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Magnetic phase diagram of type-II superconductors: From high T_c to low T_c superconductors

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ABSTRACT

Phase diagram of both high temperature superconductors and low temperature superconductors are studied within the Ginzburg–Landau approach. Due to enhanced thermal fluctuation strength, disorder effects are relatively small in high T_c superconductors and consequently can be studied analytically. The vortex glass transition line is different and well separated from both the melting line and from the so-called second peak lines. On contrary, in low temperature superconductors, the disorder effect is dominant, as the thermal fluctuation strength is very small. Peak effect appears due to cross over of the collective pinning region to stronger pinning region. The location of the peak effect is obtained. The disorder and thermal fluctuation effects on structure phase transition are also studied.

1. Introduction

Theoretical study of the phase diagram of type-II superconductors remains one of the major challenges in condensed matter physics, not only due to its importance to the application of superconductivity, but also its importance for understanding of phase transition. Vortex systems offer a unique testing ground for experimental verification of various theoretical concepts like that of the glass phase, overcooled liquid and melting. Based on the Ginzburg–Landau phenomenological theory, we use nonperturbative analytic methods to obtain various results which can be (even quantitatively) tested experimentally. The article is organized as follows: in Section 2 we consider the Ginzburg–Landau theory without disorder in Section 3 we include weak disorder effects in the model, while in Section 4 the phase diagram of the strong disordered system is studied. In Section 5, the structure transition is investigated.

2. Vortex phase diagram of a clean superconductor

The model without disorder is defined by free energy:

$$\int d^3x \frac{\hbar^2}{2m_{ab}}\left|\left(\nabla - \frac{2\pi i}{\Phi_0}A\right)\psi\right|^2 + \frac{\hbar^2}{2m_c}|\partial_z\psi|^2 - \alpha T_c(1-t)|\psi|^2 + \frac{\beta}{2}|\psi|^4, \quad (1)$$

where $\Phi_0 = hc/(2e)$, $t = T/T_c$, $A = (By, 0, 0)$. The model provides a good description of thermal fluctuations as long as $1 - t - b << 1$, where $b = H/H_{c2}$. Its thermodynamical properties turn however to be highly nontrivial, even without disorder and within the lowest Landau level (LLL) approximation, in which only the LLL mode is retained and the free energy simplifies (after rescaling):

$$g_{LLL} = \int d^3x \left[\frac{1}{2}|\partial_z\psi|^2 + a_T|\psi|^2 + \frac{1}{2}|\psi|^4\right]. \quad (2)$$

The simplified model has just one parameter – the (dimensionless) scaled temperature:

$$a_T \equiv (t + b - 1)\sqrt[3]{2/(\pi^2 b^2 t^2 \mathrm{Gi})}$$

with the Ginzburg number defined as $\mathrm{Gi} \equiv 32(\pi\lambda^2 T_c\gamma/(\Phi_0^2\xi))^2$, $\gamma^2 = m_c/m_{ab}$ the anisotropy parameter, λ the magnetic penetration

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depth and ξ the coherence length. The (effective) LLL model is applicable in a surprisingly wide range of fields and temperatures determined by the condition that the relevant excitation energy ε is much smaller than the gap between Landau levels [1]. For a very weakly disordered system like a pure single crystal sample, or even in some disordered system near phase transition temperature, the disorder effect is small, so the result of this model (not including disorder effects) can be also tested experimentally.

The solid energy can be calculated up to two loops order in perturbation theory or by Gaussian variational method, [2] while the liquid free energy can be obtained by Borel–Pade method [3] to achieve a precision of better than 1%. Comparing the liquid and solid free energy it reveals that the melting transition occurs

$$a_T^m = -9.5. \tag{3}$$

The experimental verification of this equation can be found in various experiments in YBCO type and even in low temperature type-II superconductors like Nb_3Sn [4]. Gaussian variational calculation also showed that a spinodal line for the solid, the end point of superheated solid, is given by

$$a_T^{sp} = -5. \tag{4}$$

The spinodal line was recently observed and the theoretical prediction of the spinodal line was confirmed in various experiments [5]. The line $a_T^s = -5$ therefore separates the vortex liquid region of the phase diagram into two regions: $-9.5 < a_T < -5$ is a normal vortex liquid region, and $-5 < a_T < 0$ is vortex gas region in which flux line can move freely like particles in a gas.

3. Weak disorder effect

In high temperature superconductors, disorder are relatively weak compared to the thermal fluctuation, and its effect can be taken into account analytically. The disorder can lower the melting transition line in H–T on the vortex phase diagram as the vortex liquid state can adjust to the disorder better than the vortex lattice state [1,6–8]. On the other hand, the lower temperature part of the phase diagram becomes the glass phase in which ergodicity is broken.

We begin with a simpler two dimensional case. Quenched disorder is accounted for by random components of coefficients: $m^{*-1} \to m^{*-1}[1 + w_1(x)], \alpha \to \alpha[1 + w_2(x)], \beta \to \beta[1 + w_3(x)]$ with variances p_1, p_2, and p_3, respectively. Using the replica trick $\overline{G} = -T\lim_{n\to 0}\frac{\overline{Z^n}-1}{n}$ where $\overline{A}$ is the disorder average of the physics quantity A, we arrive at the scalar field theory $\overline{Z^n} = \int \prod_{a=1}^{n} D\psi_a D\psi_a^* \exp\left(-\frac{G(\psi_a,\psi_a^*,w_i)}{T}\right)$. w_i can be averaged out and the resulting theory can be analyzed nonperturbatively via Gaussian approximation introduced in [9]. Expanding $\psi(x)$ in the basis of the LLL wavefunctions with quasi-momenta k, $\psi_a(x) \propto \int \mathrm{d}k\phi_k(x)\psi_a(k)$, the Gaussian effective free energy can be expressed via the variational matrix parameters, $m_{ab} = \langle\psi_a^*(k)\psi_b(-k)\rangle$,

$$\overline{G} \propto \sum_a \left[-\log m + a_T m - rm^2 - q(\widehat{m})^2 m^2\right]_{aa} - \frac{q}{4}\sum_{a,b} m_{ab}^4, \tag{5}$$

where the dimensionless parameters are: the 2D LLL temperature

$$a_T = -\left(bt\pi\sqrt{2\mathrm{Gi}}\right)^{-1/2}(1 - t - b),$$

with the Ginzburg number $\mathrm{Gi} = \left(\frac{8\pi e^2\lambda^2 T_c}{c^2 h^2 L_z}\right)^2$, and disorder variances $r = \frac{b^2 p_1 + (1-t)^2 p_2}{4\pi^2\xi^2\sqrt{2\mathrm{Gi}}}$, $q = \frac{4bp_3}{\pi\xi^2}$. The glass state is characterized by the loss of ergodicity and reversibility with respect to dynamic processes. This is expressed, formally, by spontaneous breaking of the replica permutation symmetry (RSB). It was shown by Parisi in the context of the spin glass theory, that the correct solution for the theory of this type is given by the subclass of the matrices m_{ab} which has a hierarchical structure and which can be parameterized by the Parisi function $m(x)$, $0 < x < 1$. In particular, the well known Edwards–Anderson (EA) glass order parameter corresponds to $m(x = 1)$. The label x reflects the hierarchy level and corresponds to the overlap between different valleys in the potential landscape. We find that in the disordered liquid (domain to the right of the irreversibility line in Fig. 1) the replica symmetric solution is stable, while in the glassy phase (the left side of the line) a nontrivial Parisi function describes a continuous replica symmetry breaking. The irreversibility line for small q is given by

$$a_T^g = 2\sqrt{2}(r^{1/2} - r^{-1/2}) + \frac{2+5r}{2\sqrt{2}r^{5/2}}q + O(q^2), \quad 2D$$

$$a_T^g = -\frac{4-r}{r^{1/3}} + \left(\frac{5}{r} + \frac{4r^{4/3}}{3}\right)q + O(q^2), \quad 3D \tag{6}$$

for the case of 2D and 3D, respectively. The glass lines are compared with transport experimental data for the 2D organic superconductor and with the 3D high T_c superconductor YBCO data in [8]. On this line the magnetization M has a cusp, while finite, and its slope $\mathrm{d}M/\mathrm{d}T$ experiences a jump. This was also recently confirmed in BSCCO [6]. For the range of parameters shown the magnetic field is very small and we have $r|_{b\to 0} \to \frac{(1-t)^2 p_2}{4\pi^2\xi^2\sqrt{2\mathrm{Gi}}}, q \to 0$. Thus we assume that q is very small. If q increases, the glass line will shift higher (parameters H_{c2}, Gi were fitted in the region in which the melting line is near T_c and the disorder effect small in [1]). Away from T_c where the disorder effects will appear, the disorder parameter r is determined from the melting line. The generic phase diagram can be found in Fig. 1.

With weak disorder as in high T_c superconductor, the melting line and glass line are usually below line $a_T = -9.5$ (the melting line of zero disorder). We will call $a_T = -9.5$ as T_X line. T_X line divides

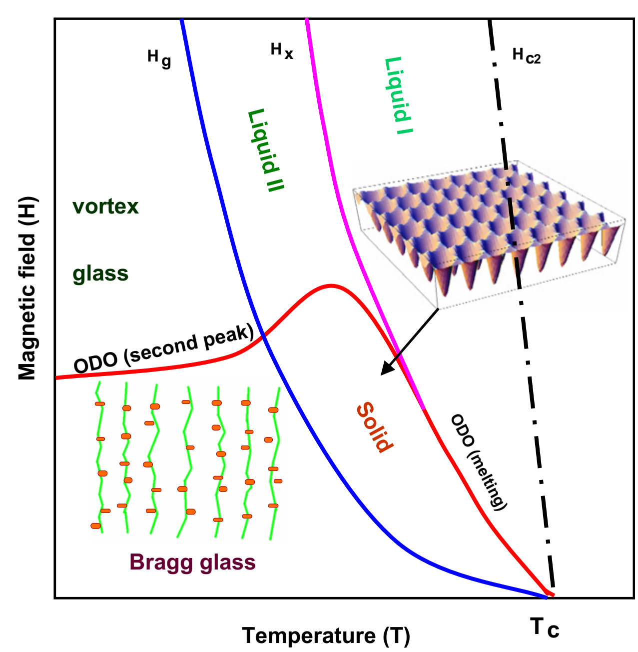

Fig. 1. Generic phase diagram of the vortex matter: The order–disorder line (red) separates the crystalline phase from the homogenous phase. The glass transition line (blue) separates the glass from the weakly pinned phases, while the pink line is a crossover between two homogeneous phases, locally pinned liquid I and essentially unpinned liquid II. The left inset shows well defined vortex lines pinned by impurities in Bragg glass region and the right inset shows the distribution of the order parameter in the Abrikosov lattice near the melting line. (For interpretation of the references in colour in this figure legend, the reader is referred to the web version of this article.)

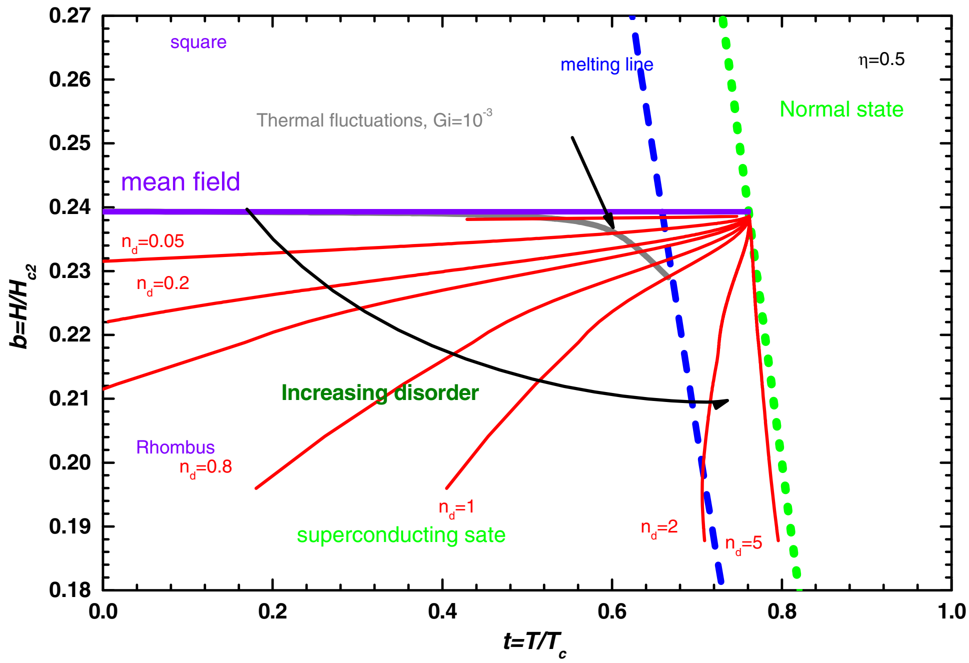

Fig. 2. The mean field structural phase transition line is located at $b_{SPT} = 0.02387/|\tilde{\eta}|$ which has square lattice beyond it. The SPT with strong thermal fluctuation, Gi = 10^{-3}, has negative slope near the H_{c2} line. And the red lines present various disorder strength which influence strongly in low temperature. The solid line is melting line which correspond to Gi = 10^{-3}. (For interpretation of the references in colour in this figure legend, the reader is referred to the web version of this article.)

liquid to two parts: liquid I above T_X line and liquid II below T_X line [10]. Liquid II has local crystal structure as the correlation length is larger than the lattice constant. Though T_X line is not a phase transition line, however it can be taken as a dynamical phase transition line as liquid II is more viscous, therefore has much less resistance than liquid I.

4. Stronger disorder

In low temperature type-II superconductors, the disorder is strong compared to the thermal fluctuation strength. The glass line will be shifted to high temperature even above line T_X line. Therefore the glass phase will be divided to two regions by T_X. Below T_X, phase has local crystalline order as shown in previous sections, therefore the pinning is collective. Above T_X, the pinning is more individual. So T_X is a crossover line from single pinning to collective pinning, and peak effect appears at line T_X. This explanation of peak effect was confirmed by experiments [11] and the peak effect appears exactly at line T_X.

As discussed above, $a_T = -5$ is the spinodal line representing a crossover of the vortex liquid into a "vortex gas". Above this line, vortices move like particles in a gas, and the flux line can not be pinned, the glass line shall be below the spinodal line. With very strong disorder as in low temperature type-II superconductor, the glass transition line shall be near to this spinodal line.

5. Structural phase transition in the vortex lattice

In this section, we will discuss the distortion of vortex lattice due to the influence of underlying anisotropy in a–b plane of material which breaks the in-plane symmetry. With the capability to improve the quality of single crystal in recently years, observation of the structural phase transition (SPT) of vortex lattice are carried out from nonmagnetic borocabide [12–16], and high T_c cuprates such as LSCO and YBCO [17]and $CeCoIn_5$ [18] via small angle neutron scattering, scanning tunneling spectroscopy and decoration experiments. Those materials have 4-fold symmetric crystal lattice, either tetragonal or cubic lattice. The typical case is to apply an external magnetic field along the crystallographic c axis which preserves the symmetry in a–b plane; for a fixed temperature, in low field the hexagonal lattice undergoes reorientation with respect to underlying crystal lattice and in high field the rhombic lattice become square lattice till normal phase.

The rhombic to square phase transition line, $H_2(T)$, observed on earlier experiments $LuNi_2B_2C$ [12–15] has a very small positive slope of the transition line in the T–H plane till it approaches the $H_{c2}(T)$ region. In some experiments [13,14], it abruptly turns up and even acquires a negative slope at high fields, while in other experiments with a closely related material YNi_2B_2C [16] it continues the gradual increase even near $H_2(T)$. However, in LaSCO, the transition line exhibits negative slope in the T–H plane.

Previous theoretical studies of SPT ignored the disorder effects, however, we find that disorder effect is in fact important for the structure phase transition in low temperature superconductors. To introduce the anisotropy effect of interested tetragonal material, we add 4-fold symmetric term based on the reason discussed in reference [19]. The gap anisotropy is well represented by two-component Ginzburg–Landau model which can be simply reduced to a one component GL with additional high derivative term:

$$H_{4\text{-fold}} = -\frac{\tilde{\eta}}{4}\left\{\left(D_x^2 - D_y^2\right)^2 - \left(D_xD_y + D_yD_x\right)^2\right\}. \tag{7}$$

The coefficient $\tilde{\eta}$ can be positive (usually in low T_c materials) or negative (usually in high T_c materials). By solving the Ginzburg–Landau equation approximately analytically, we found near H_{c2} line, the mean field SPT line is temperature independent. While thermal fluctuations influence become stronger, in perturbation approximation, the slope is increasing, see Fig. 2. While taking into account the disorder influence, the slope becomes negative and departs from the mean field STP line with increasing disorder strength. One concludes therefore that materials with strong thermal fluctuations exhibit negative slope of structural phase transition line (at least well below the melting line). When thermal fluctuations are small and disorder prevails, one expects a positive slope.

6. Conclusion

Phase diagram of both high temperature superconductors and low temperature superconductors can be effectively studied within the Ginzburg–Landau approach for fields significantly larger than H_{c1}. There are three phase transition lines separating various phases. The vortex glass transition line (replica symmetry breaking) separates pinned from unpinned phases. The order–disorder line (translation and rotation symmetry breaking) consisting the melting line and from the second peak segments separates homogeneous from crystalline phase, while the structural transition line (4-fold symmetry breaking) separated different crystalline phases.

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