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Analytical prediction of the cogging torque and torque constant of a hybrid stepping motor

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Abstract

We present an analytic method to efficiently calculate the torque constant and cogging torque of a hybrid stepping motor. The feature of the proposed method is its computational efficiency in that it is a useful tool for the motor design which is aimed at reducing the level of cogging torque or increasing the torque constant. This method is verified by comparing with the 3D finite element method and some experiments. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Torque constant; Cogging torque; FEM; Maxwell stress

1. Introduction

Since hybrid stepping motors have a complicated magnetic structure, the conventional way for predicting their torque constant and cogging torque employs the finite element method (FEM) [1], which consumes much computation time. It is then desirable to develop an analytic method that can partially replace FEM. This paper tries to meet this goal and presents an analytic method. The proposed method is to solve algebra equations which then requires only a little computation in comparison with FEM. Initially, we make some assumptions to predict the flux paths, and develop the ways to calculate the permeances of air-gap and the magnetic distribution in the motor. Finally, the motor parameters can be easily obtained. This analytic method will be verified by comparing 3D FEM and experiments.

2. Analytical prediction method

We consider a two-phase, bipolar hybrid stepping motor. First, we need to develop an analytic method for calculating magnetic flux density B_m, which requires the following assumptions:

(A1) The relative permeability of the iron is infinite.
(A2) The stator and rotor slots are simplified to a rectangular shape.
(A3) The air-gap permeance is calculated according to an assumed field pattern, where the magnetic flux lines are orthogonal to the interface of the air-gap and steel.

The equivalent electric circuit (see Ref. [2]) of the motor is depicted in Fig. 1. The total permeance is $G_t = G_m + G_U + G_L$, where G_m is the permeance of the magnet, G_U and G_L are those of motor in the upper and lower air-gap, respectively. Note that the upper air-gap means the region in the enclosure of the upper rotor stack and stator. We equally divide the upper air-gap into n distributed points so that $G_U = \sum_{k=1}^{n} G_{Uk}$. In the following, subscript "k" denotes the value at point k. According to Kirchoff's Voltage Law, the magnetic field B_{mUk} in the upper air-gap at the kth point is

$$B_{mUk} = \frac{G_{Uk}}{2G_m + G_U}\frac{B_r A_m}{dA_{gk}} \tag{1}$$

where B_r is the residual permeance of the magnet, A_m is the axial area of the magnet, and dA_{gk} is the area normal to the flux line at point k, which is equal to $R_i l_a 2\pi/n$ with l_a as the active length of rotor upper stack and R_i as the

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inner radius of the stator (see Fig. 2). By the definition of permeance, we can calculate G_m and G_{Uk} as

$$G_m = \frac{\mu_m A_m}{l_m}, \quad (2)$$

$$G_{Uk} = \mu_0 \frac{dA_{gk}}{l_k}, \quad (3)$$

where μ_m is the permeability of the magnet, l_m is the thickness of the magnet, and l_k is the length of the flux line through point k. As an example, if k is point a_1 in Fig. 2, then l_k is the length of the flux line through a_1, a_2, and a_3: $l_k = l_g + \pi r_s/2 + \pi r_r/2$ with l_g as the air-gap distance [3]. Note that B_{mLk} in the lower air-gap can be obtained in the same way. B_{mUk} and B_{mLk} are then used to predict the cogging torque and torgue constant as follows.

Each phase of a two-phase stepping motor has its own torque constant (denoted by K_{tA} and K_{tB}). The stator has N_t tooth groups so that each phase has $N_t/2$ ones. Each tooth group has N turns winding. The winding of the same phase is in series. Thus, we apply Lenz's law and obtain

$$K_{tA} = K_{eA} = f_c \frac{N_t}{2} N \times \frac{d\phi}{d\theta_r}, \quad (4)$$

where θ_r is the rotation angle, ϕ is the magnetic flux which is perpendicular to a winding turn per tooth group, and f_c is a correcting factor. Note that K_{eA} is the voltage constant of phase A and is equal to K_{tA} in the mks unit [2]. The same method applies to predicting K_{tB}. According to our experience, coil windings and flux lines have an include angle of almost 60° . This implies that the correction factor should be about 0.5, i.e., the effect of flux lines perpendicular to the coil windings is about a half of that in the ideal case.

Maxwell Stress Law [3] provides the rule to calculate the net cogging torque for any rotor displacement (cf. Fig. 2):

$$T_c = 2l_a \sum_{j=1}^{Q_s} \left[\int_0^{b_0/2} \left(-\frac{B^2_{r,a(r_s)}}{2\mu_0} \right) (R_i + r_s)\, dr_s + \int_{b_0/2}^{0} \left(\frac{B^2_{r,c(\bar{r}_s)}}{2\mu_0} \right) (R_i + \bar{r}_s)\, d\bar{r}_s \right]_j, \quad (5)$$

where k is $1, 2, \ldots, n$, b_0 is the width of the stator's slot opening, Q_s is the number of slots in the stator, and the subscripts $a(r_s)$ and $c(\bar{r}_s)$ in B_r are, respectively, the points with the distances of r_s and $\bar{r}_s$ from the right and left corners of the stator slot, respectively (cf. Fig. 2).

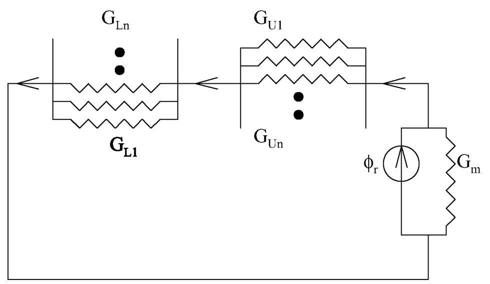

Fig. 1. The equivalent electric circuit of a stepping motor.

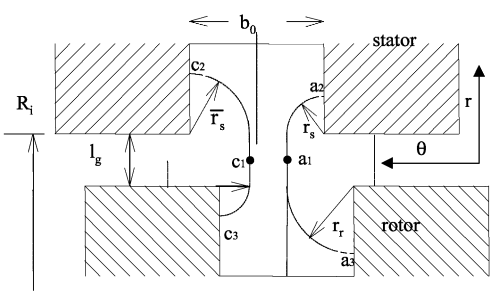

Fig. 2. The assumed flux path of motor in air-gap.

3. Comparison

The proposed analytic method for the torque constant and cogging torque will be verified by comparing with FEM and experiments. We take a two-phase motor as an example. The step angle is 1.8°, the rotor's outer diameter is 21.97 mm, the stator's inner diameter is 22.1 mm, the height of the motor is 10 mm, the energy product of Nd–Fe–B permanent is 33 MGOe, and its thickness is 2 mm. Fig. 3 shows the computed values of the B_m field in air-gap by both analytic method and FEM. It can be seen that the analytic method over-estimates B_m in higher permeance regions, but has a good agreement in lower permeance. This mismatch comes from assumption (A3), which causes errors in higher permeance regions for the case of the tooth width approximately equal to the slot width. This problem can be solved by modifying Eq. (1) for B_{mUk} in high

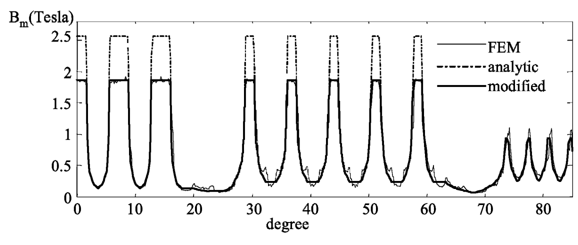

Fig. 3. Field distribution in air-gap by FEM, analytic, and modified analytic method.

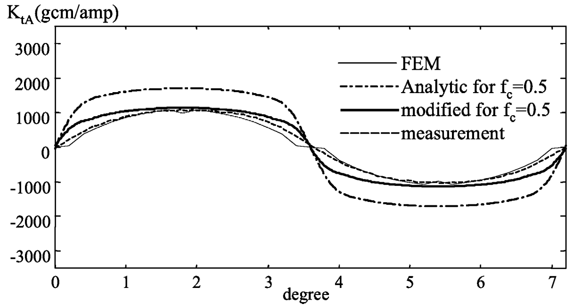

Fig. 4. Torque constant about FEM, analytic and modified analytic method, measurement.

permeance regions as

$$B_{mUk} = \frac{\bar{G}_{Uk}}{2G_m + \bar{G}_U}\frac{B_r A_m}{dA_{gk}},$$

in high permeance regions of upper air-$g$ap, (6)

where $\bar{G}_U$ and $\bar{G}_{Uk}$ are still obtained by utilizing Eq. (3) but replacing l_g with $2l_g$ for calculating the flux path length l_k. The same applies to predicting B_{mLk}, Fig. 3, indicates that the results of the modified method for B_m match those of FEM very well.

We then use the analytic method to compute the torque constants with the analytic and modified analytic values of B_m in Fig. 3. Fig. 4 compares these computed torque constants with the measurement values and those of FEM. Both the measurement and FEM results are very close. The results of Eq. (4) modified analytic method with $f_c = 0.5$ shown in Fig. 4 are then very close to the measurement values.

Fig. 5 reveals that there is a similar agreement in the tendency of cogging torque between the analytic method and FEM. According to Eq. (6), we can obtain the same value about cogging torque by analytic and modified analytic method [3]. We use torque meter to measure the peak value of cogging torque, which is from 30 to 80 gcm, it hints that analytic method (or modified analytic method) and FEM both can provide us enough information to predict the parameter. It should be remarked that the harsh profile of the FEM means only the limitation of our FEM package and does not imply that it is a worse method. Our FEM package allows us to have 10 meshes within 1.8°, otherwise a serious numerical error occurs.

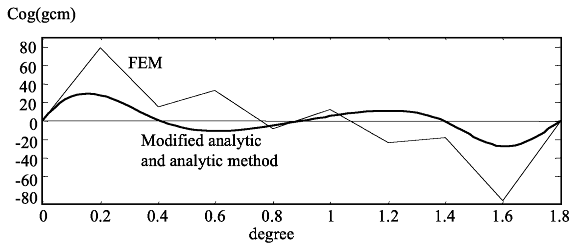

Fig. 5. Comparison of cogging torque by FEM, analytic and modified analytic method.

4. Conclusion

An analytical method for predicting hybrid stepping motor's cogging torque and torque constant has been developed and verified. Although only the two-phase motor was considered in this paper, the proposed method can be extended to n-phase hybrid stepping motor by replacing $N_t/2$ with N_t/n in Eq. (4). The technique offers acceptable accuracy in the peak value of the cogging torque and the torque constants. Based on the developed analytical model, the effects of varying the leading design parameters on the cogging torque and torque constant can be investigated.

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