Synthesis of a robust broadband duct ANC system using convex programming approach

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A robust active controller using spatially feedforward structure is proposed for broadband attenuation of noise in ducts. To meet the requirements of performance and robust stability in the presence of plant uncertainties, an H_2 cost function and an H_∞ constrain are employed in the synthesis of the controller. The design is then converted into a convex programming problem using Q-parametrization and frequency discretization. An optimal controller that satisfies the quadratic cost functions and linear inequality constraints can be found by sequential quadratic programming. The optimal controller was implemented via a digital signal processor (DSP) and verified by experiments. Experiment results showed that the system attained 16.5 dB maximal attenuation and 5.9 dB total attenuation in the frequency band 200–600 Hz. © *2002 Acoustical Society of America.* [DOI: 10.1121/1.1460926]

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I. INTRODUCTION

Active noise control (ANC) offers numerous advantages, e.g., compact size and low-frequency performance, over conventional passive technologies. Although the adaptive feedforward control, e.g., the filtered-X algorithm is widely used, fixed controllers based on optimal control and robust control are gaining research attention in the field of ANC. Many design techniques have been applied to the duct ANC problem for synthesizing fixed controllers. Hong *et al.*[1] and Wu *et al.*[2] investigated the duct ANC problem using the linear quadratic Gaussian (LQG) control. Along the same line, robust controllers are designed using combined pole placement and loop shaping method.[3] Using model matching approach, Bai and Wu[4] solved the problem via linear programming in the l_1-norm and l_2-norm vector space. In earlier research, Bai and Chen[5] developed the H_2 and H_∞ model matching principle to deal with the same problem. Moreover, to suppress narrow-band noise, an internal model-based active noise control system has been proposed.[6] In the work of Bai and Lin,[7] H_∞ robust control theory was used to compare three control structures: feedback control, feedforward control, and hybrid control in terms of performance, stability, and robustness.

In ANC applications to date, feedforward control has been a most effective approach. However, a nonacoustical reference required by feedforward control is usually unavailable in many applications. The *spatially feedforward* control structure shown in Fig. 1 appears to be a more viable approach in dealing with such situations, especially when broadband attenuation is sought. The term, spatially feedforward structure, stems from the fact that the ANC system in Fig. 1 includes an acoustic feedback path and is thus not a purely feedforward structure. In this case, stability and robustness problems will arise and the achievable performance will also be degraded. For the spatially feedforward structure, the zero spillover controller[8] or the Roure's controller[9] can be used. However, these controllers may be noncausal and require special modifications, e.g., direct truncation of impulse response, before practical implementation. Therefore, an optimization method combining an H_2 performance objective with an H_∞ constraint is proposed for the design of a spatially feedforward controller for duct. Using this approach, nominal performance can be achieved under the H_∞ constraint of robust stability.

Much work has been addressed in the literature[10–15] on the mixed H_2/H_∞ control. The controllers can be synthesized by the Riccati equation approach, the convex optimization approach or the dynamic game approach, etc. In Ref. 15, a duct ANC system was developed using mixed H_2/H_∞ control. The work incorporated an H_∞-constrained, LQG feedback controller with an internal model to attenuate a narrow-band disturbance. The controller is synthesized analytically by solving the Riccati equations. In contrast to their approach, a numerical optimization approach, convex programming, applied within a general framework originally suggested by Boyd[16] is employed in this paper for achieving broadband noise reduction in a spatially feedforward structure. Then, Q-parametrization[17,18] is used to formulate all stabilizing controllers. The resulting Q-parametrization represented by a finite impulse response (FIR) filter transforms the design problem into a convex optimization problem in frequency domain. The approach suggested by Boyd[16] is used in this work, where the optimal controller is found using *sequential quadratic programming*.[19,20] Common causes of plant uncertainties are modeling errors, measurement noises, and the perturbations in physical conditions. A comprehensive investigation on the effect of different physical conditions on ducts such as flow, temperature, lining, and radiation impedance, can be found in Ref. 21. However, in this paper, we shall focus on primarily the uncertainty resulting from modeling error on which the robust stability constraint is based.

This paper is organized as follows. First, the convex programming and Q-parametrization are presented in the

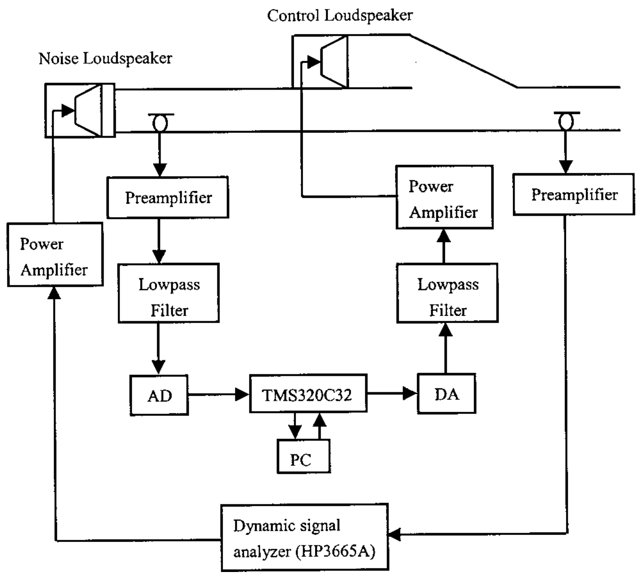

FIG. 1. Experimental setup of the ANC problem.

context of performance and robustness analysis. Then, the calculated controller is implemented as a FIR filter on a TMS320C32 digital signal processor (DSP). Finally, the proposed ANC system will be justified by experimental investigations and the results will be discussed.

II. ZERO SPILLOVER CONTROLLER AND ROURE'S CONTROLLER

Consider a standard control framework, with K the feedforward controller, G the plant, w the exogenous input, z the controlled variables, u the control force, and y the measurement as illustrated in Fig. 2. Thus, the general input–output relation of the augmented plant can be expressed as follows:

$$\begin{bmatrix} z(k) \\ y(k) \end{bmatrix} = \begin{bmatrix} G_{zw}(z) G_{zu}(z) \\ G_{yw}(z) G_{yu}(z) \end{bmatrix} \begin{bmatrix} w(k) \\ u(k) \end{bmatrix}, \tag{1}$$

where $G_{yu}(z)$, $G_{zu}(z)$, $G_{zw}(z)$, and $G_{yw}(z)$ are properly partitioned transfer matrices of G. They are represented in Figs. 3(a)–(d). The closed loop transfer function can be expressed as

$$\begin{aligned} T_{zw}(z) &= [G_{zw}(z) - (G_{zw}(z) G_{yu}(z) \\ &\quad - G_{zu}(z) G_{yw}(z)) K(z)][1 - G_{yu}(z) K(z)]^{-1} \\ &= F(z) S_0(z), \end{aligned} \tag{2}$$

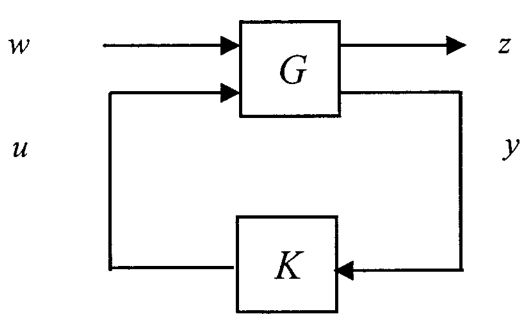

FIG. 2. The generalized control framework of the ANC system.

where

$$F(z) \equiv G_{zw}(z) - [G_{zw}(z) G_{yu}(z) - G_{zu}(z) G_{yw}(z)] K(z) \tag{3}$$

is the *spillover function* and

$$S_0(z) \equiv [1 - G_{yu}(z) K(z)]^{-1} \tag{4}$$

is the *sensitivity function*.

The block diagram of the duct system of Fig. 1 is shown in Fig. 4(a), using the notations of Eq. (1). In terms of frequency response functions, letting the spillover function represented in Eq. (3) be zero leads to the so-called *zero spillover controller*:[8]

$$K_{ZSP}(e^{j\omega}) = \frac{G_{zw}(e^{j\omega})}{G_{zw}(e^{j\omega}) G_{yu}(e^{j\omega}) - G_{zu}(e^{j\omega}) G_{yw}(e^{j\omega})}, \tag{5}$$

where the digital frequency $\omega = \Omega T$ with Ω being the analog frequency and T the sampling interval. On the other hand, the Roure's controller that is equivalent to the zero spillover controller can be obtained by dividing the numerator and the denominator of Eq. (5) by $G_{yw}(e^{j\omega})$:

$$\begin{aligned} K_{ZSP}(e^{j\omega}) &= \frac{-G_{zw}(e^{j\omega})/G_{yw}(e^{j\omega})}{G_{zu}(e^{j\omega}) - G_{yu}(e^{j\omega}) G_{zw}(e^{j\omega})/G_{yw}(e^{j\omega})} \\ &= \frac{-H_0(e^{j\omega})}{H_2(e^{j\omega}) - H_1(e^{j\omega}) H_0(e^{j\omega})} \\ &\equiv K_{\mathrm{Roure}}(e^{j\omega}), \end{aligned} \tag{6}$$

where $H_0(e^{j\omega}) \equiv G_{zw}(e^{j\omega})/G_{yw}(e^{j\omega})$ is the frequency response function between the performance microphone and the measurement microphone, $H_1(e^{j\omega}) \equiv G_{yu}(e^{j\omega})$ is the frequency response function between the measurement microphone and the control speaker, and $H_2(e^{j\omega}) \equiv G_{zu}(e^{j\omega})$ is the frequency response function between the performance microphone and the control speaker. The block diagram of the Roure's controller is shown in Fig. 4(b).

It is noted that in the Roure's controller the frequency response function $H_0(e^{j\omega})$ can be directly measured without knowing the disturbance w. The impulse response of the Roure's controller is then obtained by inverse Fourier transform of $K_{\mathrm{Roure}}(e^{j\omega})$. However, this procedure generally results in noncausal filters. Roure adopted a simple but effective approach to overcome this difficulty. He chose to truncate the noncausal part directly and implemented the controller with a FIR filter.

III. DESIGN OBJECTIVE AND CONSTRAINT

The aforementioned Roure's method provides a straightforward means to obtain an implementable (proper, stable, and causal) controller. Unlike Roure's approach that requires the closed loop transfer function $T_{zw}(z)$ be strictly zero, we seek to recast the problem into an optimization problem: find an implementable controller such that the following cost function of performance,

$$\| T_{zw}(z) W_1(z) \|_2^2, \tag{7}$$

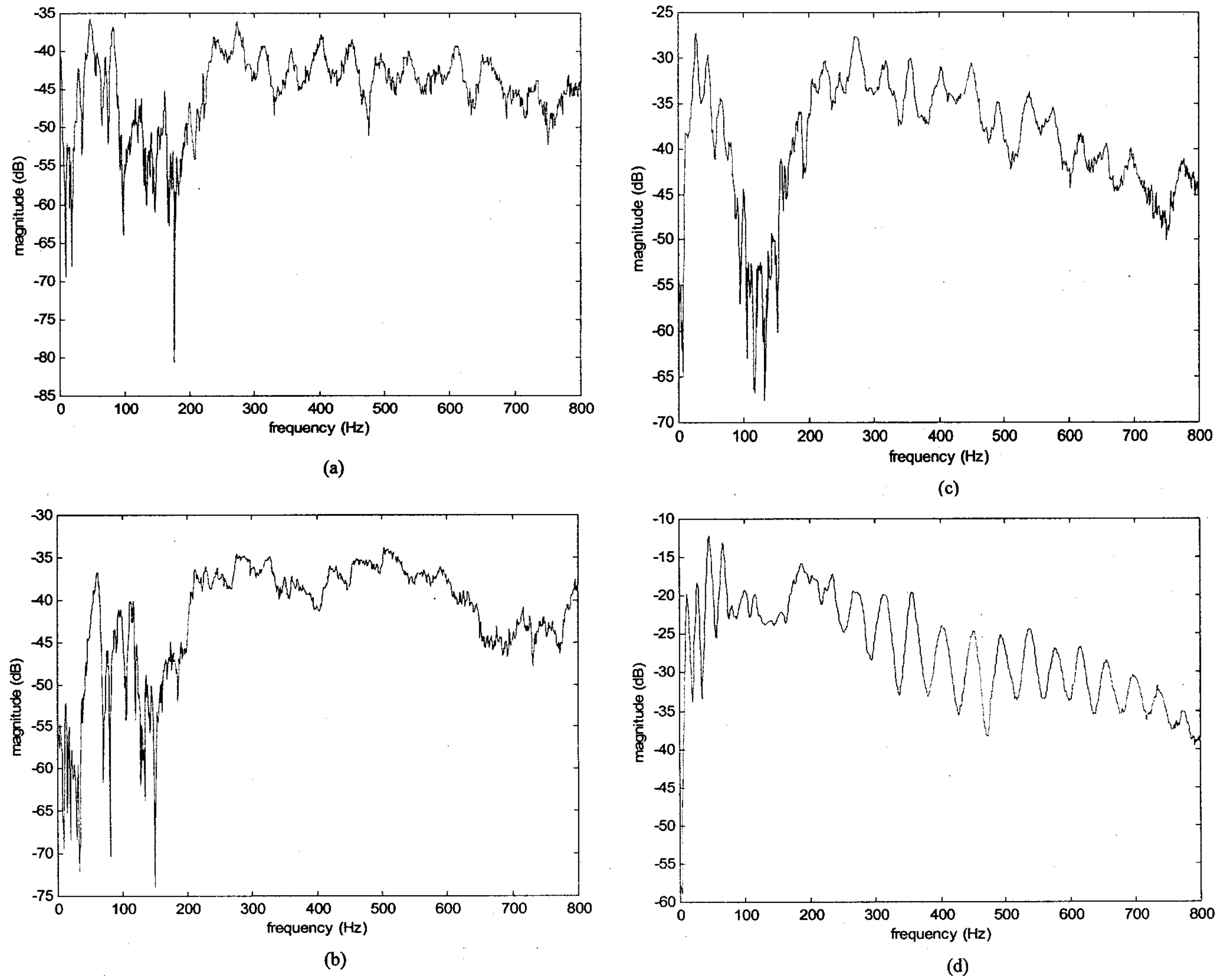

FIG. 3. The frequency responses of the nominal duct system. (a) The frequency response of G_{yu}; (b) the frequency response of G_{zu}; (c) the frequency response of G_{zw}; and (d) the frequency response of G_{yw}.

is minimized, where

$$\|T_{zw}(z)\|_2 \triangleq \left(\frac{1}{2\pi} \int_{-\pi}^{\pi} |T_{zw}(e^{j\omega})|^2 \, d\omega \right)^{1/2} \tag{8}$$

represents the two-norm[22] in discrete-time domain and $W_1(z)$ is a weighting function that is generally a low-pass function.

In addition to performance consideration, robust stability is another important issue. Robust stability is understood as the robustness of stability of the closed loop ANC system against plant uncertainties and perturbations frequently encountered in practical applications. One way of coping with system uncertainties and perturbations is to use an adaptive controller, whereas this paper adopted an alternative strategy—a robust controller which requires simpler implementation than the adaptive counterpart. To this end, a robust stability constraint is incorporated into the aforementioned optimization problem. Assume the following perturbed plant model,[23]

$$G_p(z) = G_{yu}(z)[1 + \Delta(z)], \tag{9}$$

where $G_{yu}(z)$ is the nominal plant, $\Delta(z)$ is the multiplicative uncertainty, and G_p represents the physical plant. The uncertainty $\Delta(z)$ is assumed to be bounded by $|\Delta(z)| < W_2(z)$. The uncertainty considered here is primarily the difference between the plant model and the real plant beyond the control bandwidth, which may cause excessive control output at high frequency. Therefore, it is important for the controller to be robust against this type of uncertainty. It can be shown by *small-gain theorem* that the condition of robust stability takes the form

$$\|T_0(z) W_2(z)\|_\infty < 1, \tag{10}$$

where $T_0(z) = 1 - S_0(z)$ is the *complementary sensitivity function* $[S_0(z)$ is as defined in Eq. (4)$]$ and $W_2(z)$ is a weighting function that is generally a high-pass function to be determined from the plant uncertainty.

In summary, the ANC problem can be written in the language of optimization as follows:

$$\min_K \|T_{zw}(z) W_1(z)\|_2^2 \tag{11}$$

subject to

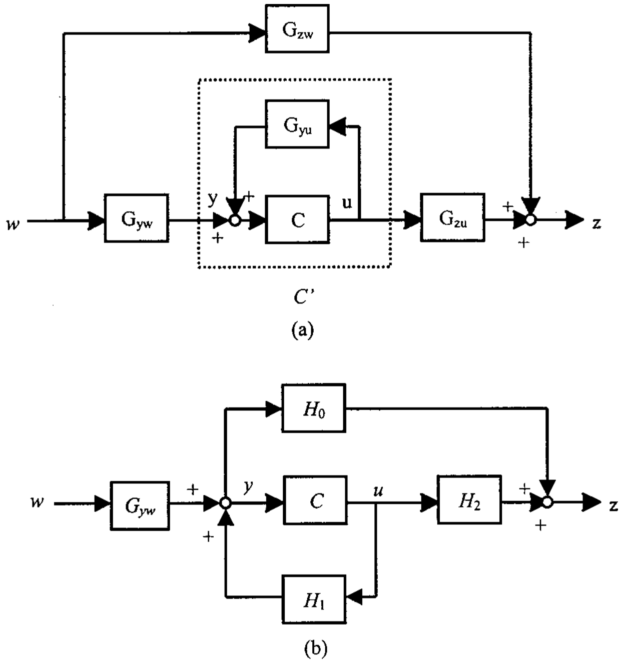

FIG. 4. Two equivalent spatially feedforward controllers. (a) Block diagram of zero spillover controller; (b) block diagram of the Roure's controller.

$$\|T_0(z)W_2(z)\|_\infty < 1. \tag{12}$$

The control problem described in Eqs. (11) and (12) is a mixed norm problem that can be solved by a convex programming technique to be presented in the next section.

IV. CONVEX PROGRAMMING USING Q-PARAMETRIZATION

In this section, the formulation in Eqs. (11) and (12) will be converted to a convex programming problem and solved by using a technique originally suggested by Boyd.[16] The key step of the conversion to a convex problem is the so-called Q-parametrization[17] to Youla's parametrization.[18] Assume that the plant G_{yu} is stable. According to the method, the controllers that stabilize the closed loop system can be parametrized by a proper and stable transfer function $Q(z)$ as follows:

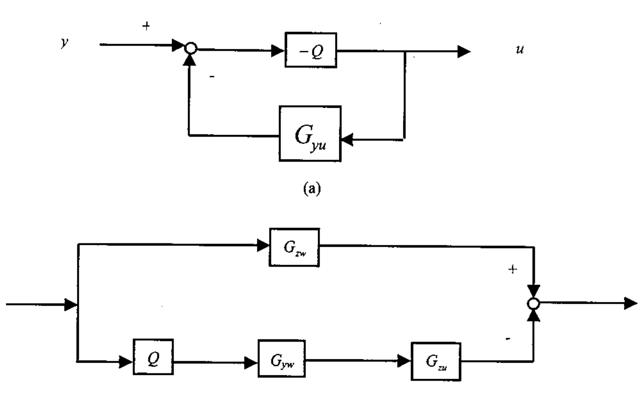

FIG. 5. Block diagrams of Q-parametrization. (a) The internal model controller; (b) the model matching problem obtained from Q-parametrization.

$$K(z) = \frac{-Q(z)}{1 - Q(z)G_{yu}(z)}. \tag{13}$$

The block diagram is shown in Fig. 5(a). More precisely, the resulting closed loop system will guarantee to be stable if the controller is selected in the form of Eq. (13) with a proper and stable $Q(z)$. The latter requirement is easily achieved in discrete time domain by choosing $Q(z)$ to be a FIR filter. In addition, the controller design in Eq. (13) is also known as the *internal model controller* because it has a "built in" nominal plant model G_{yu}.

At this point, the optimization problem has been formulated in frequency domain via Q-parametrization. To solve the problem, an approach proposed by Boyd and Helton[24] was employed in the paper. The same method was also successfully used by Rafaely[25] and Elliott who have done pioneering work to solve an ANC problem for a headrest. This method first discretizes the frequency response function by uniformly sampling the unit circle, and then solves the optimization problem by convex programming.

Substituting Eq. (13) into Eqs. (11) and (12), $T_{zw}(z)$ and $T_0(z)$ can be parametrized in terms of a proper and stable function $Q(z)$:

$$T_{zw}(z) = G_{zw}(z) - Q(z)G_{yw}(z)G_{zu}(z),$$

$$T_0(z) = Q(z)G_{yu}(z).$$

Thus minimization of $\|T_{zw}\|_2$ is tantamount to a model matching problem depicted in Fig. 5(b). Note that both $T_{zw}(z)$and $T_0(z)$ are affine functions of $Q(z)$. By frequency discretization, the cost function and constraint function of the optimization problem now take the following form:

$$\min_q \frac{1}{N} \sum_{k=0}^{N-1} |[G_{zw}(k) - Q(k)G_{yw}(k)G_{zu}(k)]W_1(k)|^2 \tag{14}$$

subject to

$$|Q(k)G_{yu}(k)W_2(k)| < 1, \quad k = 0,1,...,N-1, \tag{15}$$

where $\mathbf{q} = (q_0 q_1 \cdots q_{m-1})^T$ being the coefficient vector of the FIR filter Q with length m are the design variables and N is the number of frequency samples. Only the acoustic feedback path $G_{yu}(k)$ has to do with robust stability constraints. It should be noted that those gains of the transfer functions such as $G_{yu}(z)$, $G_{zw}(z)$, $G_{yu}(z)$, and $G_{zu}(z)$ are assumed to be time invariant for our application where the temperature is not changed much. To summarize, the optimization in Eq. (15) is comprised of the design variable $\mathbf{q}$, the cost function in quadratic form, and the constraint function in linear inequality form.

Define the following Hessian matrix[26]

$$\mathbf{H}_s = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \frac{\partial^2 f}{\partial x_n \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}. \qquad (16)$$

Because the Hessian matrix of the function in Eq. (14) is positive definite, the function is a strictly convex function.[26] Furthermore, the design specification in Eq. (15) is a convex specification because it is norm-bounded.[27] Since the cost function and the constraint function are both convex, Eqs. (14) and (15) represent a convex problem. A local minimum of a convex problem is also a global minimum.

The method used in this work to compute the optimal controller by solving the optimization problem in Eqs. (14) and (15) was sequential quadratic programming[26] which is provided as a MATLAB command *constr* in optimization toolbox. However, other programming methods such as semidefinite programming[27] can be used. Because the function *constr* was based on the sequential programming that accepts only linear constraints, the single robust stability constraint in Eq. (12) has been replaced with N linear inequality constraints in Eq. (15).

V. CONTROLLER IMPLEMENTATION AND EXPERIMENTAL INVESTIGATION

A duct made of plywood shown in Fig. 1 is used for verifying the proposed ANC method. The length of the duct is 440 cm and the cross section is 25×25 cm^2. There is 10 cm between the primary source speaker and the measurement microphone. To reduce the undesirable acoustic feedback, we use the backward control loudspeaker facing the open end of the duct. The distance between the measurement microphone and the control speaker is 235 cm to ensure causality of the controller. The distance between the control speaker and the performance microphone is 110 cm. A floating point DSP, TMS320C32 equipped with four 16-bit analog IO channels, is utilized to implement the controller. The sampling frequency is chosen to be 2 kHz. Considering the cutoff frequency of the duct (approximately 700 Hz) and the poor response of speaker at low frequency, we chose control bandwidth from 200 to 600 Hz. It should be noted that a loudspeaker should be mounted near the primary noise source for identification of the frequency response functions $G_{yw}(e^{j\omega})$ and $G_{zw}(e^{j\omega})$ because the noise source (a fan, for example) is generally unmeasurable in practice.

A digital spatially feedforward controller is designed to reject the noise in the duct. The ANC system is as shown in Fig. 1. Figure 3(a) shows the measured frequency response function $G_{yu}(e^{j\omega})$. To weight performance as in Eq. (14), the weighting function $W_1(z)$ is chosen as a low-pass filter with cutoff frequency 600 Hz and unity gain, as shown in Fig. 6(a).

To weigh the robust stability as in Eq. (15), on the other hand, the weighting function $W_2(z)$ is determined from the

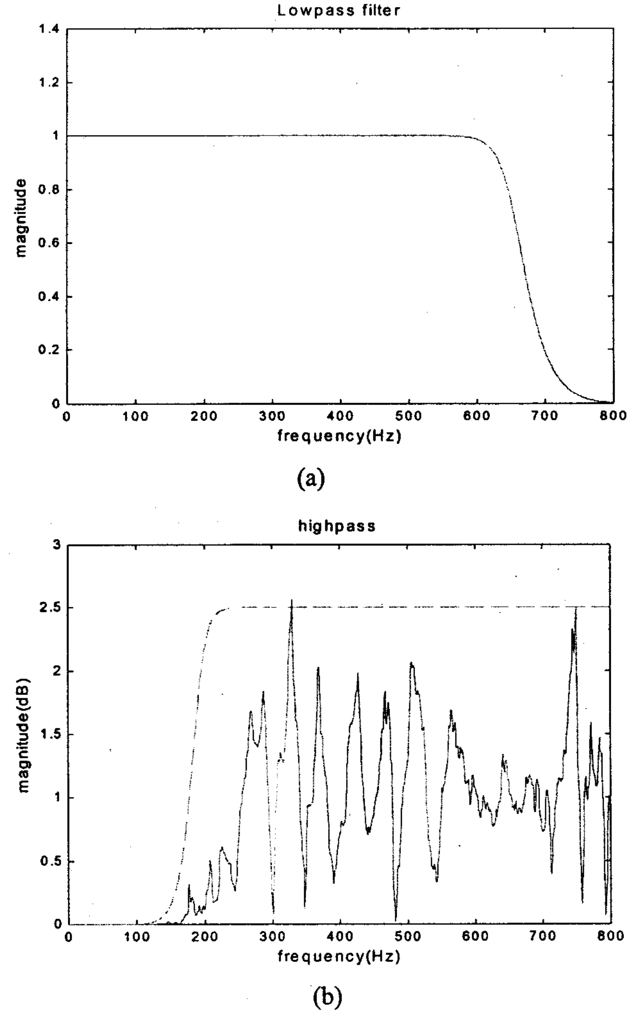

FIG. 6. Two weighting functions used in the implementation. (a) The weighting function W_1; (b) — — —, the weighting function W_2; ——, the uncertainty of the nominal plant.

uncertainty definition in Eq (9). The frequency responses of the nominal plant in 0–800 Hz and in 0–1600 Hz are measured. The former is taken as the nominal plant, while the latter the actual plant. By using the MATLAB command *invfreqz*, transfer functions of the nominal and true plant are calculated. Then the weighting function $W_2(z)$ is selected according to the following equation:

$$\left|\frac{G_p(e^{j\omega})}{G_{yu}(e^{j\omega})}-1\right| \leqslant |W_2(e^{j\omega})| \quad \forall\omega, \qquad (17)$$

where $G_p(e^{j\omega})$ is the frequency response function of the actual plant and $G_{yu}(e^{j\omega})$ is the frequency response function of the nominal plant. Figure 6(b) shows the calculated uncertainty bounded by the weighting function W_2 whch is a high-pass filter with gain 2.5 and cutoff frequency 200 Hz.

The optimization problem formulated in Eqs. (14) and (15) with the design variables $\mathbf{q}$ of 128 coefficients (initially set to be zeros) was solved numerically by using the MATLAB function *constr.* The unit circle was uniformly sampled at 2048 points. For the control bandwidth 200–600 Hz, only 801 constraints need to be considered. In the choice of parameters, sufficiently large number of frequency samples[28] N and long FIR filter Q are generally required to ensure an

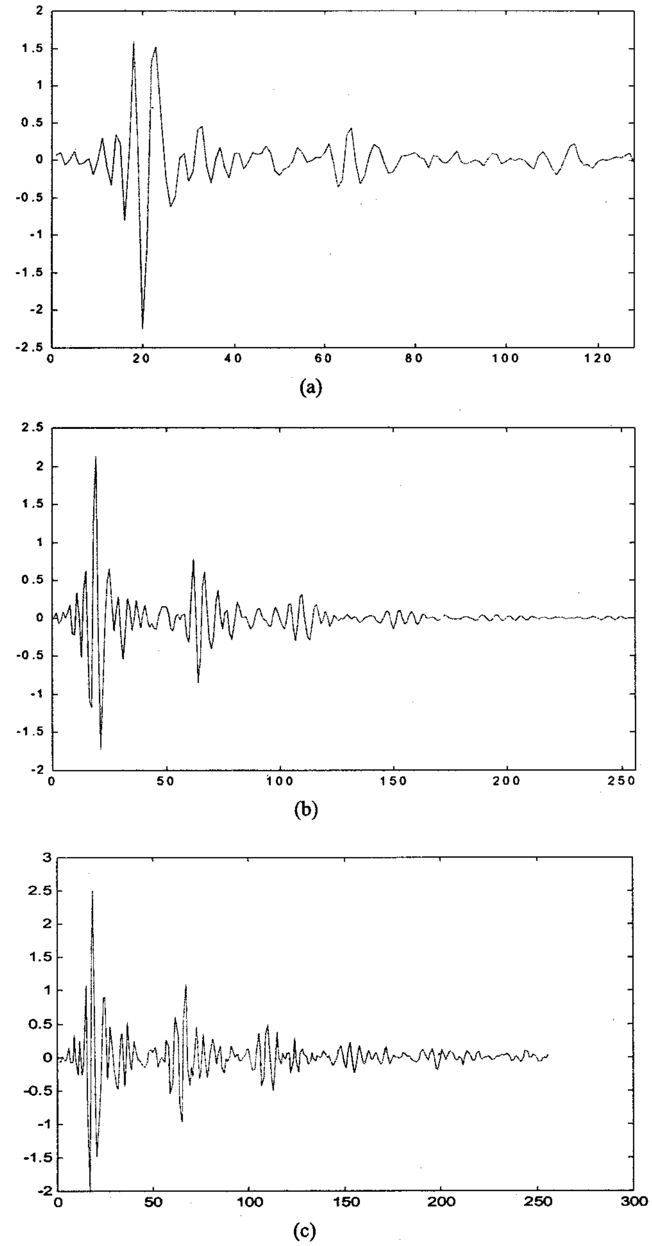

FIG. 7. The calculation result of Q-parametrization and the impulse responses of convex programming controller and Roure's controller. (a) The coefficient vector $\mathbf{q}$; (b) the impulse response of the convex programming controller; and (c) the impulse response of Roure's controller.

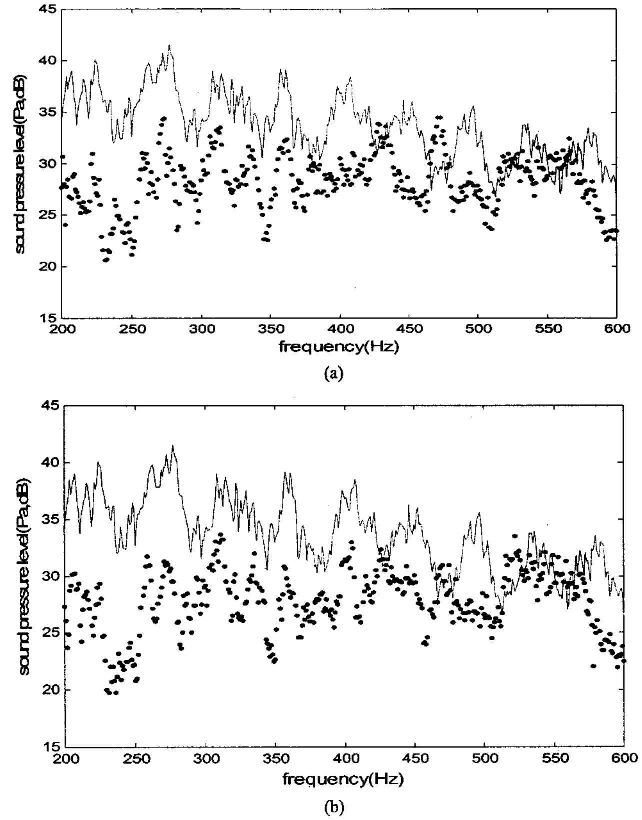

FIG. 8. Active control results of the feedforward controllers. (a) Noise reduction using Roure's controller; (b) noise reduction using the convex programming controller. ——, control off; · · · ·, control on.

accurate discrete approximation of the continuous problem. Titterton and Olkin[29] suggested increasing the length of $\mathbf{q}$ and the length of N until the impulse response of $\mathbf{q}$ sufficiently decays such that any further increase of length would not result in significant improvement.

Figure 7(a) shows the coefficient vector of the control filter $\mathbf{q}$ with 128 coefficients. As can be seen in the figure, the impulse response decays almost completely. Any further increase in filter length will not provide significant improvement. The calculated coefficient vector $\mathbf{q}$ is then substituted into Eq. (13) to obtain the optimal controller that is in turn converted to a FIR filter of length 256. Figure 7(b) shows the calculated impulse response of the controller. The resulting controller is then implemented on the platform of the DSP. An experiment is undertaken to verify the ANC system. The result of control in terms of pressure spectra measured at the performance microphone is shown in Fig. 8(b). The total attenuation within the control bandwidth is 5.9 dB.

Roure's controller is also implemented for comparison with the mixed norm controller. Roure's controller is calculated from Eq. (6), based on the measurable frequency response functions $H_0(e^{j\omega})$, $H_1(e^{j\omega})$, and $H_2(e^{j\omega})$. The impulse response of Roure's controller is shown in Fig. 7(c). Using Roure's controller, the experimental result is shown in Fig. 8(a) with a total attenuation 5.4 dB. Detailed comparison between the mixed norma controller and Roure's controller is summarized in Table I.

VI. CONCLUSIONS

This paper presents a spatially feedforward design for active control of noise in a duct. As opposed to earlier methods that compute optimal controllers analytically, this method utilizes a frequency domain convex programming technique. This approach is effective in that it guarantees to

TABLE I. Comparison of ANC methods.

Items	Roure's controller	Convex programming controller
Control bandwidth	200–600 Hz	200–600 Hz
FIR filter length	256	256
Maximal attenuation	15.5 dB	16.5 dB
Total band attenuation	5.4 dB	5.9 dB

find the global minimum solution. In comparison to conventional LQG or H_∞ design, where experience is required for choosing appropriate weighting functions, all the functions required in our method can be derived directly from measured data. The convex programming method, in conjunction with Q-parametrization and frequency discretization, proved to be useful in ANC application for several reasons. First, the chosen uncertainty model derived from measured data was found to be a reasonable model for construction of constraints in optimization. This is an advantage over other ''trial and error'' design methods such as LQG in which we are unable to determine explicitly the weighting functions from measured data. Second, the design problem is a convex problem whose solution can easily be found using commercially available software such as MATLAB optimization toolbox. Third, optimization can be focused on performance since closed-loop stability has already been ensured by Q-parametrization.

To summarize, this research has two potential contributions to the area of ANC. First, we have shown that the controller obtained from convex programming approximates well the idea spatially feedforward controller. It yields comparable performance as the zero spillover controller and the Roure controller which use a direct truncation of the noncausal impulse response of the ideal controller. The proposed approach achieves nominal performance subject to the H_∞-constraint on closed-loop robust stability. Second, the procedures involved in the analysis and implementation phases are presented in detail. Experimental verifications of the proposed ANC system were conducted for a duct problem. In particular, technical issues such as how to choose the plant uncertainty model and the performance index are addressed.

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