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Aharonov-Bohm effect in phase shifts

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Abstract. – We discuss the influence of the Aharonov-Bohm effect on phase shifts. The general integral representation of phase shifts for short-range potentials with an Aharonov-Bohm flux at the origin is given. The interesting result is that the phase shifts δ_γ will become $(n+1/2)\pi$ $(n=0,1,2,\ldots)$ as long as the flux is adjusted at the values $\Phi=(n+1/2)\Phi_0$, where $\Phi_0=(2\pi\hbar c/e)$ is the fundamental flux quantum. According to the Levison theorem, it implies that new bound states will appear at these values. This phenomenon may be useful in controlling the number of bound states in quantum well as well as the spectra of light emission.

Since the implication of the global structure of the Aharonov-Bohm (AB) effect was discovered in 1959 [1], it has had much impact in our comprehension of the foundations of quantum theory [2]. Forty years later, the AB effect has contributed to the understanding of the phenomenon of fractional quantum Hall effect [3], superconductivity [3], repulsive Bose gases [4], cosmic string [5], and $(2+1)$D gravity theories [6]. Nevertheless, we are probably still far from exhausting the full meaning of the deep global concept [7]. The general discussion of the AB effect in phase shifts is still lacking. In this paper we discuss the general influence of the AB effect on phase shifts in spherical symmetric systems. Starting from the influence of the nonintegrable phase factor [8, 9] for the AB effect in the wave function, the pure radial wave equation containing the AB effect is extracted. The general formula for phase shifts of a magnetic flux plus a short-range potential is given. As a realization, we calculate the phase shifts of a charged particle which is scattered by a square-well plus an Aharonov-Bohm flux. The obtained result is compared with the exact solution. Furthermore, an experiment is proposed which may be useful to detect whether the quantum non-local effect is instant or not.

The Schrödinger equation for a charged particle with mass m moving in a spherical symmetry potential can be expressed as

$$\left[E-H_0\left(\boldsymbol{r},\frac{\hbar}{i}\nabla\right)\right]\Psi^0_{nlk}(\boldsymbol{r})=0, \tag{1}$$

where $H_0=-\hbar^2\nabla^2/2m+V(r)$ is the system Hamiltonian and Ψ^0_{nlk} is the wave function. Due to the spherical symmetry, the angular part of the wave function can be decomposed as

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$\Psi^0_{nlk}(\boldsymbol{r}) = R_{nl}(r)Y_{lk}(\theta,\varphi)$, where Y_{lk} are the well-known spherical harmonics. Equation (1) in spherical polar coordinates can be rewritten as

$$\left\{E - \left[-\frac{\hbar^2}{2m}\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r}\right) + \frac{l(l+1)\hbar^2}{2mr^2}\right] - V(r)\right\} R_{nl}(r)Y_{lk}(\theta,\varphi) = 0. \tag{2}$$

Since any arbitrary number pair (l,k) satisfies the equation, we have

$$\sum_{l=0}^{\infty}\sum_{k=-l}^{l}\left\{E - \left[-\frac{\hbar^2}{2m}\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r}\right) + \frac{l(l+1)\hbar^2}{2mr^2}\right] - V(r)\right\} R_{nl}(r)Y_{lk}(\theta,\varphi) = 0. \tag{3}$$

For a charged particle interacting with the magnetic field, the wave function is different from the original one by a global nonintegrable phase factor [8,9]

$$\Psi_{nlk}(\boldsymbol{r},P) = \Psi^0_{nlk}(\boldsymbol{r})e^{\frac{ie}{\hbar c}\int_P^r \boldsymbol{A}(\boldsymbol{r}')\cdot\mathrm{d}\boldsymbol{r}'}, \tag{4}$$

where we have used the vector potential $\boldsymbol{A}(\boldsymbol{r}')$ to describe the magnetic field and P to represent the nonintegrable phase of the wave function which depends on the path (*e.g.*, Chapt. 10 in ref. [10]). For the Aharonov-Bohm effect under consideration, the vector potential can be described by

$$\boldsymbol{A}(\boldsymbol{x}) = \begin{cases} \frac{1}{2}B\rho\hat{e}_\varphi & (\rho < \epsilon), \\ \frac{1}{2}B\frac{\epsilon^2}{\rho}\hat{e}_\varphi = \frac{\Phi}{2\pi\rho}\hat{e}_\varphi & (\rho > \epsilon), \end{cases} \tag{5}$$

where $\rho^2 = x^2 + y^2$ is the two-dimensional radial length, $\hat{e}_\varphi$ is the unit vector of coordinate φ, ϵ is the radius of the region where magnetic field exists, and Φ is the magnetic flux which is given by $\Phi = \pi\epsilon^2 B$. The associated magnetic field lines are confined within a tube along the z-axis. Along the free region of magnetic field, the path-dependent nonintegrable phase factor is given by $\exp[-i\mu_0\int_P^\lambda \mathrm{d}\lambda'\dot{\varphi}(\lambda')]$, where $\dot{\varphi} = \mathrm{d}\varphi/\mathrm{d}\lambda'$, and $\mu_0 = -2eg/\hbar c$ is a dimensionless number defined by $\Phi = 4\pi g$. The minus sign is a matter of convention. According to the discussion of ref. [9], only the loops of the phase factor are considered, the description of the electromagnetic phenomenon is then complete. Therefore the integral $\int^\lambda \mathrm{d}\lambda'\dot{\varphi}(\lambda')$ can be written as $(2\pi\tilde{n}+\varphi)$, where $\tilde{n}$ is an integer which classifies the topological homotopy class. The magnetic interaction is therefore purely topological. The nonintegrable phase factor becomes $e^{-i\mu_0(2\tilde{n}\pi+\varphi)}$. The influence of the magnetic flux in the wave function Ψ^0 can be considered by noting the relation between the associated Legendre polynomial $P_\nu^\mu(z)$ and the Jacobi function $P_n^{(\alpha,\beta)}(z)$ [11]:

$$P_l^k(\cos\theta) = (-1)^k\frac{\Gamma(l+k+1)}{\Gamma(l+1)}(\cos\theta/2\sin\theta/2)^k P_{l-k}^{(k,k)}(\cos\theta) \tag{6}$$

as well as Poisson's summation formula $\sum_{k=-\infty}^{\infty} f(k) = \int_{-\infty}^{\infty}\mathrm{d}y\sum_{n=-\infty}^{\infty} e^{2\pi nyi}f(y)$ (*e.g.*, p. 124 in ref. [10]). We find that eq. (3) turns into [11,12]

$$\sum_{q=0}^{\infty}\sum_{k=-\infty}^{\infty}\left\{E - \left[-\frac{\hbar^2}{2m}\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r}\right) + \frac{\gamma(\gamma+1)\hbar^2}{2mr^2}\right] - V(r)\right\} R_{n,\gamma}(r)\times$$

$$\times\sqrt{\frac{(2\gamma+1)}{4\pi}\frac{\Gamma(1+q)\Gamma(\gamma+|k+\mu_0|+1)}{\Gamma^2(\gamma+1)}}(\cos\theta/2\sin\theta/2)^{|k+\mu_0|}P_q^{(|k+\mu_0|,|k+\mu_0|)}(\cos\theta)e^{ik\varphi} = 0, \tag{7}$$

where we have defined $q + |k + \mu_0| = \gamma$. We see that the influence of the AB effect in the radial wave function is completely determined by replacing the integer quantum number l by the fractional one γ. The same result was discussed within the path integral approach in refs. [11–14] which also holds for relativistic systems [11, 15]. With the help of orthogonal property of the angular part [11], it is easy to find that the radial wave function with the fixed quantum number (q, k) satisfies the wave equation

$$\left\{E - \left[-\frac{\hbar^2}{2m}\left(\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r}\right) + \frac{\gamma(\gamma+1)\hbar^2}{2mr^2}\right] - V(r)\right\}R_{n,\gamma}(r) = 0. \tag{8}$$

With the help of the definition, $u_\gamma(\kappa r) = rR_{n,\gamma}(r)$ and the reduced potential $U(r) = 2mV(r)/\hbar^2$, the corresponding radial wave equation reads

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \kappa^2 - \frac{\gamma(\gamma+1)}{r^2} - U(r)\right]u_\gamma(\kappa r) = 0, \tag{9}$$

where we define $\kappa = \sqrt{2mE/\hbar^2}$. We see that the AB effect reflects itself by a coupling to the angular momentum in the radial wave equation which converts the integer quantum number to a fractional one. For our discussions of phase shifts, we consider the particle scattering by two reduced potentials $U(r)$ and $\tilde{U}(r)$. Equation (9) implies the equation

$$\frac{\mathrm{d}}{\mathrm{d}r}W(u_\gamma, \tilde{u}_\gamma) = -\big(U - \tilde{U}\big)u_\gamma\tilde{u}_\gamma, \tag{10}$$

where the Wronskian is defined as

$$W(u_\gamma, \tilde{u}_\gamma) = u_\gamma\left(\frac{\mathrm{d}}{\mathrm{d}r}\tilde{u}_\gamma\right) - \left(\frac{\mathrm{d}}{\mathrm{d}r}u_\gamma\right)\tilde{u}_\gamma \tag{11}$$

in which $\tilde{u}_\gamma$ is the solution corresponding to the potential $\tilde{U}(r)$. For short-range potentials, *i.e.* U vanishes for $r > a$, we can divide the domain of the variable r into an internal region $(r < a)$ and an external region $(r > a)$. The exterior solution may be given by $u_\gamma(r) = r[C^1_\gamma(\kappa)j_\gamma(\kappa r) + C^2_\gamma(\kappa)n_\gamma(\kappa r)]$, where $j_\gamma(z) \equiv \sqrt{\pi/2z}J_{\gamma+1/2}(z)$ is the spherical Bessel function which is regular at the origin, *i.e.* $j_\gamma(0) = 0$, and $n_\gamma \equiv \cos[(\gamma+1)\pi]\sqrt{\pi/2z}J_{-\gamma-1/2}(z)$ is the spherical von Neumann function which is irregular at the origin [16]. With the help of the asymptotic forms of spherical Bessel functions (see [16], p. 199), we obtain

$$\begin{aligned} j_\gamma(z) &\overset{z\to\infty}{\longrightarrow} \frac{1}{z}\sin(z - \gamma\pi/2), \\ n_\gamma(z) &\overset{z\to\infty}{\longrightarrow} -\frac{2\cos\gamma\pi}{z}\cos(z + \gamma\pi/2), \end{aligned} \tag{12}$$

where for n_γ the prefactor $\cos\gamma\pi$ is chosen so that it reduces to the well-known one: $-2\cos(z - l\pi/2)/z$ when the AB effect disappears. The asymptotic behavior for u_γ $(r \to \infty)$ reads

$$u_\gamma(r) \longrightarrow \frac{1}{\kappa}[\sin(\kappa r - \gamma\pi/2 + \delta_\gamma) + \sin\delta_\gamma\cos(\kappa r + 3\gamma\pi/2)]. \tag{13}$$

In this case, we have defined the phase shifts $\tan\delta_\gamma(\kappa) = -C^2_\gamma(\kappa)/C^1_\gamma(\kappa)$, and $\sqrt{(C^1_\gamma)^2 + (C^2_\gamma)^2} = 1$. We see that the AB effect leads to the appearance of an additional term in the asymptotic form of $u_\gamma(r)$ in eq. (13). The Wronskian (11) can now be calculated and is given by

$$W(u_\gamma, \tilde{u}_\gamma) = \frac{1}{\kappa}\big[\big(\tan\delta_\gamma - \tan\tilde{\delta}_\gamma\big)(\cos 2\gamma\pi + 1)\big]. \tag{14}$$

Integrating eq. (10), we finally obtain the integral representation of phase shifts for any short-range potential with a magnetic flux

$$\big(\tan\delta_\gamma - \tan\tilde{\delta}_\gamma\big) = -\frac{\kappa}{(1+\cos 2\gamma\pi)}\int_0^\infty \mathrm{d}r\tilde{u}_\gamma(r)\big[U(r)-\tilde{U}(r)\big]u_\gamma(r). \tag{15}$$

If we let $\tilde{U}(r) = 0$, then $\tilde{u}_\gamma(r) = rj_\gamma(\kappa r)$, and we obtain

$$\tan\delta_\gamma = -\frac{\kappa}{(1+\cos 2\gamma\pi)}\int_0^\infty \mathrm{d}rr^2 j_\gamma(\kappa r)R_\gamma(r)U(r). \tag{16}$$

Let us now discuss the tendency of phase shifts when the energy of the incident particle is fixed but the potential strength is changed. Let $U(r) \to U(\alpha, r), \tilde{U}(r) \to \tilde{U}(\tilde{\alpha}, r)$, and $\mathrm{d}\alpha = (\tilde{\alpha} - \alpha)$. It is easy to obtain

$$\frac{\mathrm{d}}{\mathrm{d}\alpha}\delta_\gamma(\kappa, \alpha) = -\frac{\kappa}{(1+\cos 2\gamma\pi)}\int_0^\infty \mathrm{d}r[u_\gamma(\alpha, r)]^2\frac{\partial U(\alpha, r)}{\partial\alpha}. \tag{17}$$

It stands for the appearance of more negative phase shifts when the potential strength is increased. The tendency is the same as in the flux free case when the flux $\gamma \neq n + 1/2$ [17].

There is an interesting phenomenon arising from the magnetic flux which can be observed from eq. (16). When we regulate the magnetic flux Φ at the values of $\gamma = n + 1/2$ ($n = 0, 1, 2, \ldots$), the values of the phase shift must be $(n+1/2)\pi$ ($n = 0, 1, 2, \ldots$). From the famous Levinson theory [18, 19], we may argue that then additional bound states will appear. This possible influence of the AB effect in the number of bound states is observed in the conclusion of ref. [20]. Here we have the more quantitative result in form of eq. (16). The number of bound states will increase by one when we increase a flux quantum. We expect that the quantum well system is a good candidate for checking this anticipation.

A. *The behavior of the phase shifts for large* $\gamma \neq n + 1/2$.
For a potential of finite range, it is function-known that the wave function $R_\gamma(r)$ will differ little from the corresponding free wave j_γ when $\gamma \gg \kappa a$. Therefore, we conclude from eq. (16)

$$\tan\delta_\gamma \approx -\frac{\kappa}{(1+\cos 2\gamma\pi)}\int_0^\infty \mathrm{d}rr^2[j_\gamma(\kappa r)]^2 U(r). \tag{18}$$

Using the asymptotic form

$$j_\gamma(z \longrightarrow 0) = \frac{\sqrt{\pi}}{2}\frac{(z/2)^\gamma}{\Gamma(\gamma+3/2)}, \tag{19}$$

we have

$$\tan\delta_\gamma \approx -\frac{\frac{\pi}{4}\frac{\kappa^{2\gamma+1}}{4^\gamma\Gamma^2(\gamma+3/2)}}{(1+\cos 2\gamma\pi)}\int_0^\infty \mathrm{d}rr^{2\gamma+2}U(r). \tag{20}$$

Let us consider the square-well potential

$$U(r) = \begin{cases} -C, & r < a, \\ 0, & r > a. \end{cases} \tag{21}$$

The integral can be performed and yields

$$\tan\delta_\gamma = \frac{(Ca^2)\pi(\kappa a)^{2\gamma+1}}{4^{\gamma+1}(2\gamma+2)\Gamma^2(\gamma+3/2)(1+\cos 2\gamma\pi)}. \tag{22}$$

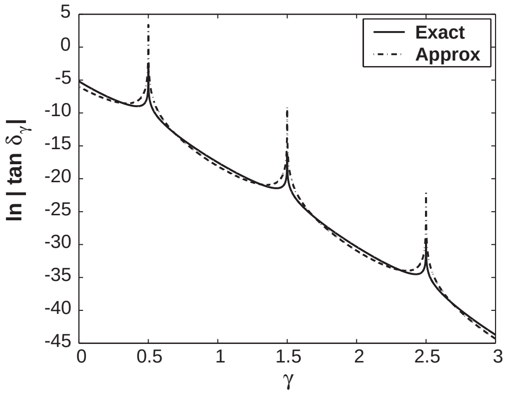

Fig. 1

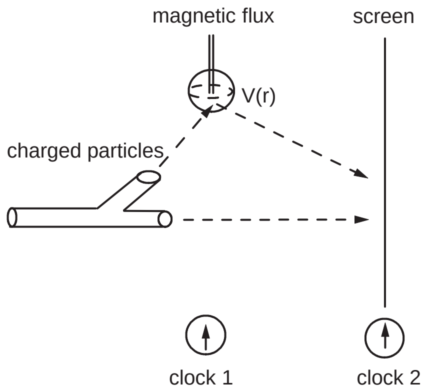

Fig. 2

Fig. 1 – The phase shifts of a square-well potential plus a magnetic flux located at the origin. The parameters $\kappa a = 0.01$, and $\alpha a = 1$ have been selected. The solid line represents the exact result obtained from eq. (28), whereas the dash-dotted line represents the approximative result obtained from eq. (22).

Fig. 2 – Two synchronized clocks are used, one located at the scattering center and another at the screen, which can test the instantaneousness of the non-local effect of phase shifts. Two cases are studied: one refers to the situation that the magnetic flux is given by $\gamma \neq n + 1/2$, and the other refers to $\gamma = n + 1/2$.

In the low-energy region, *i.e.* $\kappa a \ll 1$, the equation implies that the quantity $\tan \delta_\gamma$ falls off rapidly as γ increases. In fact, when $\gamma \gg \kappa a$, it is easy to obtain the ratio $\delta_{\gamma+1}/\delta_\gamma \approx (\kappa a)^2/(2\gamma)^2$. The phase shifts $\delta_\gamma(\kappa)$ will tend to zero (modulo π) for large γ. A similar argument for the case at high energies gives the same result, *i.e.* $\lim_{\kappa\to\infty} \delta_\gamma(\kappa) = 0$, if $\gamma \neq (n + 1/2)$, $n = 0, 1, 2, \ldots$.

B. *Scattering by a square-well plus Aharonov-Bohm effect.*
To confirm the correctness of eq. (16), let us analyze the influence of the AB effect in the square-well system. The potential reads

$$V(r) = \begin{cases} -V_0, & r < a, \\ 0, & r > a. \end{cases} \tag{23}$$

The radial wave equation which contains the AB effect is given by

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r} + \kappa^2 + \frac{2m}{\hbar^2}V_0 - \frac{\gamma(\gamma+1)}{r^2}\right]R_\gamma = 0, \quad r < a,$$
$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r} + \kappa^2 - \frac{\gamma(\gamma+1)}{r^2}\right]R_\gamma = 0, \quad r > a. \tag{24}$$

It is easy to find the solutions

$$R_\gamma(r) = C_\gamma j_\gamma(\alpha r),\ \alpha = \sqrt{\kappa^2 + \frac{2m}{\hbar^2}V_0} \quad \text{for } r < a,$$
$$R_\gamma(r) = \left[C_\gamma^1(\kappa) j_\gamma(\kappa r) + C_\gamma^2(\kappa) n_\gamma(\kappa r)\right] \quad \text{for } r > a. \tag{25}$$

The boundary conditions at $r = a$ require that the logarithmic derivative is continuous:

$$\frac{1}{R_\gamma}\frac{\mathrm{d}R_\gamma}{\mathrm{d}r}\bigg|_{r=a^-} = \frac{1}{R_\gamma}\frac{\mathrm{d}R_\gamma}{\mathrm{d}r}\bigg|_{r=a^+}. \tag{26}$$

With the help of the following identity:

$$j'_\gamma(z) = j_{\gamma-1}(z) - \frac{\gamma+1}{z}j_\gamma(z), \tag{27}$$

and the definition $\tan\delta_\gamma = -C^2_\gamma/C^1_\gamma$, eq. (26) yields

$$\tan\delta_\gamma(\kappa) = \frac{j_\gamma(\kappa a)}{n_\gamma(\kappa a)}\frac{\kappa\frac{j_{\gamma-1}(\kappa a)}{j_\gamma(\kappa a)} - \alpha\frac{j_{\gamma-1}(\alpha a)}{j_\gamma(\alpha a)}}{\kappa\frac{n_{\gamma-1}(\kappa a)}{n_\gamma(\kappa a)} - \alpha\frac{j_{\gamma-1}(\alpha a)}{j_\gamma(\alpha a)}}. \tag{28}$$

We plot the phase shifts in fig. 1, where the parameters $\kappa a = 0.01$ and $\alpha a = 1$ have been selected. The solid line represents the exact result obtained from eq. (28), whereas the dash-dotted line represents the approximate result obtained from eq. (22). We see that the approximation agrees with the exact result. The phase shifts are given by $n\pi$ except for the values of $\gamma = n + 1/2$, where the phase shifts become $(n + 1/2)\pi$. According to the Levison theory, new bound states will appear at these values which may be applied to control the number of bound states and frequencies of emission light. We expect that the phenomenon can be checked in a quantum well or a quantum dot. Another application of phase shifts influenced by the AB effect is to test the instantaneousness of the quantum non-local effect [21, 22]. As shown in fig. 2, one branch of charged particles scattered by a short-range potential plus a magnetic flux ($\gamma \neq n+1/2$) and the other one moving in the free space will interfere and show a certain interference pattern on the screen. Since the AB effect is non-local, the change of phase shifts by varying the magnetic flux from $\gamma = n$ to $\gamma = n + 1/2$ will result in an instant effect on the pattern. For example, take the distance from the scattered center to the screen to be 1 meter. During one nanosecond light will propagate 0.3 meter only, so if we change the magnetic flux from $\gamma = n$ to $\gamma = n + 1/2$ during this period, the influence of light via the usual causal manner will not touch the screen. Thus if there indeed exists fluctuation on the pattern, it should be due to the non-local influence. In fact, a single branch of charged particles scattered by the potential-flux system in fig. 2 can produce observable phase shifts due to non-local effects on the screen.

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