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COMPUTER PROGRAM FOR CALCULATING THE p-VALUE IN TESTING PROCESS CAPABILITY INDEX C_{pmk}

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SUMMARY

Many process capability indices, including C_p, C_{pk}, and C_{pm}, have been proposed to provide numerical measures on the process potential and performance. Combining the advantages of these indices, Pearn *et al.* (1992) introduced a new capability index called C_{pmk}, which has been shown to be a useful capability index for processes with two-sided specification limits. In this paper, we implement the theory of a testing hypothesis using the natural estimator of C_{pmk}, and provide an efficient Maple computer program to calculate the p-values. We also provide tables of the critical values for some commonly used capability requirements. Based on the test we develop a simple step-by-step procedure for in-plant applications. The practitioners can use the proposed procedure to determine whether their process meets the preset capability requirement, and make reliable decisions. Copyright © 2002 John Wiley & Sons, Ltd.

KEY WORDS: process capability index; testing hypothesis; critical value; p-value

1. INTRODUCTION

Process capability indices, including C_p, C_{pk}, and C_{pm}, have been proposed in the manufacturing industry to provide numerical measures on whether a process is capable of reproducing items meeting the quality requirement preset in the factory. Combining the advantages of these indices, Pearn *et al.* [1] introduced a new capability index called C_{pmk}, which has been shown to be a useful capability index for processes with two-sided specification limits. Vännman [2] constructed a unified superstructure for the four basic indices, C_p, C_{pk}, C_{pm}, and C_{pmk}. The superstructure has been referred to as $C_p(u, v)$, which can be defined as

$$C_p(u, v) = \frac{d - u|\mu - m|}{3\sqrt{\sigma^2 + v(\mu - T)^2}},$$

where μ is the process mean, σ is the process standard deviation, T is the target value preset by the product designer, $d = (USL - LSL)/2$ is half of the length of the specification interval, $m = (USL + LSL)/2$ is the mid-point between the lower and the upper specification limits (LSL and USL), and $u, v \geqslant 0$. It is easy to verify that $C_p(0, 0) = C_p$, $C_p(1, 0) = C_{pk}$, $C_p(0, 1) = C_{pm}$, and $C_p(1, 1) = C_{pmk}$, which have been defined explicitly as

$$C_p = \frac{USL - LSL}{6\sigma},$$

$$C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\},$$

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}},$$

$$C_{pmk} = \min\left\{\frac{USL - \mu}{3\sqrt{\sigma^2 + (\mu - T)^2}}, \frac{\mu - LSL}{3\sqrt{\sigma^2 + (\mu - T)^2}}\right\}$$

The index C_{pmk} is constructed [1] by combining the yield-based index C_{pk} and the loss-based index C_{pm}, taking into account the process yield (meeting the manufacturing specifications) as well as the process loss (variation from the target). When the process mean μ departs from the target value T, the reduced value of C_{pmk} is more significant than those of C_p, C_{pk}, and C_{pm}. Hence, the index C_{pmk} responds to the departure of the process mean μ from the target value T faster than the other three basic indices C_p, C_{pk}, and C_{pm}, while it remains sensitive to the changes of process variation (see [1]). We note that a process meeting the capability requirement '$C_{pk} \geqslant C$' may

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not be meeting the capability requirement '$C_{pm} \geqslant C$'. On the other hand, a process meeting the capability requirement '$C_{pm} \geqslant C$' may not be meeting the capability requirement '$C_{pk} \geqslant C$' either. The discrepancy between the two indices may be contributed to the fact that the C_{pk} index primarily measures the process yield, but the C_{pm} index focuses mainly on the process loss.

However, if the process meets the capability requirement '$C_{pmk} \geqslant C$', then the process must meet both capability requirements '$C_{pk} \geqslant C$' and '$C_{pm} \geqslant C$' since $C_{pmk} \leqslant C_{pk}$ and $C_{pmk} \leqslant C_{pm}$. According to today's modern quality improvement theory, reduction of the process loss is as important as increasing the process yield. While C_{pk} remains the more popular and widely used index, C_{pmk} is considered to be an advanced and useful index for processes with two-sided specification limits.

2. DISTRIBUTION OF THE ESTIMATED C_{pmk}

For a normally distributed process that is demonstrably stable (under statistical control), Pearn *et al.* [1] considered the maximum likelihood estimator (MLE) of C_{pmk}:

$$\hat{C}_{pmk} = \min\left\{\frac{USL - \overline{X}}{3\sqrt{S_n^2 + (\overline{X} - T)^2}}, \frac{\overline{X} - LSL}{3\sqrt{S_n^2 + (\overline{X} - T)^2}}\right\},$$

where

$$\overline{X} = \sum_{i=1}^{n} X_i/n \quad \text{and} \quad S_n^2 = \sum_{i=1}^{n} (X_i - \bar{X})^2/n$$

are the MLEs of μ and σ^2, respectively. We note that

$$S_n^2 + (\overline{X} - T)^2 = \sum_{i=1}^{n} (X_i - T)^2/n,$$

which is the major part of the denominator of $\hat{C}_{pmk}$, is the uniformly minimum variance unbiased estimator (UMVUE) of

$$\sigma^2 + (\mu - T)^2 = E[(X - T)^2]$$

in the denominator of C_{pmk}.

Under the assumption of normality, Pearn *et al.* [1] obtained the rth moment and the first two moments, as well as the mean and the variance of $\hat{C}_{pmk}$ for the common cases with $T = m$. Chen and Hsu [3] showed that the estimator $\hat{C}_{pmk}$ is consistent, and asymptotically unbiased. Vännman and Kotz [4] obtained the distribution of the estimated $C_p(u, v)$ for cases with $T = m$. Vännman [5] further provided a simplified form for the obtained distribution. The cumulative distribution function of $\hat{C}_{pmk}$ can, therefore, be expressed (using our notation) as

$$F_{\hat{C}_{pmk}}(x) = 1 - \int_0^{b\sqrt{n}/(1+3x)} G\left(\frac{(b\sqrt{n} - t)^2}{9x^2} - t^2\right) \times [\phi(t + \xi\sqrt{n}) + \phi(t - \xi\sqrt{n})]\,\mathrm{d}t \qquad (1)$$

for $x > 0$, where $b = d/\sigma$, $\xi = (\mu - T)/\sigma$, $G(\cdot)$ is the cumulative distribution function of the chi-squared distribution χ^2_{n-1}, and $\phi(\cdot)$ is the probability density function of the standard normal distribution $N(0, 1)$.

In practice, sample data must be collected in order to calculate the index value; therefore, a great degree of uncertainty may be introduced into capability assessments due to sampling errors. The approach of simply looking at the index value calculated from the given sample and then making a conclusion on whether the given process is capable or not is intuitively reasonable but not reliable because sampling errors are ignored. Taking into account the sampling errors, we implement the theory of a testing hypothesis using the natural estimator of C_{pmk}, and provide an efficient Maple computer program to calculate the p-values for making reliable decisions. This approach is similar to the one proposed by Cheng [6] for testing process capability C_{pm}. We also provide the tables of the critical values for some commonly used capability requirements. Using these tables, the practitioners may choose not to run the computer program. Based on the test we develop a simple step-by-step procedure for in-plant applications. The practitioners can use the proposed procedure to judge whether or not their process meets the preset capability requirement (capable) and runs under the desired quality condition.

3. TESTING THE PROCESS CAPABILITY

To test whether a given process is capable using the index C_{pmk}, we consider the following statistical testing hypotheses:

H_0: $C_{pmk} \leqslant C$ (process is not capable),
H_1: $C_{pmk} > C$ (process is capable).

Based on a given $\alpha(c_0) = \alpha$, the chance of incorrectly concluding an incapable process as capable, the decision rule is to reject H_0 if $\hat{C}_{pmk} > c_0$ and fail to reject H_0 otherwise.


```
> # Input parameter values LSL, USL, T, C, n, X_bar, Sn.
LSL:=2.40;   USL:=3.40;   T:=2.90;   C:=1.00;
n:=100;   X_bar:=2.865;   Sn:=0.125;
d:=(USL - LSL)/2;  ξ=(X_bar - T)/Sn;
c1:=(d - abs(X_bar - T))/(3*(Sn^2 + (X_bar - T)^2)^0.5):
# Note that c1 = c* = Cpmk_hat.
b:=(3*C*(1 + ξ^2)^0.5 + abs(ξ)):
G:=(c1,t)->stats[statevalf,cdf,chisquare[n-1]]
  ((b*n^0.5 - t)^2/(9*c1^2) - t^2):
h:=t->stats[statevalf,pdf,normald](t + ξ*n^0.5)
  +stats[statevalf,pdf,normald](t - ξ*n^0.5):
pV:=c1->int(G(c1,t)*h(t),t=0..(b*n^0.5/(1 + 3*c1))):
Estimated_Cpmk:=c1;
p_Value:=evalf(pV(c1));
```

The output is:
`LSL` := 2.40
`USL` := 3.40
`T` := 2.90
`C` := 1.00
`n` := 100
`X_bar` := 2.865
`Sn` := 0.125
`d` := 0.500000000
ξ := −0.2800000000
`Estimated_Cpmk` := 1.194075384
`p_Value` := 0.02529584382.

Figure 1.

Given values of α and C, the critical value c_0 can be obtained by solving the equation $P(\hat{C}_{pmk} \geqslant c_0 \mid C_{pmk} = C) = \alpha$ using available numerical methods. For processes with a target value set to the mid-point of the specification limits ($T = m$), the index may be rewritten as

$$C_{pmk} = \frac{d - |\mu - T|}{3\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{d/\sigma - |\xi|}{3\sqrt{1 + \xi^2}},$$

where $\xi = (\mu - T)/\sigma$.

Given $C_{pmk} = C$, $b = d/\sigma$ can be expressed as $b = 3C\sqrt{1 + \xi^2} + |\xi|$. Given a value of C (the capability requirement), the p-value corresponding to c^*, a specific value of $\hat{C}_{pmk}$ calculated from the sample data, is (by equation (1))

$$P\{\hat{C}_{pmk} \geqslant c^* \mid C_{pmk} = C\} = \int_0^{b\sqrt{n}/(1+3c^*)} G\left(\frac{(b\sqrt{n} - t)^2}{9(c^*)^2} - t^2\right) \times [\phi(t + \xi\sqrt{n}) + \phi(t - \xi\sqrt{n})]\,\mathrm{d}t \qquad (2)$$

Hence, given values of the capability requirement C, the parameter ξ, the sample size n, and risk α, the critical value c_0 can be obtained by solving the following equation:

$$\int_0^{b\sqrt{n}/(1+3c_0)} G\left(\frac{(b\sqrt{n} - t)^2}{9c_0^2} - t^2\right) \times [\phi(t + \xi\sqrt{n}) + \phi(t - \xi\sqrt{n})]\,\mathrm{d}t = \alpha \qquad (3)$$

Given values of C, n, and α, the critical value c_0 for $\xi = \xi_0$ and $\xi = -\xi_0$ is the same because equation (3) is an even function of ξ.

4. COMPUTER PROGRAM

An efficient Maple computer program is developed to calculate equation (2), to obtain the p-value for given c^*. We note that similar programs can also be written using 'Mathematica' or 'MatLab' software. The program is listed in Figure 1, with input parameters set to $LSL = 2.40$, $USL = 3.40$, $T = 2.90$, $C = 1.00$, $n = 100$, $\overline{X} = 2.865$, and $S_n = 0.125$. Here, we set $\xi = \hat{\xi} = (\overline{X} - T)/S_n$, since generally $\xi = (\mu - T)/\sigma$ is unknown. This approach is similar to

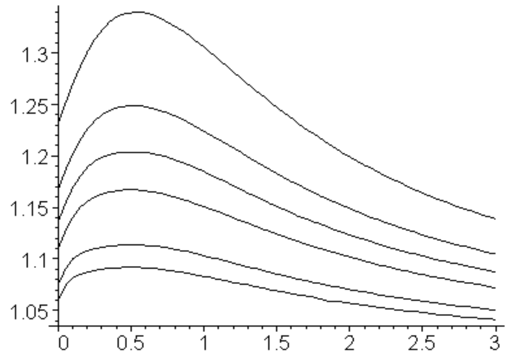

Figure 2. Plots of c_0 versus $|\xi|$ for $C_{pmk} = 1.00$, $\alpha = 0.05$, and $n = 30, 50, 70, 100\ 200, 300$ (top to bottom in plot)

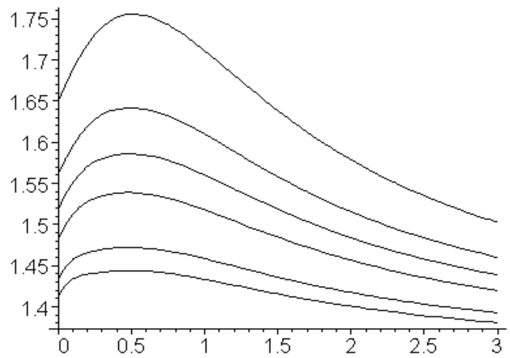

Figure 3. Plots of c_0 versus $|\xi|$ for $C_{pmk} = 1.33$, $\alpha = 0.05$, and $n = 30, 50, 70, 100\ 200, 300$ (top to bottom in plot)

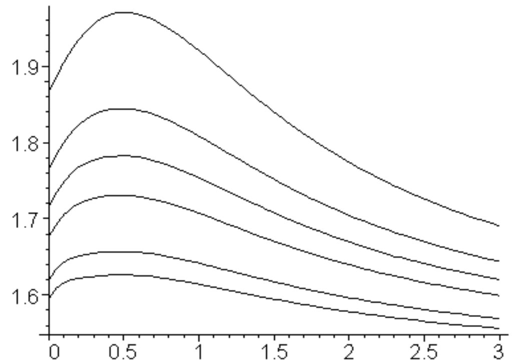

Figure 4. Plots of c_0 versus $|\xi|$ for $C_{pmk} = 1.50$, $\alpha = 0.05$, and $n = 30, 50, 70, 100\ 200, 300$ (top to bottom in plot)

the one proposed by Cheng [5] for testing the process capability index C_{pm}. On the other hand,

$$c^* = \hat{C}_{pmk} = (d - |\overline{X} - T|)/\{3[S_n^2 + (\overline{X} - T)^2]^{1/2}\}$$

can be calculated from the sample data. The program gives $\hat{\xi} = -0.28$ and $\hat{C}_{pmk} = 1.194$, and the corresponding p-value as 0.0253.

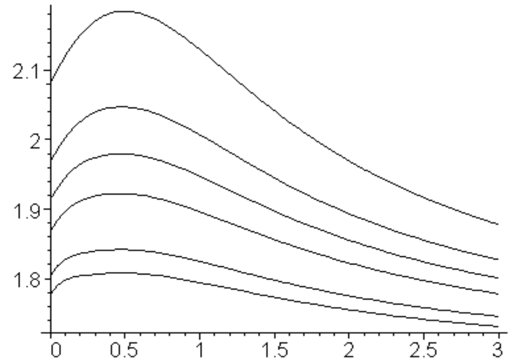

Figure 5. Plots of c_0 versus $|\xi|$ for $C_{pmk} = 1.67$, $\alpha = 0.05$, and $n = 30, 50, 70, 100\ 200, 300$ (top to bottom in plot)

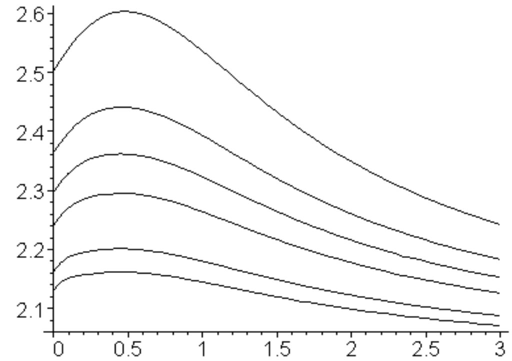

Figure 6. Plots of c_0 versus $|\xi|$ for $C_{pmk} = 2.00$, $\alpha = 0.05$, and $n = 30, 50, 70, 100\ 200, 300$ (top to bottom in plot)

5. CRITICAL VALUES c_0 AND ξ

Since the process parameters μ and σ are unknown, then the parameter $\xi = (\mu - T)/\sigma$ is also unknown, which has to be estimated in real applications, naturally by substituting μ and σ by its sample mean and sample standard deviation. Such an approach certainly would make our approach less reliable. To eliminate the need for estimating the parameter ξ, we examine the behavior of the critical values c_0 as a function of ξ. We calculate the critical values c_0 for $\xi = 0(0.05)3.00$, $n = 30, 50, 70, 100, 200, 300$, $C = 1.00, 1.33, 1.50, 1.67, 2.00$, and $\alpha = 0.01, 0.025, 0.05$. Noting that $\xi = 0(0.05)3.00$ covers a wide range of applications with process capability $C_{pmk} \geqslant 0$. We find that the critical value c_0 obtains its maximum either at $\xi = 0.50$ (for most cases), or at 0.45 (in a few cases), and the difference between the two critical values is less than 10^{-3}. Hence, for practical purposes we may solve equation (3) for $\xi = 0.50$ to obtain the required critical value for the given C, n, and α, without having to estimate the parameter ξ. We note the above result is almost impossible to prove theoretically.

Table 1. Critical values c_0 for $C = 1.00$, $n = 10(5)405$, and $\alpha = 0.01, 0.025, 0.05$

n	$\alpha = 0.01$	$\alpha = 0.025$	$\alpha = 0.05$	n	$\alpha = 0.01$	$\alpha = 0.025$	$\alpha = 0.05$
10	2.148	1.892	1.704	210	1.160	1.133	1.111
15	1.831	1.660	1.530	215	1.158	1.132	1.110
20	1.675	1.542	1.439	220	1.156	1.130	1.108
25	1.578	1.468	1.380	225	1.154	1.129	1.107
30	1.512	1.416	1.339	230	1.152	1.127	1.106
35	1.463	1.377	1.309	235	1.151	1.126	1.105
40	1.425	1.347	1.285	240	1.149	1.124	1.103
45	1.394	1.323	1.265	245	1.147	1.123	1.102
50	1.369	1.303	1.249	250	1.146	1.121	1.101
55	1.348	1.286	1.235	255	1.144	1.120	1.100
60	1.330	1.271	1.224	260	1.143	1.119	1.099
65	1.314	1.259	1.213	265	1.141	1.118	1.098
70	1.301	1.248	1.205	270	1.140	1.116	1.097
75	1.289	1.238	1.197	275	1.138	1.115	1.096
80	1.278	1.229	1.189	280	1.137	1.114	1.095
85	1.268	1.221	1.183	285	1.136	1.113	1.094
90	1.259	1.214	1.177	290	1.134	1.112	1.093
95	1.251	1.208	1.172	295	1.133	1.111	1.093
100	1.244	1.202	1.167	300	1.132	1.110	1.092
105	1.237	1.196	1.162	305	1.131	1.109	1.091
110	1.231	1.191	1.158	310	1.130	1.108	1.090
115	1.225	1.186	1.154	315	1.128	1.107	1.089
120	1.219	1.182	1.151	320	1.127	1.106	1.089
125	1.214	1.178	1.147	325	1.126	1.105	1.088
130	1.210	1.174	1.144	330	1.125	1.105	1.087
135	1.205	1.170	1.141	335	1.124	1.104	1.087
140	1.201	1.167	1.138	340	1.123	1.103	1.086
145	1.197	1.164	1.136	345	1.122	1.102	1.085
150	1.193	1.161	1.133	350	1.121	1.101	1.085
155	1.190	1.158	1.131	355	1.120	1.101	1.084
160	1.186	1.155	1.129	360	1.119	1.100	1.083
165	1.183	1.152	1.127	365	1.119	1.099	1.083
170	1.180	1.150	1.125	370	1.118	1.098	1.082
175	1.177	1.147	1.123	375	1.117	1.098	1.082
180	1.175	1.145	1.121	380	1.116	1.097	1.081
185	1.172	1.143	1.119	385	1.115	1.096	1.080
190	1.169	1.141	1.117	390	1.114	1.095	1.080
195	1.167	1.139	1.116	395	1.114	1.095	1.079
200	1.165	1.137	1.114	400	1.113	1.094	1.079
205	1.162	1.135	1.112	405	1.112	1.094	1.078

Table 2. Critical values c_0 for $C = 1.33$, $n = 10(5)405$, and $\alpha = 0.01, 0.025, 0.05$

n	$\alpha = 0.01$	$\alpha = 0.025$	$\alpha = 0.05$	n	$\alpha = 0.01$	$\alpha = 0.025$	$\alpha = 0.05$
10	2.792	2.464	2.224	210	1.530	1.497	1.469
15	2.383	2.165	1.999	215	1.528	1.495	1.467
20	2.182	2.013	1.882	220	1.525	1.493	1.465
25	2.059	1.918	1.808	225	1.523	1.491	1.464
30	1.974	1.852	1.756	230	1.520	1.489	1.462
35	1.912	1.804	1.717	235	1.518	1.487	1.460
40	1.864	1.765	1.687	240	1.516	1.485	1.459
45	1.825	1.735	1.662	245	1.514	1.483	1.458
50	1.793	1.710	1.642	250	1.512	1.482	1.456
55	1.767	1.688	1.625	255	1.510	1.480	1.455
60	1.744	1.670	1.610	260	1.508	1.478	1.454
65	1.724	1.654	1.597	265	1.506	1.477	1.452
70	1.707	1.640	1.586	270	1.504	1.475	1.451
75	1.692	1.628	1.576	275	1.503	1.474	1.450
80	1.678	1.617	1.567	280	1.501	1.473	1.449
85	1.666	1.607	1.559	285	1.499	1.471	1.448
90	1.654	1.598	1.552	290	1.498	1.470	1.447
95	1.644	1.590	1.545	295	1.496	1.469	1.445
100	1.635	1.582	1.539	300	1.495	1.467	1.444
105	1.627	1.575	1.533	305	1.493	1.466	1.443
110	1.619	1.569	1.528	310	1.492	1.465	1.442
115	1.611	1.563	1.523	315	1.490	1.464	1.442
120	1.605	1.557	1.518	320	1.489	1.463	1.441
125	1.598	1.552	1.514	325	1.488	1.462	1.440
130	1.592	1.547	1.510	330	1.486	1.461	1.439
135	1.587	1.543	1.507	335	1.485	1.459	1.438
140	1.581	1.539	1.503	340	1.484	1.458	1.437
145	1.576	1.535	1.500	345	1.483	1.457	1.436
150	1.572	1.531	1.497	350	1.481	1.456	1.435
155	1.567	1.527	1.494	355	1.480	1.456	1.145
160	1.563	1.524	1.491	360	1.479	1.455	1.434
165	1.559	1.520	1.488	365	1.478	1.454	1.433
170	1.555	1.517	1.486	370	1.477	1.453	1.432
175	1.552	1.514	1.483	375	1.476	1.452	1.432
180	1.548	1.511	1.481	380	1.475	1.451	1.431
185	1.545	1.509	1.479	385	1.474	1.450	1.430
190	1.542	1.506	1.476	390	1.473	1.449	1.430
195	1.539	1.504	1.474	395	1.472	1.449	1.429
200	1.536	1.501	1.472	400	1.471	1.448	1.428
205	1.533	1.499	1.470	405	1.470	1.447	1.428

Table 3. Critical values c_0 for $C = 1.50$, $n = 10(5)405$, and $\alpha = 0.01, 0.025, 0.05$

n	$\alpha = 0.01$	$\alpha = 0.025$	$\alpha = 0.05$	n	$\alpha = 0.01$	$\alpha = 0.025$	$\alpha = 0.05$
10	3.124	2.759	2.492	210	1.721	1.684	1.653
15	2.668	2.425	2.241	215	1.718	1.682	1.651
20	2.444	2.256	2.111	220	1.715	1.679	1.649
25	2.306	2.151	2.028	225	1.713	1.677	1.647
30	2.213	2.078	1.971	230	1.710	1.675	1.646
35	2.143	2.023	1.927	235	1.708	1.673	1.644
40	2.090	1.981	1.894	240	1.705	1.671	1.642
45	2.047	1.947	1.867	245	1.703	1.669	1.641
50	2.012	1.919	1.844	250	1.701	1.667	1.639
55	1.982	1.896	1.825	255	1.699	1.666	1.638
60	1.957	1.876	1.809	260	1.696	1.664	1.636
65	1.935	1.858	1.795	265	1.694	1.662	1.635
70	1.916	1.843	1.783	270	1.692	1.660	1.634
75	1.899	1.829	1.771	275	1.691	1.659	1.632
80	1.884	1.817	1.762	280	1.689	1.657	1.631
85	1.871	1.806	1.753	285	1.687	1.656	1.630
90	1.858	1.796	1.745	290	1.685	1.654	1.629
95	1.847	1.787	1.737	295	1.683	1.653	1.627
100	1.837	1.779	1.730	300	1.682	1.652	1.626
105	1.827	1.771	1.724	305	1.680	1.650	1.625
110	1.819	1.764	1.718	310	1.678	1.649	1.624
115	1.811	1.757	1.713	315	1.677	1.648	1.623
120	1.803	1.751	1.708	320	1.675	1.646	1.622
125	1.796	1.745	1.703	325	1.674	1.645	1.621
130	1.790	1.740	1.699	330	1.673	1.644	1.620
135	1.783	1.735	1.695	335	1.671	1.643	1.619
140	1.778	1.730	1.691	340	1.670	1.642	1.618
145	1.772	1.726	1.687	345	1.668	1.641	1.617
150	1.767	1.722	1.684	350	1.667	1.640	1.616
155	1.762	1.718	1.681	355	1.666	1.638	1.615
160	1.757	1.714	1.677	360	1.665	1.637	1.615
165	1.753	1.710	1.674	365	1.663	1.636	1.614
170	1.749	1.707	1.672	370	1.662	1.635	1.613
175	1.745	1.703	1.669	375	1.661	1.634	1.612
180	1.741	1.700	1.666	380	1.660	1.634	1.611
185	1.737	1.697	1.664	385	1.659	1.633	1.611
190	1.734	1.694	1.662	390	1.658	1.632	1.610
195	1.730	1.692	1.659	395	1.657	1.631	1.609
200	1.727	1.689	1.657	400	1.656	1.630	1.608
205	1.724	1.686	1.655	405	1.654	1.629	1.608

Table 4. Critical values c_0 for $C = 1.67$, $n = 10(5)405$, and $\alpha = 0.01, 0.025, 0.05$

n	$\alpha = 0.01$	$\alpha = 0.025$	$\alpha = 0.05$	n	$\alpha = 0.01$	$\alpha = 0.025$	$\alpha = 0.05$
10	3.457	3.054	2.760	210	1.912	1.871	1.837
15	2.953	2.686	2.483	215	1.909	1.869	1.835
20	2.705	2.499	2.339	220	1.906	1.866	1.833
25	2.554	2.383	2.249	225	1.903	1.864	1.831
30	2.451	2.303	2.185	230	1.900	1.862	1.829
35	2.375	2.243	2.138	235	1.897	1.859	1.827
40	2.316	2.197	2.101	240	1.895	1.857	1.826
45	2.269	2.160	2.072	245	1.892	1.855	1.824
50	2.231	2.129	2.047	250	1.890	1.853	1.822
55	2.198	2.103	2.026	255	1.887	1.851	1.821
60	2.171	2.081	2.008	260	1.885	1.849	1.819
65	2.147	2.062	1.993	265	1.883	1.847	1.818
70	2.126	2.045	1.979	270	1.881	1.846	1.816
75	2.107	2.030	1.967	275	1.878	1.844	1.815
80	2.091	2.017	1.956	280	1.876	1.842	1.813
85	2.076	2.005	1.947	285	1.874	1.840	1.812
90	2.062	1.994	1.938	290	1.873	1.839	1.811
95	2.050	1.984	1.930	295	1.871	1.837	1.809
100	2.039	1.975	1.922	300	1.869	1.836	1.808
105	2.029	1.967	1.915	305	1.867	1.834	1.807
110	2.019	1.959	1.909	310	1.865	1.833	1.806
115	2.010	1.952	1.903	315	1.864	1.832	1.805
120	2.002	1.945	1.898	320	1.862	1.830	1.803
125	1.994	1.939	1.893	325	1.860	1.829	1.802
130	1.987	1.933	1.888	330	1.859	1.828	1.801
135	1.980	1.927	1.883	335	1.857	1.826	1.800
140	1.974	1.922	1.879	340	1.856	1.825	1.799
145	1.968	1.917	1.875	345	1.854	1.824	1.798
150	1.962	1.912	1.871	350	1.853	1.823	1.797
155	1.957	1.908	1.868	355	1.851	1.821	1.796
160	1.952	1.904	1.864	360	1.850	1.820	1.795
165	1.947	1.900	1.861	365	1.849	1.819	1.794
170	1.942	1.896	1.858	370	1.847	1.818	1.794
175	1.938	1.893	1.855	375	1.846	1.817	1.793
180	1.934	1.889	1.852	380	1.845	1.816	1.792
185	1.930	1.886	1.849	385	1.844	1.815	1.791
190	1.926	1.883	1.847	390	1.842	1.814	1.790
195	1.922	1.880	1.844	395	1.841	1.813	1.789
200	1.919	1.877	1.842	400	1.840	1.812	1.789
205	1.915	1.874	1.840	405	1.839	1.811	1.788

Table 5. Critical values c_0 for $C = 2.00$, $n = 10(5)405$, and $\alpha = 0.01, 0.025, 0.05$

n	$\alpha = 0.01$	$\alpha = 0.025$	$\alpha = 0.05$	n	$\alpha = 0.01$	$\alpha = 0.025$	$\alpha = 0.05$
10	4.102	3.627	3.280	210	2.282	2.235	2.195
15	3.506	3.192	2.953	215	2.279	2.232	2.193
20	3.214	2.972	2.784	220	2.275	2.229	2.190
25	3.036	2.835	2.677	225	2.272	2.226	2.188
30	2.915	2.741	2.603	230	2.269	2.224	2.186
35	2.825	2.671	2.548	235	2.265	2.221	2.184
40	2.756	2.616	2.504	240	2.262	2.218	2.182
45	2.701	2.573	2.469	245	2.259	2.216	2.180
50	2.656	2.537	2.441	250	2.257	2.214	2.178
55	2.618	2.507	2.416	255	2.254	2.211	2.176
60	2.586	2.481	2.395	260	2.251	2.209	2.174
65	2.558	2.458	2.377	265	2.248	2.207	2.172
70	2.533	2.439	2.361	270	2.246	2.205	2.171
75	2.511	2.421	2.347	275	2.243	2.203	2.169
80	2.492	2.405	2.335	280	2.241	2.201	2.167
85	2.474	2.391	2.323	285	2.239	2.199	2.166
90	2.459	2.379	2.313	290	2.236	2.197	2.164
95	2.444	2.367	2.303	295	2.234	2.195	2.163
100	2.431	2.356	2.295	300	2.232	2.194	2.161
105	2.419	2.346	2.287	305	2.230	2.192	2.160
110	2.408	2.337	2.279	310	2.228	2.190	2.158
115	2.397	2.329	2.272	315	2.226	2.189	2.157
120	2.388	2.321	2.266	320	2.224	2.187	2.156
125	2.379	2.314	2.260	325	2.222	2.185	2.154
130	2.370	2.307	2.254	330	2.220	2.184	2.153
135	2.362	2.300	2.249	335	2.219	2.182	2.152
140	2.355	2.294	2.244	340	2.217	2.181	2.151
145	2.348	2.289	2.239	345	2.215	2.180	2.150
150	2.341	2.283	2.235	350	2.213	2.178	2.148
155	2.335	2.278	2.231	355	2.212	2.177	2.147
160	2.329	2.273	2.227	360	2.210	2.176	2.146
165	2.323	2.269	2.223	365	2.209	2.174	2.145
170	2.318	2.264	2.219	370	2.207	2.173	2.144
175	2.313	2.260	2.216	375	2.206	2.172	2.143
180	2.308	2.256	2.212	380	2.204	2.171	2.142
185	2.303	2.252	2.209	385	2.203	2.169	2.141
190	2.299	2.248	2.206	390	2.201	2.168	2.140
195	2.294	2.245	2.203	395	2.200	2.167	2.139
200	2.290	2.241	2.201	400	2.199	2.166	2.138
205	2.286	2.238	2.198	405	2.197	2.165	2.137

Similarly, without having to estimate the parameter ξ we can set $\xi = 0.50$ and calculate the p-value using the program provided in Section 4. The p-value for $\xi = 0.50$ is greater than the p-values for other choice of ξ in all cases. Based on the conservative p-value, the decision making is more reliable. In the example displayed in Section 4, if we set $\xi = 0.50$, then the program gives $\hat{C}_{pmk} = 1.194$ and the corresponding p-value as 0.0290 for the same input parameters: $LSL = 2.40$, $USL = 3.40$, $T = 2.90$, $C = 1.00$, $n = 100$, $\overline{X} = 2.865$, and $S_n = 0.125$.

Figures 2–6 plot the curves of c_0 versus the parameter ξ for sample size $n = 30$ (top curve 1), 50 (top curve 2), 70 (top curve 3), 100 (top curve 4), 200 (top curve 5), 300 (bottom curve), and $C = 1.00$, 1.33, 1.50, 1.67, 2.00, with $\alpha = 0.05$.

6. TESTING PROCEDURE

Tables 1–5 display the critical values c_0 for $C = 1.00$, 1.33, 1.50, 1.67, and 2.00, with sample sizes $n = 10(5)405$, and α-risk $= 0.01$, 0.025, 0.05. To judge if a given process meets the capability requirement, we first determine the value of C, the capability requirement, and the α-risk. Checking the appropriate table from Tables 1–5, we may obtain the critical value c_0 based on the given values of α-risk, C, and the sample size n. If the estimated value $\hat{C}_{pmk}$ is greater than the critical value c_0 ($\hat{C}_{pmk} > c_0$), then we conclude that the process meets the capability requirement ($C_{pmk} > C$). Otherwise, we do not have sufficient information to conclude that the process meets the present capability requirement. In this case, we would believe that $C_{pmk} \leqslant C$.

Step 1. Decide the definition of 'capable' (C, normally set to 1.00, or 1.33), and the α-risk (normally set to 0.01, 0.025, or 0.05), the chance of wrongly concluding an incapable process as capable.

Step 2. Calculate the values of $\hat{C}_{pmk}$ from the sample.

Step 3. Check the appropriate table and find the critical value c_0 based on α-risk, C, and the sample size n.

Step 4. Conclude that the process is capable ($C_{pmk} > C$) if the $\hat{C}_{pmk}$ value is greater than the critical value c_0 ($\hat{C}_{pmk} > c_0$). Otherwise, we do not have enough information to conclude that the process is capable.

REFERENCES

1. Pearn WL, Kotz S, Johnson NL. Distributional and inferential properties of process capability indices. *Journal of Quality Technology* 1992; **24**:216–231.
2. Vännman K. A unified approach to capability indices. *Statistica Sinica* 1995; **5**:805–820.
3. Chen SM, Hsu NF. The asymptotic distribution of the process capability index: C_{pmk}. *Communications in Statistics—Theory and Methods* 1995; **24**:1279–1291.
4. Vännman K, Kotz S. A superstructure of capability indices: Distributional properties and implications. *Scandinavian Journal of Statistics* 1995; **22**:477–491.
5. Vännman K. Distribution and moments in simplified form for a general class of capability indices. *Communications in Statistics—Theory and Methods* 1997; **26**(1):159–179.
6. Cheng SW. Practical implementation of the process capability indices. *Quality Engineering* 1994; **7**(2):239–259.

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