Next-to-leading-order power correction to photon-pion transition form factor

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We propose an approach to calculate the next-to-leading-order power corrections to the photon-pion transition form factor $F_{\pi\gamma}(Q^2)$. The effects of the next-to-leading-order power corrections are analyzed.

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I. INTRODUCTION

The photon-pion transition process $\gamma^*\pi\to\gamma$ provides a good example for tests of QCD. The amplitude for the transition process can be expressed as $M(\gamma^*\pi\to\gamma)=-ie^2\epsilon_{\mu\alpha\beta\lambda}P_1^\alpha P_2^\beta\epsilon^\lambda F_{\pi\gamma}(Q^2)$, where P_1 denotes the pion momentum and P_2 and ϵ^λ represent the momentum and polarization of the real photon. All information on QCD for this process is contained in the form factor $F_{\pi\gamma}(Q^2)$, with $Q^2=-(P_2-P_1)^2$ being the virtuality of the virtual photon. In this paper, we would like to investigate the effects of the next-to-leading-order (NLO) power corrections for the transition form factor. For sufficiently high energies, $Q^2\gg\Lambda_{QCD}^2$, the transition form factor can be calculated in perturbative QCD (PQCD) to have the form

$$F_{\pi\gamma}(Q^2)=4C_\pi\int_0^1 dx\frac{\phi_2(x)}{Q^2x(1-x)},\tag{1}$$

where $C_\pi=\sqrt{2}/6$ is the pion charge factor and $\phi_2(x)$ represents the leading twist (twist-2) pion distribution amplitude (DA). In the high energy limit $Q^2\to\infty$, the nonleading anomalous dimension contributions to the pion DA can be ignored and the pion DA approaches its asymptotical form $\phi_2(x)=3x(1-x)f_\pi/\sqrt{2}$. It implies that the transition form factor $F_{\pi\gamma}$ also has an asymptotical limit [1]

$$F_{\pi\gamma}(Q^2)|_{Q^2\to\infty}=\frac{2f_\pi}{Q^2},\tag{2}$$

where $f_\pi=93$ MeV is the pion decay constant. The asymptotic of the form factor is about 15% larger than the upper end of the CLEO data [2]. Many proposals, such as the inclusion of $O(\alpha_s)$ corrections into the transition form factor [3–7] and the introduction of the transverse structure for the pion DA [8,9], have been suggested to solve this discrepancy between theory and experiment. It has also been shown [2] that the data can be described by the Brodsky-Lepage interpolating formula [10] for both $Q^2\to\infty$ and $Q^2\to 0$ limits of $F_{\pi\gamma}(Q^2)$:

$$F_{\pi\gamma}^{BL}(Q^2)=\frac{2f_\pi}{s_0+Q^2},\tag{3}$$

where $s_0=8\pi^2f_\pi^2\approx 0.68\ \mathrm{GeV}^2$. This implies that the NLO power corrections, i.e. the $O(1/Q^4)$ corrections, might be important.

In this paper, we shall present a perturbative calculation for the NLO power corrections to the photon-pion transition form factor $F_{\pi\gamma}(Q^2)$. The method of calculation is related to the terminology of the collinear expansion [11,12]. The collinear expansion has the following features: (1) it preserves individual gauge invariance of the soft function and the hard function; (2) it can systematically separate the leading-order (LO) contributions from the next-to-leading-order (NLO) power corrections; (3) it can simultaneously derive different kinds of higher twist contributions from the sources: the noncollinear partons, the wrong spin projection and the higher Fock states; (4) it is a twist-by-twist expansion and also free from the twist mixing problem; (5) it is a Feynman diagram approach such that the partonic picture for higher twist contributions can be preserved.

Our main results are summarized as follows. We shall employ collinear expansion to evaluate the NLO power corrections for the process $\gamma^*\pi\to\gamma$. The NLO power corrections to $F_{\pi\gamma}(Q^2)$ involve four twist-4 pion DAs. Two of them are due to nonvanishing masses of the valence quarks of the pion. With the help of equations of motion, the number of independent twist-4 DAs is reduced from four to two. The remaining two twist-4 DAs are assumed to be asymptotic. The theoretical prediction for the scaled form factor $Q^2F_{\pi\gamma}(Q^2)$ is in good agreement with the CLEO data.

The organization of the remaining text is as follows. We describe the collinear expansion for the process $\gamma^*\pi\to\gamma$ in Sec. II. The NLO power corrections to the transition form factor $F_{\pi\gamma}(Q^2)$ are calculated in Sec. III. Section IV is devoted to conclusions.

II. COLLINEAR EXPANSION

We sketch the procedures of collinear expansion for $\gamma^*\pi\to\gamma$.

Let $M=\sigma_p(k)\otimes\phi(k)$ represent the lowest order amplitude for $\gamma^*(q)\pi(P_1)\to\gamma(P_2)$ as depicted in Figs. 1(a) and 1(b). The $\sigma_p(k)$ denotes the amplitude for partonic subprocess and the $\phi(k)$ represents the pion DA. The $\otimes$ represents the convolution integral over the loop momentum k and the traces over the color indices and spin indices. The amplitude M is expressed as

$$M=\int\frac{d^4k}{(2\pi)^4}\mathrm{Tr}[\sigma_p(k)\phi(k)],\tag{4}$$

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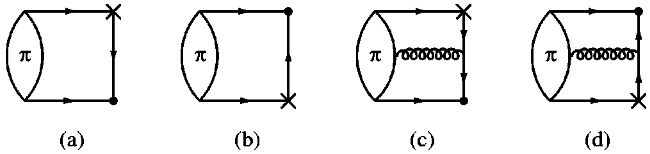

FIG. 1. The leading order Feynman diagrams for $\gamma^*\pi\to\gamma$. The cross symbol represents the vertex of the virtual photon.

where the trace Tr is run over fermion and color indices, and the pion DA $\phi(k)$ takes the form

$$\phi(k)=\int d^4y e^{ik\cdot y}\langle 0|\overline{q(0)}q(y)|\pi(P_1)\rangle. \tag{5}$$

If we assign the momentum of the final state photon in the minus light-cone direction, $P_2=Q^2n/2$, and the momentum of the initial pion in the plus light-cone direction, $P_1=p$, the leading configuration for the process is then constructed from the collinear momentum $\hat{k}=xp$ with $x=k\cdot n$ the fraction of the momentum of the pion carried by the partons. The vectors p and n represent lightlike vectors in the $+$ and $-$ directions, respectively, and satisfy conditions $p^2=n^2=0$ and $n\cdot p=1$. The first step of the collinear expansion is to make a Taylor expansion for the partonic amplitude $\sigma_p(k)$ with respect to $\hat{k}=xp$:

$$\sigma_p(k)=\overline{\sigma_p}(k=\hat{k})+(\overline{\sigma_p})_\alpha(x,x)w^\alpha_{\alpha'}k^{\alpha'}+\cdots, \tag{6}$$

where we have assumed the low energy theorem

$$\frac{\partial}{\partial k^\alpha}\sigma_p(k)|_{k=\hat{k}}=(\overline{\sigma_p})_\alpha(x,x), \tag{7}$$

and have employed notations $w^\alpha_{\alpha'}k^{\alpha'}=(k-xp)^\alpha$ and $w^\alpha_{\alpha'}=g^\alpha_{\alpha'}-p^\alpha n_{\alpha'}$. Since $\overline{\sigma_p}(x)$ is only dependent on the fractional variable x, we can recast the convolution $\overline{\sigma_p}\otimes\phi$ into the form

$$\overline{\sigma_p}\otimes\phi=\int dx\,\mathrm{Tr}[\overline{\sigma_p}(x)\phi(x)], \tag{8}$$

with

$$\phi(x)=\int\frac{d^4k}{(2\pi)^4}\int d^4y e^{ik\cdot y}\delta(x-k\cdot n)\langle 0|\overline{q(0)}q(y)|\pi\rangle. \tag{9}$$

The leading term $\overline{\sigma_p}\otimes\phi$ contains leading, next-to-leading and higher order power contributions. To separate the contributions of different power order, we can investigate the spin structures of the leading partonic amplitude $\overline{\sigma_p}$. The $\overline{\sigma_p}$ has terms proportional to $\not{n}$ and $\not{p}$. The terms proportional to $\not{n}$ would project out a collinear $q\overline{q}$ pair from the parent pion, and those terms proportional to $\not{p}$ would not vanish only when the $q\overline{q}$ pair carries noncollinear momentum. The second step is to substitute the leading partonic amplitude $\overline{\sigma_p}$ into the convolution integral with ϕ to extract the terms $\overline{\sigma_p}\otimes\phi_1$:

$$\overline{\sigma_p}\otimes\phi=\overline{\sigma_p}\otimes\phi_0+\overline{\sigma_p}\otimes\phi_1+\cdots, \tag{10}$$

where ϕ_0 and ϕ_1 denote the leading and subleading pion DAs, respectively. However, this is not the final answer. The ϕ_1 contains both short distance and long distance contributions. The short distance part of ϕ_1 arises from the noncollinear components of the loop momentum k. By the equations of motion, the noncollinear components of k will induce one quark-gluon interaction vertex $i\gamma_\alpha$ and one special propagator $i\not{n}/(2k\cdot n)$ [11]. Because the special propagator is not propagating in the minus light-cone direction, the quark-gluon interaction vertex and the special propagator should be included in the leading partonic amplitude, $\overline{\sigma_p}$. In this way, we may factorize ϕ_1 as $\phi_1\approx(\phi_1^H)_\alpha w^\alpha_{\alpha'}(\phi_1^S)^{\alpha'}$ and absorb the short distance piece $(\phi_1^H)_\alpha$ into $\overline{\sigma_p}$. It leads to the third step

$$\begin{aligned}\overline{\sigma_p}\otimes\phi_1&=\overline{\sigma_p}\otimes((\phi_1^H)_\alpha\bullet w^\alpha_{\alpha'}(\phi_1^S)^{\alpha'})\\&=(\overline{\sigma_p}\bullet\phi_1^H)_\alpha\otimes w^\alpha_{\alpha'}(\phi_1^S)^{\alpha'},\end{aligned} \tag{11}$$

where $(\phi_1^S)^{\alpha'}$ containing covariant derivative $D^{\alpha'}=i\partial^{\alpha'}-gA^{\alpha'}$ is implied. Notice that the light-cone gauge $n\cdot A=0$ assures $w^\alpha_{\alpha'}A^{\alpha'}=A^\alpha$. It is useful to write the above equation in a concise form

$$\begin{aligned}\overline{\sigma_p}\otimes\phi_1=\int dx dx_1\mathrm{Tr}[&(\overline{\sigma_p}\bullet\phi_1^H)_\alpha(x,x_1)\\&\times w^\alpha_{\alpha'}(\phi_1^S)^{\alpha'}(x,x_1)],\end{aligned} \tag{12}$$

with $\bullet$ being the matrix product. The new soft function $(\phi_1^S)^{\alpha'}(x,x_1)$ has the expression

$$\begin{aligned}(\phi_1^S)^{\alpha'}(x,x_1)=\int\frac{d^4k}{(2\pi)^4}\int\frac{d^4k_1}{(2\pi)^4}&\\\times\int d^4y\int d^4z e^{ik\cdot y}e^{i(k_1-k)\cdot z}&\\\times\delta(x-k\cdot n)\delta(x_1-k_1\cdot n)&\\\times\langle 0|\overline{q(0)}D^{\alpha'}(z)q(y)|\pi\rangle,&\end{aligned} \tag{13}$$

and the hard function $(\overline{\sigma_p}\bullet\phi_1^H)_\alpha(x,x_1)$ is defined as

$$\begin{aligned}(\overline{\sigma_p}\bullet\phi_1^H)_\alpha(x,x_1)=(i\gamma_\alpha)&\frac{-i\not{n}}{2x}\overline{\sigma_p}(x)\\&+\overline{\sigma_p}(x)(i\gamma_\alpha)\frac{i\not{n}}{2(1-x)}.\end{aligned} \tag{14}$$

The contribution from the second term of Eq. (6) should be considered, too. Similar to the treatment for terms related to $\overline{\sigma}_p$, the terms related to $(\overline{\sigma_p})_\alpha(x,x)w^\alpha_{\alpha'}k^{\alpha'}$ are necessary to be substituted into the convolution integral with ϕ_0, where we have made the approximation $\phi\approx\phi_0+\cdots$. The momentum factor k^α will be absorbed by ϕ_0 to become a coordinate derivative acting on the quark field. That is we make the transformation $k^\alpha\phi_0\equiv\phi^\alpha_{1,\partial}$ which has the expression

$$\phi^\alpha_{1,\partial}\equiv\int d^4y e^{ik\cdot y}\langle 0|\overline{q(0)}i\partial^\alpha q(y)|\pi\rangle. \tag{15}$$

Consider another contribution $M_1\approx(\overline{\sigma_p})_\alpha\otimes w^\alpha_{\alpha'}\phi^{\alpha'}_{1,A}$ from Figs. 1(c) and 1(d), where $\phi^\alpha_{1,A}$ contains gauge fields

$$\phi^\alpha_{1,A}\equiv\int d^4y d^4z e^{ik\cdot y}e^{i(k_1-k)\cdot z} \times\langle 0|\overline{q(0)}(-gA^\alpha(z))q(y)|\pi\rangle. \tag{16}$$

Note that we have employed the approximation that $(\overline{\sigma_p})_\alpha\otimes\phi^\alpha_{1,A}$ is the leading term of M_1. This comes to the fourth step:

$$(\overline{\sigma_p})_\alpha\otimes w^\alpha_{\alpha'}\phi^{\alpha'}_{1,\partial}+(\overline{\sigma_p})_\alpha\otimes w^\alpha_{\alpha'}\phi^{\alpha'}_{1,A}\equiv(\overline{\sigma_p})_1\otimes\phi^S_1, \tag{17}$$

where $\phi^\alpha_{1,\partial}+\phi^\alpha_{1,A}\equiv\phi^S_1$. We write the new quantities as

$$(\overline{\sigma_p})_1\otimes\phi^S_1\equiv\int dx\int dx_1\mathrm{Tr}[(\overline{\sigma_p})_\alpha(x,x_1)\times w^\alpha_{\alpha'}(\phi^S_1)^{\alpha'}(x,x_1)], \tag{18}$$

with ϕ^S_1 as defined before. However, it will become clear later that $(\overline{\sigma_p})_1$ vanishes as it convolutes with twist-4 DA ϕ^Γ_1 (see below definition). Up to $O(1/Q^4)$, we may drop the $(\overline{\sigma_p})_1$ term and arrive at the result

$$M+M_1\approx\overline{\sigma_p}\otimes\phi_0+(\overline{\sigma_p}\bullet\phi^H_1)\otimes\phi_1, \tag{19}$$

where ϕ_1 represents ϕ^S_1. This involves only one subleading DA ϕ_1 for the NLO power correction.

To proceed, we need to consider the factorizations of the spin indices and the color indices. For factorization of spin indices, we expand the pion DAs into their spin components by means of Fierz transformation

$$\phi_{0,1}=\sum_\Gamma\phi^\Gamma_{0,1}\Gamma, \tag{20}$$

where Γ means Dirac matrix $\Gamma=1,\ \gamma^\mu,\ \gamma^\mu\gamma_5,\ \sigma^{\mu\nu}$. The factorization of the color indices takes the convention that the color indices of the partonic amplitude are extracted from the partonic amplitude and attributed to the corresponding pion DA. The choice of the lowest twist components $\phi^\Gamma_{0,1}$ of $\phi_{0,1}$ is made by employing the power counting. Assume that the pion DA $\phi^{\mu_1\cdots\mu_F;\alpha_1\cdots\alpha_B}$ has the fermion index F and the boson index B. The fermion index F arises from the spin index factorization for $2F$ fermion lines connecting the pion DA and partonic amplitude and the boson index B denotes the n_D power of momenta in previous collinear expansion and the n_G gluon lines as $B=n_D+n_G$. Because we have attributed the large scale factor $1/Q$ into the partonic amplitude, the pion DA can only contain small scale $\Lambda\approx\Lambda_{QCD}$. Therefore, we may write

$$\phi^{\mu_1\cdots\mu_F;\alpha_1\cdots\alpha_B}=\sum_i\Lambda^{\tau_i-1}e_i^{\mu_1\cdots\mu_F;\alpha_1\cdots\alpha_B}\phi^i. \tag{21}$$

The spin polarizers e_i are composed of vectors p^μ, n^μ and $\gamma^\mu_\perp$, where $\gamma^\mu_\perp$ has superscript $\mu=1,2$. The variable τ_i represents the twist associated with ϕ^i. The restrictions over the polarizer $e_i^{\mu_1\cdots\mu_F;\alpha_1\cdots\alpha_B}$ are

$$n_{\alpha_j}e_i^{\mu_1\cdots\mu_F;\alpha_1\cdots\alpha_j\cdots\alpha_B}=0. \tag{22}$$

This is because the polarizers e_i are always projected by $w^\alpha_{\alpha'}$. The dimension of $\phi^{\mu_1\cdots\mu_F;\alpha_1\cdots\alpha_B}$ is determined by dimensional analysis

$$d(\phi)=3F+B-1, \tag{23}$$

and the maximum of the dimension of e_i can be found as

$$\max[d(e_i)]=F-\frac{1}{2}[1-(-1)^B]. \tag{24}$$

By equating the dimensions of both sides of Eq. (21), we can obtain the minimum value of τ_i

$$\tau_i^{\min}=2F+B+\frac{1}{2}[1-(-1)^B]. \tag{25}$$

It is obvious from Eq. (25) that there are only finite numbers of fermion lines, gluon lines and derivatives for a given power of $1/Q^2$.

III. $O(1/Q^4)$ CONTRIBUTIONS OF $\gamma^*\pi\to\gamma$

The lowest order diagrams are displayed in Fig. 1. By applying the collinear expansion, we can write the result as

$$M(\gamma^*\pi\to\gamma)=-ie^2\epsilon_{\mu\alpha\beta\lambda}P_1^\alpha P_2^\beta\epsilon^\lambda F_{\pi\gamma}(Q^2), \tag{26}$$

where ϵ^λ denotes the polarization vector of the final state photon. The leading-order power contribution of $F_{\pi\gamma}(Q^2)$ is calculated from Figs. 1(a) and 1(b) as

$$F^{LO}_{\pi\gamma}(Q^2)=4C_\pi\int_0^1 dx\frac{\phi_2(x)}{Q^2x(1-x)}, \tag{27}$$

where the charge factor $C_\pi=(e_u^2-e_d^2)/\sqrt{2}$ with e_u and e_d the charges of u and d quark in units of the elementary charge. The NLO power correction part of $F_{\pi\gamma}(Q^2)$ is evaluated from Fig. 2 to take the form as

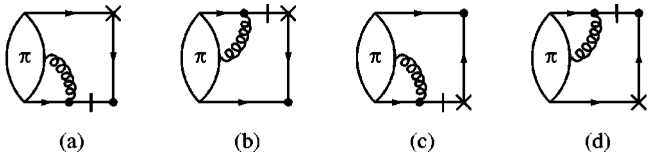

FIG. 2. The Feynman diagrams contribute to the next-to-leading-order power corrections. The propagator with one bar is the special propagator.

$$F_{\pi\gamma}^{NLO}(Q^2) = -16C_\pi \int_0^1 dx \frac{[G(x)+\tilde{G}(x)(1-2x)]}{Q^4 x(1-x)}. \tag{28}$$

The relevant DAs are expressed explicitly as follows:

$$\phi_2(x) = -i\frac{1}{4}\int_0^\infty \frac{d\lambda}{(2\pi)} \times e^{i\lambda x}\langle 0|\overline{q}(0)\gamma_5 \not{n} q(\lambda n)|\pi(P_1)\rangle, \tag{29}$$

$$G(x) = -\frac{1}{8}\epsilon_\perp^{\alpha\beta}\int_0^1 dx_1 \int_0^\infty \frac{d\lambda}{(2\pi)}\frac{d\eta}{(2\pi)} e^{i\eta(x_1-x)}e^{i\lambda x} \times \langle 0|\overline{q}(0)\gamma_\alpha D_\beta(\eta n) q(\lambda n)|\pi(P_1)\rangle, \tag{30}$$

$$\tilde{G}(x) = -\frac{i}{8}d_\perp^{\alpha\beta}\int_0^1 dx_1 \int_0^\infty \frac{d\lambda}{(2\pi)}\frac{d\eta}{(2\pi)} e^{i\eta(x_1-x)}e^{i\lambda x} \times \langle 0|\overline{q}(0)\gamma_5\gamma_\alpha D_\beta(\eta n) q(\lambda n)|\pi(P_1)\rangle. \tag{31}$$

The tensors $\epsilon_\perp^{\alpha\beta}$ and $d_\perp^{\alpha\beta}$ are defined as $\epsilon_\perp^{\alpha\beta} = \epsilon^{\alpha\beta\gamma\lambda}p_\gamma n_\lambda$ and $d_\perp^{\alpha\beta} = p^\alpha n^\beta + n^\alpha p^\beta - g^{\alpha\beta}$.

The nonvanishing valence quark mass can also contribute to the NLO power corrections. We employ the scheme that the partons involved in the partonic amplitude are massless. This does not affect the final result. By taking into account the contributions from the quark mass operator m, we get the result

$$F_{\pi\gamma}^{NLO}(Q^2)|_{m\neq 0} = -8C_\pi \int_0^1 dx \frac{[H(x)+\tilde{H}(x)(1-2x)]}{Q^4 x(1-x)}, \tag{32}$$

where two new twist-4 DAs H and $\tilde{H}$ are expressed as

$$H(x) = -\frac{i}{16}\epsilon_\perp^{\alpha\beta}\int_0^\infty \frac{d\lambda}{(2\pi)} \times e^{i\lambda x}\langle 0|\overline{q}(0) m\sigma_{\alpha\beta} q(\lambda n)|\pi(P_1)\rangle, \tag{33}$$

$$\tilde{H}(x) = -\frac{i}{4}\int_0^\infty \frac{d\lambda}{(2\pi)} \times e^{i\lambda x}\langle 0|\overline{q}(0) m\gamma_5 q(\lambda n)|\pi(P_1)\rangle. \tag{34}$$

Note that $H(x)$ and $\tilde{H}(x)$ are related to the conventional twist-3 pion DAs $\phi_\sigma(x)$ and $\phi_p(x)$ [13] by a factor m_0, the average quark mass.

The four twist-4 DAs G, $\tilde{G}$, H and $\tilde{H}$ are not independent. They are related to each other by equations of motion. After employing equations of motion, we obtain the remaining independent twist-4 DAs G' and $\tilde{G}'$:

$$G'(x) = -\frac{1}{16}\epsilon_\perp^{\alpha\beta}\int_0^1 dx_1 \int_0^\infty \frac{d\lambda}{(2\pi)}\frac{d\eta}{(2\pi)} e^{i\eta(x_1-x)}e^{i\lambda x} \times \langle 0|\overline{q}(0)\gamma_\alpha \not{D}(\eta n)\gamma_\beta q(\lambda n)|\pi(P_1)\rangle, \tag{35}$$

and

$$\tilde{G}'(x) = -\frac{i}{16}d_\perp^{\alpha\beta}\int_0^1 dx_1 \int_0^\infty \frac{d\lambda}{(2\pi)}\frac{d\eta}{(2\pi)} e^{i\eta(x_1-x)}e^{i\lambda x} \times \langle 0|\overline{q}(0)\gamma_5\gamma_\alpha \not{D}(\eta n)\gamma_\beta q(\lambda n)|\pi(P_1)\rangle. \tag{36}$$

As a result, we can recast Eq. (28) as

$$F_{\pi\gamma}^{NLO}(Q^2) = -16C_\pi \int_0^1 dx \frac{[G'(x)+\tilde{G}'(x)(1-2x)]}{Q^4 x(1-x)}. \tag{37}$$

Because of the factor $(1-2x)$ associated with $\tilde{G}'$, G' becomes dominant. The normalization of $\phi_2(x)$ is fixed from the pion weak decay $\pi\to\mu\nu$ such that $\phi_2^{AS}(x) = 3f_\pi x(1-x)/\sqrt{2}$ for the asymptotic (AS) model and $\phi_2^{CZ}(x) = 15f_\pi x(1-x)(1-2x)^2/\sqrt{2}$ for the Chernyak-Zhitnitsky (CZ) model [14]. The normalization for $G'(x)$ is, in principle, unknown. However, we can assume that it can be determined from the axial anomaly $\pi\to 2\gamma$ to yield $G'^{AS}(x) = 3\sqrt{2}\pi^2 f_\pi^3 x(1-x)$ for the AS model and $G'^{CZ}(x) = 15\sqrt{2}\pi^2 f_\pi^3 x(1-x)(1-2x)^2$ for the CZ model. We express this normalization for G' in more detail. It is known that the amplitude for $\pi^0\to 2\gamma$ is fixed by axial anomaly as

$$M(\pi^0\to 2\gamma) = -ie^2\epsilon_{\mu\alpha\beta\lambda}\epsilon^\mu(q)P_1^\alpha P_2^\beta \epsilon^\lambda(P_2)A, \tag{38}$$

where

$$A = \frac{1}{4\pi^2 f_\pi},$$

and $\epsilon(q)$ and $\epsilon(P_2)$ represent the polarization vectors for the final state photons. We first consider the AS model. Suppose that $G'(x) = 3f_\pi N x(1-x)/\sqrt{2}$ with an unknown factor N. Substituting $G'(x) = 3f_\pi N x(1-x)/\sqrt{2}$ into Eq. (37) and completing the integration over x, we obtain

$$F_{\pi\gamma}(Q^2) = 6\sqrt{2}f_\pi C_\pi \frac{1}{Q^2}\left[1-\frac{4N}{Q^2}\right]. \tag{39}$$

By extrapolating $F_{\pi\gamma}(Q^2)$ to all orders in Q^2, we then obtain

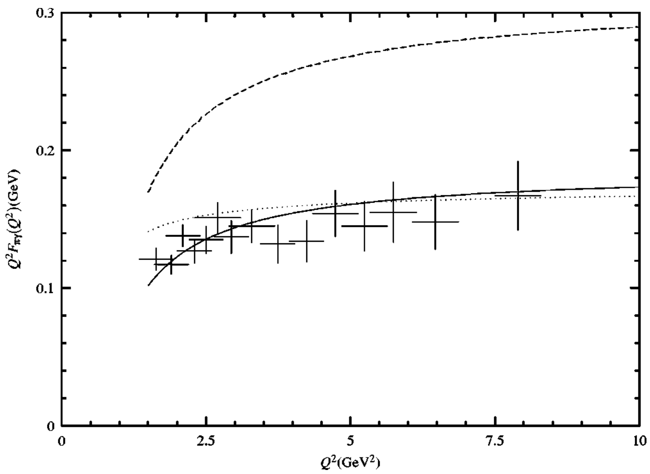

FIG. 3. The prediction of the photon-pion transition form factor $Q^2F_{\pi\gamma}(Q^2)$ with NLO power corrections is compared with the experimental data [2]. The solid and dash lines correspond to the transition form factor with the asymptotic (AS) and the Chernyak-Zhitnitsky (CZ) distribution amplitudes, respectively. The leading twist transition form factor with $O(\alpha_s)$ correction and asymptotic distribution amplitude is also displayed as a point line.

$$F_{\pi\gamma}^{all}(Q^2)=\frac{6\sqrt{2}f_\pi C_\pi}{Q^2+4N}. \tag{40}$$

The form factor $F_{\pi\gamma}^{all}(Q^2)$ approaches a constant as $Q^2\to 0$:

$$F_{\pi\gamma}^{all}(Q^2)|_{Q^2\to 0}\to\frac{3f_\pi C_\pi}{\sqrt{2}N}. \tag{41}$$

We then compare the amplitude $M(\gamma^*\pi\to\gamma)$ under the limit $Q^2\to 0$ and the amplitude $M(\pi\to 2\gamma)$ to find that $N=2\pi^2 f_\pi^2$. The normalization for the CZ model can be dealt with in the same way. The above assumption that the amplitude $M(\gamma^*\pi\to\gamma)$ and the amplitude $M(\pi\to 2\gamma)$ are identical in the limit $Q^2\to 0$ will be tested by comparing the theoretical prediction and the experimental data. If the comparison appears to be not good, this only implies that the assumption should be modified, or wrong. But our approach for deriving the NLO power correction is still applicable. Up to $O(1/Q^4)$, we write the photon-pion transition form factors

$$F_{\pi\gamma}^{AS}(Q^2)=\frac{2f_\pi}{Q^2}\left[1-\frac{8f_\pi^2}{Q^2}\right], \tag{42}$$

for the AS model, and

$$F_{\pi\gamma}^{CZ}(Q^2)=\frac{10f_\pi}{3Q^2}\left[1-\frac{8f_\pi^2}{Q^2}\right] \tag{43}$$

for the CZ model. As shown in Fig. 3, the predictions of Eqs. (42) and (43) are compared with the CLEO data [2]. The pion decay constant $f_\pi=93$ MeV has been used. The prediction from the AS model is in good agreement with the CLEO data.

As mentioned in the Introduction, the leading twist transition form factor with $O(\alpha_s)$ correction can also explain the CLEO data. The $O(\alpha_s)$ correction is available and the corrected form factor is expressed as (see e.g. [15])

$$F_{\pi\gamma}(Q^2)=\frac{2f_\pi}{Q^2}\left(1-\frac{5}{3}\frac{\alpha_s(\mu_R^2)}{\pi}\right), \tag{44}$$

where the Brodsky-Lepage-Mackenzie (BLM) scale setting $\mu_R^2\approx Q^2/9$, the AS model for the leading twist pion DA, and the usual one-loop formula for the QCD running coupling constant:

$$\alpha_s(Q^2)=\frac{4\pi}{\beta_0\ln\frac{Q^2}{\Lambda_{QCD}^2}} \tag{45}$$

have been used with $\Lambda_{QCD}=0.2$ GeV and $\beta_0=11-2/3n_f$. We compare Eq. (44) with Eq. (42) in Fig. 3. The difference between Eqs. (42) and (44) is very small for $Q^2\geqslant 3$ GeV^2.

IV. CONCLUSIONS

We have shown that the collinear expansion for $\gamma^*\pi\to\gamma$ can be systematically performed. The $O(Q^{-4})$ power corrections for $F_{\pi\gamma}(Q^2)$ have been evaluated in terms of four twist-4 DAs. The effects of the NLO power corrections have been estimated.

The other sources of power correction may also be important, such as the renormalon. The investigation of this kind of power correction is beyond the scope of this paper.

We have also limited ourselves to tree amplitudes. The factorization theorem for the NLO power corrections should be proven in order to have a confident PQCD formalism. The work for the proof of the factorization theorem is in preparation.

ACKNOWLEDGMENTS

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