Model-Based Channel Estimation for OFDM Signals in Rayleigh Fading

Ming-Xian Chang and Yu T. Su

***Abstract*—This paper proposes a robust pilot-assisted channel estimation method for orthogonal frequency division multiplexing (OFDM) signals in Rayleigh fading. Our estimation method is based on nonlinear regression channel models. Unlike the linear minimum mean-squared error (LMMSE) channel estimate, the method proposed does not have to know or estimate channel statistics like the channel correlation matrix and the average signal-to-noise ratio (SNR) per bit. Numerical results indicate that the performance of the proposed channel estimator is very close to the theoretical bit error propagation lower bound that is obtained by a receiver with perfect channel response information.**

***Index Terms*—Equalizers, frequency-division multiplexing, gain control.**

I. Introduction

PILOT-ASSISTED channel estimation for single- and multi-carrier systems has received considerable attention for many years [1]–[6]. Often, an estimate of the multiplicative channel response (CR) at pilot locations (i.e., those time or time–frequency positions where pilot symbols are inserted) is obtained first by either the least square (LS) or the linear minimum mean-squared error (LMMSE) method. Those estimated CRs (i.e., fading factors) at pilot locations are then used to estimate the CRs at data locations by linear or polynomial interpolation [3], [5]. Van de Beek *et al.* [1] suggested that small entries in the channel correlation matrix be eliminated to reduce the matrix dimension. They also proposed [2] the use of singular value decomposition (SVD) to reduce the LMMSE estimation complexity.

This letter proposes a new pilot-assisted channel estimation algorithm for orthogonal frequency division multiplexing (OFDM) signals. Our algorithm is based on a nonlinear two-dimensional (2-D) regression model for Rayleigh fading channels that characterize either broadcasting or mobile communication environments. Like those earlier proposals, we first obtain initial estimates of the CRs at those pilot locations by a simple LS method [1]. In the second stage, we divide the time–frequency plane into blocks of the same basic structure. Within each block, we find a 2-D surface function such that its weighted Euclidean distance to the LS-estimated CRs at the pilot locations is minimized. Then the CRs at other (data symbol) locations are estimated by using this regression surface function. The 2-D function takes into account the correlations of the fading process in time and frequency domains. We can also use a one-dimensional (1-D) function to model the channel variation on each subchannel. Our method is simpler than those of previous LMMSE proposals. Every symbol in each subchannel needs less than four complex multiplications in the equalization process, and no information about the channel correlation and noise power level is needed. Furthermore, although the LS method is more sensitive to noise than the LMMSE method, our second-stage algorithm is very effective in reducing the LS estimation error.

The rest of this letter is organized as follows. Section II provides a mathematical model of an OFDM system in Rayleigh fading and gives a brief description of earlier algorithms. Section III presents the new channel estimation algorithm. Section IV gives some numerical examples and related discussions, and Section V provides a short summary.

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II. System Model and Channel Estimate

Consider an OFDM system that uses multiple carriers $e^{i\omega_m t}$, $m = 1, 2, \ldots, N$ for parallel transmission. These modulated carriers are orthogonal over a symbol interval of T seconds. Without loss of generality, we shall assume that $\omega_m = 2\pi m/T$ and denote by X_{mn}, the data symbol of the mth subchannel in the nth time interval. The transmitted baseband waveform is obtained by: 1) taking the inverse discrete Fourier transform (IDFT) of $\{X_{mn}\}_{m=0}^{N-1}$; 2) making a parallel-to-serial transform; 3) inserting a cyclic prefix (guard interval) of T_g seconds so that an "extended" symbol period becomes $T_s = T + T_g$ seconds, and then 4) performing a digital-to-analog conversion.

In the receiving end, the received baseband waveform is matched-filtered and sampled at a rate of $1/T$. Removing those samples in guard intervals and performing DFT on $\{y_k\}$, we obtain

$$Y_{mn} = H_{mn}X_{mn} + N_{mn} \tag{1}$$

where $N_{mn} = N_{I,mn} + iN_{Q,mn}$ is a zero-mean complex Gaussian random variable with independent in-phase and quadrature phase components and identical variance $\mathrm{var}(N_I) = \mathrm{var}(N_Q) = N_0/2T \triangleq \sigma_n^2$. In the above model (1), we have assumed that the equivalent baseband channel is given by

$$h(t) = \sum_{j=1}^{L_p} h_j(t)\delta(t - \tau_j(t)) \tag{2}$$

where $h_j(t)$ and $\tau_j(t)$ remain constant during an extended symbol interval T_s, i.e., no interchannel interference exists. Hence,

$$H_{mn} = \sum_{j=1}^{L_p} h_j[n] \exp\left(i2\pi m \frac{\tau_j[n]}{T}\right) \tag{3}$$

represents the corresponding channel effect (response). Equation (1) implies that, if H_{mn} is perfectly known, the maximum-likelihood (ML) receiver would make its decision based on the statistic $\hat{X}_{mn} = Y_{mn}/H_{mn}$ for it is the ML solution to $|Y_{mn} - H_{mn}X_{mn}|^2 = 0$. When the true CR is not known, the receiver would need a CR estimate $\hat{H}_{mn}$ to make a symbol decision.

A common practice for estimating H_{mn} is to insert pilot symbols at some predetermined (pilot) locations in the time–frequency plane (see Fig. 2) where the time–frequency location of the mth channel and the nth time interval is denoted by (m, n). One obvious CR estimate at a pilot location (m, n) is the LS estimate [1]

$$\hat{H}_{mn,\mathrm{LS}} = \frac{Y_{mn}}{X_{mn}} = H_{mn} + \frac{N_{mn}}{X_{mn}} = H_{mn} + V_{mn} \quad (4)$$

where V_{mn} is the error term due to the presence of Gaussian noise and its conditional variance is $E[|V_{mn}|^2|X_{mn}] = 2\sigma_n^2/|X_{mn}|^2$. A more elaborate method that is capable of reducing the effect of V_{mn} is the LMMSE method [1], [2], [5]. Based on the estimated channel autocorrelation matrix $\tilde{\mathbf{R}}_{\mathbf{h}}$ and noise variance $\tilde{\sigma}_n^2$, this method results in the following CR estimates at the pilot locations [2]

$$\hat{\mathbf{h}}_{\mathrm{LMMSE}} = \tilde{\mathbf{R}}_{\mathbf{h}} \left[\tilde{\mathbf{R}}_{\mathbf{h}} + \tilde{\sigma}_n^2 (\mathbf{X}\mathbf{X}^H)^{-1}\right]^{-1} \hat{\mathbf{h}}_{\mathrm{LS}} \quad (5)$$

where $\mathbf{h}$, $\hat{\mathbf{h}}_{\mathrm{LS}}$, and $\hat{\mathbf{h}}_{\mathrm{LMMSE}}$ are, respectively, the true, LS-estimated, and LMMSE-estimated vector of CRs at pilot locations. $\mathbf{X}$ is a diagonal matrix whose diagonal elements are the pilot symbols and $\mathbf{X}^H$ represents its Hermitian.

After the CRs at pilot locations are obtained, either by the LS or LMMSE method, the CRs at data locations can be estimated by various interpolation methods [5]. Therefore, the CR estimate at a data location is also a linear combination of LS-estimated CRs at pilot locations

$$\hat{H} = \mathbf{a}^H \hat{\mathbf{h}}_{\mathrm{LS}} \quad (6)$$

where the weighting vector $\mathbf{a}$ is a function of both the pilot CR estimation algorithm and the interpolation method.

III. Estimation Based on Regression Model

The discrete CR H_{mn} can be viewed as a sampled version of the 2-D continuous complex baseband fading process $H(f, t)$; two typical examples of local H_{mn}, obtained by computer simulation, are shown in Fig. 1. We first select an operating block in the time–frequency plane in which $N_0 \times M_0$ pilot symbols are uniformly inserted at every r_f subchannel and every r_t symbol; see Fig. 2. Then the receiver models the true sampled fading process H_{mn} in this region by a quadrature surface

$$\begin{aligned} F(m,n) =& a\,m^2 + bmn + cn^2 + dm + en + f \\ =& H_{mn} + g(m,n) \end{aligned} \quad (7)$$

where $g(m,n)$ represents the modeling error. For Rician or Rayleigh fading channels, H_{mn} is a complex Gaussian process, hence $g(m,n)$ is also complex Gaussian-distributed. The frequency-domain model of the received samples (1) implies that the ML estimates of the coefficients $(a,b,c,d,e,f) \triangleq \mathbf{c}^H$ are chosen such that

$$\sum_{(m,n)\in\mathcal{P}} |Y_{mn} - \hat{F}(m,n)X_{mn}|^2 = \sum_{(m,n)\in\mathcal{P}} |X_{mn}|^2 |\hat{H}_{mn,\mathrm{LS}} - \hat{F}(m,n)|^2 \quad (8)$$

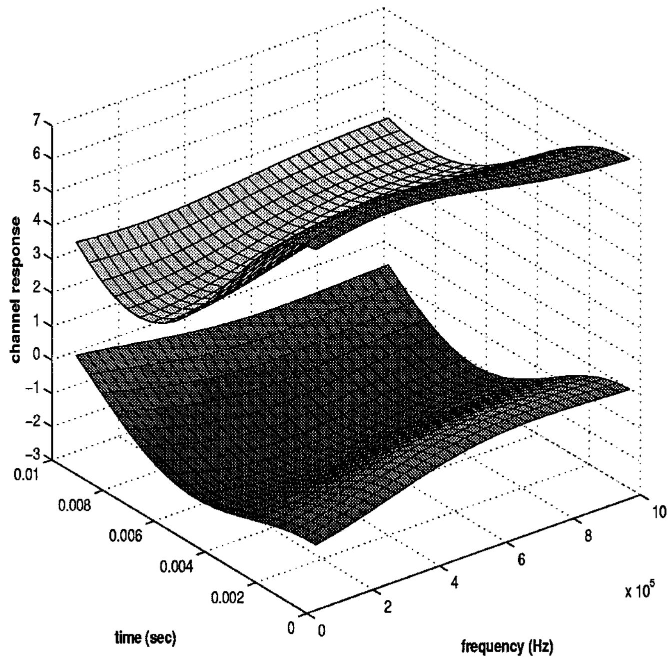

Fig. 1. Two typical OFDM channel responses. They are plotted in the same figure for the convenience of comparison. The vertical coordinate does not represent the absolute magnitude of each CR surface.

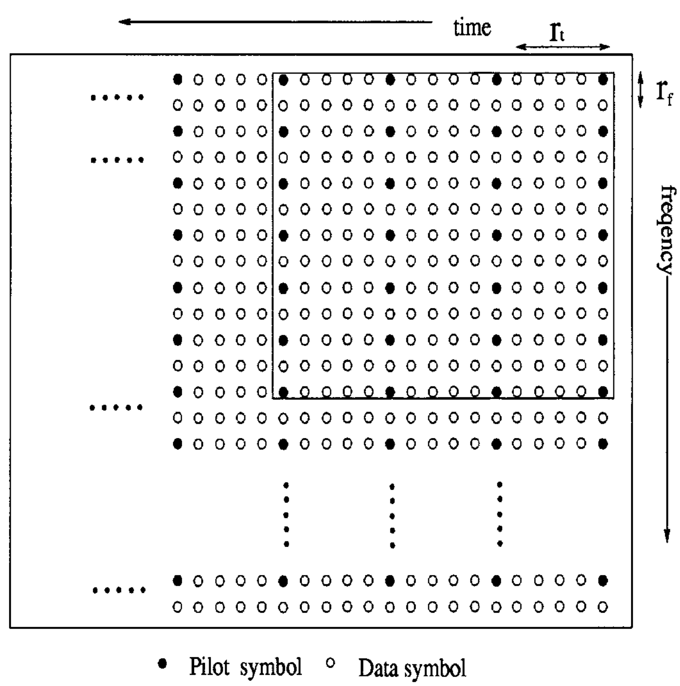

Fig. 2. A typical pilot symbol distribution in the time–frequency plane of an OFDM signal.

is minimized, where $\hat{F}(m,n)$ is the estimator of $F(m,n)$ in noise. The set $\mathcal{P}$ in (8) contains the pilot locations in the operating block (region)

$$\mathcal{P} = \left\{ (m,n) \middle| \begin{array}{l} m = 0, r_f, 2r_f, \ldots, (M_0-1)r_f \\ n = 0, r_t, 2r_t, \ldots, (N_0-1)r_t \end{array} \right\}. \quad (9)$$

Rewriting (7) as $F(m,n) = \mathbf{c}^H \mathbf{q}$, where $\mathbf{q}_{mn}^T \triangleq (m^2, mn, n^2, m, n, 1)$, we restate the problem of finding the ML solution of (8) as solving

$$\min_{\hat{\mathbf{c}}} \sum_{(m,n)\in\mathcal{P}} \left| Y_{mn} - \hat{\mathbf{c}}^H \mathbf{q}_{mn} X_{mn} \right|^2. \quad (10)$$

A. Algorithm Description

Taking the derivative of (10) with respect to $\hat{\mathbf{c}}$ and invoking the definitions

$$\mathbf{Q} \triangleq \sum_{(m,n)\in\mathcal{P}} \mathbf{q}_{mn}\mathbf{q}_{mn}^T |X_{mn}|^2, \quad \mathbf{P} \triangleq \mathbf{Q}^{-1} \quad (11)$$

$$\begin{aligned} \hat{\mathbf{b}} &\triangleq \sum_{(m,n)\in\mathcal{P}} \mathbf{q}_{mn} X_{mn} Y_{mn}^* \\ &= \sum_{(m,n)\in\mathcal{P}} \mathbf{q}_{mn} |X_{mn}|^2 \hat{H}_{mn,\mathrm{LS}}^* \end{aligned} \quad (12)$$

where X_{mn}^* is the complex conjugate of X_{mn}, we obtain the solution

$$\hat{\mathbf{c}} = \mathbf{P}\hat{\mathbf{b}} = \sum_{(m,n)\in\mathcal{P}} (\mathbf{P}\mathbf{q}_{mn}|X_{mn}|^2) \hat{H}_{mn,\mathrm{LS}}^*. \quad (13)$$

Our final CR estimate, $\hat{H}_{mn}$, for the position (m,n) is

$$\begin{aligned} \hat{H}_{mn} &= \hat{F}(m,n) = \mathbf{q}_{mn}^T \hat{\mathbf{c}}^* \\ &= \mathbf{q}_{mn}^T \sum_{(k,l)\in\mathcal{P}} (\mathbf{P}\mathbf{q}_{kl}|X_{kl}|^2) \hat{H}_{kl,\mathrm{LS}}. \end{aligned} \quad (14)$$

The above algorithm can be modified to estimate the CRs of either a single-carrier system or a subchannel of an OFDM system. This 1-D scheme models the fading process by a single-variable regression function, e.g., $F(n) = an^2 + bn + c$. The corresponding parameters are given by $\mathbf{c}^H \triangleq (a,b,c)$, $\mathbf{q}_n^T \triangleq (n^2, n, 1)$ and $\mathcal{P} = \{n | n = 0, r_t, \ldots, (N_0-1)r_t\}$, respectively.

B. Complexity Analysis

Since all time–frequency blocks have the same size and pilot symbol distribution, if all the pilots are the same, the matrix $\mathbf{P} = \mathbf{Q}^{-1}$ is fixed. Moreover, $\mathbf{q}_{kl}$ and $\mathbf{d}_{kl} \triangleq \mathbf{P}\mathbf{q}_{kl}|X_{kl}|^2$ are real constant vectors. Computing the coefficient vector, $\hat{\mathbf{c}} = \sum_{(k,l)\in\mathcal{P}} \mathbf{d}_{kl} \hat{H}_{mn,\mathrm{LS}}^*$, requires $2 \times 6 \times M_0 N_0$ real multiplications and each $\hat{H}_{mn} = \mathbf{q}_{mn}^T \hat{\mathbf{c}}^*$ requires 2×6 real multiplications. Hence, on the average, the number of real multiplications for each $\hat{H}_{mn}$ is

$$\frac{12M_0N_0 + 12(M_bN_b - M_0N_0)}{M_bN_b - M_0N_0} = 12 + \frac{12M_0N_0}{M_bN_b - M_0N_0} < 16 \quad (15)$$

if the pilot density $M_0N_0/(M_bN_b) < 0.25$, M_bN_b being the total number of (data plus pilot) symbols in a given block. In other words, less than four complex multiplications are needed for each estimate $\hat{H}_{mn}$.

Note that enlarging the operating block size or the pilot density increases only the summation terms in (13) [or (14)]; the average complexity (15) is not affected. For the LMMSE solution, however, this means increasing the dimension of the correlation matrix in (5) and the complexity in matrix inverse operation. Therefore, the operating region of the LMMSE algorithm is usually 1-D.

C. Estimation Error Analysis

A linear pilot-assisted channel estimate, including ours, has the form of (6), which can be expressed as

$$\begin{aligned} \hat{H} &= \mathbf{a}^H \hat{h}_{\mathrm{LS}} = \mathbf{a}^H \mathbf{h} + \mathbf{a}^H \mathbf{v} = H' + W \\ &= H + G + W \end{aligned} \quad (16)$$

where $\mathbf{v}$ is the vector of noise terms in (4), $H' \triangleq \mathbf{a}^H \mathbf{h}$, $G \triangleq H - H' = H - \mathbf{a}^H \mathbf{h}$, and $W = \mathbf{a}^H \mathbf{v}$. G is the error resulting from the interpolation process while W is due to the presence of channel thermal noise.

For the proposed method, the variance of W at (m,n), W_{mn}, is given by

$$\begin{aligned} E\left[|W_{mn}|^2\right] &= \sum_{(k,l)\in P} \left| \mathbf{q}_{mn}^T \mathbf{P} \mathbf{q}_{kl} X_{kl}^* \right|^2 (2\sigma_n^2) \\ &= \mathbf{q}_{mn}^T \mathbf{P} \left(\sum_{(k,l)\in\mathcal{P}} \mathbf{q}_{kl}\mathbf{q}_{kl}^T |X_{kl}|^2 \right) \mathbf{P}\mathbf{q}_{mn} (2\sigma_n^2) \\ &= \mathbf{q}_{mn}^T \mathbf{P}\mathbf{Q}\mathbf{P}\mathbf{q}_{mn} (2\sigma_n^2) \\ &= \mathbf{q}_{mn}^T \mathbf{P}\mathbf{q}_{mn} (2\sigma_n^2). \end{aligned} \quad (17)$$

Assuming all pilot symbols are the same, i.e., $X_{mn} = X$, for all $(m,n) \in \mathcal{P}$, we obtain the average (over M_0N_0 positions) variance of W_{mn}

$$\begin{aligned} 2\sigma_w^2 &\triangleq \frac{1}{M_0N_0|X|^2} \sum_{(m,n)\in\mathcal{P}} |X_{mn}|^2 \mathrm{var}(W_{mn}) \\ &= \frac{2\sigma_n^2}{M_0N_0|X|^2} \sum_{(m,n)\in\mathcal{P}} \mathbf{q}_{mn}^T \mathbf{P}\mathbf{q}_{mn} |X_{mn}|^2 \\ &= \frac{2\sigma_n^2}{M_0N_0|X|^2} \sum_{(m,n)\in\mathcal{P}} \mathrm{tr}\left(|X_{mn}|^2 \mathbf{q}_{mn}\mathbf{q}_{mn}^T \mathbf{P}\right) \\ &= \frac{2\sigma_n^2}{M_0N_0|X|^2} \mathrm{tr}(\mathbf{Q}\mathbf{P}) = \frac{6}{M_0N_0} \frac{2\sigma_n^2}{|X|^2} \end{aligned} \quad (18)$$

where $\mathrm{tr}(\mathbf{A})$ denotes the trace of the matrix $\mathbf{A}$. On the other hand, the interpolation error G, which is independent of W, is due to the modeling error $g(m,n)$ in (7). Equation (18) does imply that, for fixed pilot density (r_t, r_f), σ_w^2 can be reduced by

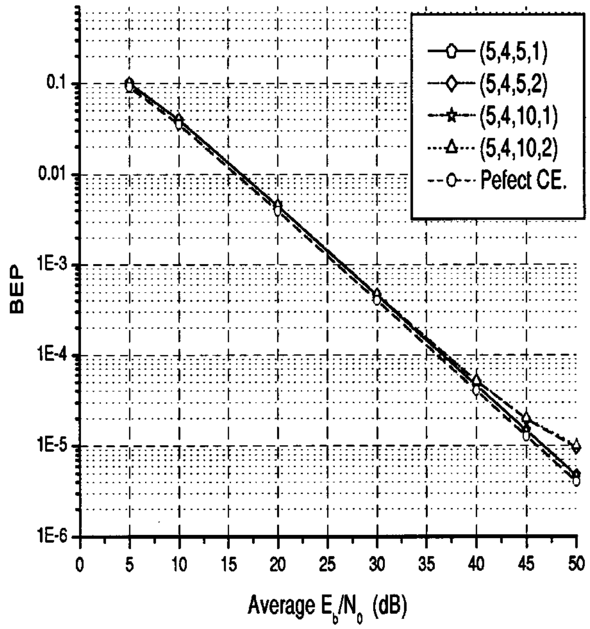

Fig. 3. BEP performance of the OFDM-16QAM system with the 2-D regression channel estimator; $f_dT = 6.9 \times 10^{-4}$.

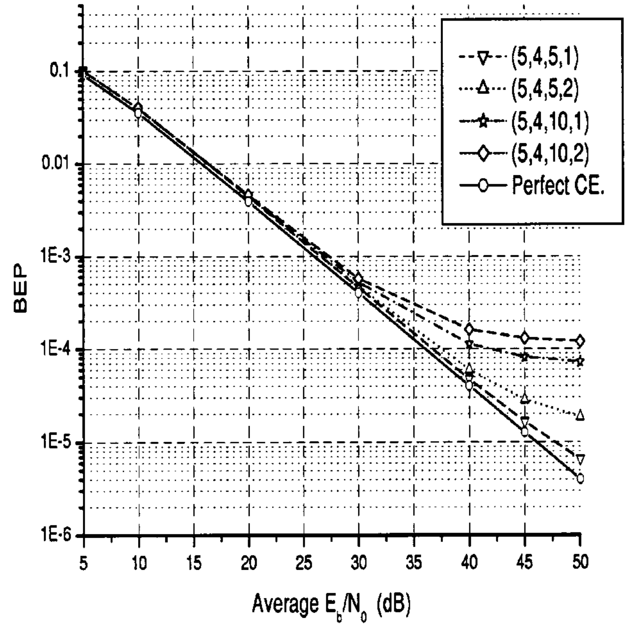

Fig. 4. BEP performance of the OFDM-16QAM system with the 2-D regression channel estimator; $f_dT = 4.16 \times 10^{-3}$.

increasing M_0N_0; however, so doing also leads to a larger operating block and hence larger $g(m,n)$. The situation becomes worse if Doppler frequency is large. On the other hand, we can cut back the modeling error by using a smaller operating block or a higher pilot density. Numerical examples supporting such conclusions are given in Section IV.

IV. Numerical Results and Discussion

Numerical examples obtained from computer simulations are provided in this section to demonstrate the effectiveness of the proposed CR estimate. We consider the multipath time-variant Rayleigh-fading channel based on (3), using $L_p = 4$, $\tau_j[n] = \tau_j$, and $h_j[n] = c_ju_j[n]$. All u_j's are independent stationary complex zero-mean Gaussian processes with unit variance while $c_j = 0.5938$, 0.7305, 0.3175, 0.1137 and $\tau_j =$ 0, 0.1, 0.5, and 1 μs, respectively.

Figs. 3 and 4 show the bit-error probability (BEP) performance of an OFDM-16QAM system whose 1-MHz bandwidth is divided into 32 subchannels. The effects of the Doppler shift and the 2-D pilot distribution (N_0, M_0, r_t, r_f) are examined. As expected, with all other system parameters fixed, the error floor is an increasing function of the normalized Doppler frequency (f_dT). Let $\overline{\gamma}_b$ denote the average SNR per bit. At lower $\overline{\gamma}_b$'s $(<30 \text{ dB})$ when the BEP performance is dictated by the noise error W [see (16)], the performance of our method is less sensitive to the pilot symbol density or number and is very close to that (curves labeled by perfect CE) of the theoretical lower bound obtained by assuming that CR is perfectly known. When $\overline{\gamma}_b$ is high, however, the modeling error G dominates the system performance, hence the performance degrades for those estimates using a larger operating block or low-density pilots.

For comparison, we consider an estimator that uses the LMMSE method and then polynomial interpolation to obtain the data CRs in each subchannel. The performance of

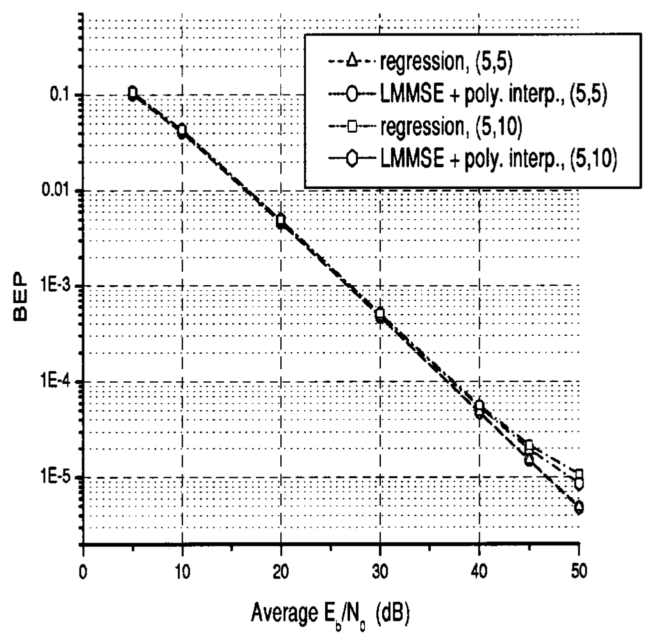

Fig. 5. BEP performance comparison for the LMMSE + polynomial-interpolation and 1-D regression estimators; $f_dT = 2.76 \times 10^{-3}$.

this estimator is plotted in Figs. 5 and 6, assuming perfect knowledge about the channel correlation matrix and SNR. The performance of our algorithm (the 1-D regression estimator in this case) that has the same operating block size and pilot distribution (N_0, r_t) is also shown in the same figure. It can be seen that both estimators yield very close BEP performance. The modeling error is an increasing function of the inter-pilot distance r_t and becomes the limiting factor at high $\overline{\gamma}_b$'s. The issue of the worst-case pilot distribution is discussed in [7]. We show the worst-case behavior of both 1-D and 2-D estimators

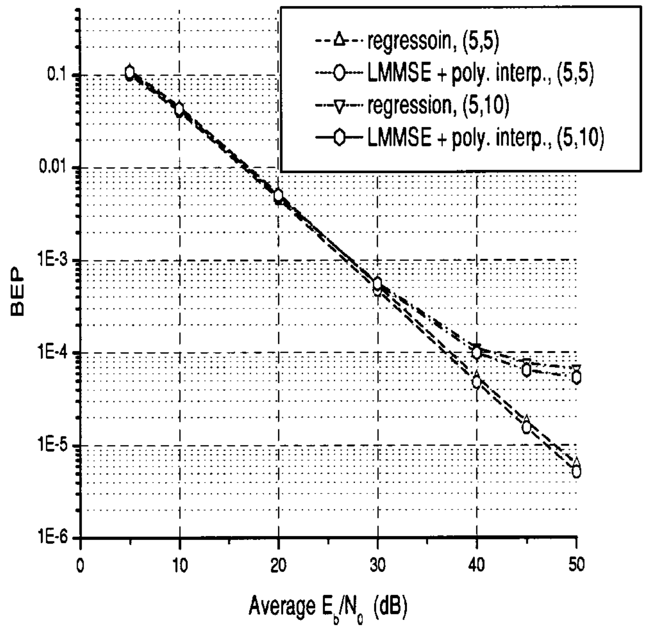

Fig. 6. BEP performance comparison for the LMMSE + polynomial-interpolation and 1-D regression estimators; $f_d T = 4.14 \times 10^{-3}$.

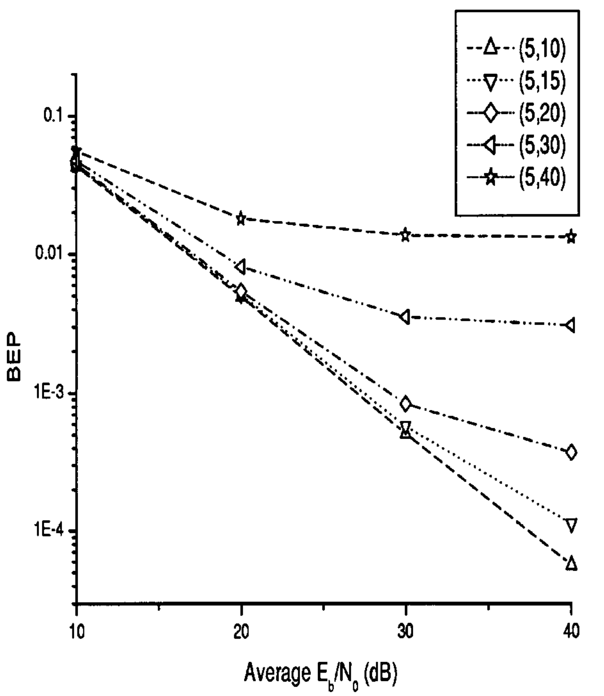

Fig. 7. BEP performance of the OFDM-16QAM system with the 1-D regression channel estimator; $f_d = 80$ Hz.

in Figs. 7 and 8. For the 2-D case, the system uses the same channel parameters as [7], i.e., 1024 subchannels and 5-MHz total bandwidth. As expected, the performance is improved as the pilot density increases. We also note that to meet a performance requirement a minimum pilot density must be in place.

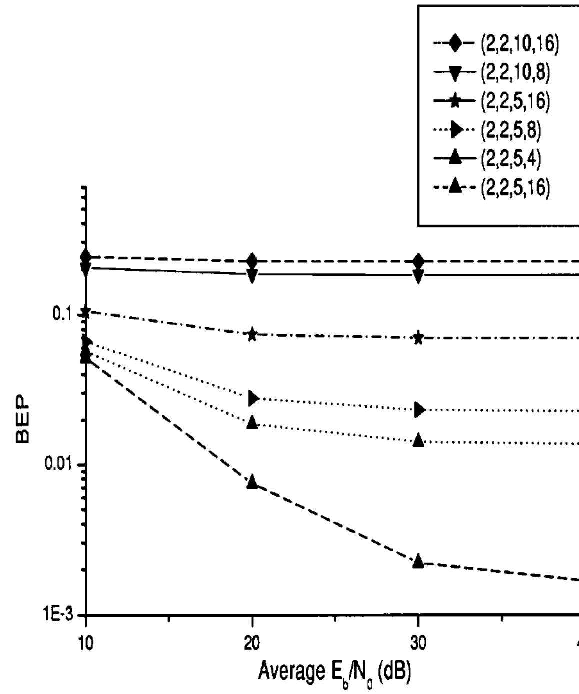

Fig. 8. BEP performance of the OFDM-16QAM system with the 2-D regression channel estimator; $f_d = 240$ Hz except for the lowest curve which assumes $f_d = 80$ Hz.

V. Conclusion

We have proposed a new robust OFDM channel estimation scheme with excellent BEP performance. The proposed scheme is based on a nonlinear regression on the local time–frequency domain; it is of low complexity and does not need channel statistics or matrix inversion. Furthermore, with a proper pilot density, it achieves performance that is not far from the theoretical lower bound, and, within a wide range of $\overline{\gamma}_b$, the performance of its 1-D version is very close to that of an LMMSE-based estimate.

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