Optimum Design for a Thermally Stable Multifinger Power Transistor With Temperature-Dependent Thermal Conductivity

Chih-Hao Liao and Chien-Ping Lee, *Fellow, IEEE*

***Abstract*—The thermal stability of multifinger bipolar transistors has been analyzed theoretically. Coupled equations are solved to study the onset of instability and its dependence on the distributions of ballasting resistors. We extended our previous work on the multiple-finger transistor thermal stability from the simple coupled thermal-electrical feedback equation to the more accurate I–V equation and taking the temperature dependence of the thermal conductivity into consideration. Transistors with three-fingers and N-fingers have been analyzed. Two design procedures, uniform current design and uniform temperature design, of the best ballasting resistor distribution for optimum thermal stability operation were developed. Using these design flows, we can design the best ballasting resistor needed for thermal stable operation under the specified current level or specified junction temperature.**

***Index Terms*—Ballasting resistor, coupling current–voltage (I–V) equations, heterojunction bipolar transistor, multifinger transistor, temperature dependent thermal conductivity, thermal effect.**

I. INTRODUCTION

BIPOLAR transistors are useful for power amplifications. With the advances in fiber communications, wireless and satellite communications, there is a strong demand on the power transistors in microwave and millimeter frequencies. GaAs-based heterojunction bipolar transistors (HBTs), because of their high-speed performance, have become the dominant devices used in these applications [1]. The transistors, when used for power applications, often have multiple fingers to spread out the current and the dissipated heat. However, because of the heat generated and the uneven heat distribution, the transistors can become unstable at high powers seriously limit the power handling capability of the transistors. When this happens, thermal runaway is observed for Si BJTs and current collapse is observed for GaAs based HBTs [2]–[5]. To prevent the thermal instability of multifinger transistors, ballasting resistors are often used. The voltage drop across these resistors compensates the build-in voltage change due to temperature rise caused by self-heating and as a result the thermal stability is improved. Many papers have devoted to the thermal modeling of transistors and the design of ballasting resistors [6]–[10].

In our previous paper [11], we developed a design procedure to determine the values of the emitter ballasting resistors needed for thermal stable operation from a set of thermal-electrical feedback equations. In this work, we extend our model to a more accurate one and take into account the temperature dependence of thermal conductivity. Two practical design procedures based on uniform current and uniform temperature are described.

In the calculation presented below, we use the same device parameters as in the previous work. They are $I_o = 6 \times 10^{-25}$ A, $R_t = 800$ °C/W, $R_{e0} + R_b/\beta = 0.9\ \Omega$. Those numbers are experimentally determined from a 3 × 40 InGaP/GaAs HBT. The current gain β is assumed to be independent of temperature, which is a valid assumption for InGaP transistors. The coupling thermal resistances are obtained by solving the heat flow equation with proper boundary conditions. The chip dimension used is $1000 \times 1000\ \mu$m^2 and the thickness is 100 μm. The finger size is 3 × 40 μm^2 and the spacing between fingers is 40 μm. The calculated coupling thermal resistances are $R_{c1} = 67.3$ °C/W and $R_{c2} = 23.7$ °C/W. And the collector voltage V_c is taken to be 6 V in the simulation. For the transistor considered above, the absolutely stable condition based on the previous design procedure is $R_{e13,\,opt} = 4.67\ \Omega$ and $R_{e2,\,ideal} = 4.93\ \Omega$.

II. TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY

The effect of temperature dependent thermal conductivity will affect the thermal stability significantly. When the thermal conductivity is a function of temperature, the steady state heat flow equation becomes

$$\nabla\left[k_{th}(T)\nabla T\right] = 0. \tag{1}$$

This equation can be solved by the Kirchoff transform through defining a linearized temperature $U(x,\, y,\, z)$ as

$$U(x,\, y,\, z) = T_A + \frac{1}{k_{th0}} \int_{T_A}^{T(x,y,z)} k_{th}(T) \cdot dT. \tag{2}$$

The heat flow equation (1) and its boundary condition can then be linearized to the same form as the temperature independent case, which is

$$\nabla^2 U = 0.$$

Manuscript received August 29, 2001; revised January 14, 2002. This work was supported in part by the National Science Council of Taiwan, R.O.C., under Contract NSC89-2218-E009-074. The review of this paper was arranged by Editor M. A. Shibib.

The authors are with the Department of Electronics Engineering, National Chiao-Tung University, Hsinchu, Taiwan, R.O.C.

Publisher Item Identifier S 0018-9383(02)04324-1.

So the solutions of $U(x, y, x)$ are linear functions of currents. For the three-finger case

$$\begin{aligned} U_1 &= T_A + V_c\left(R_t I_1 + R_{c1} I_2 + R_{c2} I_3\right) \\ U_2 &= T_A + V_c\left(R_{c1} I_1 + R_t I_2 + R_{c1} I_3\right) \\ U_3 &= T_A + V_c\left(R_{c2} I_1 + R_{c1} I_2 + R_t I_3\right). \end{aligned} \tag{3}$$

Once the temperature dependence of the thermal conductivity is known, we can solve for the actual junction temperature $T(x, y, x)$ using (2) and (3). Here, we use the temperature dependence form given in ref. [12]

$$k_{th}(T) = k_{th0}\left(\frac{T}{T_A}\right)^{-b} \tag{4}$$

where k_{th0} is the thermal conductivity at T_A and b is a fitting parameter whose value is 1.22. Using the relation of (4), the actual junction temperature $T(x, y, x)$ can be obtained as

$$T = T_A\left[1 - \frac{(b-1)}{T_A}\left(U - T_A\right)\right]^{-1/(b-1)}. \tag{5}$$

From now on, we have a very nonlinear relationship between temperature and current. This relationship will seriously affect the behavior of the thermal-electrical feedback equation, as we will see.

The thermal-electrical feedback equation which our previous work is based upon is an approximation of a more accurate expression whose accuracy has been examined by Arnold *et al.* [13]. Using a three-finger transistor as an example, the more accurate coupled current–voltage (I–V) equations with self-heating are

$$\begin{aligned} I_1 &= I_{so}\exp\left[\frac{q}{kT_1}\left(V_a - \frac{E_{g0}}{q} + \frac{\beta^*}{q}T_1 - R_{e13} I_1\right)\right] \\ I_2 &= I_{so}\exp\left[\frac{q}{kT_2}\left(V_a - \frac{E_{g0}}{q} + \frac{\beta^*}{q}T_2 - R_{e2} I_2\right)\right] \\ I_3 &= I_{so}\exp\left[\frac{q}{kT_3}\left(V_a - \frac{E_{g0}}{q} + \frac{\beta^*}{q}T_3 - R_{e13} I_3\right)\right] \end{aligned} \tag{6}$$

where the indices indicate the finger number. Is and Ts are finger current and temperature. V_a is the applied base-emitter voltage. E_{g0} is the energy gap at 0 K and β^* is a coefficient that measures the amount of energy gap shrinkage as temperature increases. R_{e13} and R_{e2} are the emitter resistances of the side fingers and the center finger, respectively. And I_{so} is the junction temperature independent saturation current defined as

$$I_{so} = I_o \exp\left[\frac{(E_{g0} - \beta^* T_A)}{kT_A}\right].$$

T_A is the ambient temperature or the heat sink temperature that is 300 K in our simulation. The ideality factor is set to unity for simplicity. From the definition of the thermal-electrical feedback coefficient ϕ, we can obtain an expression as [14]

$$\phi_i = -\left.\frac{\partial V_a}{\partial T}\right|_{I_i = const.} = \frac{\beta^*}{q} - \frac{k}{q}\ln\frac{I_i}{I_{so}} \tag{7}$$

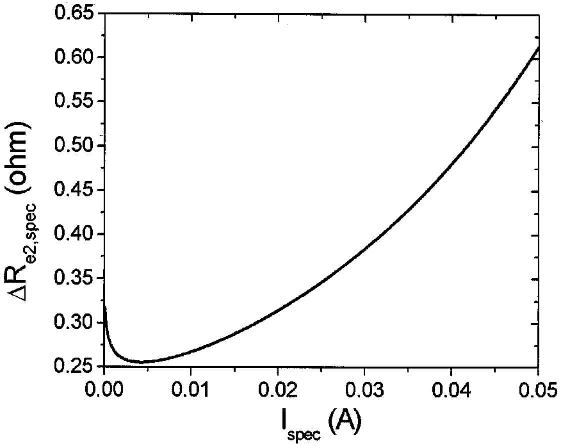


Fig. 1. Simulation result of $\Delta R_{e2,\,spec}$ is a function of I_{spec} for a three-finger transistor when the thermal conductivity is temperature dependent.

where i is the index of finger. The junction temperature for the three fingers are given by (3) and (5). By setting all currents the same, i.e., $I_1 = I_2 = I_3 = I$, and using the expression of (7), a relationship between R_{e13} and R_{e2} can be obtained from (6), (3), and (5) as

$$\begin{aligned} R_{e2,\,ideal} &= \frac{I_1}{I_2} R_{e13} + \frac{\phi_2 T_2 - \phi_1 T_1}{I_2} \\ &= R_{e13} + \frac{\phi \cdot (T_2 - T_1)}{I} \\ &= R_{e13} + \Delta R_{e2,\,ideal}. \end{aligned} \tag{8}$$

This distribution is no longer a constant but is a function of the current or the temperature of all fingers. Fig. 1 shows the simulated results of $\Delta R_{e2,\,ideal}$ versus the current per finger. As the current increases, we need a higher $\Delta R_{e2,\,ideal}$ to guarantee identical currents in all fingers. This fact is also true for the N finger case. It is interesting to point out that the linearized temperature U has an upper limit because the junction temperature T will go to infinity as U increases to a certain value. It happens when the second term in the bracket of (5) is equal to unity. And because of (3), the currents of the fingers also have a theoretical upper limit and the hottest finger, the center finger under normal operation, decides the limit. That is

$$I_{\max} = \frac{T_A}{(b-1)\cdot V_c(R_t + 2R_{c1})}. \tag{9}$$

Substituting the parameters used before, we have $I_{\max} = 243$ mA. For the N-finger case, the upper limit becomes

$$I_{\max} = \frac{T_A}{(b-1)\cdot V_c\left[\max\limits_{m\in\{1,2,\ldots,N\}}\left(\sum\limits_{n=1}^{N} R_{mn}\right)\right]} \tag{10}$$

where R_{mn} is the thermal resistance matrix element. Because of the existence of this upper limit, setting the current to infinity when deriving the eigenvalue equations in our previous work is not valid, now. Even so, the basic idea of these equations is still correct. All the change we need to do is, instead of setting the current to infinity, setting the current to a specific current level.

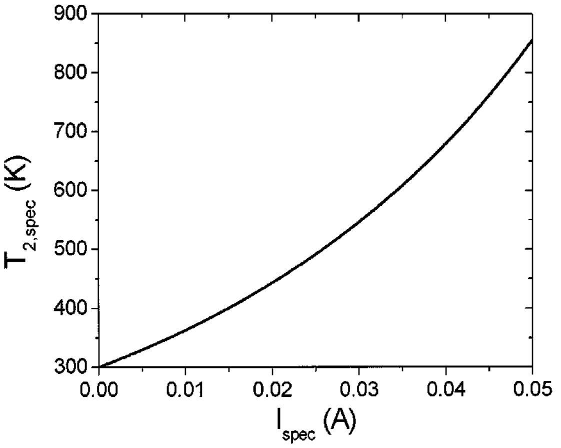


Fig. 2. Simulation result of $T_{2,\,spec}$ is a function of I_{spec} for a three-finger transistor that the thermal conductivity is temperature dependent.

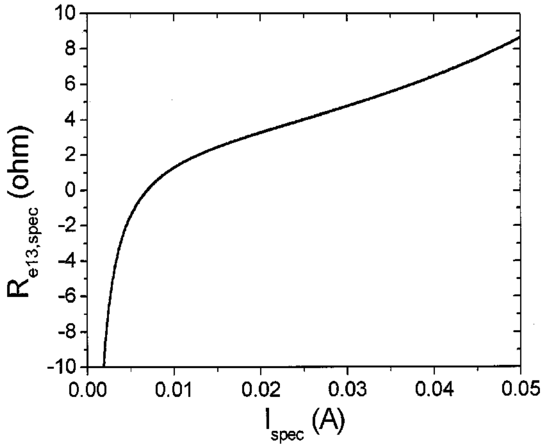


Fig. 3. Simulation result of $R_{e13,\,spec}$ is a function of I_{spec} for a three-finger transistor when the thermal conductivity is temperature dependent.

III. Uniform Current Design

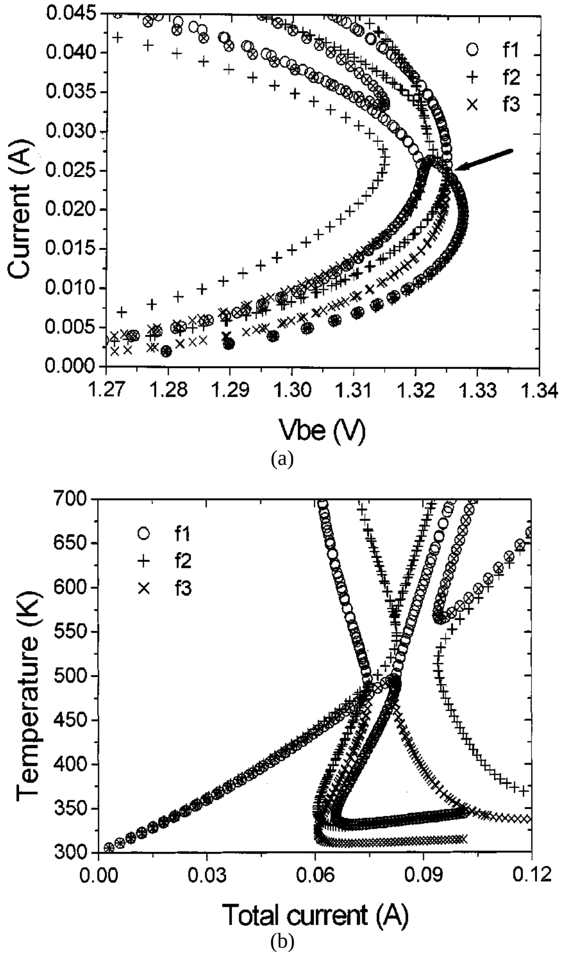
We can solve the optimum resistance needed for stable operation under the specific current level, I_{spec}. This is so-called uniform current design. Once I_{spec} is given, we can determine the junction temperature $T_{1,\,spec}$, $T_{2,\,spec}$, $T_{3,\,spec}$ of each finger from (3) and (5) and ϕ_{spec} is obtained by substituting I_{spec} into (7). Then the ideal emitter ballasting resistance distribution $R_{e2,\,spec}$ and $\Delta R_{e2,\,spec}$ can be determined from (8). But a reasonable I_{spec} is determined by the highest junction temperature that is $T_{2,\,spec}$ in the three finger case. Fig. 2 shows the relationship between $T_{2,\,spec}$ and I_{spec} in our simulation. We can see that the current level cannot exceed 26 mA if the junction temperatures need to be kept below 500 K. Because of the temperature dependence of the thermal conductivity, the derivatives of temperatures respect to base-emitter voltage become

$$\begin{aligned} T_1' &= V_c\left(R_t I_1' + R_{c1} I_2' + R_{c2} I_3'\right)\left(\frac{T_{1,\,spec}}{T_A}\right)^b \\ T_2' &= V_c\left(R_{c1} I_1' + R_t I_2' + R_{c1} I_3'\right)\left(\frac{T_{2,\,spec}}{T_A}\right)^b \\ T_3' &= V_c\left(R_{c2} I_1' + R_{c1} I_2' + R_t I_3'\right)\left(\frac{T_{3,\,spec}}{T_A}\right)^b. \end{aligned} \tag{11}$$

Combining (11) with the derivative of (6) respect to V_a and using the requirement that the solutions of the current derivatives are not unique, the eigenvalue equation becomes (12), as shown at the bottom of the next page, where

$$\begin{aligned} \theta_1 &= \left(\frac{T_{1,\,spec}}{T_A}\right)^b & \Lambda_1 &= \frac{kT_{1,\,spec}}{q}\,\frac{1}{I_{spec}} \\ \theta_2 &= \left(\frac{T_{2,\,spec}}{T_A}\right)^b & \Lambda_2 &= \frac{kT_{2,\,spec}}{q}\,\frac{1}{I_{spec}} \\ \theta_3 &= \left(\frac{T_{3,\,spec}}{T_A}\right)^b & \Lambda_3 &= \frac{kT_{3,\,spec}}{q}\,\frac{1}{I_{spec}}. \end{aligned}$$

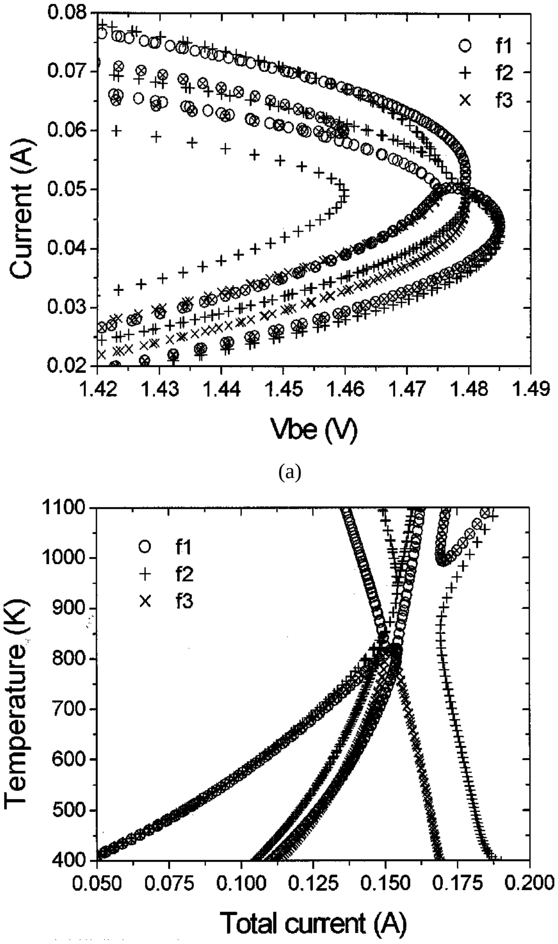
There are three solutions for R_{e13} as before. Fig. 3 shows the simulation results of the second largest solution of R_{e13}, namely, $R_{e13,\,spec}$, as a function of the current per finger if the currents of all fingers are identical for the mentioned three-finger transistor. We can find that R_{e13} is negative if I_{spec} is below 6.34 mA. That means the transistor is unconditional stable under this current level. If we set $I_{spec} = 25$ mA and use (8) and (12) to determine $R_{e2,\,spec}$ and $R_{e13,\,spec}$, the I–V and I–T curves are shown in Fig. 4(a) and (b) with $R_{e13,\,spec} = 4.43\ \Omega$ and $R_{e2,\,spec} = 4.77\ \Omega$. Before the



(a)

(b)

Fig. 4. Simulation result of a three-finger transistor with $R_{e13} = R_{e13,\,spec} = 4.43\ \Omega$, $R_{e2} = R_{e2,\,spec} = 4.77\ \Omega$ under $I_{spec} = 25$ mA. (a) I–V curves (the arrow marks the unstable point) and (b) the temperature as function of total current. (f2 is the center finger, f1 and f3 are the side fingers.) The temperature dependence of thermal conductivity is taking into account.



(a)

(b)

Fig. 5. Simulation result of a three-finger transistor with $R_{e13} = R_{e13,\,spec} = 9.19$, $R_{e2} = R_{e2,\,spec} = 9.80\ \Omega$ under $I_{spec} = 50$ mA. (a) I–V curves and (b) the temperature as function of total current. (f2 is the center finger, f1 and f3 are the side fingers.) The temperature dependence of thermal conductivity is taking into account.

unstable point is reached, the currents of all fingers are roughly equal but not exactly the same. It is mainly because of the current dependent nature of (8). So the ballasting resistor of the center finger cannot cancel the excess thermal coupling resistance between the side finger and the center finger in all current range. We can find that the unstable point, as shown in Fig. 4(a) marked by an arrow, is just at 25 mA as we specified. At this current level, the temperature is near but below 500 K that is still in a reasonable range. When the unstable point is reached, there are two unstable modes. One is that the center finger starts to dominate most of the current. The other is that one of the side fingers dominates. It seems like those two unstable points described under linear temperature model in our previous work merge together. In other words, every finger has the opportunity to become the hot finger. This is quit different from the traditional brief. Fig. 5(a) and (b) show the I–V and I–T curves for $I_{spec} = 50$ mA. Although the current can be stable before 50 mA is reached, the temperatures of all fingers exceed 800 K at this high current level. It is not reasonable to operate a device at such a high temperature because of the device operation or reliability considerations.

We list the formulas of the general N-finger case as follows. Once I_{spec} is given

$$\mathbf{T_{spec}} = T_A\left[1-\frac{(b-1)}{T_A}V_c\mathbf{R_c}\mathbf{I_{spec}}\right]^{-1/(b-1)}. \tag{13}$$

Here $\mathbf{I_{spec}}$ is the current vector that satisfies $I_1 = I_2 = \cdots = I_N = I_{spec}$, $\mathbf{T_{spec}}$ is the junction temperature vector, and $\mathbf{R_c}$ is the thermal resistance matrix. The ideal emitter ballasting resistor distribution (8) becomes

$$\begin{aligned}\mathbf{R_{e,spec}} &= R_{e1} + \frac{\phi_{spec}\cdot(\mathbf{T_{spec}} - T_1)}{I_{spec}}\\ &= R_{e1} + \Delta\mathbf{R_{e,spec}}.\end{aligned} \tag{14}$$

Here, $\mathbf{R_{e,spec}}$ is the ideal emitter ballasting resistor distribution vector and $\boldsymbol{\Delta}\mathbf{R_{e,spec}}$ is the emitter resistor difference vector under I_{spec}. And then, the eigenvalue equation (12) becomes

$$\det[\phi_{spec}V_c\,\mathrm{diag}(\boldsymbol{\theta})\mathbf{R_c} - \mathrm{diag}(\boldsymbol{\Lambda} - \Delta\mathbf{R_{e,spec}}) - R_{e1}\mathbf{II}] = 0 \tag{15}$$

where

$$\boldsymbol{\theta} = \left(\frac{\mathbf{T_{spec}}}{T_A}\right)^b \qquad \boldsymbol{\Lambda} = \frac{k\mathbf{T_{spec}}}{q}\,\frac{1}{I_{spec}}.$$

$\det(A)$ is the determinant of matrix A, $\mathrm{diag}(v)$ means a diagonal matrix with vector v as its diagonal elements, and $\mathbf{II}$ is the identity matrix. Now, we have a modified version for the design procedure of the ballasting resistors for an N-finger transistor when the temperature dependent thermal conductivity is considered. We conclude as follows.

1) The thermal resistance matrix $\mathbf{R_c}$ is determined by measurement using test structures or by a three-dimensional (3-D) simulation.
2) We specify the highest or the center finger junction temperature $T_{center,\,spec}$ from device operation or reliability considerations and solve for the corresponding specific current level I_{spec} from (13).
3) The ideal emitter ballasting resistor distribution under I_{spec} can be obtained by (14). Even when this distribution is satisfied, the currents of all fingers will not be exactly

$$\begin{vmatrix} \phi_{spec}V_cR_t\theta_1 - \Lambda_1 - R_{e13} & \phi_{spec}V_cR_{c1}\theta_1 & \phi_{spec}V_cR_{c2}\theta_1 \\ \phi_{spec}V_cR_{c1}\theta_2 & \phi_{spec}V_cR_t\theta_2 - \Lambda_2 - R_{e13} - \Delta R_{e2,\,spec} & \phi_{spec}V_cR_{c1}\theta_2 \\ \phi_{spec}V_cR_{c2}\theta_3 & \phi_{spec}V_cR_{c1}\theta_3 & \phi_{spec}V_cR_t\theta_3 - \Lambda_3 - R_{e13} \end{vmatrix} = 0. \tag{12}$$

identical because of the temperature dependence nature of the ideal distribution in (14).

4) Then, by solving the eigenvalue equation of (15), the second largest real positive solution is the optimum value of the emitter ballasting resistance $R_{e1,\,spec}$ for stable operation up until I_{spec} is reached. This value is the smallest resistor needed for the stable operation.

IV. UNIFORM TEMPERATURE DESIGN

Besides requiring uniform current distribution, we can require the temperature of all fingers identical, too. As shown in Figs. 4(b) and 5(b), the center finger will be hotter than the side fingers when approaching the unstable point under the requirement that the currents of all fingers must be the same at that point. If we increase the value of the ballasting resistor of the center finger, the current and temperature of this finger will decrease. There will be a certain value for the ballasting resistor of the center finger that will result in identical temperature of all fingers. Because the temperature difference is small, the increment of the ballasting resistor is minor. We can expect that the behavior of the curves will not change too much under the uniform temperature design. But we still need a procedure to decide the distribution of the ballasting resistors under temperature consideration. Here, as the last section, we take the temperature dependence of the thermal conductivity into account to generalize our discussion. First, we rewrite (3) as

$$\begin{aligned} I_1 &= A_{t1}\Delta U_1 + A_{c1}\Delta U_2 + A_{c2}\Delta U_3 \\ I_2 &= A_{c1}\Delta U_1 + A_{t2}\Delta U_2 + A_{c1}\Delta U_3 \\ I_3 &= A_{c2}\Delta U_1 + A_{c1}\Delta U_2 + A_{t1}\Delta U_3 \end{aligned} \tag{16}$$

where

$$\Delta U_i = U_i - T_A$$

$$\begin{bmatrix} A_{t1} \\ A_{t2} \\ A_{c1} \\ A_{c2} \end{bmatrix} = \frac{1}{V_c(R_t - R_{c2})(R_t^2 + R_t R_{c2} - 2R_{c1}^2)} \cdot \begin{bmatrix} R_t^2 - R_{c1}^2 \\ R_t^2 - R_{c2}^2 \\ R_{c1}R_{c2} - R_t R_{c1} \\ R_{c1}^2 - R_t R_{c2} \end{bmatrix}.$$

Then, setting the temperature of all fingers identical, that is, $T_1 = T_2 = T_3 = T$, which also results in $U_1 = U_2 = U_3 = U$ according to (5), the ideal emitter ballasting resistor distribution under uniform temperature condition can be obtained as

$$\begin{aligned} R_{e2,\,ideal} &= \frac{I_1}{I_2}\left(R_{e13} + \frac{(\phi_2 - \phi_1)T}{I_1}\right) \\ &= \frac{I_1}{I_2}\left(R_{e13} + \Delta R_{e2,\,ideal}\right) \\ &= \frac{A_{t1} + A_{c1} + A_{c2}}{A_{t2} + 2A_{c1}} R_{e13} \\ &\quad + \frac{(\phi_2 - \phi_1)T}{(A_{t2} + 2A_{c1})(U - T_A)}. \end{aligned} \tag{17}$$

Similar to (8), this ideal distribution is temperature or current dependent. So, we need to set a specific temperature level, T_{spec}, instead of setting a specific current level as the last section. The following argument is very similar to the last section. Once T_{spec} is given, we can get

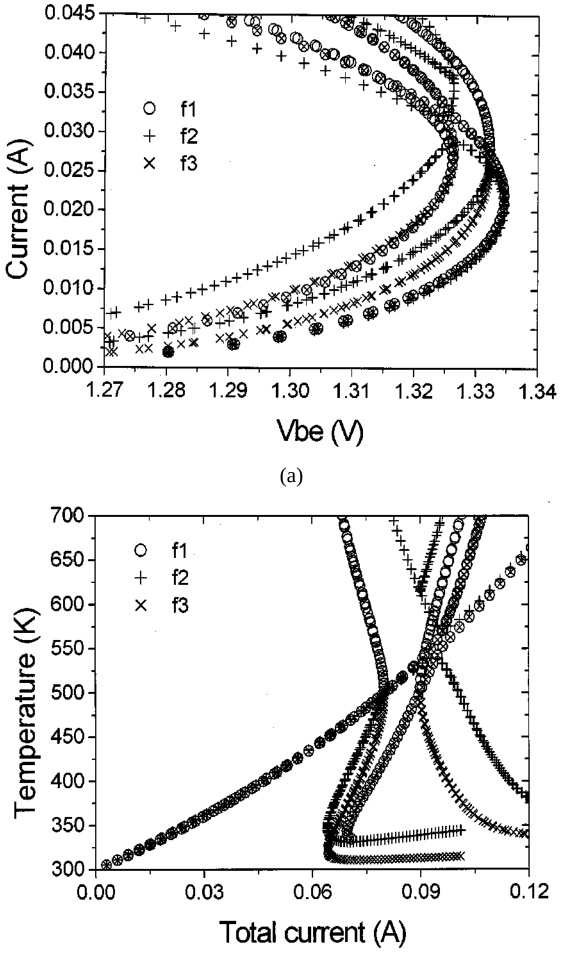
$$U_{spec} - T_A = \frac{T_A}{b-1}\left[1 - \left(\frac{T_{spec}}{T_A}\right)^{-(b-1)}\right] \tag{18}$$

and then we can obtain $I_1 = I_{1spec}$, $I_2 = I_{2spec}$, and $I_3 = I_{3spec}$ by substituting $U = U_{spec}$ into (16) and $R_{e2,\,ideal} = R_{e2,\,spec}$, $\Delta R_{e2,\,ideal} = \Delta R_{e2,\,spec}$, by substituting $U = U_{spec}$ into (17). Based on earlier results the eigenvalue (12) becomes (19), shown at the bottom of the next page, where

$$\begin{aligned} r &= \frac{I_{2,\,spec}}{I_{1,\,spec}} \quad & \phi_1 &= \frac{\beta^*}{q} - \frac{k}{q}\ln\frac{I_{1,\,spec}}{I_{so}} \quad & \Lambda_1 &= \frac{kT_{spec}}{q}\,\frac{1}{I_{1,\,spec}} \\ & & \phi_2 &= \frac{\beta^*}{q} - \frac{k}{q}\ln\frac{I_{2,\,spec}}{I_{so}} \quad & \Lambda_2 &= \frac{kT_{spec}}{q}\,\frac{1}{I_{2,\,spec}} \\ \theta &= \left(\frac{T_{spec}}{T_A}\right)^b \quad & \phi_3 &= \frac{\beta^*}{q} - \frac{k}{q}\ln\frac{I_{3,\,spec}}{I_{so}} \quad & \Lambda_3 &= \frac{kT_{spec}}{q}\,\frac{1}{I_{3,\,spec}}. \end{aligned}$$

There are also three solutions for R_{e13} and we only need the second largest real positive solution as $R_{e13,\,spec}$. If we set $T_{spec} = 500$ K, $R_{e13,\,spec} = 4.75\ \Omega$ and $R_{e2,\,spec} = 5.15\ \Omega$, the I–V and I–T curves are shown in Fig. 6(a) and (b). The total current of the unstable point is 80 mA comparing to 75 mA of $I_{spec} = 25$ mA for the identical current case. Although the total current under uniform temperature consideration is about 6.5% larger than that under uniform current consideration, the ballasting resistances are about 7.5% larger. It seems that the uniform current design is better. But because that the difference is small and the uniform temperature design is more straightforward, we think these two designs are both useful.

$$\frac{1}{r}\begin{vmatrix} \phi_1 \cdot V_c R_t\theta - \Lambda_1 - R_{e13} & \phi_1 \cdot V_c R_{c1}\theta & \phi_1 \cdot V_c R_{c2}\theta \\ r \cdot \phi_2 \cdot V_c R_{c1}\theta & r \cdot (\phi_2 \cdot V_c R_t\theta - \Lambda_2) - \Delta R_{e2,\,spec} - R_{e13} & r \cdot \phi_2 \cdot V_c R_{c1}\theta \\ \phi_3 \cdot V_c R_{c2}\theta & \phi_3 \cdot V_c R_{c1}\theta & \phi_3 \cdot V_c R_t\theta - \Lambda_3 - R_{e13} \end{vmatrix} = 0. \tag{19}$$



(a)

(b)

Fig. 6. Simulation result of a three-finger transistor with $R_{e13} = R_{e13,\,spec} = 4.75\ \Omega, R_{e2} = R_{e2,\,spec} = 5.15\ \Omega$ under $T_{spec} = 500$ K. (a) I–V curves and (b) the temperature as function of total current (f2 is the center finger, f1 and f3 are the side fingers.) The temperature dependence of thermal conductivity is taking into account.

Finally, we derive the formulas of the general N-finger case as follows. Once T_{spec} is given, U_{spec} is obtained from (18) and

$$\mathbf{I_{spec}} = \mathbf{A}(\mathbf{U_{spec}} - T_A) \tag{20}$$

where

$$\mathbf{A} = (V_c\mathbf{R_c})^{-1}$$

$\mathbf{U_{spec}}$ is the linearized temperature vector that satisfies $U_1 = U_2 = \cdots = U_N = U_{spec}$. From (7), the thermal-electrical feedback coefficient vector is

$$\boldsymbol{\varphi} = \frac{\beta^*}{q} - \frac{k}{q}\ln\left(\frac{1}{I_{so}}\mathbf{I_{spec}}\right). \tag{21}$$

Then, the ideal emitter ballasting resistor distribution (17) becomes

$$\begin{aligned}\mathbf{R_{e,spec}} &= \mathbf{r}\left[R_{e1} + \frac{T_{spec}}{I_1}(\boldsymbol{\varphi} - \phi_1)\right]\\ &= \mathbf{r}\,(R_{e1} + \Delta\mathbf{R_{e,spec}})\end{aligned} \tag{22}$$

where

$$\mathbf{r} = \frac{1}{I_{1,\,spec}}\mathbf{I_{spec}}.$$

Following the procedure as before, the eigenvalue equation (19) becomes:

$$\begin{aligned}\det[\mathrm{diag}(\mathbf{r}\cdot\boldsymbol{\varphi})(V_c\mathbf{R_c}\theta - \mathrm{diag}(\boldsymbol{\Lambda})) - \mathrm{diag}(\Delta\mathbf{R_{e,spec}})\\ -R_{e1}\mathbf{II}] = 0\end{aligned} \tag{23}$$

where

$$\boldsymbol{\Lambda} = \frac{kT_{spec}}{q}\frac{1}{\mathbf{I_{spec}}}.$$

The optimum value of the emitter ballasting resistance $R_{e1,\,spec}$ for stable operation up to T_{spec} can be solved by (23). The design procedure of the ballasting resistors for an N-finger transistor with temperature-dependent thermal conductivity under uniform temperature consideration is similar to that of last section. We list it as follows.

1) The thermal resistance matrix $\mathbf{R_c}$ is determined by measurement using test structures or by a 3-D simulation.
2) We specify the highest junction temperature T_{spec} from device operation or reliability consideration and solve the corresponding specific current level $\mathbf{I_{spec}}$ from (18) and (20).
3) The ideal emitter ballasting resistor distribution under T_{spec} can be obtained from (21) and (22).
4) Then, by solving the eigenvalue equation of (23), the second largest real positive solution is the optimum value of the emitter ballasting resistance $R_{e1,\,spec}$ for stable operation up until T_{spec} is reached. This value is the smallest resistor needed for the stable operation.

V. CONCLUSION

We extended our previous work on the multiple-finger transistor thermal stability from the simple thermal-electrical feedback equation to the more accurate I–V equation and taking the temperature dependence of the thermal conductivity into account. Transistors with three fingers and N-fingers have been analyzed. Two design flows, uniform current design, and uniform temperature design, of the best ballasting resistor distribution for optimum thermal stability operation were developed. Using these design flows, we can design the best ballasting resistor needed for thermal stability operation under the specified current level or the specified junction temperature.

REFERENCES

[1] B. Bayraktaroglu and J. A. Higgins, "HBTs for microwave power applications," in *Current Trends in Heterojunction Bipolar Transistors*, M. F. Chang, Ed. Singapore: World Scientific, 1996.
[2] R. H. Winkler, "Thermal properties of high power transistors," *IEEE Trans. Electron Devices*, vol. ED-14, pp. 260–264, May 1967.
[3] J. A. Higgins, "Thermal properties of power HBTs," *IEEE Trans. Electron Devices*, vol. 40, p. 2171, Dec. 1993.
[4] W. Liu, S. Nelson, D. Hill, and A. Khatibzadeh, "Current gain collapse in microwave multi-finger heterojunction bipolar transistors," *IEEE Trans. Electron Devices*, vol. 40, pp. 1917–1927, Nov. 1993.

[5] L. L. Liou, B. Bayraktaroglu, and C. I. Huang, "Thermal instability analysis of multiple-finger microwave AlGaAs/GaAs heterojunction bipolar transistors," in *IEEE MTT-S Microwave Symp. Tech. Dig.*, 1993, p. 281.

[6] G. B. Gao, H. Morkoc, and M. F. Chang, "Heterojunction bipolar transistor design for power applications," *IEEE Trans. Electron Devices*, vol. 39, pp. 1987–1996, Sept. 1992.

[7] W. Liu, A. Khatibzadeh, J. Sweder, and H. Chau, "The use of base ballasting to prevent the collapse of current gain in AlGaAs/GaAs heterojunction bipolar transistors," *IEEE Trans. Electron Devices*, vol. 43, pp. 245–250, Feb. 1996.

[8] C. W. Kim, H. Goto, and K. Honjo, "Thermal behavior depending on emitter finger and substrate configurations in power heterojunction bipolar transistors," *IEEE Trans. Electron Devices*, vol. 45, p. 441, Feb. 1998.

[9] L. L. Liou and B. Bayraktaroglu, "Thermal stability analysis of AlGaAs/GaAs heterojunction bipolar transistors with multiple emitter fingers," *IEEE Trans. Electron Devices*, vol. 41, p. 629, May 1994.

[10] G. B. Gao, M. S. Unlu, H. Morkoc, and D. L. Blackburn, "Emitter ballasting resistor design for the current handling capability of AlGaAs/GaAs power heterojunction bipolar transistors," *IEEE Trans. Electron. Devices*, vol. 38, pp. 185–196, Feb. 1991.

[11] C.-H. Lee, C.-P. Liao, N. L. Wang, and B. Lin, "Optimum design for a thermally stable multifinger power transistor," *IEEE Trans. Electron Devices*, vol. 49, pp. 902–908, May 2002.

[12] L. L. Liou, J. L. Ebel, and C. I. Huang, "Thermal effects on the characteristics of AlGaAs/GaAs heterojunction bipolar transistors using two-dimensional numerical simulation," *IEEE Trans. Electron Devices*, vol. 40, pp. 35–43, Jan. 1993.

[13] R. P. Arnold and D. S. Zoroglu, "A quantitative study of emitter ballasting," *IEEE Trans. Electron Devices*, vol. ED-21, pp. 385–391, July 1974.

[14] W. Liu, "Thermal coupling in 2-finger heterojunction bipolar transistors," *IEEE Trans. Electron Devices*, vol. 42, pp. 1033–1038, June 1995.

Chih-Hao Liao was born in Taoyuan, Taiwan, R.O.C., in 1974. He received the B.S. degree in physics from National Taiwan University, Taipei, in 1996, and is currently pursuing the Ph.D. degree in electronics engineering at National Chiao Tung University, Hsinchu, Taiwan.

His current research interests include modeling, characterization, and fabrication of HBTs.

Chien-Ping Lee (M'80–SM'94–F'00) received the B.S. degree in physics from National Taiwan University, Taipei, Taiwan, R.O.C., in 1971, and the Ph.D degree in applied physics from the California Institute of Technology, Pasadena, in 1978.

After graduation, he worked for Bell Laboratories and then Rockwell International until 1987. While at Rockwell, he was Department Manager in charge of developing high-speed semiconductor devices. He became a Professor at National Chiao-Tung University (NCTU), in 1987. He was also appointed Director of the Semiconductor Research Center and later the first Director of the National Nano Device Laboratory. Currently, he is the Director of the Nano Science and Technology Center, NCTU. He is well recognized in the field of semiconductor research and is an expert in compound semiconductor devices. He was the pioneer in the development of optoelectronic integrated circuits (OEIC), high electron mobility transistors (HEMTs), and ion-implanted MESFETs. His current interest includes semiconductor nano structures, quantum devices, spintronics,and heterjunction bipolar transistors. He has several patents and more than 260 publications. He has graduated 19 Ph.D students and more than 30 master degree students.

Dr. Lee is the founding Chair of the IEEE LEOS Taipei chapter and has also served in the EDS Taipei chapter. He has organized and served in several international conferences. He was awarded the Engineer of the Year award from Rockwell in 1982, the Best Teacher Award from the Ministry of Education in 1993, the Outstanding Engineering Professor Award from the Chinese Institute of Engineers in 2000, the Outstanding Research Award from the National Science Council in 1993, 1995, and 1997, the Outstanding Scholar Award from the Foundation for the Advancement of Outstanding Scholarship in 2000, and the Academic Achievement Award from the Ministry of Education in 2001.