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Ultrashort bragg soliton in a fiber bragg grating

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Abstract

The propagation of a nonlinear ultrashort pulse in a photonic bandgap structure is investigated by using the finite-difference time-domain method. The simulation results show that an ultrashort pulse near the bandgap edge can propagate through a nonlinear fiber Bragg grating, even if the broadband spectrum of this ultrashort pulse overlaps the whole forbidden band of the grating. It is also shown that the time delay of such an ultrashort solitary wave is proportional to its detuning wavelength from the exact Bragg resonance. © 2002 Published by Elsevier Science B.V.

Keywords: Finite-difference time-domain method; Photonic bandgap; Bragg soliton; Gap soliton; Fiber Bragg grating

1. Introduction

Gap solitons are solitary waves propagating in a nonlinear photonic bandgap (PBG) structure [1]. The exact analytic solution to describe such a nonlinear pulse has been obtained from the nonlinear coupled-mode equations (NLCMEs). By using the multiple scale method [2], the NLCMEs can be reduced to the nonlinear Schrödinger equation (NLSE). Soliton solutions to this approximated NLSE are called Bragg solitons. Bragg solitons exist near the PBG edge and have been widely discussed both in theory [3–7] and experiment [8,9]. It has been demonstrated that a Bragg soliton can propagate through a fiber Bragg grating (FBG) [8]. The experimental results are in very good agreement with the NLSE model.

One of the attractive characteristics of a Bragg soliton is the reduction of its group velocity. The experiments have shown that such a soliton-like pulse with 80-ps width can travel with the velocity as low as 70% of the light speed in an unprocessed fiber. Thus all optical buffer based on the slow propagation of a Bragg soliton is an ongoing challenge. Moreover, nonlinear compression for optical pulses by using FBGs is also an interested subject associated with the Bragg soliton propagation. Such research may result in applying solitary propagation in FBGs to the practical all-optical communication system. However, to investigate the dynamics of a Bragg soliton, the models of the NLCMEs and the NLSE have the drawback: The NLSE is derived from the NLCMEs under the low-intensity limit. This limitation restricts a Bragg soliton to a broad pulse, but for a high-speed lightwave system an ultrashort pulse is more practical and necessary.

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In this paper, we use the finite-difference time-domain (FD–TD) method to study the nonlinear ultrashort pulse in a PBG structure. The FD–TD method can directly simulate Maxwell's equations. Hence it provides a robust simulation theory to investigate the characteristics of a Bragg soliton without any approximation. It is shown that a nonlinear ultrashort pulse near the bandgap edge still can propagate through a FBG, even if the broad spectrum of this ultrashort pulse overlaps the whole forbidden band of the FBG. The propagating dynamics of such an ultrashort solitary wave is numerically studied and presented.

2. Simulation theory

We consider an electromagnetic field with the electric component E_z polarized along the x-axis and the magnetic component H_y polarized along the y-axis. Such an electromagnetic field propagates along the x direction in a medium, which is assumed to be isotropic and non-dispersive. Maxwell's curl equations for this problem are written as:

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu_0}\frac{\partial E_z}{\partial x}, \tag{1}$$

$$\frac{\partial D_z}{\partial t} = \frac{\partial H_y}{\partial t}, \tag{2}$$

$$D_z = \varepsilon_0 \varepsilon_r(x) E_z + P_z^{\mathrm{NL}}, \tag{3}$$

where μ_0 is vacuum permeability, ε_r is vacuum permittivity, $\varepsilon_\infty(x)$ is the relative material permittivity, D_z is the electric induced polarization including the linear and nonlinear contributions of the medium, and P_z^{NL} is the nonlinear polarization regarding the Kerr nonlinearity. On the basis of the FD–TD method, the finite difference equations for Eqs. (1) and (2) are:

$$H_y|_{i+1/2}^{n+1/2} = H_y|_{i+1/2}^{n-1/2} + \frac{\Delta t}{\mu_0 \Delta x}\left(E_z|_{i+1}^{n} - E_z|_{i}^{n}\right), \tag{4}$$

$$D_z|_{i}^{n+1} = D_z|_{i}^{n} + \frac{\Delta t}{\Delta x}\left(H_y|_{i+1/2}^{n+1/2} - H_y|_{i-1/2}^{n+1/2}\right), \tag{5}$$

where Δt and Δx are the finite difference intervals in the temporal and spatial domain, respectively. The procedures of the FD–TD approach are described in the following. First Eq. (4) is used to determine $H_y|_{i+1/2}^{n+1/2}$ from the previous values of $H_y|_{i+1/2}^{n-1/2}$, $E_z|_{i+1}^{n}$ and $E_z|_{i}^{n}$. Second $D_z|_{i}^{n+1}$ determined by using Eqs. (5) from the previous values of $D_z|_{i}^{n}$, $H_y|_{i+1/2}^{n+1/2}$ and $H_y|_{i-1/2}^{n+1/2}$. Finally the resulting $D_z|_{i}^{n+1}$ are substituted into Eq. (3) to determine $E_z|_{i}^{n+1}$ under the Newton iterative procedure:

$$E_z^{\langle p+1\rangle} = \frac{D_z|^{n+1}}{\varepsilon_0[\varepsilon_r(x) + \chi^{(3)}|E_z^{\langle p\rangle}|^2]}, \tag{6}$$

where $\chi^{(3)}$ is the third-order susceptibility, p is zero or positive integral, and $E_z^p = E_z^n$ for $p = 0$.

To investigate Bragg solitons in a one-dimensional PBG medium, we consider a uniform FBG with the relative material permittivity $\varepsilon_\infty(x) = n(x)^2$, where

$$n(x) = n_0 + \Delta n \cos\left(\frac{2\pi x}{\Lambda}\right). \tag{7}$$

Here n_0 is the linear refractive index at the central wavelength of the electric field, Δn is the magnitude of the periodic index variations, and Λ is the grating period with respect to the Bragg wavelength λ_B via $\Lambda = \lambda_B/2n_0$. It is noticed that during the FD–TD process, there is no constraint on the quantity of Δn and the apodized profile of the grating. Thus the FD–TD method is more suitable than the NLCMEs and the NLSE model to investigate the dynamics of a nonlinear pulse in a

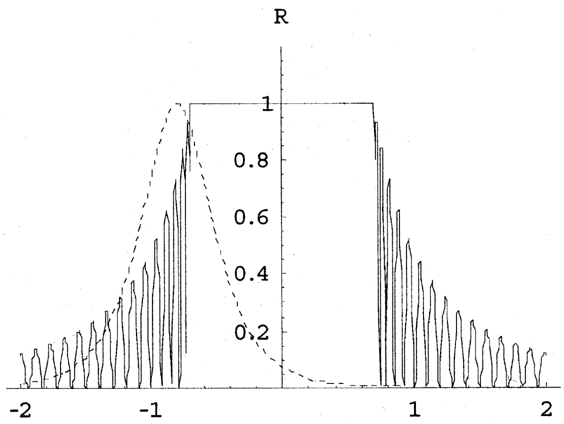

Fig. 1. The reflectivity (solid curve) of the uniform FBG and the broadband spectrum (dotted curve) of the incident pulse as functions of the wavelength detuning $\Delta\lambda = \lambda - \lambda_B$ from the exact Bragg resonance.

realistic rectangular waveguide grating or an apodized FBG. Such nonuniform gratings have been widely discussed for pulse compression and all-optical delay line based on the mechanisms of the Bragg soliton. Nevertheless, in the present paper, we focus our attention on how an ultrashort pulse evolves in a uniform FBG with Kerr nonlinearity.

3. Numerical results and discussions

The solid curve in Fig. 1 shows the reflectivity $R(\Delta\lambda)$ of the uniform FBG in our simulation. The linear refractive index of this FBG is $n_0 = 1.5$ and the index variation is $\Delta n = 9 \times 10^{-4}$. The central wavelength of this reflectivity is $\lambda_B = 1.55$ μm. The dotted curve in Fig. 1 shows the spectrum of the

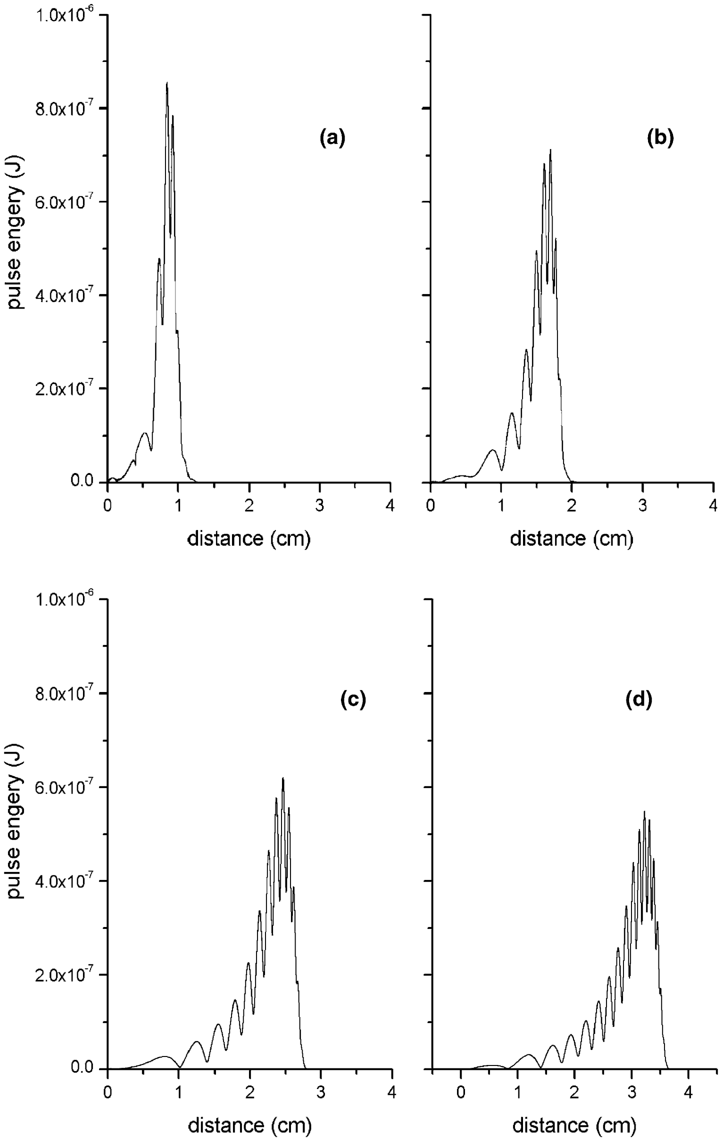

Fig. 2. Monitoring the propagation of a low-amplitude pulse in the fiber grating. The FD–TD method gives the snapshot of the propagating pulse at $t = 15$ ps, (2) $t = 60$ ps, (3) $t = 105$ ps, and (4) $t = 150$ ps.

adopted incident pulse with a hyperbolic-secant pulse shape initially. The full width at half maximum (FWHM) of this pulse is assumed to be $T_f = 5.28$ ps; likewise the central wavelength of this incident pulse is located at $\lambda_0 = 1.5492$ μm. Both of the FBG reflectivity and the pulse spectrum are shown as functions of the Bragg wavelength detuning $\Delta\lambda = \lambda - \lambda_B$. Furthermore, the range with $R(\Delta\lambda) = 1$ exhibits the forbidden band of such a PBG structure. Obviously, the initial pulse spectrum exceeds the PBG edge and even overlaps the whole forbidden band. We emphasize that because of the low-intensity limit for Bragg solitons, the previously demonstrated experiments and simulations have not yet clarified the propagation of a nonlinear pulse with such a broadband spectrum. We use FD–TD method to examine the dynamics of this nonlinear ultrashort pulse beyond the low-intensity limit. By choosing a uniform FD–TD space resolution $\Delta x = 50$ nm, the numerical phase error is limited to about 3.6×10^{-5}, which is much smaller than the dispersion due to the PBG structure. Fig. 2 shows the evolution of the incident pulse with low peak power $P = 1.4 \times 10^{-2}$ W propagating through the FBG. The Kerr coefficient of this FBG is $\chi^{(3)} = 1.97 \times 10^{-9}$ W^{-1}, in which absorbing the effective core area in three dimensions and the grating length is $L = 38$ mm. After the initial pulse is put into this FBG, Figs. 2(a)–(d) show the pulse shapes at $t = 15$ ps, $t = 60$ ps, and $t = 150$ ps, respectively. One can see that the shapes of this pulse are asymmetrically broadened as a consequence of its broadband spectrum. The evolution of the peak power and the spatial width of the pulse versus the propagating distance are shown in Fig. 3. During the propagation, the pulse undergoes the large quadratic grating dispersion. Such a large dispersion is produced by the interference among the multi-layers of the grating. To balance this quadratic grating dispersion, we have to increase the peak power of the initial pulse.

Fig. 4 shows the evolution of the nonlinear ultrashort pulse with peak power $P = 1.4 \times 10^5$ W. Figs. 4(a)–(d) represent the pulse shapes at $t = 15$ ps, $t = 60$ ps, $t = 105$ ps, $t = 150$ ps, respectively. It is shown that the peak power and the pulse width are changed very little during the

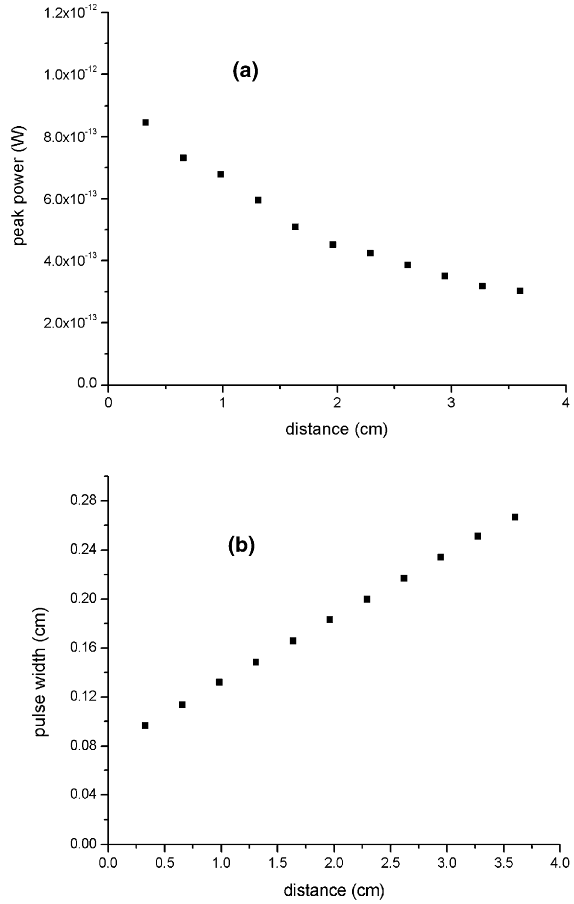

Fig. 3. (a) Peak power and (b) spatial pulse width of the low-amplitude pulse versus the propagating distance.

propagation. Hence the balance between the nonlinearity and the quadratic grating dispersion leads to a soliton-like pulse. Fig. 5 explicitly shows the evolution of the peak power and the spatial width versus the propagating distance. The numerical results show that the incident hyperbolic-secant pulse becomes quasi-stable. The pulse adjusts its amplitude and duration periodically because of the interaction between the nonlinearity and the quadratic grating dispersion. The soliton periodic L_s for such a solitary wave can be defined [10] by the nonlinear length L_{nl} via $L_s = \pi L_{nl}/2 = \pi/(2\gamma \cdot P)$. Therefore the soliton-like wave propagates about 16.7 soliton periods. Another notable characteristic of this solitary wave is its propagating delay with respect to the propa-

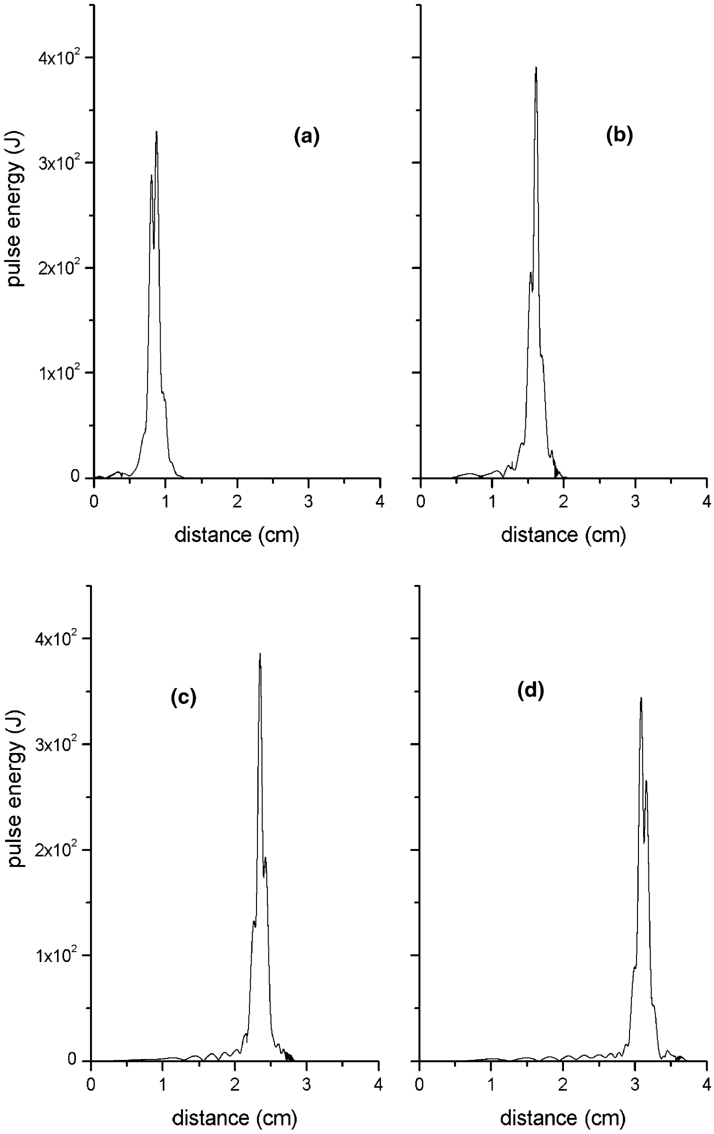

Fig. 4. Monitoring the propagation of the soliton-like pulse in the FBG. The FD–TD method gives the snapshot of the propagating pulse at (1) $t = 15$ ps, (2) $t = 60$ ps, (3) $t = 105$ ps, and (4) $t = 150$ ps.

gating time of the light in an unprocessed fiber. For the above hyperbolic-secant pulse with carrier frequency $\lambda_0 = 1.5492$ μm, the delay after propagating through the grating with length $L = 38$ mm is 42 ps. This delay corresponds to the soliton's group velocity as low as 72% of the light speed in an unprocessed fiber. The group velocity of our adopted ultrashort pulse is very close to that of the Bragg solitons demonstrated previously in the experiment [8]. The spatial width of the pulse in grating is smaller than the one in the unprocessed fiber. It results from the incidence from the normal group-velocity medium into slow group-velocity medium. However, the spatial length of an optical cycle is unchanged because of the constant average refractive index. It is found that the pulse in

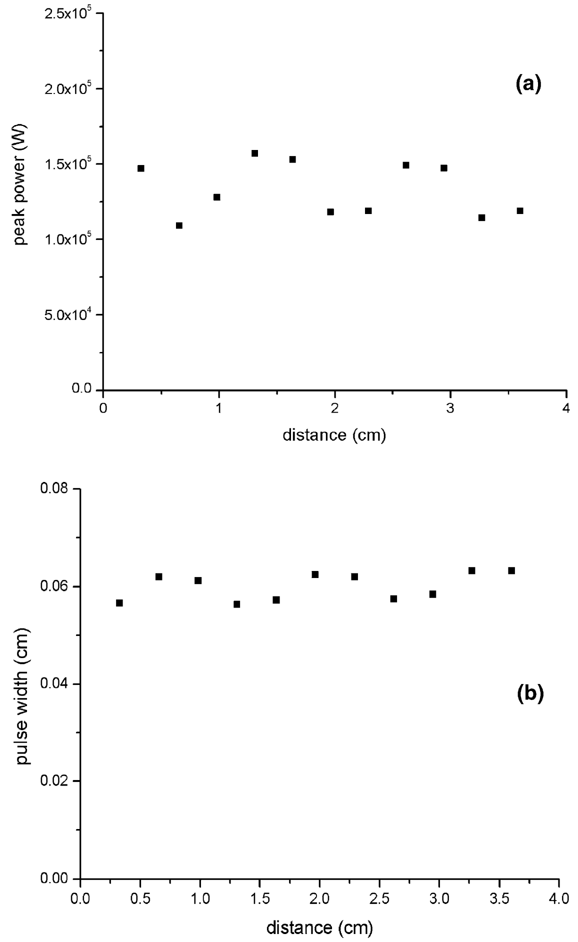

Fig. 5. (a) Peak power and (b) spatial pulse width of the soliton-like pulse versus the propagating distance. It is shown that the pulse adjusts its amplitude and duration periodically because of the interaction between the nonlinearity and the quadratic grating dispersion.

grating contains less optical cycles than the pulse in the unprocessed fiber. It could be clarified that the pulse is slowing down by periodic medium not by the linear refractive index. Fig. 6 further displays the time delay versus the carrier wavelength of the ultrashort Bragg soliton. One can see that the delay is linearly proportional to the Bragg detuning wavelength. Note that both of the NLCMEs and the NLSE model cannot predict such a relation between the time delay and the carrier wavelength detuning of an ultrashort Bragg soliton. Consequently, it would be useful to apply

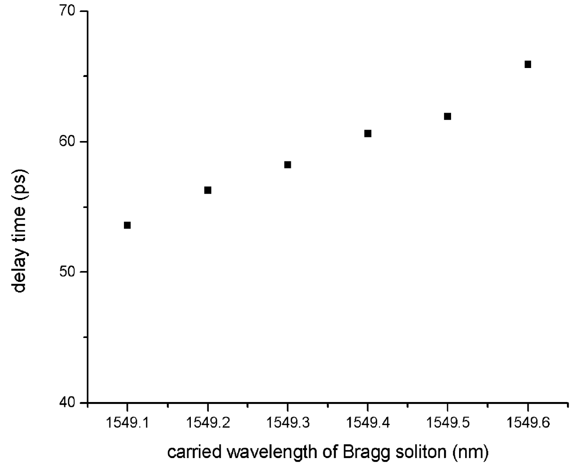

Fig. 6. Time delay versus different carried wavelength of the ultrashort Bragg soliton.

the FD–TD method to estimate the group velocity of an ultrashort Bragg soliton, especially for designing an all-optical buffer in practical high-speed communication systems.

4. Conclusion

We have applied the FD–TD method to investigate the nonlinear ultrashort pulse in a fiber Bragg grating. The FD–TD method can directly simulate Maxwell's equation and inherently computes the bi-directional electromagnetic field without using any approximation. As a result, our study numerically confirms that an ultrashort solitary wave near the bandgap edge still could propagate through a nonlinear PBG structure, even if its broadband spectrum overlaps the forbidden gap of the PBG medium. The propagating dynamics that has not yet been clarified by the NLCMEs and the NLSE model is explicitly shown on the basis of the FD–TD method. The present simulations demonstrate that the low-intensity pulse does not yield the NLSE soliton-like propagation in Kerr nonlinear Bragg gratings. Such propagation is only ensured by high-power pulses (10^5 W). This imposes severe limitations on the use of NLSE Bragg solitons for optical communications. By contrast, solitons can propagate via nearresonant self-induced transparency in resonantly absorbing Bragg reflectors with arbitrarily low intensities and are therefore much more suitable for telecommunications [11,12]. Furthermore, the FD–TD method shows that the time delay of an ultrashort Bragg soliton is linearly proportional to the Bragg detuning wavelength. It would be useful to apply the FD–TD method to design an all-optical delay line in a realistic high-speed telecommunication system.

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