Distortionless Pulse-Train Propagation in a Nonlinear Photonic Bandgap Structure Doped Uniformly With Inhomogeneously Broadening Two-Level Atoms

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***Abstract*—The pulse propagation in a one-dimensional nonlinear photonic bandgap (PBG) structure doped uniformly with inhomogeneously broadening two-level atoms is investigated. The Maxwell–Bloch equations describing pulse propagation in such a uniformly doped PBG structure are derived first and further reduced to effective nonlinear coupled-mode equations. An exact analytic pulse-train solution to these effective coupled-mode equations is obtained. Such a distortionless pulse-train solution is given by sinusoidal functions with a DC background and a modulated phase. Numerical examples of the distortionless pulse train in a silica-based PBG structure doped uniformly with Lorentzian line-shape two-level atoms are shown.**

***Index Terms*—Maxwell–Bloch equations, photonic bandgap structure, self-induced transparency.**

I. Introduction

THE DISTORTIONLESS propagation of light through an optical resonance medium has been widely discussed since McCall and Hahn discovered self-induced transparency (SIT) [1]. The SIT is characterized by the continuous absorption and reemission of electromagnetic radiation from the resonant atoms. Thus the optical pulse propagates through the medium without loss and distortion. Because of the SIT effect, the group velocity of such a coherent pulse depends on the pulsewidth and is much less than the speed of light in the host medium. Furthermore, the SIT effect is described by the Maxwell–Bloch equations, which have distortionless pulse-train solutions given by the Jacobi elliptic functions [2]–[4]. Such pulse-train propagation results from the energy of resonant atoms periodically oscillating between the ground state and upper state. In particular, when the Jacobi elliptic modulus is unity, the pulse-train solutions are reduced to single-pulse solutions of hyperbolic secant functions. These single pulse solutions are called SIT solitons. Both SIT solitons and periodic pulse trains have been observed in the experiments [5], [6].

More recently, a photonic bandgap (PBG) structure doped with resonant atoms has drawn considerable attention [7]–[15]. In the meantime, Aközbek and John have investigated the fundamental work on SIT solitary waves in PBG materials doped uniformly with resonant atoms [16]. For example, they have found single pulse solutions for frequency detuned far from Bragg resonance and frequency detuned near the PBG edge. However, the SIT analytic solution suitably for general frequency detuning and general phase modulation in a uniformly doped PBG medium has never been found. In this paper, we study the SIT in a nonlinear PBG structure doped uniformly with inhomogeneously broadening two-level atoms. After neglecting the high-order spatial harmonics of the material polarization, we show that the Maxwell–Bloch equations can be reduced to effective nonlinear coupled-mode equations (NLCMEs). Analytic distortionless pulse-train solutions to these effective NLCMEs are obtained. It is found that even if the carrier frequency of the pulse train is inside the forbidden band, the pulse trains can propagate through the PBG structure and obey the general SIT phase modulation effect.

The paper is organized as follows: In Section II, the Maxwell–Bloch equations governing the optical pulse propagating in a uniformly doped PBG structure are derived by keeping the second derivative of electromagnetic field with respect to the propagation distance. Because this second derivative is considered, our model involves the SIT-induced quadratic dispersion due to the slow-light propagation. We also take into account the material dispersion and Kerr nonlinearity of the host medium. In Section III, we solve the Bloch equations and subsequently reduce the Maxwell–Bloch equations to effective NLCMEs. The effective NLCMEs describe that pulse propagation through a uniformly doped PBG structure is equivalent to that through an effective PBG structure without dopants. In Section IV, we solve the effective NLCMEs and obtain exact pulse-train solutions given by the sinusoidal functions. It is also shown that such a pulse train obeys the general SIT phase modulation effect. In Section V, we numerically study the characteristics of the pulse trains by assuming the inhomogeneously broadening line shape of the resonant atoms is Lorentzian. In Section VI, we compare our results with the previous research and conclude this paper.

II. Maxwell–Bloch Equations

We consider a one-dimensional (1-D) Bragg grating formed in a host medium with Kerr nonlinearity. The periodic variations of the refractive index inside the grating region is assumed to be [17]

$$\tilde{n}(\omega) = n(\omega) + n_2|\mathbf{E}|^2 + n_a \cos(2\beta_g z) \tag{2.1}$$

where $\mathbf{E}$ is the electric field in the medium, $n(\omega)$ is the frequency-dependent refractive index, n_2 is the Kerr nonlinear-index co-

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efficient, n_a is the magnitude of the periodic index variations, and β_g is the grating wave number. The two-level atoms with the resonant frequency ω_r are uniformly embedded in this Kerr host medium. From Maxwell's equations, the wave equation describing light propagation in such a medium can be written as

$$\nabla^2\mathbf{E} - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} - \mu_0\frac{\partial^2\mathbf{P}}{\partial t^2} = \mu_0\frac{\partial^2\mathbf{P}_R}{\partial t^2} \tag{2.2}$$

where c is the velocity of light in vacuum, μ_0 is the vacuum permeability, $\mathbf{P}$ is the electric induced polarization including the linear and nonlinear contributions of the host medium, and $\mathbf{P}_R$ is the resonant polarization due to the two-level atoms. In Fourier domain, (2.2) becomes

$$\nabla^2\tilde{\mathbf{E}} + \tilde{n}(\omega)^2\frac{\omega^2}{c^2}\tilde{\mathbf{E}} = -\mu_0\omega^2\tilde{\mathbf{P}}_R \tag{2.3}$$

where $\tilde{\mathbf{E}}$ is the Fourier transform of $\mathbf{E}$, and $\tilde{\mathbf{P}}_R$ is the Fourier transform of $\mathbf{P}_R$. The electric field $\mathbf{E}$ propagating along the z direction in such a doped nonlinear PBG structure can be expressed as

$$\begin{aligned}\mathbf{E}(\mathbf{r},t) &= \frac{1}{2}\hat{x}\left[E(\mathbf{r},t)e^{-i\omega_B t} + \text{c.c.}\right]\\ &= \frac{1}{2}\hat{x}F(x,y)\left\{\left[E_+(z,t)e^{i(\beta_g z-\omega_B t)}\right.\right.\\ &\qquad \left.\left. + E_-(z,t)e^{i(-\beta_g z-\omega_B t)}\right] + \text{c.c.}\right\}\end{aligned} \tag{2.4}$$

where c.c. stands for complex conjugate, $\hat{x}$ is the polarization unit vector of the light assumed to be linearly polarized along the x axis, $F(x,y)$ is the transverse modal distribution, E_+ and E_- are the slowly varying envelopes of the forward and Bragg scattering fields, and ω_B is the Bragg frequency. In addition, the macroscopic resonant polarization $\mathbf{P}_R$ caused by the dopants is written as

$$\begin{aligned}\mathbf{P}_R(\mathbf{r},t) &= \frac{1}{2}\hat{x}[P(\mathbf{r},t)e^{-i\omega_B t} + \text{c.c.}]\\ &= \frac{1}{2}\hat{x}F(x,y)\left\{\left[P_+(z,t)e^{i(\beta_g z-\omega_B t)}\right.\right.\\ &\qquad \left.\left. + P_-(z,t)e^{i(-\beta_g z-\omega_B t)}\right] + \text{c.c.}\right\}\end{aligned} \tag{2.5}$$

where P_+ and P_- correspond to the slowly varying polarization envelopes induced by E_+ and E_-, respectively. For simplicity, the quantities of $n_2|\mathbf{E}|^2$ and n_a are assumed to be much smaller than the refractive index $n(\omega)$ of the host medium, so that they can be treated as perturbations for expanding $\tilde{n}(\omega)^2$ in (2.3). After substituting (2.1), (2.4), and (2.5) into (2.3), we can convert the resulting equations to time domain by following the perturbation theory of distributed feedback [17], but keeping the second derivative of electromagnetic field with respect to z. Consequently, the time-domain coupled-mode equations describing pulse propagation in a uniformly doped PBG structure are written as

$$\begin{aligned}&\frac{\partial^2 E_\pm}{\partial z^2} \pm 2i\beta_o\frac{\partial E_\pm}{\partial z} + 2i\beta_o\beta_1\frac{\partial E_\pm}{\partial t} - \left(\beta_1^2 + \beta_0\beta_2\right)\frac{\partial^2 E_\pm}{\partial t^2}\\ &\quad + 2\beta_0[\Gamma(|E_\pm|^2 + 2|E_\mp|^2)E_\pm + \delta\beta_0 E_\pm + \kappa E_\mp]\\ &\quad + \mu_0\omega_B^2\left(P_\pm + \frac{2i}{\omega_B}\frac{\partial P_\pm}{\partial t}\right) = 0\end{aligned} \tag{2.6}$$

where β_j $(j = 0, 1, 2)$ are determined by the mode-propagation constant $\beta(\omega) \equiv (\omega/c)n(\omega)$ via $\beta_j = d^j\beta/d\omega^j|_{\omega=\omega_B}, \delta\beta_0 = \beta_0 - \beta_g$ implies the wave number detuning from the exact Bragg resonance, $\kappa = \pi n_a/\lambda$ is the linear coupling coefficient, $\Gamma = n_2\omega_B/(cA_{\text{eff}})$ is the Kerr nonlinearity coefficient, and the transverse mode function is averaged out by introducing the effective core area

$$A_{\text{eff}} = \frac{\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|F(x,y)|^2\,dx\,dy\right]^2}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|F(x,y)|^4\,dx\,dy}.$$

In arriving at (2.6), we expand $\beta(\omega)^2$ in a Taylor series $\beta(\omega)^2 \approx \beta_0^2 + 2\beta_0\beta_1(\omega-\omega_B) + (\beta_0\beta_2 + \beta_1^2)(\omega-\omega_B)^2$ for converting (2.3) with $\partial^2E_\pm/\partial z^2$ terms to time domain [18]. In the literature, the wave equation for pulse propagation is usually derived by neglecting the second derivative of electromagnetic field with respect to z. However, for the SIT slow-light propagation, the $\partial^2E_\pm/\partial z^2$ terms can be comparable to the other terms in (2.6). This effect will be justified in the following derivation. Therefore, we keep the $\partial^2E_\pm/\partial z^2$ terms in our equation.

We now consider the atomic Bloch equations. If the relaxation times of the polarization and population difference are long compared with the pulsewidth, the relaxtion effects of the two-level system can be ignored. Therefore, under the rotating wave approximation, the electric field and the macroscopic polarization satisfy the Bloch equations

$$\frac{\partial}{\partial t}P = -i\Delta\omega P + i\frac{\mu}{\hbar}WE \tag{2.7a}$$

$$\frac{\partial}{\partial t}W = -i\frac{\mu}{2\hbar}(EP^* - E^*P) \tag{2.7b}$$

where $\Delta\omega$ is defined by $\Delta\omega = \omega_r - \omega_B$, $W = \mu(N_1 - N_2)$ is the macroscopic population difference multiplied by the transition matrix element μ between the ground state (N_1) and upper state (N_2) of the two-level system. The complex envelopes $E_\pm$ and $P_\pm$ can be further written as

$$E_\pm(z,t) = a_\pm(z,t)\exp[i\varphi_\pm(z,t)] \tag{2.8a}$$

$$P_\pm(z,t) = [U_\pm(z,t) + iV_\pm(z,t)]\exp[i\varphi_\pm(z,t)] \tag{2.8b}$$

where $a_\pm(z,t)$ are real envelopes, $\varphi_\pm(z,t)$ are phase functions, $U_\pm(z,t)$ correspond to the dispersion (in phase) induced by the resonant atoms, and $V_\pm(z,t)$ correspond to the absorption (in quadrature) caused by the resonant atoms. Moreover, the Bloch vectors $(u_\pm, v_\pm, w)$ relate the macroscopic polarization and population difference as follows:

$$(U_\pm, V_\pm, W) = \int_{-\infty}^{\infty}(u_\pm, v_\pm, w)g(\Delta\omega)d(\Delta\omega) \tag{2.9}$$

where $u_\pm(\Delta\omega, z, t), v_\pm(\Delta\omega, z, t)$, and $w(\Delta\omega, z, t)$ are the components of polarization and population difference contributed from the atoms with frequency $\Delta\omega$ detuned from ω_B, and $g(\Delta\omega)$ is the normalized inhomogeneous-broadening line-shape function. To keep a closed set of Bloch equations, we assume that

$$\varphi_\pm(z,t) = \phi(z,t) \pm \psi(z,t) \tag{2.10a}$$

$$w(z,t) = w_0 + 2w_1\cos[2\psi(z,t) + 2\beta_g z]. \tag{2.10b}$$

After we substitute (2.4), (2.5), and (2.8)–(2.10) into (2.7), the Bloch equations are expressed as

$$\frac{\partial u_\pm}{\partial t} = \left(\Delta\omega + \frac{\partial \varphi_\pm}{\partial t}\right) v_\pm \tag{2.11a}$$

$$\frac{\partial v_\pm}{\partial t} = -\left(\Delta\omega + \frac{\partial \varphi_\pm}{\partial t}\right) u_\pm + \frac{\mu}{\hbar}(a_\pm w_0 + a_\mp w_1) \tag{2.11b}$$

$$\frac{\partial w_0}{\partial t} = -\frac{\mu}{\hbar}(a_+ v_+ + a_- v_-) \tag{2.11c}$$

$$\frac{\partial w_1}{\partial t} = -\frac{\mu}{2\hbar}(a_+ v_- + a_- v_+). \tag{2.11d}$$

In (2.11), we have neglected the terms oscillating as $\exp(\pm i3\beta_g z)$ to get a closed set of equations. Strictly speaking, we have ignored all higher-order spatial terms of material polarization oscillating with multiples of the light wavenumber. In a periodic structure, such higher-order terms might be formed by beating of two counterpropagating and phase-modulated (chirped) waves. This process in general produce many spatial harmonics of material polarization which are converging very slowly. Nevertheless, in this paper we is devoted to finding an exact solution propagating along one-direction and exciting no spatial harmonic of material polarization [16].

III. Effective NLCMEs for Maxwell–Bloch Equations

In this section, we show how to reduce the Maxwell–Bloch equations to effective NLCMEs for pulse propagating in a nonlinear PBG structure doped uniformly with inhomogeneously broadening two-level atoms. In order to obtain the analytic solution we assume $v_\pm(\Delta\omega, z, t)$ are in factorized forms [4], [16], [19]

$$v_\pm(\Delta\omega, z, t) = v_1^\pm(0, z, t) f(\Delta\omega) \tag{3.1}$$

where $f(\Delta\omega)$ is known as the dipole spectra-response function and is normalized as $f(0) = 1$. Integrating (2.11a), we have

$$u_\pm(\Delta\omega, z, t) = [u_1^\pm(z,t) + u_2^\pm(z,t)\Delta\omega] f(\Delta\omega) \tag{3.2}$$

where $u_1^\pm$ and $u_2^\pm$ are defined as

$$\frac{\partial u_1^\pm}{\partial t} \equiv v_1^\pm \frac{\partial \varphi_\pm}{\partial t} \tag{3.3a}$$

$$\frac{\partial u_2^\pm}{\partial t} \equiv v_1^\pm. \tag{3.3b}$$

Similarly, by integrating (2.11c) and (2.11d), we obtain

$$w_0(\Delta\omega, z, t) = w_i - [w_0^+(z,t) + w_0^-(z,t)] f(\Delta\omega) \tag{3.4a}$$

$$w_1(\Delta\omega, z, t) = -\frac{1}{2}[w_1^+(z,t) + w_1^-(z,t)] f(\Delta\omega). \tag{3.4b}$$

In (3.4), w_i is the initial population difference and it is assumed in the ground state of the two-level system, i.e., $w_i = N_D\mu$, where $N_D = N_1 + N_2$ is the doping concerntration of the resonant atoms. Likewise $w_0^\pm$ and $w_1^\pm$ are defined as

$$\frac{\partial w_0^+}{\partial t} \equiv \frac{\mu}{\hbar} a_+ v_1^+ \quad \text{and} \quad \frac{\partial w_0^-}{\partial t} \equiv \frac{\mu}{\hbar} a_- v_1^- \tag{3.5a}$$

$$\frac{\partial w_1^+}{\partial t} \equiv \frac{\mu}{\hbar} a_+ v_1^- \quad \text{and} \quad \frac{\partial w_1^-}{\partial t} \equiv \frac{\mu}{\hbar} a_- v_1^+. \tag{3.5b}$$

Substituting (3.1)–(3.5) into (2.11b), we have

$$\begin{aligned}\frac{\partial v_1^\pm}{\partial t} =& -\frac{\partial \varphi_\pm}{\partial t} u_1^\pm - \frac{\mu}{\hbar}(w_0^+ + w_0^-) a_\pm \\ &+ \left[-\frac{\mu(w_1^+ + w_1^-)}{2\hbar} a_\mp - \left(u_1^\pm + \frac{\partial \varphi_\pm}{\partial t} u_2^\pm\right)\Delta\omega \right. \\ &\left. - u_2^\pm \Delta\omega^2 + \frac{\mu w_i}{\hbar}\frac{a_\pm}{f(\Delta\omega)}\right].\end{aligned} \tag{3.6}$$

The terms in square brackets in (3.6) should be independent of $\Delta\omega$. Therefore, we have

$$f(\Delta\omega) = (1 + c_1\Delta\omega + c_2\Delta\omega^2)^{-1} \tag{3.7}$$

and

$$u_1^\pm = -\frac{\partial \varphi_\pm}{\partial t} u_2^\pm + c_1 \frac{\mu}{\hbar} a_\pm w_i \tag{3.8a}$$

$$u_2^\pm = c_2 \frac{\mu}{\hbar} a_\pm w_i \tag{3.8b}$$

where c_1 and c_2 are the constants to be determined. Substituting (3.3b) and (3.8) into (3.1), (3.2), (3.4), and (3.5), we obtain

$$u_\pm = \left[c_1 - \left(\frac{\partial \varphi_\pm}{\partial t} - \Delta\omega\right) c_2\right] \frac{\mu}{\hbar} w_i f(\Delta\omega) a_\pm(z,t) \tag{3.9a}$$

$$v_\pm = c_2 \frac{\mu}{\hbar} w_i f(\Delta\omega) \frac{\partial}{\partial t} a_\pm(z,t) \tag{3.9b}$$

$$w_0 = w_i - \frac{c_2}{2}\left(\frac{\mu}{\hbar}\right)^2 w_i f(\Delta\omega)[a_+^2(z,t) + a_-^2(z,t)] \tag{3.9c}$$

$$w_1 = -\frac{c_2}{2}\left(\frac{\mu}{\hbar}\right)^2 w_i f(\Delta\omega)[a_+(z,t) a_-(z,t)]. \tag{3.9d}$$

Then substituting (3.9) into (2.11a) and (2.11b), we have

$$\frac{c_1}{c_2}\frac{\partial a_\pm}{\partial t} = 2\frac{\partial a_\pm}{\partial t}\frac{\partial \varphi_\pm}{\partial t} + \frac{\partial^2 \varphi_\pm}{\partial t^2} a_\pm \tag{3.10a}$$

$$\begin{aligned}\frac{\partial^2 a_\pm}{\partial t^2} =& \left[\frac{1}{c_2} - \frac{c_1}{c_2}\left(\frac{\partial \varphi_\pm}{\partial t}\right) + \left(\frac{\partial \varphi_\pm}{\partial t}\right)^2\right] a_\pm \\ &- \frac{\mu^2}{2\hbar^2}(a_\pm^2 + 2a_\mp^2) a_\pm.\end{aligned} \tag{3.10b}$$

Equations (3.10) not only describe the coupling nature between the envelopes and phases of the forward and Bragg scattering fields in the PBG structure, but also lead to the general SIT chirping equation, which will be studied in the next section. Now we show that using (3.10) can reduce the Maxwell–Bloch equations to effective NLCMEs. Using (2.8)–(2.10), (3.9), and (3.10), we obtain

$$\frac{\partial^2 E_\pm}{\partial t^2} = i\frac{c_1}{c_2}\frac{\partial E_\pm}{\partial t} + \frac{1}{c_2} E_\pm - \frac{1}{2}\left(\frac{\mu}{\hbar}\right)^2 (|E_\pm|^2 + 2|E_\mp|^2) E_\pm \tag{3.11a}$$

$$\begin{aligned}\mu_0\omega_B^2 &\left(P_\pm + \frac{2i}{\omega_B}\frac{\partial P_\pm}{\partial t}\right) \\ &= s\left[\left(c_1 - \frac{2}{\omega_o}\right) I_1 + c_2 I_2\right] E_\pm \\ &\quad + isc_2\left(I_1 + \frac{2}{\omega_o} I_2\right)\frac{\partial E_\pm}{\partial t} \\ &\quad + s\frac{c_2}{\omega_o} I_1 \left(\frac{\mu}{\hbar}\right)^2 (|E_\pm|^2 + 2|E_\mp|^2) E_\pm\end{aligned} \tag{3.11b}$$

where s is $s=\mu_o\omega_o^2(\mu/\hbar)w_i$, and two integral constants I_1 and I_2 are defined by

$$I_1=\int_{-\infty}^{\infty} f(\Delta\omega)g(\Delta\omega)\,d(\Delta\omega) \tag{3.12a}$$

$$I_2=\int_{-\infty}^{\infty} \Delta\omega f(\Delta\omega)g(\Delta\omega)\,d(\Delta\omega). \tag{3.12b}$$

Substituting (3.11) into (2.6), we obtain

$$\frac{\partial^2 E_\pm}{\partial z^2}\pm 2i\beta_o\frac{\partial E_\pm}{\partial z}+2i\beta_o\beta_1^e\frac{\partial E_\pm}{\partial t}+2\beta_0[\delta\beta_e E_\pm+\kappa E_\mp + \Gamma_e(|E_\pm|^2+2|E_\mp|^2)E_\pm]=0 \tag{3.13}$$

where the effective parameters are

$$\beta_1^e=\beta_1-\left(\beta_1^2+\beta_0\beta_2\right)\frac{c_1}{2\beta_o c_2}+\frac{sc_2}{2\beta_o}\left(I_1+\frac{2}{\omega_B}I_2\right) \tag{3.14a}$$

$$\Gamma_e=\Gamma+\left(\beta_1^2+\beta_0\beta_2\right)\frac{\mu^2}{4\beta_0\hbar^2}+\frac{sc_2}{2\beta_0\omega_B}I_1\frac{\mu^2}{\hbar^2} \tag{3.14b}$$

$$\delta\beta_e=\delta\beta_0+\frac{s}{2\beta_0}\left[\left(c_1-\frac{2}{\omega_B}\right)I_1+c_2I_2-\frac{\beta_1^2+\beta_0\beta_2}{sc_2}\right]. \tag{3.14c}$$

Consequently, we have reduced (2.6) to the effective NLCMEs (3.13) including the $\partial^2E_\pm/\partial z^2$ terms. The effective NLCMEs describe that pulse propagation through a uniformly doped PBG structure is equivalent to that through an effective PBG structure without dopants. Equations (3.10) and (3.13) are referred to as the Bloch-NLCMEs. It is noticed that the analytic solutions describing pulse propagation in a doped nonlinear PBG structure have to satisfy the Bloch-NLCMEs.

IV. Exact Analytic Solutions to Bloch-NLCMEs

Now, we start solving the Bloch-NLCMEs to obtain analytic solutions. By making the moving coordinate transformation $\tau=t-z/v_g$ and $\xi=z$, (3.13) are rewritten to the forms:

$$\frac{\partial^2 E_\pm}{\partial \xi^2}-\frac{2}{v_g}\frac{\partial^2 E_\pm}{\partial\tau\partial\xi}\pm 2i\beta_o\frac{\partial E_\pm}{\partial\xi} + 2i\beta_o\left(\beta_1^e\mp\frac{1}{v_g}\right)\frac{\partial E_\pm}{\partial\tau}+\frac{1}{v_g^2}\frac{\partial^2E_\pm}{\partial\tau^2}+2\beta_0\delta\beta_eE_\pm + 2\beta_0[\Gamma_e(|E_\pm|^2+2|E_\mp|^2)E_\pm+\kappa E_\mp]=0. \tag{4.1}$$

To solve the Bloch-NLCMEs, we seek the solutions of the forms

$$E_\pm(z,t)=a_\pm(\tau)\exp[i\varphi_\pm(\tau)+i\Delta\beta_0\xi]. \tag{4.2}$$

Equations (4.2) indicate that the phase functions are $\phi(z,t)=\phi(\tau)+\Delta\beta_0\xi$ and $\psi(z,t)=\psi(\tau)$, where $\Delta\beta_0$ represents the change of the propagation constant due to the resonant atoms and Kerr nonlinearity. The coefficient of $\partial^2E_\pm/\partial\tau^2$ terms in (4.1) exhibits the induced quadratic dispersion. It is known that the pulse velocity can be greatly reduced by the SIT. Thus the induced dispersion increases when the group velocity is reduced. We will numerically show in the next section that the group velocity is much less than the speed of light in the bare Kerr-host medium. Substituting (3.11a) and (4.2) into (4.1), and using the new variables of optical field $\bar{E}_\pm(\tau)=a_\pm(\tau)\exp[i\varphi_\pm(\tau)]$ to express (4.1), we have

$$+i\left(\bar\beta_1\mp\frac{1}{v_g}\right)\frac{\partial\bar E_\pm}{\partial\tau}+(\delta\bar\beta_0\mp\Delta\beta_0)\bar E_\pm + \bar\Gamma(|\bar E_\pm|^2+2|\bar E_\mp|^2)\bar E_\pm+\kappa\bar E_\mp=0 \tag{4.3}$$

where $\bar\Gamma=\Gamma_e-\mu^2/(4\beta_0v_g^2\hbar^2)$, $\bar\beta_1=\beta_1^e+c_1/(2\beta_0c_2v_g^2)-\Delta\beta_0/(v_g\beta_0)$, and $\delta\bar\beta_0=\delta\beta_e+1/(2\beta_0c_2v_g^2)-\Delta\beta_0^2/2\beta_0$. The forms of (4.3) are equivalent to the general NLCMEs that have moving gap-soliton solutions describing distortionless pulse propagation through an undoped PBG structure [20], [21]. However, we will show in the following that such gap soliton solutions cannot satisfy (3.10). Separating the real parts and imaginary parts of (4.3), we obtain the following differential equations:

$$\left(\bar\beta_1\mp\frac{1}{v_g}\right)\frac{\partial a_\pm}{\partial\tau}\mp\kappa a_\mp\sin(2\psi)=0 \tag{4.4a}$$

$$-\left(\bar\beta_1\mp\frac{1}{v_g}\right)\frac{\partial\varphi_\pm}{\partial\tau}a_\pm+(\delta\bar\beta_0\mp\Delta\beta_0)a_\pm + \bar\Gamma a_\pm^3+2\bar\Gamma a_\mp^2a_\pm+\kappa a_\mp\cos(2\psi)=0. \tag{4.4b}$$

Equations (4.4) possess two first integrals [21]

$$\beta_{1n}a_+^2+\beta_{1p}a_-^2=c_0 \tag{4.5a}$$

$$\beta_{1p}\beta_{1n}\cos(2\psi)a_+a_-=\frac{\beta_{1n}}{\kappa}\left(\Delta\beta_0\bar\beta_1-\frac{\delta\bar\beta_0}{v_g}\right)a_+^2 + \frac{\bar\Gamma}{4\kappa}\beta_{1n}\beta_{3n}a_+^4 + \frac{\bar\Gamma}{4\kappa}\beta_{1p}\beta_{3p}a_-^4+c_3 \tag{4.5b}$$

where c_0 and c_3 are integration constants, and $\beta_{1p}=\bar\beta_1+1/v_g$, $\beta_{1n}=\bar\beta_1-1/v_g$, $\beta_{3p}=\bar\beta_1+3/v_g$ and $\beta_{3n}=\bar\beta_1-3/v_g$. Substituting (4.4a) and (4.5a) into (4.5b), we have a differential equation for a_+^2:

$$\left(c_0\beta_{1p}\beta_{1n}^2a_+^2-\beta_{1p}\beta_{1n}^3a_+^4\right)-\frac{\beta_{1p}^2\beta_{1n}^4}{4\kappa^2}\left[\frac{\partial(a_+^2)}{\partial\tau}\right]^2 = \left[\left(c_3+c_0^2\frac{\bar\Gamma}{4\kappa}\frac{\beta_{3p}}{\beta_{1p}}\right)+\frac{\bar\Gamma}{4\kappa}\beta_{1n}\left(\beta_{3n}+\frac{\beta_{3p}}{\beta_{1p}}\beta_{1n}\right)a_+^4 + \frac{\beta_{1n}}{\kappa}\left(\Delta\beta_0\bar\beta_1-\frac{\delta\bar\beta_0}{v_g}-\frac{1}{2}c_0\bar\Gamma\frac{\beta_{3p}}{\beta_{1p}}\right)a_+^2\right]^2. \tag{4.6}$$

By defining $S=a_+^2$, (4.6) can be expressed as

$$\frac{(\dot S)^2}{4}+\gamma_0+\gamma_1S+\gamma_2S^2+\gamma_3S^3+\gamma_4S^4=0 \tag{4.7}$$

where the expressions of γ_m $(m=0,1,2,3,4)$ in (4.7) will be discussed later. Equation (4.7) can describe the motion of a solitary wave by analogy with that of a classical particle moving in a potential. Therefore, analytic solutions to (4.7) have been extensively studied for grating solitons and SIT solitons. The well-known solutions include single-pulse solitary waves (for $\gamma_0 = \gamma_1 = 0$) [20], [21], single-pulse solitons (for $\gamma_0 = \gamma_1 = \gamma_4 = 0$) and Jacobi elliptic soliton-trains (for $\gamma_4 = 0$) [2], [4], [16]. From the viewpoint of a mechanical analogy, a single-pulse solution corresponds to a particle being released from a zero potential position and stopping at $S = 0$; moreover, a pulse-train solution corresponds to a particle oscillating between two positions. Notice that all the above solutions are obtained under $\gamma_2 \neq 0$ and $\gamma_3 \neq 0$. However, for a uniformly doped nonlinear PBG structure, the Bloch-NLCMEs constrain the quantities of γ_3. Indeed, we subsequently show that γ_3 has to be zero for exact solutions to (4.6). Using $\partial/\partial t = \partial/\partial\tau$ and integrating (3.10a), we obtain

$$\frac{\partial\varphi_\pm}{\partial\tau} = -\Omega + \frac{c_4^\pm}{a_\pm^2} \tag{4.8}$$

where $\Omega = -c_1/(2c_2)$ and $c_4^\pm$ are integration constants. Equations (4.8) describe the general phase modulation, or pulse chirping in the SIT. The constant Ω indicates that the carrier frequency of the optical field is shifted to $\omega_B + \Omega$ by the SIT effect. Likewise the instantaneous shifted frequency is inversely proportional to the pulse intensity. The general chirping relation has been studied for the SIT in a nonlinear medium without PBG structure [4]. Substituting (4.8) and (4.5a) into (3.10b), and then integrating the resulting equation for a_+^2, we obtain

$$\frac{(\dot{S})^2}{4} + (c_4^+)^2 - c_5 S + \left(\Omega^2 + \frac{\mu^2}{\hbar^2}\frac{c_0}{\beta_{1p}} - \frac{1}{c_2}\right)S^2 + \frac{1}{2}\left(\frac{\mu}{\hbar}\right)^2\left(\frac{1}{2} - \frac{\beta_{1n}}{\beta_{1p}}\right)S^3 = 0 \tag{4.9}$$

where c_5 is another integration constant. Comparing (4.6), (4.7) and (4.9), we undergo the constraint $\gamma_3 = 0$ resulting from $\gamma_4 = 0$. These constraints lead to

$$\bar{\beta}_1 = 3/v_g \quad \text{and} \quad \bar{\Gamma} = 0. \tag{4.10}$$

Hence γ_0, γ_1 and γ_2 are determined as

$$\gamma_0 = \frac{1}{256}\kappa^2 c_3^2 v_g^6 = (c_4^+)^2 \tag{4.11a}$$

$$\gamma_1 = \frac{1}{64}\kappa c_3(3\Delta\beta_0 - \delta\bar{\beta}_0)v_g^4 - \frac{1}{16}c_0\kappa^2 v_g^3 = -c_5 \tag{4.11b}$$

$$\gamma_2 = \frac{1}{64}(3\Delta\beta_0 - \delta\bar{\beta}_0)^2 v_g^2 + \frac{1}{8}\kappa^2 v_g^2 = \Omega^2 + \frac{\mu^2}{4\hbar^2}c_0 v_g - \frac{1}{c_2}. \tag{4.11c}$$

Since γ_3 has to be zero, the exact single-pulse and Jacobi pulse-train solutions cannot exist for the Bloch-NLCMEs equations. Nevertheless, (4.9) can be integrated to yield

$$\int_{a_+(0)}^{a_+} \frac{a_+ da_+}{\sqrt{-a_+^4 - (\gamma_1 a_+^2 + \gamma_0)/\gamma_2}} \equiv \int_{a_+(0)}^{a_+} \frac{a_+ da_+}{\sqrt{P_4(a_+)}} = \sqrt{\gamma_2}\tau. \tag{4.12}$$

Because $\gamma_0 > 0$ and $\gamma_2 > 0$ from (4.11), we have to restrict $\gamma_1 < 0$ ($c_5 > 0$) for $P_4(a_+) > 0$. The solution to (4.12) depends on the roots of $P_4(a_+)$. Thus we assume $P_4(a_+) = (a_p^2 - a_+^2)(a_+^2 - a_q^2)$, where $0 \le a_q \le a_+(\tau) \le a_p$. Then (4.12) have an exact solution written by

$$a_+(\tau) = \sqrt{a_p^2 - \left(a_p^2 - a_q^2\right)\sin^2(\sqrt{\gamma_2}\tau)} \tag{4.13a}$$

where $a_p^2 = c_5/2\gamma_2 + \sqrt{(c_5/2\gamma_2)^2 - \gamma_0/\gamma_2}$ and $a_q^2 = c_5/2\gamma_2 - \sqrt{(c_5/2\gamma_2)^2 - \gamma_0/\gamma_2}$. From (4.5a) and (4.8), $a_-(\tau)$, $\varphi_+(\tau)$ and $\varphi_-(\tau)$ are obtained as follow:

$$a_-(\tau) = \frac{1}{\sqrt{2}}\sqrt{\left(\frac{1}{2}c_0 v_g - a_p^2\right) + \left(a_p^2 - a_q^2\right)\sin^2(\sqrt{\gamma_2}\tau)} \tag{4.13b}$$

$$\varphi_+(\tau) = -\Omega\tau + \frac{c_4^+}{a_p a_q\sqrt{\gamma_2}}\tan^{-1}\left[\frac{a_q}{a_p}\tan(\sqrt{\gamma_2}\tau)\right] + \Psi_+ \tag{4.13c}$$

$$\varphi_-(\tau) = -\Omega\tau + \frac{4c_4^-}{\sqrt{\gamma_2\left(c_0 v_g - 2a_p^2\right)\left(c_0 v_g - 2a_q^2\right)}} \cdot \tan^{-1}\left[\sqrt{\frac{c_0 v_g - 2a_q^2}{c_0 v_g - 2a_p^2}}\tan(\sqrt{\gamma_2}\tau)\right] + \Psi_- \tag{4.13d}$$

where Ψ_+ and Ψ_- are integration constant. As a result, we have obtained exact analytic solutions to the Bloch-NLCMEs. These solutions demonstrate that both the forward and Bragg scattering fields are modulated periodically with a period $T_p = \pi/\sqrt{\gamma_2}$. These periodic pulse trains propagate distortionlessly in the same direction. Thus the group velocity of the Bragg scattering field is in the direction opposite to its phase velocity. Such distortionless propagation results from the two-level atoms periodically absorbing energy from one part of the wave and then returning the energy to an adjacent part [3]. Likewise these coherent photon-atom interactions balance with the grating dispersion and Kerr nonlinearity. Therefore, SIT occurs and the optical field overcomes the forbidden band and passes through the PBG structure.

V. Numerical Study of the Pulse-Trains

To study the distortionless pulse trains, we have to further derive all undetermined constants. For the following discussions, we restrict our attention to the optical field without carrier frequency shift, i.e., $\Omega = 0$. In addition, the inhomogeneous broadening line shape of the resonant atoms is assumed to be Lorentzian

$$g(\Delta\omega) = \frac{\Delta\omega_a}{2\pi}\frac{1}{\Delta\omega^2 + (\Delta\omega_a/2)^2} \tag{5.1}$$

where $\Delta\omega_a = 2\pi f_a$ is the full-width at half-maximum (FWHM) of $g(\Delta\omega)$. Substituting (5.1) and (3.7) into the difinitions of I_1 and I_2 in (3.12), we find that $I_1 = 2/(\sqrt{c_2}\Delta\omega_a)$ and $I_2 = 0$ for $\Delta\omega_a \gg 1/\sqrt{c_2}$ [19]. From (3.14) and (4.10), both c_2 and $\Delta\beta_0$ are first obtained as a function of the group velocity v_g:

$$c_2 = \frac{\Delta\omega_a^2\omega_0^2}{16s^2\mu^4 v_g^4}\left[4\hbar^2 v_g^2\beta_0\Gamma - \mu^2 + v_g^2\mu^2\left(\beta_1^2 + \beta_0\beta_2\right)\right]^2 \tag{5.2a}$$

$$\Delta\beta_0 = \frac{1}{4}v_g\left[\beta_1^2\omega_0 + \beta_0\left(4\beta_1 + \beta_2\omega_0 + \frac{4\hbar^2\Gamma\omega_0}{\mu^2}\right)\right] - \frac{\omega_0}{4v_g} - 3\beta_0. \tag{5.2b}$$

Substituting (4.10) and (4.13) into (4.4b), and then comparing the expressions of $\cos(2\psi)$ derived from the upper- and lower-sign equation of (4.4b), we have

$$\delta\bar{\beta}_0 = \frac{\Delta\beta_0}{3} = \left(\frac{1}{2\beta_0 c_2}\right)\frac{1}{v_g^2} + \delta\beta_e - \frac{\Delta\beta_0^2}{2\beta_0} \tag{5.3a}$$

$$c_4^- = \frac{1}{12}c_0\Delta\beta_0 v_g^2 + \frac{c_4^+}{2}. \tag{5.3b}$$

By substituting (5.2b) into (5.3a), not only $\delta\bar{\beta}_0$ is expressed as functions of the group velocity v_g, but also the group velocity is determined for given parameters of the medium. Consequently, the pulse-train period $T_p = \pi/\sqrt{\gamma_2}$ and the integration constant c_0 are further obtained from (4.11c). On the other hand, in order to obtain the integration constants $c_3, c_4^{\pm}$, and c_5, we define the total intensity of the forward and Bragg scattering field as follows [22]:

$$I_T \equiv a_+(\tau)^2 + a_-(\tau)^2 = I_0 + I_m\cos^2(\sqrt{\gamma_2}\tau) \tag{5.4}$$

where $I_0 = c_0 v_g/4 + a_q^2/2$ is the background intensity and $I_m = (a_p^2 - a_q^2)/2$ is the modulated intensity of the total field. The relationship between the optical power and optical intensity is $P_T = (n_0/2)(\sqrt{\varepsilon_0/\mu_0})A_{\text{eff}}I_T$, where ε_0 is the vacuum permittivity, n_0 is the refractive index at the Bragg wavelength of the host medium. In this paper, we are interested in the contrast between the total field and its modulated amplitude. Thus we define the contrast $\eta = I_m/(I_0 + I_m)$. Then the integration constants $c_3, c_4^{\pm}$, and c_5 can be determined for a given η by using the expressions of I_0, I_m, a_p and a_q in conjunction with (4.11). Moreover, for the constant phase parameters Ψ_+ and Ψ_-, we can assume $\Psi_+ = \Psi_- = 0$ for chirped pluses $(c_4^{\pm} \neq 0)$. However, for an unchirped pulse train, the forward and Bragg scattering field have to satisfy a constant phase difference $\Psi_+ - \Psi_- = l\pi/4$ (l: odd number) obtained from (4.4a). Such an unchirped case leads to $\Delta\beta_0 = 0$ and $c_3 = 0$ from (4.4b) and (4.11a), respectively. Therefore, $a_\pm$ are reduced to

$$a_+(\tau) = a_p|\cos(\sqrt{\gamma_2}\tau)| \tag{5.5a}$$

$$a_-(\tau) = a_p|\sin(\sqrt{\gamma_2}\tau)|/\sqrt{2}. \tag{5.5b}$$

These results lead to $I_0 = c_0 v_g/4 = a_p^2/2$ that cannot be zero for the existence of the pulse train. Consequently, there must be a dc term in the total intensity of the pulse train.

In order to illustrate the pulse trains, we use the following material parameters: The Kerr host medium is a silica-based material with $n_0 = 1.45, A_{\text{eff}} = 50\ \mu\text{m}^2$, $\beta_0 = 5.9 \times 10^6\ \text{m}^{-1}$, $\beta_1 = 4.8 \times 10^{-9}$ s/m, $\beta_2 = -20$ ps^2/km and $n_2 = 1.2 \times 10^{-22}$ m^2/V^2 at 1550-nm wavelength region. The coupling coefficient of the Bragg grating is $\kappa = 10$ cm^{-1} corresponding to the index vibration $n_a = 0.0006$ at the Bragg wavelength $\lambda_B = 1553$ nm. Here, we focus the frequency on the exact Bragg resonance, i.e., $\delta\beta_0 = 0$ cm^{-1}. For the two-level atoms, we assume that $\mu = 1.4 \times 10^{-32}$ c.m, $\Delta f_a = 1472$ GHz and $N_D = 8.0 \times 10^{24}$ m^{-3} corresponding to the typical 1000 ppm doping concentration of erbium atoms. By using the above parameters, Fig. 1 shows the envelopes and phases of the optical forward field (solid curves) and Bragg scattering field (dashed curves) as functions of a scaled time $\sqrt{\gamma_2}\tau$. The resulting group velocity is $v_g = 1.29 \times 10^6$ m/s being consistent with the assumption of $\Delta\omega_a \gg 1/\sqrt{c_2}$. The group velocity of this forward propagating energy is substantially less than the speed of light in the bare Kerr-host medium because both SIT and Bragg scattering slow down the light. Fig. 2 shows the total power of such slow light as

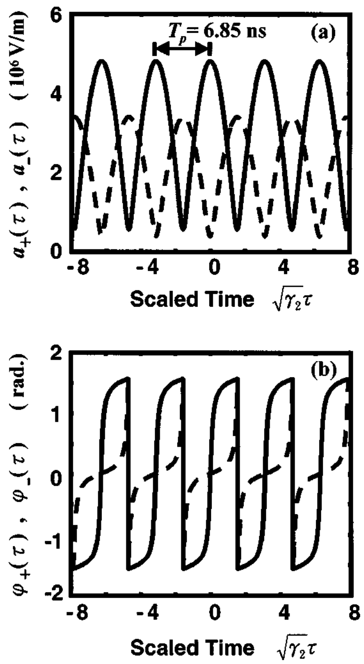

Fig. 1. (a) Envelopes and (b) phases of the optical forward field (solid curves) and Bragg scattering field (dashed curves) as functions of a scaled time $\sqrt{\gamma_2}\tau$. The pulse-train period is $T_p = \pi/\sqrt{\gamma_2} = 6.85$ ns; and the group velocity is $v_g = 1.29 \times 10^6$ m/s resulting in a appropriate $\Delta\beta_0 = -0.00169$ m^{-1} and corresponding to $1/250$ of the speed of light in the bare Kerr-host medium.

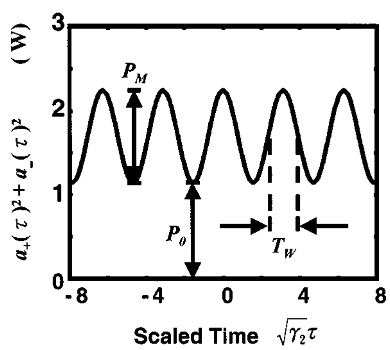

Fig. 2. Total power of the pulse train as a function of a scaled times $\sqrt{\gamma_2}\tau$. For $c_5 > 0$, the contrast η has to satisfy $\eta < 0.5$. Hence we choice $\eta = 0.49$ so that the background optical power is $P_0 = 1.10$ W and the amplitude of the modulated power is $P_M = 1.14$ W. Each pulsewidth is $T_W = 0.5T_p = 3.425$ ns (FWHM).

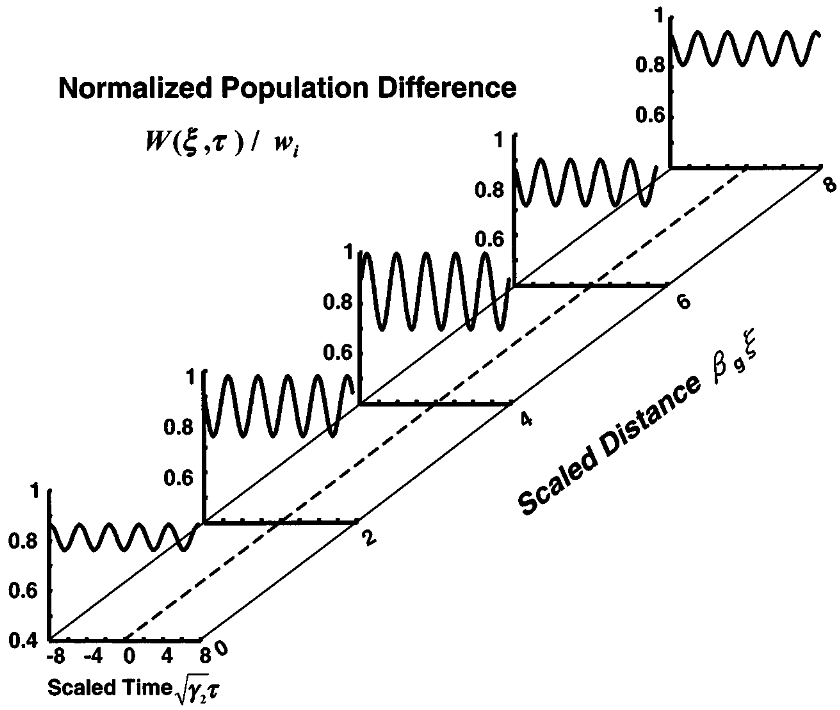

Fig. 3. Normalized population difference as a function of a scaled time $\sqrt{\gamma_2}\tau$ and a scaled distance $\beta_g\xi$. Clearly, the magnitude of the population difference is modulated periodically with the same period $T_p = 6.85$ ns; likewise the population difference varies periodically along the propagating distance.

a function of $\sqrt{\gamma_2}\tau$. The peak power is $P_0 + P_M = 2.24$ W and occurs when $\varphi_+(\tau) = \varphi_-(\tau) = 0$. Furthermore, the components of the Bloch vectors $(u_\pm, v_\pm, w)$ are obtained by substituting (4.13) into (3.9). Fig. 3 shows the normalized population difference as a function of $\sqrt{\gamma_2}\tau$ and $\beta_g\xi$. Clearly, the magnitude of the population difference is modulated periodically with the period T_p, and the population difference varies periodically along the propagating distance. Such energy exchanges between the upper state and the lower state lead to the distortionless pulse trains, even if the central frequency of the optical envelopes is inside the forbidden band. Finally, we emphasize that the pulse-train period $T_p = 6.85$ ns corresponds to each pulsewidth (FWHM) $T_W = 0.5T_p = 3.425$ ns. Recalling that for deriving (2.11), we assume the relaxation times of the resonant atoms are long compared with the pulsewidth T_W. However, at 4.2 K, the relaxation times of erbium atoms are $T_1 = 10$ ms $(\gg T_W)$ for the population difference, and $T_2 = 10$ ns $(> T_W)$ for the polarizations. From an experimental viewpoint, the total duration of a realistic finite pulse train launched into a uniformly doped PBG structure should be much less than the atomic relaxation times because the pulse-train solution has a dc background. Fig. 4 shows the individual pulsewidth as a function of the coupling coefficient for the contrast $\eta \approx 0.49$. When the coupling coefficient exceeds 80 cm^{-1}, each pulsewidth can be smaller than 0.5 ns. Therefore, a uniformly doped PBG structure with a larger coupling coefficient is more suitable for observing our numerical prediction without atomic relaxation processes. The silica-based PBG structure with such a large coupling coefficient can be fabricated on a silicon-on-insulator (SOI) structure [23], [24]. Note that the propagation constant and the group velocity for Fig. 4 remain unchanged because they are dominated by Kerr nonlinearity and doping concentration for the contrast $\eta \approx 0.49$. However, the peak power of the SIT pulse train for $\kappa > 80$ cm^{-1} exceeds 144.46 W. Consequently, it will be interesting to investigate the suitable materials and conditions to reduce the peak power required for the pulse train for experimentally studying the distortionless propagation.

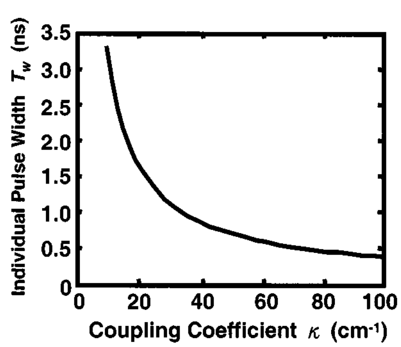

Fig. 4. Individual pulsewidth as a function of the coupling coefficient.

VI. Discussion and Conclusion

In this paper, we adopt the uniformly doped PBG model to study SIT pulse-train propagation. However, in contrast with [16], our model is more general. 1) In our uniformly doped PBG model, we derive the Maxwell–Bloch equation by keeping the $\partial^2 E_\pm/\partial z^2$ terms. In [16], the authors emphasize that they have neglected the linear contribution to the dispersion relation arising from the two-level atoms. Hence the allowed concentration of dopant atoms are limited. In [19], it has been found that SIT could induce an additional negative dispersion that cannot be predicted by the SIT theory without the second-order spatial derivative of the electromagnetic field. Since the Maxwell–Bloch equation with the $\partial^2 E_\pm/\partial z^2$ terms can reduce to Bloch-NLCMEs, these effective NLCMEs completely involve the SIT-induced negative dispersion and the effective grating dispersion. 2) The phase functions of the forward and Bragg scattering fields are assumed to be identical in [16]. On the contrary, we consider general phase functions written as $\varphi_\pm(z,t) = \phi(z,t) \pm \psi(z,t)$ for the fields; likewise the population difference are assumed to be $w = w_0 + 2w_1 \cos[2\psi(z,t) + 2\beta_g z]$. This general consideration of the phase functions makes that the phase modulation effects of the forward and Bragg scattering fields both satisfy the general SIT chirping equation. Although our model is more general than that in [16], the population difference $w = w_0 + 2w_1 \cos[2\psi(z,t) + 2\beta_g z]$ is a hypothesis for neglecting the infinite hierarchy of equations related to the successive spatial harmonics resulting from the population difference and polarization in the Bloch equations. To avoid such an assumption, a resonantly absorbing photonic crystal and a periodically doped PBG medium have been investigated. A vast family of SIT soliton in these two different resonance bandgap media has also been found [11]–[15].

In contrast to the single pulse solutions in [16], we focus our studies on the exact pulse-train solutions to the Bloch-NLCMEs. Notice that the Jacobi elliptic pulse-train solutions to the Maxwell–Bloch equations for a resonance medium without PBG structure have been theoretically studied [4] and experimentally demonstrated [6]. However, our model involves considering a resonance medium with resonant atoms embedded in a PBG structure. It is well known that a PBG structure has a forbidden band for optical energy, but the SIT provide a mechanism to make it possible that an optical pulse train can pass through the PBG medium. The pulse trains in a uniformly doped nonlinear PBG structure are given by the sinusoidal functions with background intensity. Because the PBG medium is transparent for the SIT, the research with respect to optical pulse propagation in a doped nonlinear PBG structure has attracted much interest. It has been further suggested that a doped nonlinear PBG structure could be applied to high sensitivity optical filter, pulse reshaping devices and optical switching devices for optical computing, optical interconnection and optical communication system [7]–[16]. It is our hope that our model can accurately estimate the associated medium parameters and the initial condition of the input optical field for designing such a uniformly doped device. In addition, it would be useful to study how to excite the pulse trains in a real doped PBG medium and what are the impacts of the relaxation effects on the stability of the pulse trains. These subjects would lead to practical applications of uniformly doped PBG structures in the vast area of lightwave systems.

In summary, we have derived the Bloch-NLCMEs to model the SIT effect in a 1-D nonlinear PBG structure doped uniformly with inhomogeneously broadening two-level atoms. We have found the exact analytic pulse-train solutions to the Bloch-NLCMEs. These pulse-train solutions are described by the sinusoidal functions with DC background. Their phases obey the general SIT phase modulation effect. Furthermore, because both SIT and Bragg scattering slow down the light, the pulse-train group velocity can be substantially less than the speed of light in a bare nonlinear medium. Numerical examples of the SIT pulse train in a silica-based PBG structure doped uniformly with Lorentzian line-shape two-level atoms are shown. It is found that even if the carrier frequency of the pulse train is inside the forbidden band, the pulse trains can propagate through the PBG structure. Namely, the SIT pulse train renders the PBG structure transparent.

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