46) can also be applied to the case with loop corrections. With the Fourier transformation for $\mathbf{l}_\perp$, the Feynman rule for the radiative gluon cross over the final state cut should be modified with an extra phase factor $e^{i\mathbf{l}_\perp\cdot\mathbf{b}}$. The upper and lower limits of z are chosen as $z_{\rm max}=1$ and $z_{\rm min}=x$. The lower limit $z_{\rm min}=x$ is from the jet function. The upper limit $z_{\rm max}=1$ is chosen to fill the kinematical gap between M_b and M. The Fourier transformation of Eq. (46) into the impact $\mathbf{b}$ space then takes the form

$$\frac{1}{\Gamma^{(0)}}\frac{d^3\Gamma}{dxdydy_0d\cos\theta_l}=\frac{M^2}{2}\int_x^1 dz\int\frac{d^2\mathbf{b}}{(2\pi)^2}\tilde{S}(z,\mathbf{b},\mu)\times\tilde{J}(z,P_c^-,\mathbf{b},\mu)H(z,P_c^-,\mu). \quad (48)$$

To deal with the collinear and soft divergences resulting from the radiative corrections for the massless parton inside the jet, the resummation technique is necessary and these divergences can be resummed into a Sudakov form factor [12]. The jet function is then reexpressed into the form

$$\tilde{J}(z,P_c^-,b,\mu)=\exp[-2s(P_c^-,b)]\tilde{J}(z,b,\mu), \quad (49)$$

where $\exp[-2s(P_c^-,b)]$ is the Sudakov form factor. The renormalization group (RG) invariant Sudakov exponent has the expression up to one loop accuracy

$$\begin{aligned}s(P_c^-,b)=&\frac{A^{(1)}}{2\beta_1}\hat{q}\ln\left(\frac{\hat{q}}{\hat{b}}\right)+\frac{A^{(2)}}{4\beta_1^2}\left(\frac{\hat{q}}{\hat{b}}-1\right)-\frac{A^{(1)}}{2\beta_1}(\hat{q}-\hat{b})\\&-\frac{A^{(1)}\beta_2}{4\beta_1^2}\hat{q}\left[\frac{\ln(2\hat{b})+1}{\hat{b}}-\frac{\ln(2\hat{q})+1}{\hat{q}}\right]\\&-\left[\frac{A^{(2)}}{4\beta_1^2}-\frac{A^{(1)}}{4\beta_1}\ln\left(\frac{e^{2\gamma-1}}{2}\right)\right]\ln\left(\frac{\hat{q}}{\hat{b}}\right)\\&+\frac{A^{(1)}\beta_2}{8\beta_1^3}[\ln^2(2\hat{q})-\ln^2(2\hat{b})]\end{aligned} \quad (50)$$

with the variables

$$\hat{q}=\ln\left(\frac{P_c^-}{\Lambda}\right),\quad \hat{b}=\ln\left(\frac{1}{b\Lambda}\right). \quad (51)$$

We choose the QCD scale $\Lambda=\Lambda_{\rm QCD}$ to have the value 0.2 GeV in the numerical analysis in Sec. IV. The other factors are defined as

$$\beta_1=\frac{33-2n_f}{12},$$

$$\beta_2=\frac{153-19n_f}{24},$$

$$A^{(1)}=\frac{4}{3},$$

$$A^{(2)}=\frac{67}{9}-\frac{\pi^2}{3}-\frac{10}{27}n_f+\frac{8}{3}\beta_1\ln\left(\frac{e^{\gamma}}{2}\right). \quad (52)$$

The scale invariance of the differential decay rate in Eq. (48) and the Sudakov form factor in Eq. (49) requires the functions $\tilde{J},\tilde{S}$, and H to obey the following renormalization group (RG) equations:

$$\begin{aligned}\mathcal{D}\tilde{J}(b,\mu)&=-2\gamma_q\tilde{J}(b,\mu),\\\mathcal{D}\tilde{S}(b,\mu)&=-\gamma_S\tilde{S}(b,\mu),\\\mathcal{D}H(P_c^-,\mu)&=(2\gamma_q+\gamma_S)H(P_c^-,\mu),\end{aligned} \quad (53)$$

with

$$\mathcal{D}=\mu\frac{\partial}{\partial\mu}+\beta(g)\frac{\partial}{\partial g}. \quad (54)$$

$\gamma_q=-\alpha_s/\pi$ is the quark anomalous dimension in axial gauge, and $\gamma_S=-(\alpha_s/\pi)C_F$ is the anomalous dimension of $\tilde{S}$. After solving Eq. (53), we obtain the evolution of all the convolution factors in Eq. (48),

$$\begin{aligned}\tilde{J}(z,P_c^-,b,\mu)&=\exp\left[-2s(P_c^-,b)-2\int_{1/b}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q[\alpha_s(\bar{\mu})]\right]\tilde{J}(z,b,1/b),\\\tilde{S}(z,b,\mu)&=\exp\left[-\int_{1/b}^{\mu}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_S[\alpha_s(\bar{\mu})]\right]f(z,b,1/b),\\H(z,P_c^-,\mu)&=\exp\left[-\int_{\mu}^{P_c^-}\frac{d\bar{\mu}}{\bar{\mu}}\{2\gamma_q[\alpha_s(\bar{\mu})]+\gamma_S[\alpha_s(\bar{\mu})]\}\right]H(z,P_c^-,P_c^-).\end{aligned} \quad (55)$$

In the above solutions, we set the $1/b$ as an IR cutoff for single logarithm evolution.

For the initial soft function $f(z,b,1/b)$, we shall keep the intrinsic b dependence by $f(z,b,1/b)\approx f(z,b)$. The b dependence in $f(z,b)$ can support a way to explore the mechanism for the polarization. We assume $f(z,b)$ to have the form

$$f(z,b)=f(z)e^{-\Sigma(b)}, \quad (56)$$

and take an ansaze for parametrizing $\exp[-\Sigma(b)]$ as

$$e^{-\Sigma(b)}=e^{-tM^2b^2} \quad (57)$$

with an unknown parameter t. The reason for the above parametrization for the b dependent part of $f(z,b)$ will become clear later. To avoid double counting for the contributions from transverse degrees of freedom, we need some modifications for the factorization formula. For the end point regime where the Sudakov suppression dominates, we employ the approximation

$$f(z,b)=f(z), \tag{58}$$

while for other regimes which are not under the control of the Sudakov suppression, we take into account the intrinsic b dependence of $f(z,b)$

$$f(z,b)=e^{-tM^2b^2}f(z). \tag{59}$$

We make further approximations such that $\tilde{J}(z,b,1/b)=\tilde{J}^{(0)}(z,b)$, and $H(z,P_c^-,P_c^-)=H^{(0)}(z,P_c^-)$.

Combining the above results, we arrive at the factorization formula as

$$\frac{1}{\Gamma^{(0)}}\frac{d^4\Gamma}{dxdydy_0d\cos\theta} =M^2\int_x^1 dz\int_0^\infty \frac{bdb}{4\pi}\tilde{J}^{(0)}(z,b)H^{(0)}(z,P_c^-)e^{-S(P_c^-,b)} \times\begin{cases} f(z) & \text{for } x \text{ in end point regimes}\\ \exp[-tM^2b^2]f(z) & \text{for } x \text{ in other regimes,}\end{cases} \tag{60}$$

where

$$S(P_c^-,b)=2s(P_c^-,b)-\int_{1/b}^{P_c^-}\frac{d\bar{\mu}}{\bar{\mu}}[2\gamma_q(\bar{\mu})+\gamma_S(\bar{\mu})]. \tag{61}$$

The parameter t will be determined by experiment. From practical calculations, we find that the above difference between the distribution function with and without intrinsic transverse momentum contributions is very small. Therefore we shall include the factor $\exp[-tM^2b^2]$ for the entire range of x in the numerical analysis.

The integration range of the impact parameter b will affect the determination of t. We now discuss this point. The Sudakov form factor gives strong suppression over large b for $b>1/\Lambda_{\rm QCD}$, where the perturbative calculations can no longer be applicable since $\alpha_s(1/b)>1$. Therefore the Sudakov suppression guarantees that the main contributions are from small $b<1/\Lambda_{\rm QCD}$. However, there are singular points that happen at $1/b\sim\Lambda_{\rm QCD}$ at which the strong coupling constant α_s becomes divergent. The infrared (IR) renormalon arise in such a scenario. It is well known [13–15] that the IR renormalon implies the need for the introduction of a nonperturbative function to make the physical quantity well-defined. Therefore the estimation of the IR renormalons can give the information about the distribution function. It is equivalent to reconsidering the evolution factor for the distribution function contained in the Sudakov form factor [i.e., the γ_S term in Eq. (61)] [16,17]

$$A_{\rm IR}=4\pi C_F\int\frac{d^4l}{(2\pi)^4}\frac{v_\mu v_\nu}{(v\cdot l)^2}2\pi\delta(l^2)\alpha_s(\mathbf{l}_\perp^2)e^{i\mathbf{l}_\perp\cdot\mathbf{b}}N^{\mu\nu} \tag{62}$$

where v is the velocity of the heavy quark and $N^{\mu\nu}$ the gluon polarization tensor in the axial gauge $A^+-A^-=0$. We have put the argument of $\alpha_s(\mathbf{l}_\perp^2)$ as $\mathbf{l}_\perp^2$. We then express $\alpha_s(\mathbf{l}_\perp^2)$ as

$$\alpha_s(\mathbf{l}_\perp^2)=\pi\int_0^\infty d\sigma\exp\left[-2\sigma\beta_1\ln\left(\frac{\mathbf{l}_\perp}{\Lambda_{\rm QCD}}\right)\right], \tag{63}$$

and substitute it into the amplitude $A_{\rm IR}$ to yield

$$A_{\rm IR}=C_F\int_0^\infty d\sigma\left(\frac{b\Lambda_{\rm QCD}}{2}\right)^{2\sigma\beta_1}\frac{\Gamma(-\sigma\beta_1)}{\Gamma(1+\sigma\beta_1)} \tag{64}$$

with Γ being the gamma function. There are poles contained in the $\Gamma(-\sigma\beta_1)$. The small $\sigma\sim0$ pole of $\Gamma(-\sigma\beta_1)$ results in the anomalous dimension. There are also poles for $\sigma\beta_1=1,2,3,\ldots$. This implies that, as $\sigma=1/\beta_1,2/\beta_1,\ldots$, we have the IR renormalons with corresponding corrections of $b^2,b^4,\ldots$. Since IR renormalons lead to corresponding singularities, we need to introduce nonperturbative functions to absorb these divergences. The leading contributions from the IR renormalons are of the form $\exp[(\Lambda_{\rm QCD}b)^2]$. Thus we at least need a nonperturbative function with the form like $\exp[-Cb^2]$. This is the way we employed to parametrize the intrinsic b dependent part of $f(z,b)$. In summary, we can choose the integration range of b as $0\leqslant b\leqslant 1/\Lambda_{\rm QCD}$.

Let us now discuss how to parametrize $T_0(k)$ defined in Eq. (43). As discussed in previous paragraphs, at leading order, $T_0(k)$ is expanded in the form

$$T_0(k)=\frac{1}{4}P_b\cdot n\not{p}f(z)-\frac{1}{4}S_b\cdot n\not{p}\gamma_5 g(z). \tag{65}$$

The unpolarized and polarized distribution functions, $f(z)$ and $g(z)$, are defined as

$$f(z)=\int\frac{d\lambda}{2\pi}e^{i(1-z)\lambda}\langle v,S|\overline{h_v}(0)\not{n}h_v(\lambda n)|v,S\rangle \tag{66}$$

and

$$g(z)=\int\frac{d\lambda}{2\pi}e^{i(1-z)\lambda}\langle v,S|\overline{h_v}(0)\not{n}\gamma_5 h_v(\lambda n)|v,S\rangle. \tag{67}$$

It is easy to show that $f(z)$ and $g(z)$, in the heavy quark limit, share a common matrix element which could be described by a universal distribution function, $f_{\Lambda_b}(z)$. This just reflects the heavy quark spin symmetry. We adopt the distribution function proposed in [12] in the form

$$f_{\Lambda_b}(z)=\frac{Nz^2(1-z)^2}{[(z-a)^2+\epsilon z]^2}\theta(1-z). \tag{68}$$

The parameters N, a, and ϵ are fixed by the first three moments of $f_{\Lambda_b}(z)$:

$$\int_0^1 f_{\Lambda_b}(z)dz=1,$$

$$\int_0^1 dz(1-z)f_{\Lambda_b}(z)=\bar{\Lambda}/M+\mathcal{O}(\Lambda_{\rm QCD}^2/M^2),$$

$$\int_0^1 dz(1-z)^2 f_{\Lambda_b}(z)=\frac{\bar{\Lambda}^2}{M^2}+\frac{2}{3}K_b+\mathcal{O}(\Lambda_{\rm QCD}^3/M^3), \tag{69}$$

where $\bar{\Lambda}=M-M_b$ and K_b is to parametrize the matrix element

$$K_b=-\frac{1}{2M}\langle\Lambda_b|\bar{h}_v(0)\frac{(iD)^2}{2M^2}h_v(0)|\Lambda_b\rangle. \tag{70}$$

By substituting the inputs

$$M=5.641\ \text{GeV},\quad M_b=4.776\ \text{GeV},$$

$$K_b=0.012\pm 0.0026, \tag{71}$$

into Eq. (69), we determine the parameters N, a, and ϵ to be

$$N=0.106\,15,\quad a=1,\quad \epsilon=0.004\,13. \tag{72}$$

For simplicity we shall omit the subscript of $f_{\Lambda_b}(z)$ in the following text. Finally, one should note that the second moment of the structure function implies large z that is consistent with previous discussions in determining the leading contributions.

III. DIFFERENTIAL DECAY RATES

In this section we construct four models based on the factorization formula Eq. (60). The models are the quark model (QM), the modified quark model (MQM), the parton model (PM), and the modified parton model (MPM). The charged lepton and antineutrino spectra for the decay $\Lambda_b \to X_c l \bar{\nu}$ in the quark model are expressed as

$$\frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma^{\rm T}_{\rm QM}}{dx d\cos\theta}=\begin{cases}\dfrac{x_l^2}{6}[(3-2x_l)-P\cos\theta_l(1-2x_l)] & \text{for } l\\[2ex] \dfrac{x_{\bar{\nu}}^2}{6}x_{\bar{\nu}}(1-x_{\bar{\nu}})(1-P\cos\theta_{\bar{\nu}}) & \text{for } \bar{\nu},\end{cases} \tag{73}$$

where P and $\cos\theta_{l(\bar{\nu})}$ denote the polarization and cosine of the angle $\theta_{l(\bar{\nu})}$ between the third components of the lepton (antineutrino) momentum and the Λ_b spin vector.

By taking into account Sudakov suppression from the resummation of large radiative corrections, and substituting $f(z,b)=\delta(1-z)\exp[-tM^2b^2]$, $H^{(0)}=(x_l-y)[(y_0-x_l)-P\cos\theta_l(y_0-x_l-2y/x_l)]$ and the Fourier transform of $J^{(0)}=\delta(P_c^2)$ with $P_c^2=M^2(1-y_0+y-p_\perp^2/M_B^2)$ into Eq. (60), we derive the lepton spectrum in the modified quark model. The spectrum is, after integrating Eq. (60) over z and y_0, described by

$$\frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma_{\rm MQM}}{dx_l d\cos\theta_l}=M\int_0^x dy\int_0^{1/\Lambda}db\, e^{[-t^{\rm MQM}M^2b^2]}e^{-S(P_c^-,b)}(x_l-y)\eta$$
$$\times\left\{\left[(1+y-x_l)-P\cos\theta_l\left(1+y-x_l-2\frac{y}{x_l}\right)\right]\right.$$
$$\left.\times J_1(\eta Mb)-\left(\frac{2}{Mb}\eta J_2(\eta Mb)-\eta^2 J_3(\eta Mb)\right)(1-P\cos\theta_l)\right\}, \tag{74}$$

where $P_c^- = (1-y/x_l)M/\sqrt{2}, \eta = \sqrt{(x_l-y)(1/x_l-1)}$, and J_1, J_2, and J_3 are the Bessel functions of order 1, 2, and 3, respectively. Note that we have made an approximation by substituting $\exp[-tM^2b^2]$ for the end point regimes. We also need the antineutrino spectrum in the modified quark model as

$$\frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma_{\rm MQM}}{dx_{\bar\nu}d\cos\theta_{\bar\nu}} = M\int_0^x dy\int_0^{1/\Lambda} db e^{[-t^{\rm MQM}M^2b^2]} \times e^{-S(P_c^-,b)}x_{\bar\nu}(1-x_{\bar\nu})\eta' J_1(\eta' Mb) \times(1-P\cos\theta_{\bar\nu}) \quad (75)$$

with $\eta' = \sqrt{(x_{\bar\nu}-y)(1/x_{\bar\nu}-1)}$.

The charged lepton spectrum in the parton model is obtained by adopting $H^{(0)} = (x_l-y)\{[y_0-x_l-(1-z)y/x_l] - P\cos\theta_l[y_0-x_l-(1+z)y/x_l]\}$ and $P_c^2 = M^2[1-y_0+y-(1-z)(1-y/x_l)]$. After integration over y_0, we then derive

$$\frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma_{\rm PM}}{dx_l d\cos\theta_l} = \int_0^{x_l}dy\int_{x_l}^1 dz f(z)(x_l-y)\left[(y+z-x_l) - P\cos\theta_l\left(y+z-x_l-2z\frac{y}{x_l}\right)\right]. \quad (76)$$

In the same way, the antineutrino spectrum can be written down

$$\frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma_{\rm PM}}{dx_{\bar\nu}d\cos\theta_{\bar\nu}} = \int_0^{x_{\bar\nu}}dy\int_{x_{\bar\nu}}^1 dz f(z)x_{\bar\nu}(z-x_{\bar\nu})(1-P\cos\theta_{\bar\nu}). \quad (77)$$

The charged lepton spectrum in the modified parton model takes into account large perturbative corrections and nonperturbative intrinsic contributions with the expression as

$$\frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma_{\rm MPM}}{dx_l d\cos\theta_l} = M\int_0^{x_l}dy\int_{x_l}^1 dz\int_0^{1/\Lambda}db e^{[-t^{\rm MPM}M^2b^2]}e^{-S(P_c^-,b)}f(z)(x_l-y)\eta \times\left\{\left[(z+y-x_l)-P\cos\theta_l\left(z+y-x_l-2z\frac{y}{x_l}\right)\right]J_1(\eta Mb) - \left(\frac{2}{Mb}\eta J_2(\eta Mb)-\eta^2 J_3(\eta Mb)\right)(1-P\cos\theta_l)\right\}, \quad (78)$$

with $\eta = \sqrt{(x-y)(z/x_l-1)}$. The antineutrino spectrum in the modified parton model is also easily derived as

$$\frac{1}{\Gamma^{(0)}}\frac{d^2\Gamma_{\rm MPM}}{dx_{\bar\nu}d\cos\theta_{\bar\nu}} = M\int_0^{x_{\bar\nu}}dy\int_{x_{\bar\nu}}^1 dz\int_0^{1/\Lambda}db e^{-S(P_c^-,b)}e^{-t^{\rm MPM}M^2b^2}f(z)x_{\bar\nu}(z-x_{\bar\nu})\eta' J_1(\eta' Mb)(1-P\cos\theta_{\bar\nu}). \quad (79)$$

with $\eta' = \sqrt{(x_{\bar\nu}-y)(z/x_{\bar\nu}-1)}$.

IV. NUMERICAL RESULT

The Λ_b's produced in ALEPH, DELPHI, and OPAL experiments are highly boosted in the laboratory frame. For the relativistic Λ_b's, the forward-backward asymmetry of a decay product can be directly expressed in terms of a shift in the average value of its energy. The charged lepton also carried a residual sensitivity to the Λ_b polarization. Because neither the Λ_b four-momentum nor the lepton four-momentum can be fully reconstructed in the experiments, the ALEPH and DELPHI experiments proposed to measure the Λ_b polarization, P, through the variable y suggested in [11]

$$y = \frac{\langle E_l\rangle}{\langle E_{\bar\nu}\rangle}. \quad (80)$$

However, there still exist many uncertainties suffered from experimental procedures on extracting the energy spectra. It requires normalizing the measured y with an unpolarized simulated $y^{\rm MC}(0)$. Therefore the experimentally measured quantity is the ratio

$$\mathrm{R} = \frac{y(P)}{y^{\rm MC}(0)}. \quad (81)$$

ALEPH and DELPHI determined the polarization by comparing the measured value of the ratio R with the Monte Carlo simulation $\mathrm{R}^{\rm MC}(P)$ with varying P. The experimental results are $\mathrm{R}=1.12\pm0.10$ and $P=-0.23^{+0.24}_{-0.20}$(stat) for ALEPH and $\mathrm{R}=1.21^{+0.16}_{-0.14}$ and $P=-0.49^{+0.32}_{-0.30}$(stat) for DELPHI, respectively. Theoretically, the Λ_b polarization can be best defined in the rest frame. It is instructive to rewrite y in terms of average variables in the rest frame

TABLE I. The values of the Λ_b polarization are predicted from the quark model, the parton model, the modified quark model, and the modified parton model by employing the ALEPH and DELPHI experiments. The ALEPH and DELPHI experimental results are also shown for comparison.

$P_{\rm QM}$	$P_{\rm PM}$	$P_{\rm MQM}$	$P_{\rm MPM}$	$P_{\rm EXP}$	R	Experiment
$-0.23^{+0.19}_{-0.17}$	$-0.23^{+0.19}_{-0.17}$	$-0.24^{+0.20}_{-0.17}$	$-0.24^{+0.20}_{-0.17}$	$-0.23^{+0.24}_{-0.20}$	1.12 ± 0.1	ALEPH
$-0.38^{+0.24}_{-0.24}$	$-0.38^{+0.24}_{-0.24}$	$-0.39^{+0.25}_{-0.24}$	$-0.39^{+0.25}_{-0.24}$	$-0.49^{+0.32}_{-0.30}$	$1.21^{+0.16}_{-0.14}$	DELPHI

$$y=\frac{\langle E_l^*\rangle+P\langle P_l^*(P=-1)\rangle}{\langle E_\nu^*\rangle+P\langle P_\nu^*(P=-1)\rangle}, \tag{82}$$

where the star average variables are evaluated with $P=-1$. The average variables can be calculated from the formula

$$\langle a\rangle=\int\int a\,\frac{d^2\Gamma}{\Gamma^{(0)}dxd\cos\theta}\,dxd\cos\theta \tag{83}$$

by employing different models for the differential decay rate. It is much simplified in calculations of these average quantities, if the charged lepton and antineutrino average quantities are evaluated by their corresponding differential decay rates. From these relations we can determine P in terms of R as

$$P=\frac{\langle E_l^*\rangle\langle E_\nu^*\rangle(1-{\rm R})}{\langle E_l^*\rangle\langle P_{\bar\nu}^*\rangle{\rm R}-\langle E_{\bar\nu}^*\rangle\langle P_l^*\rangle}. \tag{84}$$

We first compare the difference between the experimentally determined polarization $P^{\rm EXP}$ and the theoretically evaluated polarization $P^{\rm TH}$ in the four models QM, PM and MQM, MPM with parameters $t^{\rm MQM}=t^{\rm MPM}=0$. The result is shown in Table I. We can see that the theoretical polarizations are close to the ALEPH polarization $P^{\rm ALEPH}=-0.23$ but have a large deviation from the DELPHI polarization $P^{\rm DELPHI}=-0.49$. Among different model evaluations with one R, their differences are very small. This implies that nonperturbative effects from the distribution function over longitudinal momentum fraction and perturbative effects from Sudakov suppression are not important in determining the polarization.

We now turn on the parameters $t^{\rm MQM}$ and $t^{\rm MPM}$ to find out their values from experiments. It is interesting to note that the ratio $z_l=\langle P_l^*\rangle/\langle E_l^*\rangle$ is model dependent but the ratio $z_{\bar\nu}=\langle P_{\bar\nu}^*\rangle/\langle E_{\bar\nu}^*\rangle$ is almost the same for all models. Using these two z variables, we can rewrite Eq. (84) as

$$P=\frac{1-{\rm R}}{z_{\bar\nu}{\rm R}-z_l}. \tag{85}$$

Since $z_{\bar\nu}\approx1/3$ for all models, we can further simplify the above equation into

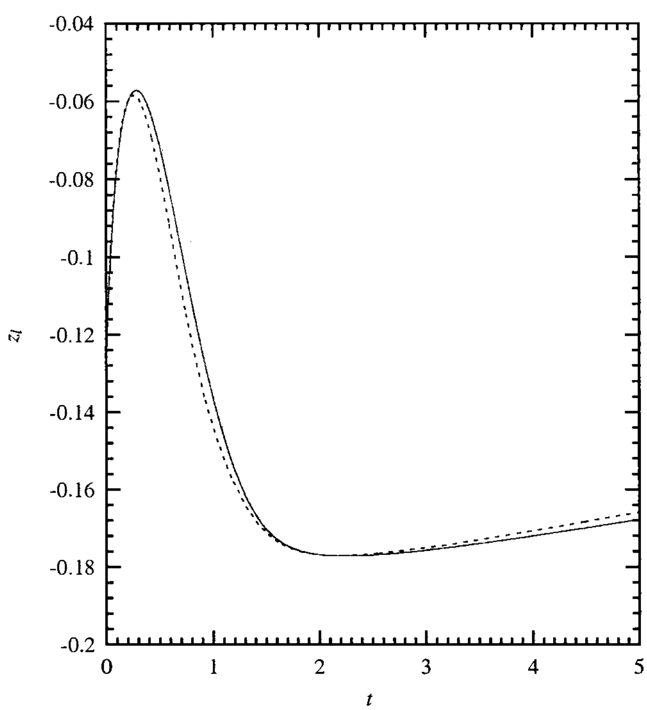

FIG. 1. Plot of z_l vs t. The modified quark model (solid line) and modified parton model (dashed line) are shown.

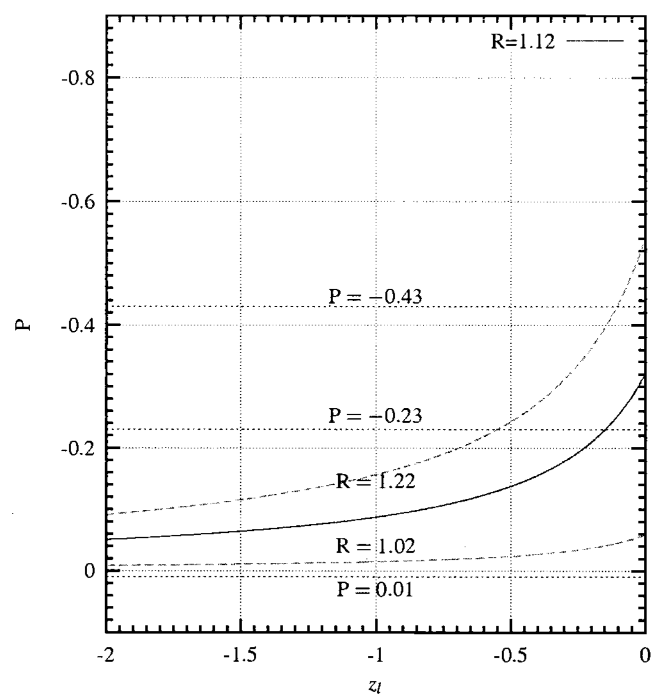

FIG. 2. The plot of P vs z_l is shown by employing the ALEPH ratio $\rm R=1.12\pm0.10$. The experimental polarization $P=-0.23^{+0.24}_{-0.20}$ is also shown to indicate the allowed range of z_l.

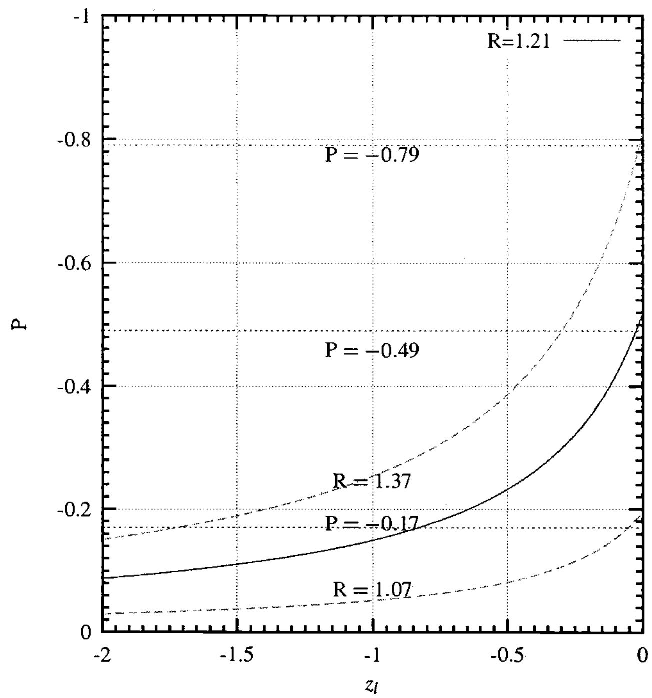

FIG. 3. The plot of P vs z_l is shown by employing the DELPHI ratio $\mathrm{R}=1.21^{+0.16}_{-0.14}$. The experimental polarization $P=-0.49^{+0.32}_{-0.30}$ is also shown to indicate the allowed range of z_l.

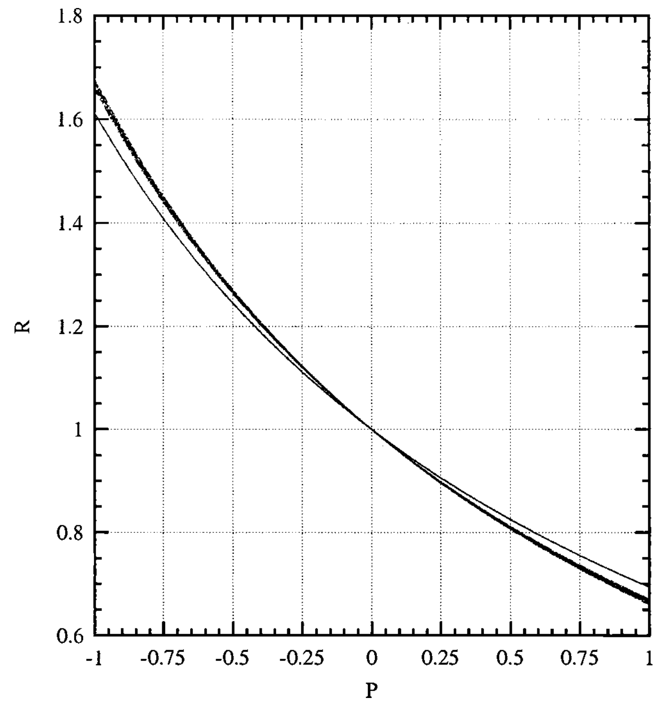

FIG. 4. Plot of R vs P. The ALEPH and DELPHI Monte Carlo simulations (solid line) and the theoretical prediction (band line) are shown.

$$P=\frac{3(1-\mathrm{R})}{\mathrm{R}-3z_l}. \tag{86}$$

Theoretically, the z_l range would be model dependent. By varying the value of t, we can easily change z_l. This is because the suppressions from the contributions of intrinsic transverse momentum are modeled by parameter t. Considering MQM and MPM and plotting the z_l-t relation in Fig. 1, we can find that there is an upper bound for z_l as $z_l\leqslant -0.05$ with $t\sim 0.3$, and a lower bound for z_l as $z_l\geqslant -0.18$ with $t\sim 2$. The reason for existing the upper and lower bounds for z_l is as follows. The fluctuations from the Bessel function in the differential decay rates would prevent the suppression of t from becoming large and small. In the end, there exist upper and lower bounds for z_l. We also hope that t should be less than unity and close to zero to make the perturbative calculation reliable. Thus the z_l bound should be $-0.12\leqslant z_l\leqslant -0.05$ with the corresponding bound for t, $0\leqslant t\leqslant 0.3$. Since, in the z_l-t plot, the differences between the MPM and MQM are very small, we shall not distinguish t^{MQM} and t^{MPM}.

We now discuss the extraction of z_l from experiments. We first plot the behaviors of P with respect to z_l for $\mathrm{R}^{\mathrm{ALEPH}}=1.12\pm 0.10$ (ALEPH) and $\mathrm{R}^{\mathrm{DELPHI}}=1.21^{+0.16}_{-0.14}$ (DELPHI) in Figs. 2 and 3, respectively. By applying the experimental bounds for P, we can extract from Fig. 2 the z_l range of $-\infty\leqslant z_l\leqslant -0.105$ for ALEPH and from Fig. 3 the range of $-1.75\leqslant z_l\leqslant -0.02$ for DELPHI. We now discuss that the possible constraint over z_l can be obtained from the OPAL experiment. The OPAL Collaboration employed a comparison between the measured $y3=E_{\bar{\nu}}/E_l$ and the Monte Carlo simulated y_3^{MC} to determine the polarization $P=-0.56^{+0.20}_{-0.13}(\mathrm{stat})$. Applying the OPAL P to the DELPHI and ALEPH experiments, we obtain $-0.6\leqslant z_l\leqslant -0.1$ for DELPHI and $-0.55\leqslant z_l\leqslant -0.105$ for ALEPH. We summarize the above discussions for the determination of the z_l in Table II. In the models we are considering, the lower bound for z_l cannot be smaller than -0.12. We then assume that the range of z_l can be obtained by combining the experimental and theoretical bounds. We thus have the z_l range of $-0.12\leqslant z_l\leqslant -0.105$, and the corresponding t range of $0\leqslant t\leqslant 0.05$.

As a consistent check, we can write R in terms of P and z_l as

$$\mathrm{R}=\frac{3(1+Pz_l)}{(3+P)}. \tag{87}$$

TABLE II. The bounds of z_l are obtained from the ALEPH, DELPHI, and OPAL experiments, and from the theory. OPAL1 represents the combination of the OPAL and the ALEPH experiments, and OPAL2 the combination of the OPAL and the DELPHI experiments.

ALEPH	DELPHI	OPAL1	OPAL2	THEORY
$-\infty\leqslant z_l\leqslant -0.105$	$-1.75\leqslant z_l\leqslant -0.02$	$-0.55\leqslant z_l\leqslant -0.105$	$-0.6\leqslant z_l\leqslant -0.1$	$-0.12\leqslant z_l\leqslant -0.05$

By this equation, we can parametrize the Monte Carlo simulation ratios $R^{MC}(P)$'s of ALEPH and DELPHI. We find that the value of $z_l \sim -0.075$ can be used for both experiments in a good approximation within 5%–10%. In Fig. 4 we compare the $R-P$ plots for $z_l \sim -0.075$ and for $-0.12 \leqslant z_l \leqslant -0.105$. The experimental bounds for ratio R can give constraints over P. The combination of ALEPH and DELPHI experiments gives the range of P as $-0.79 \leqslant P \leqslant -0.05$, while our analysis results in $-0.73 \leqslant P \leqslant -0.05$. The difference between these two bounds of P can be reduced by including higher order corrections for the theory, such as the mass corrections, etc.

V. CONCLUSION

In this paper we have constructed four models based on the PQCD factorization formula for $\Lambda_b \to X_c l \bar{\nu}$. We used these models to investigate the physics implied by the ALEPH, OPAL, and DELPHI experiments. We found that these experiments can be understood from theoretical models, the modified quark model and modified parton model. These two models contain intrinsic transverse momenta for partons, which are nonperturbative and parametrized by an exponential form with a parameter t. The parameter t relates to the variable $z_l = \langle P_l^*(P=-1) \rangle / \langle E_l^* \rangle$ with $\langle P_l^*(P=-1) \rangle$ and $\langle E_l^* \rangle$ the average momentum and energy of charge lepton in the rest frame of Λ_b baryon. We found that the ratio $R = y(P)/y(0)$ can be approximately expressed in terms of P and z_l. Using experimental results, we then determined the ranges of z_l and t.

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