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A multiprocess performance analysis chart based on the incapability index C_{pp}: an application to the chip resistors

W.L. Pearn [a,*], C.H. Ko [a], K.H. Wang [b]

[a] *Department of Industrial Engineering and Management, National Chiao Tung University, 1001 Ta Hsueh Road, Hsin Chu 30050, Taiwan, ROC*

[b] *Department of Applied Mathematics, National Chung Hsin University, Taiwan, ROC*

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Abstract

Statistical process control charts, such as the $\bar{X}$, R, S^2, S, and MR charts, have been widely used in the manufacturing industry for controlling/monitoring process performance, which are essential tools for any quality improvement activities. Those charts are easy to understand, which effectively communicate critical process information without using words and formula. In this paper, we introduce a new control chart, called the C_{pp} multiple process performance analysis chart (MPPAC), using the incapability index C_{pp}. The C_{pp} MPPAC displays multiple processes with the departure, and process variability relative to the specification tolerances, on one single chart. We demonstrate the use of the C_{pp} MPPAC by presenting a case study on some resistor component manufacturing processes, to evaluate the factory performance. © 2002 Elsevier Science Ltd. All rights reserved.

1. Introduction

Process capability indices (PCIs) have been widely used in various manufacturing industries, to provide numerical measures on process potential and process performance. The two most commonly used process capability indices are C_p and C_{pk} introduced by Kane [1]. These two indices are defined in the following:

$$C_p = \frac{\text{USL} - \text{LSL}}{6\sigma},$$

$$C_{pk} = \min\left\{\frac{\text{USL} - \mu}{3\sigma}, \frac{\text{LSL} - \mu}{3\sigma}\right\},$$

where USL and LSL are the upper and the lower specification limits, respectively, μ is the process mean, and σ is the process standard deviation. The index C_p measures the process variation relative to the production tolerance, which reflects only the process potential. The index C_{pk} measures process performance based on the process yield (percentage of conforming items) without considering the process loss (a new criteria for process quality championed by Hsiang and Taguchi [2]). Taking into the consideration of the process departure (which reflects the process loss), Chan et al. [3] developed the index C_{pm}, which measures the ability of the process to cluster around the target. The index C_{pm} is defined as:

$$C_{pm} = \frac{\text{USL} - \text{LSL}}{6\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{d}{3\sqrt{\sigma^2 + (\mu - T)^2}},$$

where T is the target value, and $d = (\text{USL} - \text{LSL})/2$ is half of the length of the specification interval (LSL, USL).

Based on the index C_{pm}, Greenwich and Jahr-Schaffrath [4] introduced an incapability index, called C_{pp}, which is a simple transformation of the Taguchi index C_{pm}. The index C_{pp} is defined as:

$$C_{pp} = \left(\frac{1}{C_{pm}}\right)^2 = \left(\frac{\mu - T}{D}\right)^2 + \left(\frac{\sigma}{D}\right)^2,$$

where $D = d/3$. Some commonly used values of C_{pp}, 9.00 (process is incapable), 4.00 (process is incapable), 1.00 (process is normally called capable), 0.57 (process is normally called satisfactory), 0.44 (process is normally called good), and 0.25 (process is normally called super), and the corresponding C_{pm} values are listed in Table 1.

* Corresponding author. Tel.: +886-35-714261; fax: +886-35-722392.

E-mail address: roller@cc.nctu.edu.tw (W.L. Pearn).

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Table 1
Some commonly used C_{pp} and equivalent C_{pm}

C_{pp}	C_{pm}
9.00	0.33
4.00	0.50
1.00	1.00
0.57	1.33
0.44	1.50
0.25	2.00

Table 2
Some commonly used precision requirements

Quality condition	Precision requirement
Capable	$0.56 \leqslant C_{ip} \leqslant 1.00$
Satisfactory	$0.44 \leqslant C_{ip} \leqslant 0.56$
Good	$0.36 \leqslant C_{ip} \leqslant 0.44$
Excellent	$0.25 \leqslant C_{ip} \leqslant 0.36$
Super	$C_{ip} \leqslant 0.25$

If we denote the first term $(\mu - T)^2/D^2$ as C_{ia}, and the second item σ^2/D^2 as C_{ip}, then C_{pp} can be rewritten as $C_{pp} = C_{ip} + C_{ia}$. The sub-index C_{ip} measures the relative variability, which has been referred to as the imprecision index. Some commonly used values of C_{ip}, 1.00, 0.56, 0.44, 0.36, and 0.25, and the corresponding quality conditions are listed in Table 2. Note that those values of C_{ip} are equivalent to $C_p = 1.00$, 1.33, 1.50, 1.67, and 2.00 respectively, covering a wide range of the precision requirements used for most real-world applications.

On the other hand, the sub-index C_{ia} measures the relative departure, which has been referred to as the inaccuracy index. The advantage of using the index C_{pp}, is that it provides an uncontaminated separation between information concerning the process precision and process accuracy. The separation suggests a direction the practitioners may consider on the process parameters to improve the process quality.

Based on the sub-indices C_{ip} and C_{ia}, we introduce a control chart called the C_{pp} multiple process performance analysis chart (MPPAC), using the incapability index C_{pp}. The C_{pp} MPPAC displays multiple processes with the relative departure, and process variability relative to their specification tolerances on one single chart. We demonstrate the use of the C_{pp} MPPAC by presenting a case study taken from a resistor component manufacturing company located on an Industrial Park in Taiwan, to evaluate the factory performance.

2. Estimation of C_{ip}, C_{ia}, C_{pp}

2.1. Estimation of C_{ip}

To estimate the process imprecision, we consider the natural estimator $\hat{C}_{ip}$ defined in the following, where the sample standard deviation S_{n-1} is calculated as $S_{n-1} = \left[\sum(X_i - \bar{X})^2/(n-1)\right]^{1/2}$, which is the conventional estimator of the process standard deviation σ,

$$\hat{C}_{ip} = \frac{1}{n-1}\sum_{i=1}^{n}\frac{\left(X_i - \bar{X}\right)^2}{D^2} = \frac{S_{n-1}^2}{D^2}.$$

The natural estimator $\hat{C}_{ip}$ can be rewritten as:

$$\hat{C}_{ip} = \frac{C_{ip}}{n-1}\frac{(n-1)\hat{C}_{ip}}{C_{ip}} = \frac{C_{ip}}{n-1}\sum_{i=1}^{n}\frac{\left(X_i - \bar{X}\right)^2}{\sigma^2}.$$

If the process characteristic is normally distributed, Pearn and Lin [5] showed the natural estimator $\hat{C}_{ip}$ distributed as $[C_{ip}/(n-1)]\chi^2_{n-1}$, where χ^2_{n-1} is a chi-square distribution with $(n-1)$ degrees of freedom. Pearn and Lin [5] showed that the natural estimator $\hat{C}_{ip}$ is the uniformly minimum-variance unbiased estimate (UMVUE) of C_{ip}, which is consistent, and asymptotically efficient. Pearn and Lin [5] also showed that the statistic $\sqrt{n}(\hat{C}_{ip} - C_{ip})$ converges to $N(0, 2C_{ip}^2)$ in distribution. Thus, in real-world applications, using $\hat{C}_{ip}$ which has all desired statistical properties as an estimate of C_{ip}, would be reasonable.

Note that by multiplying the constant $c_n = (n-1)/n$ to the UMVUE $\hat{C}_{ip}$, we can obtain the maximum likelihood estimate (MLE) of C_{ip}. Pearn and Lin [5] showed that the MLE $\hat{C}'_{ip}$ is consistent, asymptotically unbiased and efficient. They also showed that the statistic $\sqrt{n}(\hat{C}'_{ip} - C_{ip})$ converges to $N(0, 2C_{ip}^2)$ in distribution.

Since the constant $c_n < 1$, then the MLE $\hat{C}'_{ip} = c_n\hat{C}_{ip}$ underestimates C_{ip} but with smaller variance. In fact, we may calculate the mean square error MSE $(\hat{C}'_{ip}) = [(2n-1)/n^2](C_{ip})^2$. Hence, MSE $(\hat{C}_{ip}) - \mathrm{MSE}(\hat{C}'_{ip}) = [(3n-1)/n^2(n-1)](C_{ip})^2 > 0$, for all sample size n. Therefore, the MLE $\hat{C}'_{ip}$ has a smaller mean square error than that of the UMVUE $\hat{C}_{ip}$, hence is more reliable, particularly, for short production run applications (such as accepting a supplier providing short production runs in QS-9000 certification). For short run applications (with $n \leqslant 35$) we recommend using the MLE $\hat{C}'_{ip}$ rather than the UMVUE $\hat{C}_{ip}$. For other applications with sample sizes $n > 35$, the difference between the two estimators is negligible (less than 0.52%).

2.2. Estimation of C_{ia}

For the process inaccuracy index C_{ia}, we consider the natural estimator $\hat{C}_{ia}$ defined as the following

$$\hat{C}_{ia} = \frac{(\bar{X} - T)^2}{D^2},$$

where the sample mean $\bar{X} = \sum_{i=1}^{n} X_i/n$ is the conventional estimator of the process mean μ. We note that the estimator $\hat{C}_{ia}$ can be written as the following:

$$\hat{C}_{ia} = \frac{C_{ip}}{n}\frac{n\hat{C}_{ia}}{C_{ip}} = \frac{C_{ip}}{n}\frac{n(\bar{X}-T)^2}{\sigma^2}.$$

If the process characteristic follows the normal distribution, then the estimator $\hat{C}_{ia}$ is distributed as $[C_{ip}/n]\chi_1^2(\delta)$, where $\chi_1^2(\delta)$ represents a non-central chi-square distribution, which has one degree of freedom with non-centrality parameter $\delta = n(\mu - T)^2/\sigma^2 = nC_{ia}/C_{ip}$. We note that since the estimator $\hat{C}_{ip}$ is a function of S_{n-1} only, and the estimator $\hat{C}_{ia}$ is a function of $\bar{X}$ only, then the two estimators $\hat{C}_{ip}$ and $\hat{C}_{ia}$ are statistically independent.

Since $\bar{X}$ is the MLE of μ, then by the invariance property of the MLE, the natural estimator $\hat{C}_{ia}$ is the MLE of C_{ia}. Noting that $E(\hat{C}_{ia}) = C_{ia} + (C_{ip}/n)$, and $E(\hat{C}_{ip}) = C_{ip}$, the corrected estimator $\tilde{C}_{ia} = \hat{C}_{ia} - (\hat{C}_{ip}/n)$ must be unbiased for C_{ia}. Pearn and Lin [5] showed that $\tilde{C}_{ia}$ is the UMVUE of C_{ia}, which is consistent and asymptotically efficient, and that $\sqrt{n}(\tilde{C}_{ia} - C_{ia})$ converges to $N(0, 4C_{ip}C_{ia})$ in distribution. Thus, in real-world applications using the UMVUE $\tilde{C}_{ia}$, which has all desired statistical properties, as an estimate of C_{ia} would be reasonable.

We note that the MLE $\hat{C}_{ia}$ has smaller variance than the UMVUE $\tilde{C}_{ia}$. But, we can show that $\mathrm{MSE}(\tilde{C}_{ia}) = 4C_{ip}C_{ia}/n + [2/n(n-1)](C_{ip})^2$, and so $\mathrm{MSE}(\tilde{C}_{ia}) - \mathrm{MSE}(\hat{C}_{ia}) = [(3-n)/n^2(n-1)](C_{ip})^2$, which is less than 0 for $n \geqslant 4$. Therefore, the UMVUE $\tilde{C}_{ia}$ has a smaller mean square error than that of the MLE $\hat{C}_{ia}$, and is more reliable for applications with $n \geqslant 4$.

2.3. Estimation of C_{pp}

To estimate the process incapability C_{pp}, a combined measure of process imprecision and process inaccuracy, we consider the natural estimator $\hat{C}_{pp}$ defined as the following, which also can be rewritten as a function of C_{ip}.

$$\hat{C}_{pp} = \frac{1}{n}\sum_{i=1}^{n}\frac{\left(X_i - \bar{X}\right)^2}{D^2} + \frac{\left(\bar{X} - T\right)^2}{D^2} = \frac{C_{ip}}{n}\frac{n\hat{C}_{pp}}{C_{ip}} = \frac{C_{ip}}{n}\sum_{i=1}^{n}\frac{(X_i - T)^2}{\sigma^2}.$$

If the process characteristic follows the normal distribution $N(\mu, \sigma^2)$, then the estimator $\hat{C}_{pp}$ is distributed as $[C_{ip}/n]\chi_n^2(\delta)$, where $\chi_n^2(\delta)$ is a non-central chi-square distribution with n degrees of freedom and non-centrality parameter $\delta = n(\mu - T)^2/\sigma^2 = nC_{ia}/C_{ip}$.

If the process characteristic follows the normal distribution, Pearn and Lin [5] showed that $\hat{C}_{pp}$ is the MLE, which is also the UMVUE of C_{pp}. We also can show that the estimator $\hat{C}_{pp}$ is consistent, asymptotically efficient, and that $\sqrt{n}(\hat{C}_{pp} - C_{pp})$ converges to $N(0, 2C_{ip}C_{ia} + 2C_{ip}C_{pp})$ in distribution. Since the estimator has all the desired statistical properties, in practice using $\hat{C}_{pp}$ to estimate process incapability would be reasonable.

3. The C_{pp} MPPAC

Many statistical control charts, such as $\bar{X}$, R, S^2, S, and MR charts, have been widely used in monitoring and controlling process quality. Those charts, however, are applicable only for single processes (one process at a time). Thus, using those charts in multi-process environment can be a difficult and time-consuming task for the supervisor or shop engineer to analyze each individual chart to evaluate the overall status of shop process control activity.

The MPPAC can be used to evaluate the performance of a single process as well as multi-processes; to set the priorities among multiple processes for quality improvement, and indicate if reducing the variability, or the departure of the process mean should be the focus; to provide an easy way to quantify the process improvement by comparing the locations on the chart of the processes before and after the improvement effort. The MPPAC is an effective tool for communicating between the product designer, the manufacturers, the quality engineers, and among the management departments.

Based on the definition, $C_{pp} = (\mu - T)^2/D^2 + \sigma^2/D^2$, we first set $C_{pp} = k$, for various k values, then a set of (μ, σ) values satisfying the equation: $(\mu - T)^2 + \sigma^2 = kD^2$ can be plotted on the contour (a curve) of $C_{pp} = k$. These contours are semicircles centered at $\mu = T$ with radius $\sqrt{k}D$. The more capable the process, the smaller the semicircle is. We plot the six contours on the C_{pp} MPPAC for the six C_{pp} values listed in Table 1, as shown in Fig. 1. On the C_{pp} MPPAC, we note that:

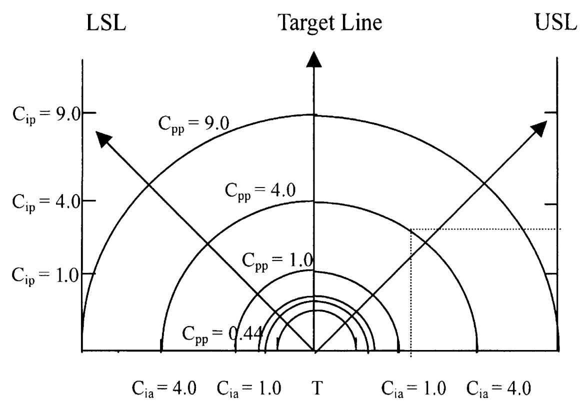

Fig. 1. The C_{pp} MPPAC.

1. As the point gets closer to the target, the value of the C_{pp} becomes smaller, and the process performance is better.
2. For the points inside the semicircle of contour $C_{pp} = k$, the corresponding C_{pp} values are smaller than k. For the points outside the semicircle of contour $C_{pp} = k$, the corresponding C_{pp} values are greater than k.
3. For processes with fixed values of C_{pp}, the points within the two 45° lines envelop, the variability is contributed mainly by the process variance.
4. For processes with fixed values of C_{pp}, the points outside the two 45° lines envelop, the process variability is contributed mainly by the process departure.
5. The perpendicular line and parallel line through the plotted point intersecting the horizontal axis and vertical axis at points represent its C_{ia} and C_{ip}, respectively.
6. The distance between T and the point, which the perpendicular line through the plotted point intersecting the horizontal axis, denote the departure of process mean from target.
7. The distance between T and the point, which the parallel line through the plotted point intersecting the vertical axis, denote the process variance.

4. An application

In the following, we consider a resistor manufacturing process. Resistor is an electronic passive component commonly used on electronic circuits, providing the function of reducing the current, voltage, as well as releasing the heat. Eight standard precisions, with their required tolerances, are displayed in Table 3 (see Chen [6], Chen [7], and Wang [8]).

We consider the following case taken from *Cinetech*, a factory located on an industrial park making chip resistors. We investigated 15 specific types of chip resistors widely used on the personal computers, televisions, and other audio and video electronic devices with different resistance specifications. A random sample of size 100 is taken from each of the fifteen resistor manufacturing processes. Their resistance specifications are displayed in Table 4. The calculated sample mean, sample variance, and the index values of C_{pp}, C_{ia}, and C_{ip} are shown as Table 5.

Fig. 2 plots the C_{pp} MPPAC for the fifteen processes listed in Table 5. We analyze the process points in Fig. 2, and obtain the following summary of the quality condition.

1. The plotted point H is very close to the contour $C_{pp} = 4$, it indicates that the process has a low capability. Since the point H is close to the target line, it indicates that the poor capability is mainly contributed by the process variation. Thus, it calls for an immediate quality improvement action to reduce the process variance.

Table 3
Code and tolerance

Level	Tolerance (%)
I	±0.1
II	±0.25
III	±0.5
IV	±1.0
V	±2.0
VI	±5.0
VII	±10
VIII	±20

Table 4
The resistance specifications

Code	Ω	Tolerance (%)	USL	LSL
A	220	±5	231.00	209.00
B	10	±5	10.50	9.50
C	1	±1	1.01	0.99
D	5 K	±2	5.10	4.90
E	1.5 K	±2	1.53	1.47
F	2 M	±1	2.02	1.98
G	10 M	±2	10.20	9.80
H	100	±0.1	100.10	99.90
I	10	±0.5	10.05	9.95
J	470 K	±2	479.40	460.60
K	180	±0.25	180.45	179.55
L	22 K	±1	22.22	21.78
M	0.3	±10	0.33	0.27
N	68	±5	71.40	64.60
O	33 K	±2	33.66	32.34

Table 5
The calculated statistics

Process	$\bar{X}$	S	C_{ia}	C_{ip}	C_{pp}
A	223.031	3.252	0.68	0.79	1.47
B	10.102	0.126	0.38	0.57	0.95
C	0.996	0.003	1.82	0.92	2.74
D	5.011	0.040	0.10	1.43	1.54
E	1.505	0.008	0.27	0.71	0.98
F	1.992	0.003	1.44	0.20	1.64
G	10.011	0.030	0.02	0.20	0.23
H	100.012	0.060	0.13	3.24	3.37
I	10.009	0.012	0.29	0.52	0.81
J	468.058	3.492	0.38	1.24	1.63
K	180.200	0.120	1.78	0.64	2.42
L	21.905	0.045	1.68	0.38	2.05
M	0.298	0.009	0.04	0.81	0.85
N	68.958	0.906	0.71	0.64	1.35
O	32.850	0.250	0.46	1.29	1.76

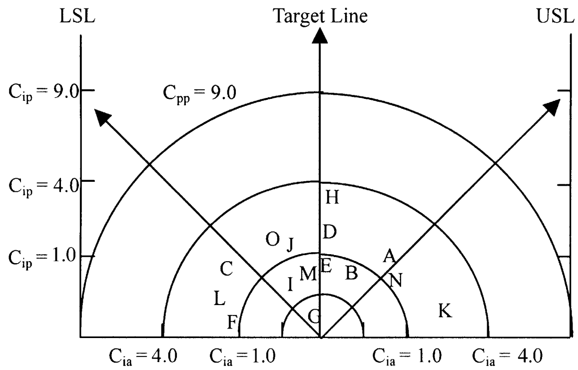

Fig. 2. The C_{pp} MPPAC for the example.

2. The plotted points D, J, and O lie outside of the contour $C_{pp} = 1$, it indicates their C_{pp} are higher than 1. Since these points all lie inside the two 45° line envelope range, it indicates that their C_{ip} must be higher than their C_{ia}. Thus, reducing their process variance has higher priority than reducing the process departure.
3. The plotted points C, F, K, and L lie outside of the contour $C_{pp} = 1$ and the two 45° line envelope range, their C_{ia} must be higher than their C_{ip}. Quality improvement efforts for these processes should be first focused on reducing the departure of process mean from the target value.
4. The plotted points A and N are close to the two 45° lines, and are outside the contour of $C_{pp} = 1$. It indicates that the variability of those processes is contributed equally by the mean departure and process variance.
5. The plotted points B, E, I, and M lie inside the contour of $C_{pp} = 1$, it means their C_{pp} are lower than 1. Capabilities of these processes are considered to be satisfactory. But they will be the candidates for lower priority quality improvement efforts.
6. Process G is very close to T and its C_{pp} is small, so the process G is considered performing well.

5. Conclusions

In this paper, we introduced a new control chart, called the C_{pp} MPPAC, using the incapability index C_{pp}. The C_{pp} MPPAC displays multiple processes with the mean departure, and process variability relative to the specification tolerances, on one single chart. We demonstrated the use of the C_{pp} MPPAC by presenting a case study on some resistor manufacturing processes, to evaluate the factory performance. The C_{pp} MPPAC chart is an efficient tool for the shop supervisors and engineers to evaluate the overall status of shop process control activity. The C_{pp} MPPAC provides critical information regarding process conditions and useful to quality improvement activity.

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