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# A model of non-homogeneous damped electromagnetic wave and heat equation in ferrite materials

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**Abstract**

This study uses a closely coupled model to treat the core loss of ferrite by the combination of non-homogeneous damped electromagnetic wave and heat equation. The heat dissipation of ferrites is caused by the core loss, which is a summation of magnetic, dielectric and eddy current losses. Explicit finite difference method solves the coupled equations to calculate core loss and compares it with the measured results. Those results show that this method can be used to analyze electromagnetic and thermal field with temperature dependence of ferrites. © 2002 Elsevier Science B.V. All rights reserved.

*Keywords:* Magneto-thermal effect; Explicit finite difference method; Loss

## 1. Introduction

The iron loss of ferrite is directly converted into heat and causes the rise in temperature. There are so many different models to present the core loss phenomenon. The conceptual work on loss is developed into three parts [1–3], hystersis loss, eddy current loss and residual loss. Sakaki [4–7] presented the equivalent loss resistance of magnetic cores and developed the dynamic power loss concept. The dominant parameters of core loss are electrical conductivity, permeability and permittivity. These studies used the finite element method solved Maxwell's equations and the magnetic loss, dielectric loss and eddy current loss which had been calculated separately depending on the electromagnetic field distribution on the thermal field. Nelson [8] presented a quasi-steady thermal analysis of a two-dimensional model of coupled magneto-thermal equations for soft ferrites.

The inference of loss in ferrite is not only the exciting field but also the thermal dependent properties. Thus it is important to put both exciting conditions and the function of thermal dependent properties into the calculation of magneto-thermal modeling in order to describe the loss behaviors of ferrites. This paper tries to use a coupled modeling of transient electromagnetic and thermal distribution with the exciting frequencies ranging from 100kHz to 10 MHz. The exciting condition is to keep constant the product of frequency and flux density ($f \times B$). The core loss of samples is also measured and compared with the calculated results.

## 2. Modeling

### 2.1. Electromagnetic field equations

Maxwell's curl equations in differential form for ferrite materials, which are homogeneous, linear, isotropic, source-free and lossy medium, are considered. Due to circular symmetry, the magnetic field **H** is a function of $(r, z, t)$ and in $\theta$-direction as shown in Fig. 1. Here, $\sigma$ is the electric conductivity. $\mu', \mu'', \varepsilon', \varepsilon''$ are the real and imaginary parts of complex permeability and permittivity, respectively.

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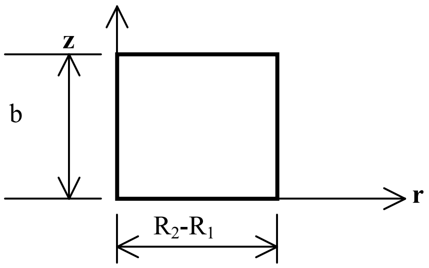


Fig. 1. The cross-sectional area coordination.

The equation is written as

$$\frac{\partial^2 H}{\partial z^2}+\frac{\partial^2 H}{\partial r^2}=\mu'\sigma\frac{\partial H}{\partial t}+\varepsilon'\mu'\frac{\partial^2 H}{\partial t^2}. \quad (1)$$

Assuming that the total field is composed of the induced $H_I$ and exciting $H_E = NI/2\pi r$, the non-homogeneous damped wave equation is written as

$$\frac{\partial^2 H_I}{\partial z^2}+\frac{\partial^2 H_I}{\partial r^2}+\Psi(r,t)=\mu'\sigma\frac{\partial H_I}{\partial t}+\varepsilon'\mu'\frac{\partial^2 H_I}{\partial t^2}, \quad (2)$$

where $\Psi(r,t)$ is the non-homogeneous term due to $H_E$ and the Dirichlet conditions on boundaries.

The electric field $E$ is

$$\nabla\times H=\varepsilon'\frac{\partial E}{\partial t}+\sigma E. \quad (3)$$

### 2.2. Iron loss calculation

There are three expressions of losses in a ferrimagnetic material. In fact, the electromagnetic material parameters in complex form are equations of polynomial function of temperature. These are denoted as magnetic permeability $\mu(T)=\mu'(T)-j\mu''(T)$, electric permittivity $\varepsilon(T)=\varepsilon'(T)-j\varepsilon''(T)$ and electrical conductivity $\sigma=\sigma(T)$. These losses are

$$P_M=\oiint_v \mu''\omega\|H\|^2\,dv, \quad (4)$$

$$P_D=\oiint_v \varepsilon''\omega(\|E_r\|^2+\|E_z\|^2)\,dv, \quad (5)$$

$$P_E=\oiint_v \sigma(\|E_r\|^2+\|E_z\|^2)\,dv. \quad (6)$$

The total loss $P_C$ is the summation of magnetic loss ($P_M$), dielectric loss ($P_D$) and eddy current loss ($P_E$)

$$P_C=P_M+P_D+P_E. \quad (7)$$

### 2.3. Thermal field

The unsteady two-dimensional heat equation is written as

$$\lambda\left(\frac{\partial^2 T}{\partial r^2}+\frac{\partial^2 T}{\partial z^2}\right)+P_C=\rho C_p\frac{\partial T}{\partial t}, \quad (8)$$

where $\rho$(4900 kg/m$^3$) is the density, $C_P$(0.72 kJ/(kg K)) is the specific heat, $\lambda$(3.5 W/(m K)) is the thermal conductivity and $\alpha^2=\lambda/(\rho c_p)$. The heat source $q$ is due to the iron loss, which is the consequence of the varying material properties in an electromagnetic field. Air convection boundary conditions are on the outer surface and no other heat flux occurs across the surface. Finally, the heat equation can be solved by integration of the equation in space and time intervals.

### 2.4. Numerical calculating steps

An explicit finite difference method solves the coupled mathematical equations.

*Step* 1: Assign an exciting frequency, calculating number ($N$) and count number ($n$).

*Step* 2: Initial conditions are $n=0$, temperature $=T_0$, time $=0$, $\mu=\mu(T_0)$, $\varepsilon=\varepsilon(T_0)$ and $\sigma=\sigma(T_0)$.

*Step* 3: Solve electromagnetic field $H$, $E$ and losses.

*Step* 4: Solve thermal field and generate a new temperature distribution $T(n+1)$.

*Step* 5: The criterion is $[T(n+1)T(n)]/T(n)\leqslant 0.001°C$, then stop the calculation and go to Step 1 of next frequency. Otherwise continue to the next time step $n+1$ (Step 6).

*Step* 6: The new materials properties are fixed due to the new temperature distribution, then Step 3.

*Step* 7: End.

## 3. Experimental procedure

Measured samples were prepared by conventional powder metallurgical process. The powders contain $Fe_2O_3$, MnO and ZnO in a molar ratio of 54.2 : 37.3 : 8.5 with the addition of 450 ppm of CaO and 150 ppm of $SiO_2$. The powders were pressed into toroids of 20 mm (OD) × 10 mm (ID) × 5 mm ($t$) and disks of 11 mm ($\varphi$) × 2.5 mm ($t$). The green compacts were sintered at 1200°C for 3 h in air and cooled through three equilibrium conditions, with an oxygen partial pressure of 0.5%(HF5), 1%(HF1)and 3%(HF3), down to 900°C then cooled with nitrogen.

Iron loss and permeability are measured by Ryowa MMS-0375 Iron Loss Measuring System [5] at constant product of frequency and flux density ($fB = 25$ kHz T) from 100 kHz to 10 MHz. Resistivity $\rho$, permittivity $\varepsilon$ are measured with HP4192A by InGa electrodes in the same frequency range.

## 4. Result and discussion

Fig. 2 shows the resistivity, with frequency, of samples with different $O_2$% to make the $\rho_{HF5} < \rho_{HF1} < \rho_{HF3}$ and to conform with Koops' [9] model so that they can be find the permittivity for calculation.

Fig. 3 shows that the iron loss and permeability vary with frequency of HF1 in both calculated and measured results. It is shown that those results are consistent but the frequency is above 8 MHz, Since the samples reveal large damping of magnetic properties in this frequency range.

Table 1 shows the loss rate of magnetic, conductive and dielectric loss of computer-simulated values and measured results of samples in the same frequencies. This shows that the magneto-thermal model is consistent with the measured results in the frequency lower than 8 MHz.

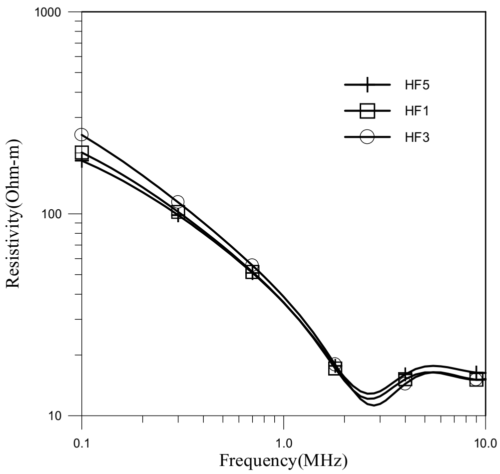


Fig. 2. The resistivity of HF5, HF1 and HF3

Fig. 4 shows the steady state temperature distribution of HF1 calculated by this model. It is reasonable and can be used to predict the temperature distribution of power ferrites.

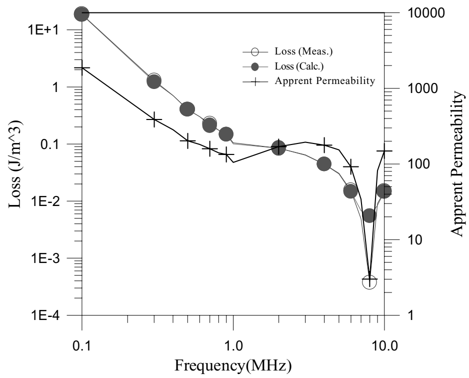


Fig. 3. The iron loss varies with frequency of HF1.

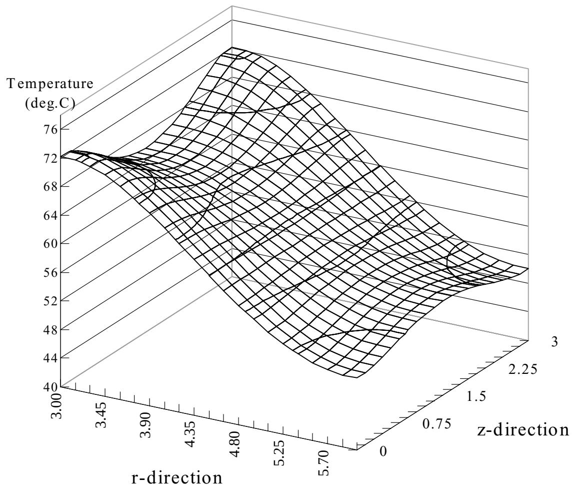


Fig. 4. Steady state temperature distribution of HF1, 200 kHz, 125 mT.

Table 1
Rate of magnetic, conductive and dielectric loss of computer-simulated values of samples

| | $F$ (MHz) | $P_M$ (J/m$^3$) | $P_D$ (J/m$^3$) | $P_C$ (J/m$^3$) | Cal. (J/m$^3$) | Meas. (J/m$^3$) |
|---|---|---|---|---|---|---|
| | 0.03 | 0.1912 | 0.1271 | 6.0905 | 6.4088 | 6.4327 |
| HF5 | 3.0 | 0.0497 | 0.0023 | 0.0064 | 0.0584 | 0.0585 |
| | 9.0 | 0.0020 | 0.0048 | 0.0011 | 0.0079 | 0.0081 |
| | 0.03 | 0.3712 | 0.1225 | 5.7342 | 6.2279 | 6.2771 |
| HF1 | 3.0 | 0.0579 | 0.0025 | 0.0051 | 0.0655 | 0.0662 |
| | 9.0 | 0.0023 | 0.0049 | 0.0009 | 0.0081 | 0.0087 |
| | 0.03 | 0.4308 | 0.1221 | 5.5006 | 6.0535 | 6.1547 |
| HF3 | 3.0 | 0.0633 | 0.0028 | 0.0040 | 0.0701 | 0.0712 |
| | 9.0 | 0.0018 | 0.0045 | 0.0011 | 0.0074 | 0.0079 |

## 5. Conclusion

A new method for analyzing electromagnetic and thermal fields with temperature dependence of the material properties has been presented. A finite difference method is used to solve the coupled mathematical equations of electromagnetic and thermal fields. The calculation result shows the main loss is conductive loss due to the large exciting field at low frequency in the range 100kHz–1 MHz. Magnetic loss will mainly dominate the iron loss at a frequency between 1.0 and 6MHz. Dielectric loss dominates the iron loss at a high frequency of 6–10 MHz. This calculation also gets the electromagnetic field and temperature distribution of sample at the same time. It was shown that this magneto-thermal model is a useful tool for the power ferrite design.

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