Optimum Design for a Thermally Stable Multifinger Power Transistor

Chih-Hao Liao, Chien-Ping Lee, *Fellow, IEEE*, N. L. Wang, and B. Lin

***Abstract*—The thermal stability of multifinger bipolar transistors has been analyzed theoretically. Coupled equations are solved to study the onset of instability and its dependence on the distributions of ballasting resistors. Analytical expressions were derived for the emitter ballasting distribution for optimum stable operation. Compared to conventional methods with uniform ballasting, the optimized design can significantly increase the stable operating current of the transistor. An absolutely stable operating condition is also derived. At this condition, the device never becomes unstable.**

***Index Terms*—Ballasting resistor, coupling current–voltage (I–V) equations, heterojunction bipolar transistor (HBT), multifinger transistor, thermal effect.**

I. Introduction

BIPOLAR transistors are useful for power amplifications. With the advances in fiber communications, wireless and satellite communications, there is a strong demand on the power transistors in microwave and millimeter frequencies. GaAs-based heterojunction bipolar transistors (HBTs), because of their high-speed performance, have become the dominant devices used in these applications [1]. The transistors, when used for power applications, often have multiple fingers to spread out the current and the dissipated heat. However, because of the heat generated and the uneven heat distribution, the transistors can become unstable at high powers seriously limiting the power handling capability of the transistors. When this happens, thermal runaway is observed for Si BJTs and current collapse is observed for GaAs-based HBTs [2]–[5].

To prevent the thermal instability of multifinger transistors, ballasting resistors are often used. The voltage drop across these resistors compensates the built-in voltage change due to temperature rise caused by self-heating and as a result the thermal stability is improved. Many papers have devoted to the thermal modeling of transistors and the design of ballasting resistors [6]–[10]. In real device implementation, each finger of the transistor is connected in series to a ballasting resistor, which can be either a part of the transistor made from epilayers or an external thin-film resistor. The fingers and the ballasting resistors connected to the fingers are usually identical to one another. However, because of the nonuniform heat dissipation, it has been realized that the uniform layout traditionally used for the fingers is not ideal for thermal stability. Instead, the use of a nonuniform distribution in the ballasting resistors [11], the emitter finger sizes [12] and the spacing [13], [14] between fingers has been proposed to improve the thermal stability. But the remaining question is: what are the optimum distributions for the ballasting resistors? A nonoptimized design can easily over correct the problem and even make the problem worse.

In this paper, we solve the basic coupled current–voltage (I–V) equations. We found that there is an ideal distribution for the ballasting resistors. Significant improvement in thermal stability can be obtained when the ideal distributions are used. Simple analytical formulas for the ideal distributions of the emitter ballasting resistors to achieve the highest stable operation current are derived. With the ideal distribution, we have also found an optimum emitter ballasting resistance for absolutely thermal stable operation condition where the device never becomes unstable. Base on the above results; a design procedure for multifinger transistors is developed.

II. Ideal Emitter Ballasting Resistance Distribution: Nonuniform Ballasting Resistors

In a multifinger transistor, the fingers are thermally coupled, which results in a nonuniform temperature distribution with the fingers near the center hotter than the fingers on the sides even before the transistors become unstable. To examine the thermal behavior of those fingers, we first use a three-finger configuration for analysis because the coupled equations can be easily solved and it can illustrate the nonuniform temperature distribution for multiple fingers. The behavior of transistors with number of fingers greater than three will be discussed later. Liu *et al.* have solved a two-finger problem [15], but it can not reflect the uneven distribution of current and temperature of multifingers due to thermal coupling.

The coupled I–V equations for the three identical fingers when self-heated are [2]

$$\begin{aligned} I_1 &= I_o \exp\left\{\frac{q}{kT_A}\left[V_a + \phi\cdot(T_1 - T_A) - R_{e13}I_1\right]\right\}\\ I_2 &= I_o \exp\left\{\frac{q}{kT_A}\left[V_a + \phi\cdot(T_2 - T_A) - R_{e2}I_2\right]\right\}\\ I_3 &= I_o \exp\left\{\frac{q}{kT_A}\left[V_a + \phi\cdot(T_3 - T_A) - R_{e13}I_3\right]\right\} \end{aligned} \tag{1}$$

where I_1 and I_3 are the current flowing through the two side fingers, and I_2 is the current of the center finger, T_1 and T_3 are the temperature of the two side fingers, T_2 is the temperature

Manuscript received August 30, 2001; revised January 14, 2002. This work was supported in part by the National Science Council of Taiwan, R.O.C., under Contract NSC 89-2218-E009-074. The review of this paper was arranged by Editor M. A. Shibib.

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Publisher Item Identifier S 0018-9383(02)04323-X.

of the center finger, V_a is the applied base-emitter voltage, ϕ is the emitter junction build-in potential change per unit temperature rise, and T_A is the ambient temperature or the heat sink temperature that is 300 K in our simulation. R_{e13} and R_{e2} are the emitter resistances of the side fingers and the center finger, respectively. We need to keep in mind that these R_e's consist of three parts: the intrinsic emitter resistance R_{e0}, the ballasting resistance R_{bb}, and the contribution from the base resistance R_b/β (β is the current gain). Here we have assumed a symmetric structure where R_e is the same for the two side fingers. Notice that the ideality factor in the I–V equations is set to unity for simplicity. The junction temperature rises for the three fingers are related to the power consumption of each finger by

$$\begin{aligned} T_1 &= T_A + V_c\left(R_t I_1 + R_{c1} I_2 + R_{c2} I_3\right) \\ T_2 &= T_A + V_c\left(R_{c1} I_1 + R_t I_2 + R_{c1} I_3\right) \\ T_3 &= T_A + V_c\left(R_{c2} I_1 + R_{c1} I_2 + R_t I_3\right). \end{aligned} \tag{2}$$

Here, R_t is the thermal resistance of each finger and R_c's are the coupling thermal resistances between these identical fingers. V_c is the collector voltage.

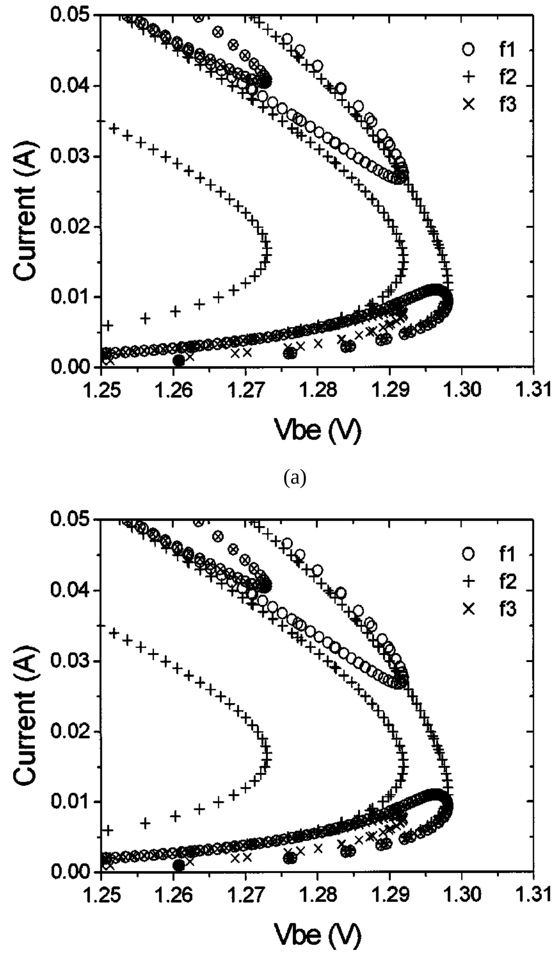
Because the current of all fingers must be the same under the stable operation condition, we can obtain a relationship between R_{e13} and R_{e2} from (1) and (2) by setting the current of all fingers identical, i.e., $I_1 = I_2 = I_3 = I$. That is

$$\begin{aligned} R_{e2,ideal} &= R_{e13} + \phi \cdot V_c\left(R_{c1} - R_{c2}\right) \\ &= R_{e13} + \Delta R_{e2,ideal}. \end{aligned} \tag{3}$$

This relationship gives the ideal emitter ballasting resistor distribution of a three-identical-finger transistor. In order to get the stable operation condition, the difference between the emitter resistances of the center finger and the side fingers should be $\Delta R_{e,ideal} = \phi \cdot V_c(R_{c1} - R_{c2})$. This is the necessary condition for achieving identical currents in all three fingers. Otherwise, the current of each finger is decided by the total effect of the positive feedback term of ΔT's and the negative feedback term of R_e's, and the currents of the fingers will never be the same. The basic idea of ideal distribution is to design a proper R_{e2} to compensate the additional thermal coupling resistance between the center finger and the side fingers. Let us first show the simulation result of a three-finger transistor with each finger having the following parameters: $I_o = 6 \times 10^{-25}$ A, $R_t = 800\,^\circ$C/W, $R_{e0} + R_b/\beta = 0.9\ \Omega$, $\phi = 1$ mV/°C. Those numbers are experimentally determined from a 3×40 InGaP/GaAs HBT by fitting the measured I–V of a single-finger transistor with (1). The current gain β is assumed to be independent of temperature, which is a valid assumption for InGaP transistors. With those numbers, we then solved the steady state heat flow equation [12], [13]

$$\nabla^2 T = 0 \tag{4}$$

with proper boundary conditions to obtain the coupling thermal resistance. The chip dimension used in simulation is $1000 \times 1000\ \mu$m^2 and the thickness is 100 μm. The finger size is $3 \times 40\ \mu$m^2 and the spacing between fingers is 40 μm. The calculated coupling thermal resistances are $R_{c1} = 67.3\,^\circ$C/W and $R_{c2} = 23.7^\circ$ C/W. And the collector voltage V_c is taken



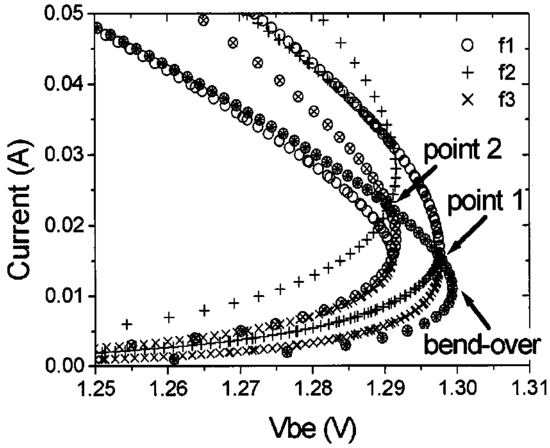
(a)

(b)

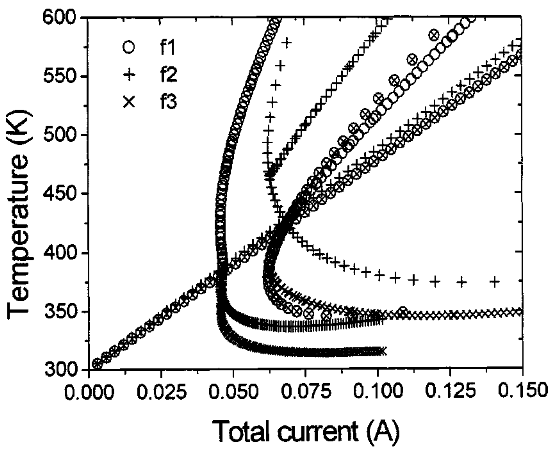
Fig. 1. Simulation results of a three-finger transistor with $R_{e13} = R_{e2} = 3\ \Omega$ (a) I–V curves and (b) the temperature as function of total current (f2 is the center finger, f1 and f3 are the side fingers).

to be 6 V in the simulation. Let us first consider a case that each finger has a total R_e of 3 Ω or a ballasting resistor of 2.1 Ω. Fig. 1(a) and (b) show the calculated I–V curves and the temperature of each finger as a function of total current. There are totally four sets of solutions for (1). Three sets of solutions are shown in the figure. The solution with $I_3 > I_1$ is not shown because the behavior is identical to the case with $I_1 > I_3$ due to the layout symmetry. The first set shows that the currents in the fingers are nearly the same before they reach the unstable point. At the unstable point, the currents in the side fingers drops while that of the center finger continue to increase. The temperatures also start to deviate from each other after the unstable point. The other sets of solutions, however, show that one or both of the side finger current is always much larger than the center finger current in all the voltage range and the curves are separated from those of the first set of solution. In other words, it is almost impossible to go into these situations if the transistor is powered up from low voltages.

Now we increase the emitter resistance of the center finger to the ideal value of $R_{e2,ideal} = 3.26\ \Omega$ calculated by (3), but keep that of the side fingers at 3 Ω. In other words, the ballasting resistor for the center finger is 2.36 Ω, while that for the side fingers is 2.1 Ω. The I–V curves and the temperature distribution

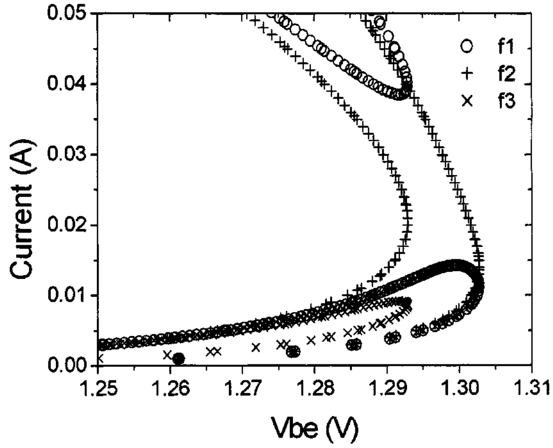


(a)

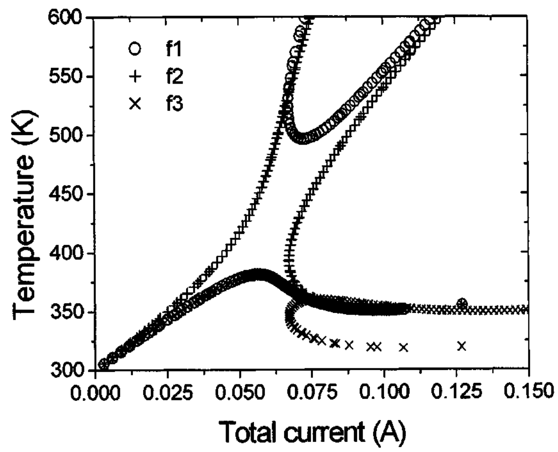


(b)

Fig. 2. Simulation results of a three-finger transistor with $R_{e13} = 3\ \Omega$, $R_{e2} = R_{e2,ideal} = 3.26\ \Omega$ (a) I–V curves ("bend-over" marks the bend-over point, "point 1" marks the first unstable point and "point 2" marks the second unstable point) and (b) the temperature as function of total current (f2 is the center finger, f1 and f3 are the side fingers).



(a)



(b)

Fig. 3. Simulation results of a three-finger transistor with $R_{e13} = R_{e2} = 3.4\ \Omega$ (a) I–V curves and (b) the temperature as function of total current (f2 is the center finger, f1 and f3 are the side fingers).

become those shown in Fig. 2(a) and (b). We can see that there is a set of solutions that has $I_1 = I_2 = I_3$ in all voltage range. However, other solutions give unstable results after the current exceeds certain value. There are two intersection points corresponding to the onsets of two different unstable modes. The first one, "point 1" in Fig. 2(a), which occurs at a lower current, is the onset point when the currents in all fingers start to differ. The second point, "point 2" in Fig. 2(a), corresponds to that when I_2 starts to differ from I_1 and I_3 but $I_1 = I_3$. Although a stable solution with $I_1 = I_2 = I_3$ exists, it is almost impossible for a transistor to pass the first intersection point and remains stable because in practical situations any small deviation from ideal will cause the device to fall into instability.

Fig. 3(a) and (b) shows the calculated I–V curves and the temperatures of the fingers of a transistor with uniform ballasting at $R_e = 3.4\ \Omega$. It is worthwhile to point out that although this value is higher than the combination of $R_{e13}/R_{e2} = 3/3.26\ \Omega$, the thermal stability, as shown in Fig. 3(a) and (b), is worse than that shown in Fig. 2(a) and (b). This is different from the traditional belief that a higher emitter resistance always provides better thermal stability. The distribution of the ballasting resistors is also important. The reason that the thermal stability is improved by a proper combination of R_{e13}/R_{e2} is very simple. The cause of thermal instability is originated from the discrepancies in current and in temperature of the fingers. If we can make them very close to one another, the thermal stability will be improved.

The current levels of the intersection points are affected by the value of R_{e13}. If we increase R_{e13} to 4 Ω but keep $R_{e2} - R_{e13} = 0.26\ \Omega$, the curves become those shown in Fig. 4. The first unstable point moves up and the second point is out of the simulation range. The device can go to a higher current before it becomes unstable. This is understandable because the increased positive feedback provided by the higher emitter resistance compensates the negative feedback caused by the temperature rise and therefore causing the transistor to become more stable. However, if we over design the ballasting resistors, the performance of the device suffers. So an optimal design of R_{e13} is very important.

III. Optimum Emitter Ballasting Resistance for Absolutely Thermal Stable Operation

From the simulation results presented earlier, obviously there exists an optimum value of R_{e13} that the unstable point happens at the infinite current level. Taking the derivative of (1) and (2)

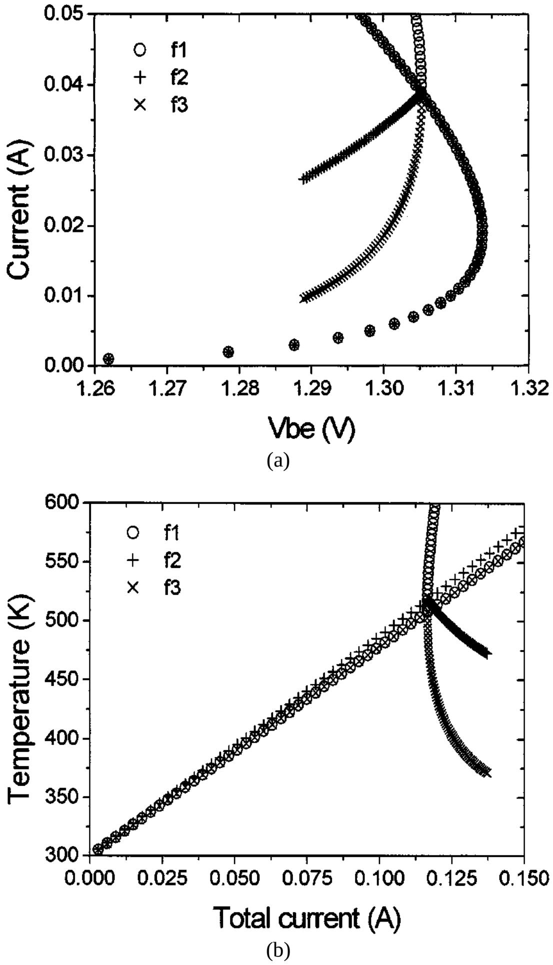


Fig. 4. Simulation results of I–V curves for a three-finger transistor with $R_{e13} = 4\ \Omega$, $R_{e2} = R_{e2,ideal} = 4.26\ \Omega$ (a) I–V curves and (b) the temperature as function of total current. (f2 is the center finger, f1 and f3 are the side fingers).

with respect to V_a, and using the ideal emitter resistor distribution shown in (3), we obtain

$$\begin{aligned} I_1' &= I\frac{q}{kT_A}\left(1+\phi\cdot T_1' - R_{e13}I_1'\right)\\ I_2' &= I\frac{q}{kT_A}\left(1+\phi\cdot T_2' - R_{e2,ideal}I_2'\right)\\ I_3' &= I\frac{q}{kT_A}\left(1+\phi\cdot T_3' - R_{e13}I_3'\right) \end{aligned} \tag{5}$$

where

$$\begin{aligned} T_1' &= V_c\left(R_t I_1' + R_{c1}I_2' + R_{c2}I_3'\right)\\ T_2' &= V_c\left(R_{c1} I_1' + R_t I_2' + R_{c1}I_3'\right)\\ T_3' &= V_c\left(R_{c2} I_1' + R_{c1} I_2' + R_t I_3'\right). \end{aligned} \tag{6}$$

We first substitute $I_1' = I_2' = I_3' = I' = \infty$ into (5) and (6) to calculate the bend-over current I_{bend}, which is the point that the I–V curve starts to bend over. This point, which is traditionally considered as the onset point for the device to become unstable, can be easily found by the I–V equations by requiring $dI/dV_a = \infty$. The current at this point is

$$I_{bend} = \frac{\frac{kT_A}{q}}{\phi\cdot V_c\left(R_t + R_{c1} + R_{c2}\right) - R_{e13}}. \tag{7}$$

And then, combining (5) and (6), we get (8), shown at the bottom of the page, where

$$\lambda = \frac{kT_A}{q}\frac{1}{I}.$$

As shown in Fig. 2(a), the unstable point can be viewed as the intercept point of two I–V curves, which are both the solutions of (1). The slopes of the I–V curves at the unstable point are not unique. We can use this requirement in (8) to solve for λ. The requirement demands are seen in (9), shown at the bottom of the page. This is an eigenvalue problem, and the solutions give conditions for the unstable points. The eigenvalues and the eigenvectors are

$$\begin{aligned} \lambda_1 &= \phi\cdot V_c\left(R_t + R_{c1} + R_{c2}\right) - R_{e13}\\ \Rightarrow I_{\lambda 1} &= \frac{\frac{kT}{q}}{\phi\cdot V_c\left(R_t + R_{c1} + R_{c2}\right) - R_{e13}}\\ I_1' &= I_2' = I_3'\\ \lambda_2 &= \phi\cdot V_c\left(R_t - R_{c2}\right) - R_{e13}\\ \Rightarrow I_{\lambda 2} &= \frac{\frac{kT}{q}}{\phi\cdot V_c\left(R_t - R_{c2}\right) - R_{e13}}\\ I_1' &= -I_3',\ I_2' = 0\\ \lambda_3 &= \phi\cdot V_c\left(R_t - R_{c1}\right) - R_{e2,ideal}\\ \Rightarrow I_{\lambda 3} &= \frac{\frac{kT}{q}}{\phi\cdot V_c\left(R_t - R_{c1}\right) - R_{e2,ideal}}\\ I_1' &= I_3',\ I_2' = -2I_1' \end{aligned} \tag{10}$$

$$\begin{bmatrix} \phi\cdot V_c R_t - R_{e13} - \lambda & \phi\cdot V_c R_{c1} & \phi\cdot V_c R_{c2}\\ \phi\cdot V_c R_{c1} & \phi\cdot V_c R_t - R_{e2,ideal} - \lambda & \phi\cdot V_c R_{c1}\\ \phi\cdot V_c R_{c2} & \phi\cdot V_c R_{c1} & \phi\cdot V_c R_t - R_{e13} - \lambda \end{bmatrix}\begin{bmatrix} I_1'\\ I_2'\\ I_3'\end{bmatrix} = \begin{bmatrix} -1\\ -1\\ -1\end{bmatrix}. \tag{8}$$

$$\begin{vmatrix} \phi\cdot V_c R_t - R_{e13} - \lambda & \phi\cdot V_c R_{c1} & \phi\cdot V_c R_{c2}\\ \phi\cdot V_c R_{c1} & \phi\cdot V_c R_t - R_{e2,ideal} - \lambda & \phi\cdot V_c R_{c1}\\ \phi\cdot V_c R_{c2} & \phi\cdot V_c R_{c1} & \phi\cdot V_c R_t - R_{e13} - \lambda \end{vmatrix} = 0 \tag{9}$$

One should note that the lowest unstable current solution is $I_{\lambda 1} = I_{bend}$ and the corresponding eigenvector is $I_1' = I_2' = I_3'$. Since the slopes are the same, the currents of all fingers will keep the same after this point. This is simply the bend-over point, not really an unstable point. The other two solutions give the real unstable points where the slopes of the I–V curves of different fingers are different and the currents of the fingers start to deviate from one another. One can find that the three solutions given earlier have the sequence $I_{\lambda 1} < I_{\lambda 2} < I_{\lambda 3}$. At the first real unstable point, which happens at $I_{\lambda 2}$, the I–V curves of the side fingers have opposite slopes and the center finger has zero slope. This means that the currents of the two side fingers start to deviate from each other at this point and the current of the center finger reaches its maximum value if the unstable operation occurs. The currents of the three fingers are all different after this point. One of the side fingers, not the center one, will become the hottest and the other will become the coldest one. At the second unstable point, which occurs at $I_{\lambda 3}$, the I–V curves of the side fingers have the same slopes, while the center finger has opposite slope to those of the side fingers. In other words, at this point, the current of the center finger starts to deviate from that of the side fingers, which have the same current. At this point, there can be two unstable modes. In one mode, the center finger becomes hot and the side fingers become cold and in the other mode, the center finger becomes cold and the side fingers become hot.

We would like to point out that the lowest unstable current, $I_{\lambda 2}$, is actually higher than the bend-over current of the I–V curves. So even the I–V curve of the transistor bends over into the negative resistance region, the device stays stable until $I_{\lambda 2}$ is reached. Before this point, the currents of all the fingers are the same and there is no solution allowed for different I_1, I_2 and I_3. Substituting those parameters used in simulation into (10), we obtain $I_{\lambda 1} = 11.0$ mA, $I_{\lambda 2} = 15.6$ mA, and $I_{\lambda 3} = 22.7$ mA corresponding to "bend-over," "point 1," and "point 2" marked in Fig. 2(a).

Examining (10), we can see that if the denominators go to zero, there are three relationships giving conditions for $I_{\lambda 1}$, $I_{\lambda 2}$ and $I_{\lambda 3}$ to go to infinity. They are

$$R_{e13,no\text{-}bend} = \phi \cdot V_c (R_t + R_{c1} + R_{c2}) \tag{11}$$

$$R_{e13,opt} = \phi \cdot V_c (R_t - R_{c2}) \tag{12}$$

$$\begin{aligned} R_{e13,opt2} &= \phi \cdot V_c (R_t - R_{c1}) - \Delta R_{e2,ideal} \\ &= \phi \cdot V_c (R_t - 2R_{c1} + R_{c2}). \end{aligned} \tag{13}$$

Equation (11) is the conventional absolutely stable condition. At this condition, the bend-over point goes to infinity, and the I–V curve never bends over. But as mentioned earlier, it is more appropriate to consider (12) as the absolutely stable condition if the ideal emitter ballasting resistor distribution of (3) is satisfied. At this condition, the currents of all the fingers are identical in the whole operation voltage and current range. In other words, the device will never become unstable even though the bend-over happens. The condition of (13) only lets the second unstable point to go to infinity and is not enough to stabilize the device.

For the transistor considered earlier, the absolutely stable condition is $R_{e13,opt} = 4.67\ \Omega$ and $R_{e2,ideal} = 4.93\ \Omega$. The no bend-over condition, however, has $R_{e2,no\text{-}bend} = 5.35\ \Omega$. Fig. 5(a) and (b) shows the I–V curve and the temperature of each finger in the absolutely stable condition. The center finger has a slightly higher temperature than the side fingers but the currents never differ. Although all the I–V curves have negative resistance as long as $R_{e13,opt}$ is smaller than $R_{e2,no\text{-}bend}$, the equations do not allow solutions with different current in the fingers. So, even the I–V curve turns over, the transistor is stable because the currents in all the fingers are identical.

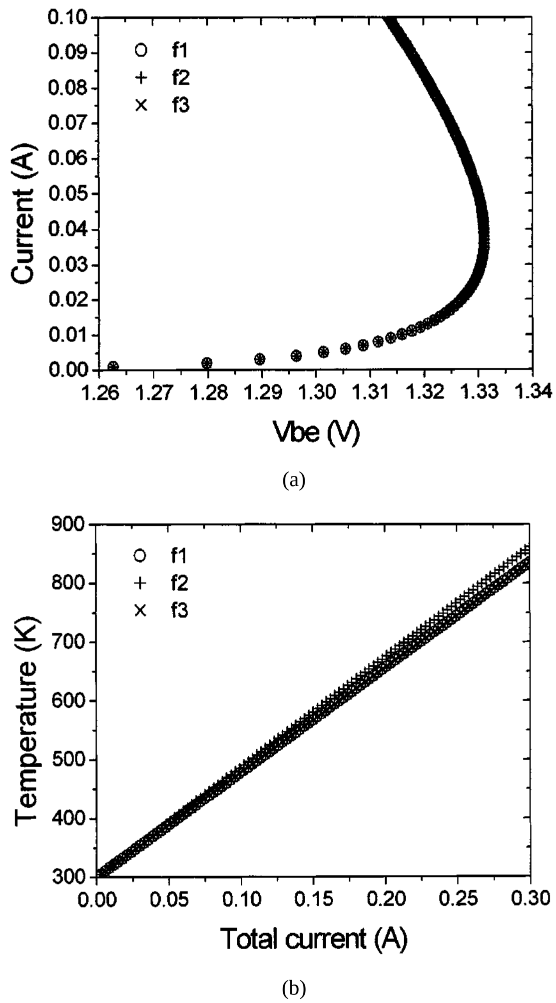


Fig. 5. Simulation results of a three-finger transistor with $R_{e13} = R_{e13,opt} = 4.67\ \Omega$, $R_{e2} = R_{e2,ideal} = 4.93\ \Omega$ (a) I–V curves and (b) the temperature as function of total current (f2 is the center finger, f1 and f3 are the side fingers).

IV. N-FINGER TRANSISTOR, THE GENERAL CASE

Similarly, we can extend the previous discussion of the three-finger case to the N-finger case. First, we label these fingers from left to right, as shown in Fig. 6. Then we can rewrite the N current variables and the N temperature rising variables of all fingers as one current vector $\mathbf{I}$ and one temperature vector $\mathbf{T}$, respectively, as

$$\mathbf{I} = (I_1 \quad I_2 \quad I_3 \quad \cdots \quad I_{N-1} \quad I_N)^T$$

$$\mathbf{T} = (T_1 \quad T_2 \quad T_3 \quad \cdots \quad T_{N-1} \quad T_N)^T$$

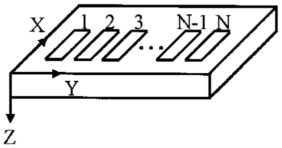


Fig. 6. Schematic diagram of an N-finger transistor chip.

The emitter resistances of the finger are written as a diagonal matrix $\mathbf{R_e}$, which is

$$\mathbf{R_e} = \mathrm{diag}\,(R_{e1} \quad R_{e2} \quad R_{e3} \quad \ldots \quad R_{e,N-1} \quad R_{eN}) = \begin{bmatrix} R_{e1} & 0 & 0 & \cdots & \cdots & 0 \\ 0 & R_{e2} & 0 & & & \vdots \\ 0 & 0 & R_{e3} & & & \vdots \\ \vdots & & & \ddots & & \vdots \\ \vdots & & & & R_{e,N-1} & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & R_{eN} \end{bmatrix}$$

where diag(v) means a diagonal matrix with vector v as its diagonal element. We can also define the thermal resistance matrix $\mathbf{R_c}$ as in (14), shown at the bottom of the page. Here, R_{cnn} is the thermal resistance of the n-th finger, and R_{cnm} is the coupling thermal resistance between the n-th and the m-th fingers. Then, the coupled I–V equations for an N-identical-finger transistor with self-heating become

$$\mathbf{I} = I_o \exp\left\{\frac{q}{kT_A}\left[V_a + \phi\cdot(\mathbf{T}-T_A) - \mathbf{R_e I}\right]\right\} \tag{15}$$

$$\mathbf{T} - T_A = V_c \mathbf{R_c I}. \tag{16}$$

Follow the same procedure as the three-finger case; the ideal emitter ballasting resistor distribution of N-fingers can be obtained as

$$\mathbf{R_{e,ideal}} = \begin{bmatrix} R_{e1} \\ R_{e2} \\ R_{e3} \\ \vdots \\ R_{e,N-1} \\ R_{eN} \end{bmatrix} = R_{e1} + \phi\cdot V_c \sum_{n=1}^{N} \begin{bmatrix} R_{c1n} \\ R_{c2n} \\ R_{c3n} \\ \vdots \\ R_{cN-1,n} \\ R_{cNn} \end{bmatrix} - \phi\cdot V_c \sum_{n=1}^{N} R_{c1n} = R_{e1} + \mathbf{\Delta R_{e,ideal}}. \tag{17}$$

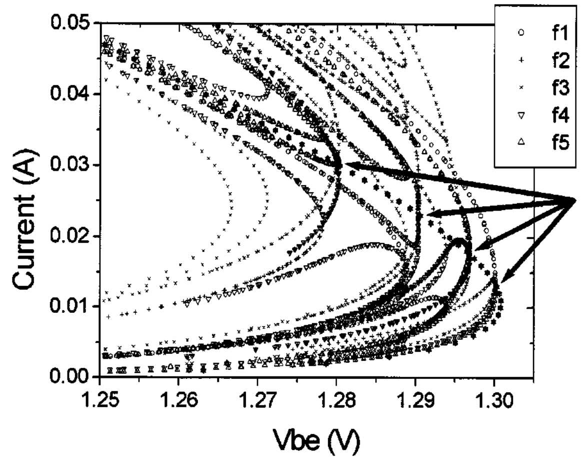


Fig. 7. Simulation results of I–V I–V curves for a five-finger transistor with $R_{e1} = 3\ \Omega$, $R_{e234} = R_{e,ideal} = 3.38\ \Omega/3.46\ \Omega/3.38\ \Omega$ (f1 is the first finger, f2 is the second finger, f3 is the third finger, f4 is the fourth finger, and f5 is the fifth finger). The arrows mark the unstable points.

Here, $\mathbf{R_{e,ideal}}$ is the ideal emitter ballasting resistor distribution vector, and $\mathbf{\Delta R_{e,ideal}}$ is the emitter resistor difference vector. Taking the derivative of (15) and (16) with respect to V_a and using the relationship of the ideal emitter resistor distribution in (17), the bend-over current can be obtained by requiring $dI/dV_a = \infty$ as

$$I_{bend} = \frac{\frac{kT_A}{q}}{\phi\cdot V_c \sum\limits_{n=1}^{N} R_{c1n} - R_{e13}} \tag{18}$$

and the eigenvalue equation becomes

$$\det\left[\phi\cdot V_c \mathbf{R_c} - diag\,(\mathbf{R_{e,ideal}}) - \lambda\cdot\mathbf{II}\right] = 0 \tag{19}$$

where λ has the same meaning as that defined in (8), $\mathbf{II}$ is the identity matrix, and $\det(A)$ is the determinant of matrix A. From (19), we can obtain N solutions for λ. One is the bend-over solution and the other $N-1$ solutions are the unstable points. Because only the real positive solutions have physical meanings, the number of the unstable points is less than or equal to $N-1$. Using a five-identical-finger transistor as an example and assuming the same chip dimension, finger size, and finger spacing as before, we can see that the I–V curves, shown in Fig. 7, have four unstable points under the ideal emitter ballasting resistor distribution, and $R_{e1} = 3\ \Omega$. This is consistent with the theoretical prediction of $5-1$ unstable points. If we use the emitter resistor difference vector $\mathbf{\Delta R_{e,ideal}}$ defined in (17), then by requiring $\lambda = 0$ as $1 \to \infty$, (19) becomes

$$\det\left[\phi\cdot V_c \mathbf{R_c} - diag\,(\mathbf{\Delta R_{e,ideal}}) - R_{e1}\mathbf{II}\right] = 0. \tag{20}$$

$$\mathbf{R_c} = \begin{bmatrix} R_{c11} & R_{c12} & R_{c13} & \ldots & R_{c1,N-1} & R_{c1N} \\ R_{c21} & R_{c22} & R_{c23} & \ldots & R_{c2,N-1} & R_{c2N} \\ R_{c31} & R_{c32} & R_{c33} & \ldots & R_{c3,N-1} & R_{c3N} \\ \vdots & & & & & \vdots \\ R_{cN-1,1} & R_{cN-1,2} & R_{cN-1,3} & \ldots & R_{cN-1,N-1} & R_{cN-1,N} \\ R_{cN1} & R_{cN2} & R_{cN3} & \ldots & R_{cN,N-1} & R_{cNN} \end{bmatrix} \tag{14}$$

This is the absolutely stable condition. The value of the optimum emitter ballasting resistance can be obtained by solving the eigenvalue equation of (20). There are N solutions of R_{e1}. The largest real positive solution corresponds to the no-bend-over condition and the second largest one, $R_{e1,opt}$ is the optimum stable condition, at which the first unstable point goes to infinity. So, we have established a design procedure to determine the ideal values of the emitter ballasting resistors for N-finger transistors. The procedure is summarized below.

1) Once the thermal resistance matrix $\mathbf{R_c}$ in (14) is determined by measurement using test structures or by a three-dimensional (3-D) simulation, the ideal emitter ballasting resistor distribution can be obtained by (17). Only when this distribution is satisfied will the currents of all fingers be exactly identical.
2) Then, by solving the eigenvalue equation of (20), the optimum value of the emitter ballasting resistance $R_{e1,opt}$ for absolutely stable condition is determined by choosing the second largest real positive solution. This value is the smallest resistor needed for the absolute stable operation. There is no need for the transistor to have a higher R_{e1}. The over design will degrade the device performance. However, if R_{e1} is insufficient, the first unstable point will not go to infinity, and unstable operation will occur.

V. Conclusion

Multiple-finger transistors with nonuniform distribution of ballasting resistors have been analyzed. Analytical formulas for the best ballasting resistor distribution for optimum thermal stability operation were derived. Comparing to the conventional method of using uniform ballasting resistors, the new schemes with optimized design can result in a significant increase in the stable device operating current. With the optimum ballasting resistor distribution, it is possible to achieve absolutely stable operation, where the device never becomes unstable.

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N. L. Wang, photograph and biography not available at the time of publication.

B. Lin, photograph and biography not available at the time of publication.