ELSEVIER

Fuzzy Sets and Systems 126 (2002) 367–376

FUZZY sets and systems

www.elsevier.com/locate/fss

A fuzzy seasonal ARIMA model for forecasting

Fang-Mei Tseng[a,*], Gwo-Hshiung Tzeng[b]

[a]Department of Finance, Hsuan Chuang University, No. 48 Hsuan Chuang Rd, Hsin-chu, Taiwan
[b]Energy and Environmental Research Group, Institute of Management of Technology, College of Management, National Chiao Tung University, No. 1001 Ta-hsieh Rd, Hsin-chu, Taiwan

Received 20 August 1999; received in revised form 20 November 2000; accepted 3 January 2001

Abstract

This paper proposes a fuzzy seasonal ARIMA (FSARIMA) forecasting model, which combines the advantages of the seasonal time series ARIMA (SARIMA) model and the fuzzy regression model. It is used to forecast two seasonal time series data of the total production value of the Taiwan machinery industry and the soft drink time series. The intention of this paper is to provide business which are affected by diversified management with a new method to conduct short-term forecasting. This model includes both interval models with interval parameters and the possible distribution of future value. Based on the results of practical application, it can be shown that this model makes good forecasts and is realistic. Furthermore, this model makes it possible for decision makers to forecast the best and worst estimates based on fewer observations than the SARIMA model.

Keywords: SARIMA; Fuzzy regression; Fuzzy SARIMA; Fuzzy time series; Time series

1. Introduction

Modern enterprises are confronted with new technologies and fierce competition worldwide. The environment is becoming progressively more dynamic and is expanding globally. These factors make decision-making difficult and critical. Effective forecasting is fundamental to future technology development and customer demand, and time series is one of the methods we can use for prediction. The seasonal time series ARIMA (SARIMA) model was initially presented by Box–Jenkins [1] and was successfully used in forecasting economic, marketing, social problems, etc. While this model has the advantage of accurate forecasting over short periods, it also has the limitation that at least 50 and preferably 100 observations or more should be used [1]. In addition, this model uses the concept of measurement error to deal with the deviations between estimators and observations, but the data it uses are precise values that do not include measurement errors. Tanaka [14]. Tanaka and Ishibuchi [15] and Tanaka et al. [16] suggested the use of fuzzy regression to solve the fuzzy environment problem and avoid modeling error. This model is basically an interval prediction model with the disadvantage that the prediction interval can be very wide if extreme values are present. An application that uses fuzzy regression to fuzzy time series analysis was presented by Watada

* Corresponding author.

0165-0114/02/$ - see front matter
PII: S0165-0114(01)00047-1

[11], but this model did not include the concept of the Box–Jenkins's model [1]. Tseng et al. [17] proposed the fuzzy ARIMA (FARIMA) method which uses the fuzzy regression method to fuzzify the parameters of the ARIMA model, although this model did not deal with the problem of seasonality. This paper is an extension of Tseng's work [17], in which we combine the advantages of the SARIMA$(p,d,q)(P,D,Q)_s$ model and the fuzzy regression model to develop the fuzzy SARIMA (FSARIMA) method.

From the results of practical application, the proposed method makes good forecasts and appears to be the most appropriate tool. Its advantages are as follows:

(a) To provide the decision makers with insight regarding the possible best and worst estimates.
(b) The required number of observations is less than required by the SARIMA model (at least 50 and preferably more than 100 observations [1]).

This paper is organized as follows: Concepts of the SARIMA model and the fuzzy regression model are reviewed in Section 2. In Section 3, the FSARIMA model is formulated and proposed. The FSARIMA model is applied to forecast the production value of Taiwan machinery industry and the soft drink time series [9] in Section 4, and finally conclusions are discussed in Section 5.

2. Review of the seasonal ARIMA model and the fuzzy regression model

A time series $\{Z_t \mid t = 1,2,\ldots,k\}$ is generated by a SARIMA$(p,d,q)(P,D,Q)_s$ process with mean μ of the Box–Jenkins's model [1] if

$$\varphi(B)\Phi(B^s)(1-B)^d(1-B^s)^D(Z_t-\mu) = \theta(B)\Theta(B^s)a_t, \qquad (1)$$

where p,d,q,P,D and Q are integers, and s is periodicity,

$$\varphi(B) = 1-\varphi_1 B-\varphi_2 B^2-\cdots-\varphi_p B^p,$$
$$\Phi(B^s) = 1-\Phi_1 B^s-\Phi_2 B^{2s}-\cdots-\Phi_P B^{Ps},$$

$\theta(B)=1-\theta_1 B-\theta_2 B^2-\cdots-\theta_q B^q$ and $\Theta(B^s)=1-\Theta_1 B^s-\Theta_2 B^{2s}-\cdots-\Theta_Q B^{Qs}$ are polynomials in B of degrees p,q,P, and Q, B is the backward shift operator, d is the number of regular differences; D is the number of seasonal differences, and Z_t denotes observed value of time series data, $t=1,2,\ldots,k$.

The SARIMA model formulation includes four steps:

(a) Identification of the SARIMA(p,d,q) $(P,D,Q)_s$ structure: use autocorrelation function (ACF) and partial autocorrelation function (PACF) to develop the rough function.
(b) Estimation of the unknown parameters.
(c) Goodness-of-fit tests on the estimated residuals.
(d) Forecast future outcomes based on the known data.

The a_t, which are the estimated residuals at each time period, should be independent and identically distributed as normal random variables with mean 0 and variance σ^2. The roots of $\varphi(Z)=0$ and $\theta(Z)=0$ should all lie outside the unit circle. If possible, at least 50 and preferably 100 observations or more should exist in the SARIMA model. In the real world, however, the environment is uncertain and there are rapid changes, so we usually must forecast future situations using little data in a short time-span, and it is hard to verify that the data has a normal distribution. Thus, the assumption of the SARIMA model has limitations. The current model uses the concept of measurement error to deal with the deviations between estimators and observations, these data are precise values and do not include measurement errors. This is the same as the basic concept of fuzzy regression model as suggested by Tanaka et al. [16].

The basic concept of fuzzy regression is that the residuals between estimators and observations are not produced by measurement errors, but rather by the parameter uncertainty in the model, and the possibility distribution is used to deal with real observations.

The following is a generalized model of fuzzy linear regression:

$$Y = \beta_0+\beta_1 x_1+\cdots+\beta_n x_n = \sum_{i=0}^{n}\beta_i x_i = \boldsymbol{x}'\beta, \qquad (2)$$

where $x_0=1$, $\boldsymbol{x}$ is the vector of independent variables; superscript $'$ denotes the transformation operation; n is the number of variables and β_i represents the ith parameter of the model.

Instead of using crisp, fuzzy parameters β_i in the form of L-type fuzzy numbers of Dubois and Prade

[3], $(\alpha_i, c_i)_L$, possibility distribution is

$$\mu_{B_i}(\beta_i) = L\{(\alpha_i - \beta_i)/c_i\}, \tag{3}$$

where L is a function type. Fuzzy parameters in the form of triangular fuzzy numbers are used

$$\mu_{B_i}(\beta_i) = \begin{cases} 1 - \dfrac{|\alpha_i - \beta_i|}{c_i}, & \alpha_i - c_i \leqslant \beta_i \leqslant \alpha_i + c_i, \\ 0 & \text{otherwise}, \end{cases} \tag{4}$$

where $\mu_{B_i}(\beta_i)$ is the membership function of the fuzzy set which is represented by parameter β_i, α_i is the center of the fuzzy number, and c_i is the width or spread around the center of the fuzzy number.

Through the extension principle, the membership function of the fuzzy number $Y_t = \boldsymbol{x}_t'\beta$ can be defined by the membership function using pyramidal fuzzy parameter β as follows:

$$\mu_Y(y_t) = \begin{cases} 1 - |y_t - \boldsymbol{x}_t'\alpha| / \boldsymbol{c}'|\boldsymbol{x}_t| & \text{for } \boldsymbol{x}_t \neq \boldsymbol{0}, \\ 1 & \text{for } \boldsymbol{x}_t = \boldsymbol{0},\ y_t = 0, \\ 0 & \text{for } \boldsymbol{x}_t = \boldsymbol{0},\ y_t \neq 0, \end{cases} \tag{5}$$

where α and $\boldsymbol{c}$ denote vectors of model parameter values and spreads, respectively, for all model parameters, t is the number of observations, $t = 1, 2, \ldots, k$.

Finally, the method uses the criterion of minimizing the total vagueness, S, defined as the sum of individual spreads of the fuzzy parameters of the model.

$$\text{minimize} \quad S = \sum_{t=1}^{k} \boldsymbol{c}'|\boldsymbol{x}_t|. \tag{6}$$

At the same time, this approach takes into account the condition that the membership value of each observation y_t is greater than an imposed threshold h, $h \in [0, 1]$. This criterion simply expresses the fact that the fuzzy output of the model should 'cover' all the data points $y_1, y_2, \ldots, y_k$ to a certain h level. The choice of the h level value influences the widths c of the fuzzy parameters

$$Y(y_t) \geqslant h \quad \text{for } t = 1, 2, \ldots, k. \tag{7}$$

The index t refers to the number of nonfuzzy data used in constructing the model. Then the problem of finding the fuzzy regression parameters was formulated by Tanaka and Ishibuchi [15] as a linear programming problem:

$$\text{minimize} \quad S = \sum_{t=1}^{k} \boldsymbol{c}'|\boldsymbol{x}_t|$$

subject to

$$\begin{aligned} &\boldsymbol{x}_t'\alpha + (1-h)\boldsymbol{c}'|\boldsymbol{x}_t| \geqslant y_t, \quad t = 1, 2, \ldots, k, \\ &\boldsymbol{x}_t'\alpha - (1-h)\boldsymbol{c}'|\boldsymbol{x}_t| \leqslant y_t, \quad t = 1, 2, \ldots, k, \\ &\boldsymbol{c} \geqslant \boldsymbol{0}, \end{aligned} \tag{8}$$

where $\boldsymbol{\alpha}' = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ and $\boldsymbol{c}' = (c_0, c_1, \ldots, c_n)$ are vectors of unknown variables and S is the total vagueness as previously defined.

Watada [18] suggested the use of fuzzy time series analysis, which is formulated by possibility regression model but does not include the concept of Box–Jenkins [1] model. Also in the Watada model, the weight of the objective function does not contain criteria which may be somewhat subjective. We propose the FSARIMA model, which uses the criterion of fuzzy regression model for its formulation and improves the limitations in Watada's model.

3. Model formulation

In the previous section, the parameter of SARIMA $(p,d,q)(P,D,Q)_s$, $\varphi_1, \varphi_2, \ldots, \varphi_p$, $\Phi_1, \Phi_2, \ldots, \Phi_P$, $\theta_1, \theta_2, \ldots, \theta_q$ and $\Theta_1, \Theta_2, \ldots, \Theta_Q$ are all crisp values. The SARIMA model is a precise forecasting model for short time periods, although it is limited by the large amount of historical data required. However, we usually have to forecast future situations using limited amounts of data in a short span of time. So this model addresses the limitations of real world applications. This model uses the concept of measurement error to deal with the difference between estimators and observations, but these data are correct values that do not include measurement errors. Instead of using crisp, fuzzy parameters, $\tilde{\varphi}_1, \tilde{\varphi}_2, \ldots, \tilde{\varphi}_p$, $\tilde{\Phi}_1, \tilde{\Phi}_2, \ldots, \tilde{\Phi}_P$, $\tilde{\theta}_1, \tilde{\theta}_2, \ldots, \tilde{\theta}_q$ and $\tilde{\Theta}_1, \tilde{\Theta}_2, \ldots, \tilde{\Theta}_Q$, in the form of triangular fuzzy numbers are used. Finally, a_t has a fuzzy parameter $\tilde{\gamma}$.

A FSARIMA$(p,d,q)(P,D,Q)_s$ model is described by a fuzzy function with fuzzy parameter

$$\tilde{\varphi}(B)\tilde{\Phi}(B^s)W_t = \tilde{\beta}_0 + \tilde{\theta}(B)\tilde{\Theta}(B^s)a_t, \tag{9}$$

$$W_t = (1-B)^d(1-B^s)^D Z_t, \tag{10}$$

$$\begin{aligned}\tilde{W}_t = {} & \tilde{\beta}_0 + \sum_{i=1}^{p}\tilde{\varphi}_i W_{t-i} + \sum_{i=1}^{P}\tilde{\Phi}_i W_{t-is} \\ & - \sum_{i=1}^{p}\varphi_i\Phi_1 W_{t-s-i} - \sum_{i=1}^{p}\varphi_i\Phi_2 W_{t-2s-i} \\ & - \cdots - \sum_{i=1}^{p}\varphi_i\Phi_P W_{t-Ps-i} + \tilde{\gamma}a_t \\ & - \sum_{i=1}^{q}\tilde{\theta}_i a_{t-i} - \sum_{i=1}^{Q}\Theta_i a_{t-is} \\ & + \sum_{i=1}^{q}\tilde{\theta}_i\Theta_1 a_{t-s-i} + \sum_{i=1}^{q}\tilde{\theta}_i\Theta_2 a_{t-2s-i} \\ & + \cdots + \sum_{i=1}^{q}\tilde{\theta}_i\Theta_Q a_{t-Qs-i},\end{aligned} \tag{11}$$

where Z_t is observations, $\tilde{\varphi}_1,\tilde{\varphi}_2,\ldots,\tilde{\varphi}_p$, $\tilde{\Phi}_1,\tilde{\Phi}_2,\ldots,\tilde{\Phi}_P,\tilde{\gamma},\tilde{\theta}_1,\tilde{\theta}_2,\ldots,\tilde{\theta}_q$ and $\tilde{\Theta}_1,\tilde{\Theta}_2,\ldots,\tilde{\Theta}_Q$ are fuzzy numbers. The center value of $\tilde{\gamma}$ is 1. Eq. (11) is modified as

$$\begin{aligned}\tilde{W}_t = {} & \tilde{\beta}_0 + \sum_{i=1}^{p}\tilde{\beta}_i W_{t-i} + \sum_{i=1}^{P}\tilde{\beta}_{p+i} W_{t-is} \\ & - \sum_{j=1}^{P}\sum_{i=1}^{p}\tilde{\beta}_i\tilde{\beta}_{p+j} W_{t-js-i} + \tilde{\beta}_{p+P+1}a_t \\ & - \sum_{i=1}^{q}\tilde{\beta}_{p+P+1+i}a_{t-i} \\ & - \sum_{i=1}^{Q}\tilde{\beta}_{p+P+q+1+i}a_{t-is} + \sum_{j=1}^{Q}\sum_{i=1}^{q} \\ & \tilde{\beta}_{p+P+1+i}\tilde{\beta}_{p+P+q+1+j}a_{t-js-i}.\end{aligned} \tag{12}$$

Fuzzy parameters in the form of triangular fuzzy numbers are used

$$\mu_{\tilde{B}_i}(\beta_i) = \begin{cases} 1-|\alpha_i-\beta_i|/c_i & \text{if } \alpha_i - c_i \leqslant \beta_i \leqslant \alpha_i + c_i, \\ 0 & \text{otherwise,}\end{cases} \tag{13}$$

where $\mu_{\tilde{B}_i}(\beta_i)$ is the membership function of the fuzzy set that represents parameter β_i, α_i is the center of the fuzzy number, and c_i is the width or spread around the center of the fuzzy number.

From the extension principle [19], Laarhoven and Pedrycz [8] use Dubois and Prade's [3] approximation formula to get fuzzy multiplication

$$A_i \otimes A_j \cong (c_i c_j, a_i a_j, b_i b_j),$$

where $A_i = (c_i, a_i, b_i)$ and $A_j = (c_j, a_j, b_j)$ are triangular fuzzy numbers.

In Eq. (12), the fuzzy multiplication of $\tilde{\beta}_i\tilde{\beta}_{p+j}$ is given as

$$\tilde{\beta}_i\tilde{\beta}_{p+j} = (c_i c_{p+j}, \alpha_i\alpha_{p+j}, c_i c_{p+j}). \tag{14}$$

Using fuzzy parameters β_i in the form of triangular fuzzy numbers the membership of W in Eq. (12) is given as

$$\mu_{\tilde{W}}(W_t) = \begin{cases} 1-|W_t - E_t|/F_t & \text{for } W_t \neq 0,\ a_t \neq 0, \\ 0 & \text{otherwise,}\end{cases} \tag{15}$$

where

$$\begin{aligned}E_t = {} & \alpha_0 + \sum_{i=1}^{p}\alpha_i W_{t-i} + \sum_{i=1}^{P}\alpha_{p+i}W_{t-is} \\ & - \sum_{j=1}^{P}\sum_{i=1}^{p}\alpha_i\alpha_{p+j}W_{t-js-i} + a_t \\ & - \sum_{i=1}^{q}\alpha_{p+P+i}a_{t-i} - \sum_{i=1}^{Q}\alpha_{p+P+q+i}a_{t-is} \\ & + \sum_{j=1}^{Q}\sum_{i=1}^{q}\alpha_{p+P+i}\tilde{\alpha}_{p+P+q+j}a_{t-js-i},\end{aligned} \tag{16}$$

$$\begin{aligned}F_t = {} & c_0 + \sum_{i=1}^{p}c_i|W_{t-i}| + \sum_{i=1}^{P}c_{p+i}|W_{t-is}| \\ & + \sum_{j=1}^{P}\sum_{i=1}^{p}c_i c_{p+j}|W_{t-js-i}| + c_{p+P+1}|a_t| \\ & + \sum_{i=1}^{q}c_{p+P+1+i}|a_{t-i}| + \sum_{i=1}^{Q}c_{p+P+q+1+i}|a_{t-is}| \\ & + \sum_{j=1}^{Q}\sum_{i=1}^{q}c_{p+P+1+i}c_{p+P+q+1+j}|a_{t-js-i}|.\end{aligned} \tag{17}$$

Simultaneously, Z_t represents the tth observation, and h-level is the threshold value representing the degree to which the model should satisfy all the data points $Z_1, Z_2, \ldots, Z_t$. A choice of the h value influences the widths c of the fuzzy parameters

$$Z(Z_t) \geqslant h \quad \text{for } t = 1, 2, \ldots, k. \tag{18}$$

The index t refers to the number of nonfuzzy data used for constructing the model. On the other hand, the fuzziness S included in the model is defined by Eq. (17).

The problem of finding the fuzzy seasonal ARIMA parameters was formulated as the following linear programming problem:

$$\begin{aligned}
&\text{minimize } \; S = \sum_{t=1}^{k} F_t \\
&\text{subject to} \\
&\quad E_t + (1-h)F_t \geqslant W_t, \quad t = 1, 2, \ldots, k, \\
&\quad E_t - (1-h)F_t \leqslant W_t, \quad t = 1, 2, \ldots, k, \\
&\quad c_i \geqslant 0 \quad \text{for all } i = 1, 2, \ldots, p + P + q + Q + 1.
\end{aligned} \tag{19}$$

In the SARIMA model, if possible, at least 50 and preferably 100 observations or more should be used [1]. This makes the number of the LP constraint functions to be twice the number of the observations, leading the LP model to be more complex. The method we develop here to combine the advantages of two methods is comprised of two basic phases that let the number of constraint functions remain the same as the number of observations (the concept derived by Savic and Pedrycz [10]):

Phase I: Determining the type of FSARIMA model by using observations. Fitting the SARIMA model by using the available information about the center points of the observations, i.e., input data is considered nonfuzzy. The results of Phase I, the optimum solution of the parameter, $\alpha^* = (\alpha_1, \alpha_2, \ldots, \alpha_{p+P+q+Q})$, is used as one of the input data sets in Phase II.

Phase II: Determining the minimal fuzziness by using the same criterion as in Eq. (18), but without α being a vector of decision variable. The FSARIMA model is an interval prediction model. The model is

$$\tilde{W}_t = \text{Eq. (16)} \pm \text{Eq. (17)}. \tag{20}$$

In order that the model include all possible conditions, when data include a significant difference, the FSARIMA processes it as possibly happening, so as to produce c_j with a wide spread. Ishibuchi and Tanaka [7] suggest deleting the data around the model upper bound and lower bound, and then formulating the fuzzy regression model. Here the FSARIMA model also uses the same method.

There are some constraints when we use the FSARIMA model, as follows:

(a) The FSARIMA model cannot be used to forecast the future value when the p and P equal zero. This is because the white noise is the residual of the actual value and it does not exist in the future.
(b) The FSARIMA model cannot be used when the linear programming of Phase II has the situation that all of the spreads are equal to zero.

4. Experimental results

In the following, the performance of the FSARIMA model is compared with other models using two seasonal time series: the total production value of Taiwan machinery industry, and the sales volume of soft drinks quoted from Montgomery [9]. In Section 4.1, the results of the total production value of Taiwan machinery industry are described. Section 4.2 describes the soft drink time series. Section 4.3 describes the differences between SARIMA, FSARIMA and Watada's fuzzy time series.

4.1. Production value of machinery industry in Taiwan

The machinery industry in Taiwan has made steady progress over the past decade, playing a critical supporting role as the foundation of Taiwan's overall manufacturing industries. In addition, it is itself a major exporting industry. Projecting into the future, the potential of the machinery industry is almost limitless, considering the postulations of automation, systemization, and precision for the integral manufacturing industry, and the backgrounds of current machinery industry is as follows [5,6]:

(a) The development of the machinery industry began from the assembly of parts, and the repair and maintenance of machinery then maturing into one of the main supplier countries of world machinery industry. For example, the scale of exports of fans and blowers has already topped all others, while carpentry machines, sewing machines, and office machines are being ranked third.

(b) Taiwan leads the world in exports of ventilation fans, which are increasingly important devices in today's information industry. In addition, the development of precision machinery, semiconductor manufacturing equipment, high-tech anti-pollution equipment, and crucial machinery parts are being actively sponsored under government promotion. By taking advantage of the domestic market to push for the localization of equipment, as well as enhancing the proportion of domestically made crucial parts, thorough research and development for the precision machinery industry can be established. In addition, a strategic alliance system can be set up in coordination with other industries in order to expand overseas markets, and to facilitate the cooperation with partners for international technology and infrastructure investment.

(c) With the development of the semiconductor industry, the industry for manufacturing semiconductor equipment provides vast business opportunities for Taiwan. However, in view of domestic input, the level of our skill is restricted to a certain peripheral ability on the later part of the manufacturing process, thus the stage is considered embryonic as we are far short of the manufacturing equipment to design and the ability to manufacture. However, Taiwan's government is now vigorously pushing to turn Taiwan into a machinery manufacturing center for the Asia–Pacific region, actively promoting the development of IC manufacturing equipment, wafer slicing and packaging equipment, as well as detection equipment for semiconductors, it is hoped that a self-production rate of 25% for this kind of equipment can be achieved in five years. With the employment of the domestic market to push for the localization policy of such machinery, there are abundant possibilities for development.

(d) Of those manufacturers dedicated to the machinery industry, 95% of them are of medium and small enterprises; in 95% of these enterprises the number of their staff employed is less than 100. With regard to business turnover, an approximate 90% of the entrepreneurs have an annual turnover of less than NT$50 million.

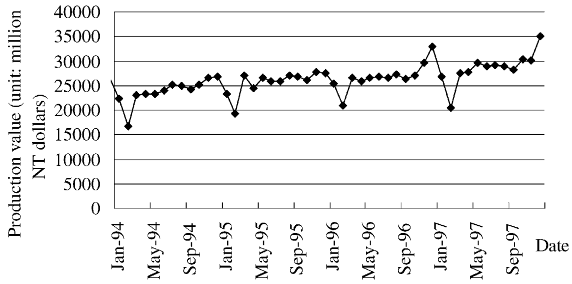

Fig. 1. Total production value of Taiwan Machinery Industry (Jan. 1994 to Dec. 1997).

(e) The modes of production within the industry vary, but most of them involve assembly of parts produced elsewhere.

(f) The entrepreneurs in the machinery industry are mainly focused on overseas markets, with 60–70% of their market share in Mainland China, the United States, and South-east Asia. And the rate of reliance on imports remains high since the rate of self-containment is less than 30%. Thus, the expansion of domestic market waits to be explored under active government initiative.

(g) In terms of production value, the machinery industry comprises as much as 5% of the manufacturing industry, but its expansion growth in export value is far less than that of the integral manufacturing industry.

From the preceding data, it can be seen that, although there are very strong future business opportunities in the machinery industry, there might not be any immediate breakthrough in this area. However, Taiwan's government is now vigorously pushing to turn Taiwan into a machinery manufacturing center in the Asia Pacific region with active drives being rendered to promote IC manufacturing equipment, wafer slicing, packaging equipment, etc. According to the above description, the forecasting of the total production value of Taiwan's machinery industry is suitable for time series forecasting. As Fig. 1 shows, the time series data of the total production value of Taiwan's machinery industry in the period from January 1994 to December 1997 showed strong seasonality and growth trends. This experiment used a data set from January

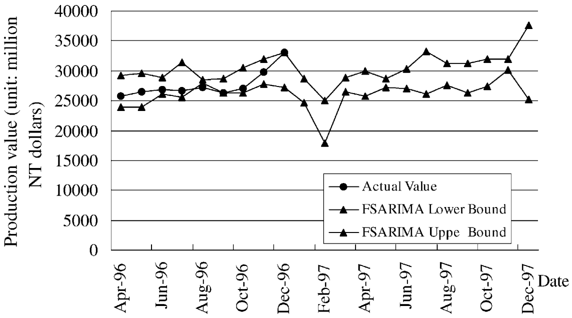

Fig. 2. The result of the estimated value of the FSARIMA model.

1994 to December 1996 to build the models, and forecast for the period from January 1997 to December 1997.

The FSARIMA model formulation is as follows:

Phases I: *Fitting the* SARIMA $(p,d,q)(P,D,Q)_s$ *model*. Using SAS package software, we acquire the best model of the production value of machinery industry which is SARIMA$(1,1,0)(0,1,1)_{12}$,

$$W_t = -0.2588W_{t-1} + a_t - 0.73997a_{t-12},$$

$$W_t = Z_t - Z_{t-1} - Z_{t-12} + Z_{t-13}. \tag{21}$$

Phase II: *Determining the minimal fuzziness*. From Phase I, we can get the FSARIMA model as

$$W_t = \langle\alpha_1, c_1\rangle W_{t-1} + \langle 1, c_2\rangle a_t + \langle\alpha_2, c_2\rangle a_{t-12},$$

$$W_t = Z_t - Z_{t-1} - Z_{t-12} + Z_{t-13}. \tag{22}$$

Set $(\alpha_0, \alpha_1) = (-0.259, -0.740)$ and $h = 0$. The following linear interval model is obtained and its results are shown in Fig. 2.

$$W_t = \langle -0.259, 0.548\rangle W_{t-1} + \langle 1, 0\rangle a_t + \langle -0.740, 1.640\rangle_{a_{t-12}},$$

$$W_t = Z_t - Z_{t-1} - Z_{t-12} + Z_{t-13}. \tag{23}$$

Using Eq. (23), we forecast the future production value of the machinery industry over the next six months, with the results as shown in Fig. 2. We find that the predictions are clarified.

At the same time, we apply the SARIMA model and Watada's fuzzy time series model [18] to forecast the production value of the machinery industry, and show this in Table 1.

Based on the empirical results of this application, we find that the prediction interval of the FSARIMA model is narrower than the 95% confidence interval of the SARIMA model and Watada's fuzzy time series model. From this application we can see that FSARIMA is the most effective method to forecast the production value of the machinery industry.

4.2. The soft drinks time series

In order to demonstrate the performance of the FSARIMA model, the authors applied the models to another time series, the monthly sales volume of soft drinks from Montgomery's book *Forecasting*

Table 1
Forecasted production value of the machinery industry by using the SARIMA model, the FSARIMA model and Watada's fuzzy time series model

Date	Actual value	SARIMA $(1,1,0)(0,1,1)_{12}$		FSARIMA		Watada's fuzzy time series	
		95% CI Lower bound	95% CI Upper bound	Lower bound	Upper bound	Lower bound	Upper bound
Jan-97	26910	15133	38288	24690	28731	18542	9010
Feb-97	20489	9558	33316	17930	24943	13763	24231
Mar-97	27489	15284	39969	26453	28800	20273	30741
Apr-97	27669	15072	405586	25672	29959	19283	29751
May-97	29737	14789	41076	27160	28705	20284	30752
Jun-97	29053	15143	42200	27110	30233	20389	30857
Jul-97	29279	15795	43603	26166	33232	20684	31152
Aug-97	29020	15177	43715	27622	31270	21184	31652
Sep-97	28251	14120	43371	26336	31155	20544	31012
Oct-97	30288	14671	44617	27338	31949	20852	31320
Nov-97	30188	15742	46368	30128	31981	22773	33241
Dec-97	35099	15736	470275	25194	37570	23866	34334

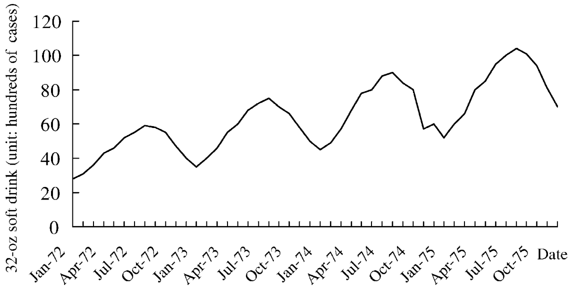

Fig. 3. Monthly sales of a 32-oz drink in hundreds of cases.

and Time Series Analysis, [9, p. 364]. The time series demonstrates growth trend and seasonality, as is shown in Fig. 3.

The FSARIMA model formulation is as follows:

Phase I: *Building the SARIMA model*. The time series data was pre-processed using logarithmic transformation, first-order regular differencing, and first-order seasonal differencing in order to stabilize the variance and remove the growth trend and seasonality. The authors used the SAS statistical package to formulate the SARIMA model. Akaike Information Criterion (AIC) [1,20] was used to determine the best model. The derived model is $\mathrm{ARIMA}(1,1,0)(0,1,0)_{12}$, and the equation is

$$(1+0.73B)(1-B)(1-B^{12})Z_t = a_t. \tag{24}$$

Phase II: *Determining the minimal fuzziness*. Let $w_t = Z_t - Z_{t-1} - Z_{t-12} + Z_{t-13}$, we get the model that is in Eq. (24). The results are shown in Table 2.

$$\tilde{w}_t = \langle 1, 1.63\rangle a_t + \langle -0.73, 0.80\rangle w_{t-1}. \tag{25}$$

Based on the empirical results of this application, we find that the prediction interval of the FSARIMA model is narrower than the 95% confidence interval of the SARIMA model.

The basic concept of SARIMA is used to formulate the model and find α^* by FSARIMA. The output of FSARIMA is fuzziness leading to the assumption of white noise (a_t). This make FSARIMA require fewer observations than SARIMA. In the real world, the environment is uncertain, and we must use limited amounts of data to forecast future situations in a short time. Under these circumstances, FSARIMA is more satisfactory than SARIMA. There are several aspects which indicate that FSARIMA is the most appropriate tool, as follows:

(a) It provides the decision makers the best and worst possible situations.
(b) The required observations are less than those required by the SARIMA model (prefer more than 100).

4.3. Comparison between SARIMA, FSARIMA and fuzzy time series

A comparison between SARIMA, FSARIMA and fuzzy time series [18] models are described as following and are shown in Table 3.

(a) The theoretical foundation of the SARIMA method is founded on the probability distribution of statistics, and the relationship between input and output is of precise function. In addition, the input and output information can only be managed through the relationship of their information, thus massive amounts of information would be required. A method of this kind is beneficial for observing information that has trendal growth and seasonal cycles, with merely acceptable cost and expense.
(b) The FSARIMA method is revised from the SARIMA method, and is based on the concept of fuzzy regression, as the residual difference between the prediction value and observation value is brought about because of the uncertainty of parameter, in other words, the parameter is a fuzzy number. Also, the probability distribution of the coefficient can be obtained from the distribution of information, thus the input of the model is of a certain domain embracing observation value, that is FSARIMA is a kind of prediction for the domain. Such a method is able to deal with the information observed, if that information is endowed with trendal growth and periodic cycle, and its cost is not excessive.
(c) Watada's fuzzy time series uses the theory of fuzzy method, employs probability or fuzzy information management with regard to the information observed, and defines them as fuzzy numbers. Furthermore, the time series model is construed as the fuzzy function of time, while the parameter is construed as the fuzzy number. This method is capable of dealing with

Table 2
Forecasted sales value of soft drinks using the SARIMA and the FSARIMA model

Date	Actual value	SARIMA $(1,1,0)(0,1,1)_{12}$		FSARIMA	
		95% CI Lower bound	95% CI Upper bound	Lower bound	Upper bound
Jan-75	52	39.12	55.68	35.09	91.62
Feb-75	60	49.32	60.54	39.54	106.38
Mar-75	66	55.502	71.99	42.60	126.94
Apr-75	80	69.002	83.26	50.74	157.46
May-75	85	69.59	100.55	51.62	172.64
Jun-75	95	76.19	95.73	57.60	201.46
Jul-75	100	74.39	107.78	58.69	218.15
Aug-75	104	81.74	113.94	61.86	240.64
Sep-75	101	61.84	117.78	58.99	242.01
Oct-75	94	58.56	116.666	54.86	235.31
Nov-75	81	32.38	108.636	45.68	205.55
Dec-75	70	42.68	99.36	38.10	179.00

Table 3
Comparison between SARIMA, FSARIMA and Watada's fuzzy time series models

	SARIMA	FSARIMA	Fuzzy time series
Theory	Probability of statistic	Fuzzy regression	Fuzzy regression
The relationship of input and output	Previous function	Fuzzy function	Fuzzy function
Forecasted interval	Provide confidence interval	Provide possibility distribution	Provide possibility distribution
Treatment of trend	Yes	Yes	Yes
Treatment of seasonal cycle	Yes	Yes	Yes
Number of observations	At least 50 and preferably 100 or more	Less than SARIMA	Less than SARIMA
Unit cost of forecast	Low	Low	Low

information observations that have trendal growth as well as periodic cycles, and only an insignificant amount of observation is required for it.

5. Conclusions

Based on the basic concepts of the SARIMA model and Tanaka's fuzzy regression model, we combine the advantages of these two methods to present a new method (FSARIMA) and apply it to forecast the production value of Taiwan's machinery industry and the sales volume of the soft drink. From the empirical results of the production value of Taiwan's machinery industry, we find that the prediction capability of the SARIMA model and the FSARIMA model are both rather encouraging but the preference of the fuzzy time series is not. In addition, the interval of the proposed method is narrower than SARIMA, and all of the three models have the capacity to handle growth trends and seasonal cycles, with the unit cost of forecasting being relatively low. From the empirical results of the sales volume of soft drinks, we find that the prediction capability of the FSARIMA model is rather encouraging and the interval of the proposed method is narrower than SARIMA.

In the real world, the environment is uncertain and we must use limited amount of data to provide future forecasts in a short period. For this kind of situation, the FSARIMA is more satisfactory than the SARIMA. The FSARIMA appears to be the most appropriate tool according to the following two primary advantages:

(a) It provides the decision makers with best and worst possible situations.
(b) The number of required observations is less than required by the SARIMA model.

References

[1] G.P. Box, G.M. Jenkins, Time Series Analysis: Forecasting and Control, Holden-Day, San Francisco, CA, 1976.
[2] S.M. Chen, Forecasting enrollments based on fuzzy time series, Fuzzy Sets and Systems 81 (3) (1996) 311–319.
[3] D. Dubois, H. Prade, Operations on fuzzy numbers, Internat. J. Systems Sci. 9 (1978) 613–629.
[4] D. Dubois, H. Prade, in: Theory and Applications, Fuzzy Sets and Systems, Academic Press, New York, 1980.
[5] Institute of Mechanical Industry in Industrial Technology Research Institute, 1997, Present Position and Trend Analysis of Mechanical Industry, Research Report in Industrial Technology Research Institute, 1998.
[6] Institution for Economic Research, Taiwan Year Book of Mechanical Industry, Research Report in Institution for Economic Research, 1996.
[7] H. Ishibuchi, H. Tanaka, Interval regression analysis based on mixed 0–1 integer programming problem, J. Japan Soc. Industrial Engng. 40 (5) (1988) 312–319.
[8] P.J.M. Laarhoven, W. Pedrycz, A fuzzy extension of Saaty's priority theory, Fuzzy Sets and Systems 11 (1983) 229–241.
[9] Montgomery, D.C.L.A. Johnson, J.S. Gardiner, Forecasting and Time Series Analysis, McGraw-Hill, New York, 1990, p. 364.
[10] D.A. Savic, W. Pedrycz, Evaluation of fuzzy linear regression models, Fuzzy Sets and Systems 39 (1) (1991) 51–63.
[11] Q. Song, B.S. Chissom, Fuzzy time series and its models, Fuzzy Sets and Systems 54 (3) (1993) 269–277.
[12] Q. Song, B.S. Chissom, Forecasting enrollments with fuzzy time series—Part I, Fuzzy Sets and Systems 54 (1) (1993) 1–9.
[13] Q. Song, B.S. Chissom, Forecasting enrollments with fuzzy time series—Part II, Fuzzy Sets and Systems 62 (1) (1994) 1–8.
[14] H. Tanaka, Fuzzy data analysis by possibility linear models, Fuzzy Sets and Systems 24 (3) (1987) 363–375.
[15] H. Tanaka, H. Ishibuchi, Possibility regression analysis based on linear programming, fuzzy regression analysis, in: J. Kacprzyk, M. Fedrizzi (Eds.), Omnitech Press, Warsaw and Physica-Verlag, Heidelberg, 1992, pp. 47–60.
[16] H. Tanaka, S. Uejima, K. Asai, Linear regression analysis with fuzzy model, IEEE Trans. Systems, Man Cybernet. 12 (6) (1982) 903–907.
[17] F.M. Tseng, G.H. Tzeng, H.C. Yu, B.J.C. Yuan, Fuzzy ARIMA model for forecasting the foreign exchange market, Fuzzy Sets and Systems 118 (2000) 9–19.
[18] J. Watada, Fuzzy time series analysis and forecasting of sales volume, fuzzy regression analysis, in: J. Kacprzyk, M. Fedrizzi (Eds.), Omnitech Press, Warsaw and Physica-Verlag, Heidelberg, 1992, pp. 211–227.
[19] L.A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning, Parts 1–3, Inform. Sci. 8 (1975) 199–249; 8 (1975) 301–357; 9 (1976) 43–80.
[20] H. Akaike, Maximum likelihood identification of Gaussian auto-regressive moving average models, Biometrika 60 (1973) 255–266.