Dynamic Optimal Groundwater Management with Inclusion of Fixed Costs

Chin-Tsai Hsiao[1] and Liang-Cheng Chang[2]

Abstract: Obtaining optimal solutions for groundwater resources planning problems, while simultaneously considering both fixed costs and time-varying pumping rates, is a challenging task. Application of conventional optimization algorithms such as linear and nonlinear programming is difficult due to the discontinuity of the fixed cost function in the objective function and the combinatorial nature of assigning discrete well locations. Use of conventional discrete algorithms such as integer programming or discrete dynamic programming is hampered by the large computational burden caused by varying pumping rates over time. A novel procedure that integrates a genetic algorithm (GA) with constrained differential dynamic programming (CDDP) calculates optimal solutions for a groundwater resources planning problem while simultaneously considering fixed costs and time-varying pumping rates. The GA determines the number and locations of pumping wells with operating costs then evaluated using CDDP. This study demonstrates that fixed costs associated with installing wells significantly impact the optimal number and locations of wells.

DOI: 10.1061/(ASCE)0733-9496(2002)128:1(57)

CE Database keywords: Genetic algorithm; Constrained differential dynamic programming; Groundwater management; Fixed cost.

Introduction

Groundwater, as an underground reservoir, is a valuable water resource with many diverse domestic, agricultural, and industrial uses. Owing to its importance, ensuring the sustainable use of groundwater has been extensively studied (Gorelick 1983; Lin and Yang 1991; Yeh 1992; Pezeshk et al. 1994; Takahashi and Peralta 1995). Many optimization techniques have been employed in the planning stages of groundwater management, including linear programming (Aquado and Remson 1974; Molz and Bell 1977), nonlinear programming (Murtagh and Saunders 1982; Gorelick et al. 1984; Ahlfeld et al. 1988a; b), mixed-integer programming (Rosenwald and Green 1974), genetic algorithms (McKinney and Lin 1994; Wang and Zheng 1998), and differential dynamic programming (DDP) (Jones et al. 1987). Among these methods, DDP significantly reduces the dimensionality difficulties associated with nonlinear dynamic groundwater management problems (Jones et al. 1987, Chang et al. 1992; Culver and Shoemaker 1992).

Genetic algorithms (GAs) are heuristic programming methods capable of locating near-global optimal solutions for complex problems (Goldberg 1989). A single GA cycle, known as a ''generation,'' includes three genetic operators: reproduction, crossover, and mutation. GAs have found diverse applications in water resources management. Wardlaw and Sharif (1999) evaluated a GA for optimal reservoir system operations. Morshed and Kaluarachchi (2000) introduced three potential methods to enhance GAs. The fitness reduction method (FRM), search bound sampling method (SBSM), and optimal resource allocation guideline (ORAG). According to the results, an FRM increases the efficiency of a GA in handling constraints; an SBSM enhances the accuracy of a GA in solving problems with fixed costs; and an ORAG enhances the reliability of a GA by providing some convergence guarantee for a given computational resource. Earlier, Mckinney and Lin (1994) optimized groundwater management using a GA, and Cieniawski et al. (1995) addressed the problem of how to select a system of monitoring wells with a GA.

For groundwater management, total cost generally includes well installation (fixed costs) and pumpage (operating costs). Since the fixed cost function is discontinuous, fixed costs are frequently neglected in application of gradient-based optimization algorithms. The omission of fixed costs can lead to designs that rely on a large number of wells pumping at small rates over long time periods (McKinney and Lin 1995). Therefore, considering fixed costs significantly affects the optimal design of groundwater withdrawal systems, particularly when planning periods are short. Although a GA can easily incorporate the fixed costs associated with a groundwater management system, it is not conducive to dynamic optimization over time-varying variables (Culver and Shoemaker 1997).

The temporal nature of water-resource systems requires any simulation or optimization model to be dynamic in order to yield satisfactory results, unless the input assumptions justify a static system (Taghavi et al. 1994). For groundwater supply, the demand for groundwater may vary over time, particularly when the aquifer is operated in conjunction with the surface water system (Philbrick and Kitanidis 1998; Basagaoglu and Marino 1999). In addition, hydraulic head of the groundwater system may also vary seasonally. To accommodate these situations, time-varying pumping rates are required.

[1]Graduate Student, Dept. of Civil Engineering, National Chiao Tung Univ., 1001 TA Hsueh Rd., Hsinchu, Taiwan 300, ROC. E-mail: cthsiao@chang.cv.nctu.edu.tw

[2]Associate Professor, Dept. of Civil Engineering, National Chaio Tung Univ., 1001 TA Hsueh Rd., Hsinchu, Taiwan 300, ROC. E-mail: lcchang@chang.cv.nctu.edu.tw

Note. Discussion open until June 1, 2002. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on November 19, 1999; approved on April 24, 2001. This paper is part of the ***Journal of Water Resources Planning and Management***, Vol. 128, No. 1, January 1, 2002. ©ASCE, ISSN 0733-9496/2002/1-57–65/$8.00+$.50 per page.

The writers are unaware of any investigations reported in the scientific literature that simultaneously consider the fixed costs of installation wells and operating costs of time-varying pumping rates. Culver and Shoemaker (1997) applied quasi-Newtonian differential dynamic programming (QNDDP) to optimal groundwater reclamation, for which the treatment capital cost was related linearly to the extraction rate. However, their investigation did not include fixed costs for well installation. Although McKinney and Lin (1994, 1995) considered both fixed and operating costs in their objective function and applied GAs and mixed-integer nonlinear programming to solve the problem, they only assumed time-invariant pumping rates and steady-state conditions. Zheng and Wang (1999) integrated tabu search and linear programming for optimal design of groundwater remediation by accounting for both fixed and operating costs, but only time-invariant pumping rates were considered.

Wang and Zheng (1998) applied a GA and simulated annealing, coupled with the MODFLOW finite-difference groundwater flow model, for optimal groundwater remediation design over multiple management periods, including both fixed and operating costs. However, this study limited the maximum number of planning periods to four. This limitation was likely due to the exponential increase in computational expense of GAs and simulated annealing, with an increasing number of planning periods and corresponding decision variables and pumping rates. In this study, a novel management model is proposed that combines CDDP and GA to optimize groundwater basin development and management. By exploiting the advantages of both methods, the proposed model solves a groundwater supply problem that simultaneously considers both the fixed costs of well installation and the operating costs of time-varying pumping.

Formulation of Proposed Management Model

The management model contains an aquifer, that is a 2D confined system. The governing equation that describes groundwater movement is

$$\frac{\partial}{\partial x}\left(T_{xx}\frac{\partial h}{\partial x}\right)+\frac{\partial}{\partial y}\left(T_{yy}\frac{\partial h}{\partial x}\right)+\sum_{i\in I}u_i\delta(x_i,y_i)=S\frac{\partial h}{\partial t} \tag{1}$$

where h denotes the hydraulic head; T_{xx} and T_{yy} represent the principal components of transmissivity aligned along the x and y coordinate axes; I represents the set of pumping wells; S denotes the storage coefficient; u_i represents the pumping rate located at (x_i,y_i); and $\delta(x_i,y_i)$ is the Dirac delta function evaluated at (x_i,y_i). Eq. (1) is subject to the appropriate initial and boundary conditions. The simulator used herein is ISOQUAD (Pinder 1978), where the numerical solution is obtained by applying the Galerkin finite-element method for the space derivative and an implicit finite-difference scheme for the time derivative. The corresponding matrix equation (1) can be expressed as follows:

$$\left(A+\frac{B}{\Delta t}\right)h_{t+1}=\frac{B}{\Delta t}h_t+Pu_t-R \tag{2}$$

which can be further expressed as

$$h_{i+1}=Fh_t+Gu_t+z \tag{3a}$$

$$\mathbf{F}=\left(\mathbf{A}+\frac{\mathbf{B}}{\Delta t}\right)^{-1}\frac{\mathbf{B}}{\Delta t}; \tag{3b}$$

$$\mathbf{G}=\left(\mathbf{A}+\frac{\mathbf{B}}{\Delta t}\right)^{-1}\mathbf{P}; \tag{3c}$$

$$\mathbf{z}=-\left(\mathbf{A}+\frac{\mathbf{B}}{\Delta t}\right)^{-1}\mathbf{R} \tag{3d}$$

where **A**, **B**, are $n\times n$ matrices, which generally contain the hydrogeological parameters and are produced by the numerical procedure; **R** represents $n\times 1$ vectors associated with the boundary conditions; **P** is an $n\times m$ permutation matrix and is employed to map the well number onto the model global node numbering system; **F** denotes an $n\times n$ matrix; **G** represents an $n\times m$ matrix; **z** is an n vector; and n and m denote the total number of hydraulic heads and control variables, respectively. The matrix equation (3) is embedded in the management model and serves as a linear transfer function. The management model is then formulated as follows:

$$\min_{\substack{I\subset\Omega\\ u_s^i,\, i\in I,\, t=1,\ldots,T}} J(I)=\sum_{i\in I}\left\{c_1y^i(I)+\sum_{i=1}^{T}c_2u_t^i(I)[L_*^i(I)-h_{t+1}^i(I)]\right\} \tag{4}$$

subject to

$$h_{t+1}=\mathbf{F}h_t+\mathbf{G}(I)u_t(I)+z;\quad t=1,2,\ldots,T \tag{5}$$

$$h_{i+1}\geqslant h_{\min};\quad t=1,2,\ldots,T \tag{6}$$

$$\sum_{i\in t}u_t^i\geqslant d_t;\quad t=1,2,\ldots,T \tag{7}$$

$$u_{\min}^i\leqslant u_t^i(I)\leqslant u_{\max}^i;\quad t=1,2,\ldots,T,I\subset\Omega \tag{8}$$

where Ω is an index set defining all the candidate well locations within the aquifer, and I is a subset of Ω and is a possible network alternative (design). The upper index i denotes a well in the network design. The dimension of $\mathbf{G}(I)$, $u_t(I)$, $L_*(I)$, and $h_{t+1}(I)$ is adjusted according to the pattern of I. Eq. (4) represents the total cost associated with network alternative I. The first term in the objective function (4) represents the fixed costs of the well network in which c_1 represents the unit cost of well installation; $L_*^i(I)$ are the distances between the ground surface and the lower datum of the aquifer for each well; and $y^i(I)$ equal the depth of each well. The second term embodies the operational costs where c_2 denotes the cost coefficient of pumpage and is expressed as $c_2=\gamma\times c_3\times\Delta t$, with Δt the duration of pumping, γ the specific gravity, and c_3 the unit cost of electric power. The $u_t^i(I)$ are variable pumping rates at time t, and $h_{t+1}^i(I)$ denote hydraulic heads for each node at time $t+1$. The expression $L_*^i(I)-h_{t+1}^i(I)$ simply represents drawndown at pumping well i. Eq. (5), as derived from Eq. (3), represents the system dynamics relation in the optimization. Eq. (6) defines lower limits on hydraulic head to avoid damage caused by overpumping. Eq. (7) represents the requirement that total demand for groundwater supply must be satisfied. The upper limits of Eq. (8) denote the capacity of each well, while the lower limits can be applied to avoid well installation that has small pumping rates, which are obviously infeasible.

The groundwater management model defined by Eqs. (4)–(8) has two key elements. First, the search for optimal network alternatives is a discrete combinatorial optimization problem, thereby prohibiting the application of general gradient-based algorithms. Second, the system is dynamic and continuous, as indicated by Eq. (5) for each network alternative, and may cause excessive computational loads when applying a discrete-based algorithm

J. Water Resour. Plann. Manage. 2002.128:57-65.

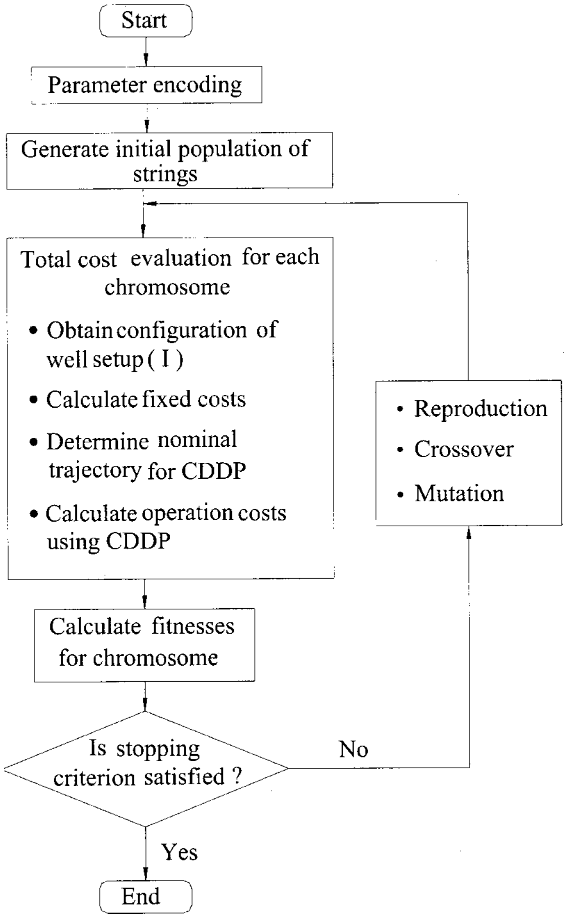

Fig. 1. Flowchart for groundwater management model

such as integer programming or discrete dynamic programming. Notably, combining the two elements makes the problem difficult to solve using conventional schemes.

Integration of GA and CDDP

When locations of pumping wells are predetermined, the CDDP algorithm is an efficient procedure for determining optimal pumping rates for each well. However, the optimal pumping rates are not a complete optimization in the management model because the number of wells and locations are prespecified. In terms of the problems with fixed costs where the number of wells and locations are considered as decision variables, CDDP has difficulty in solving the problem owing to the discontinuity of the fixed cost function. Therefore, this study integrates GA and CDDP to develop the groundwater management model defined by Eqs. (4)–(8), as illustrated in Fig. 1. The algorithm is a simple GA with CDDP embedded in the total cost evaluation and exhibits two key features. First, the discrete nature of searching for optimal well location network alternatives is accomplished by the GA. Second, the CDDP algorithm is used to calculate optimal operating costs for time-varying pumping associated with each network alternative (chromosome). These features are clarified by a sequential explanation of the algorithm as follows:

Step 0: Initialization

Encode the network alternatives as chromosomes and randomly generate an initial population. The population size in this study ranges from 50 to 100. A chromosome, I, is a binary string representing a network alternative, as indicated by Fig. 2. The length of the binary string is the total number of candidate wells. Each bit within the binary string corresponds to a candidate well. For a particular network alternative represented by a chromosome, a well is selected when the value of the corresponding bit (i) is one. Since the selection of wells is binary in nature, the encoding and decoding of the chromosome is straightforward. For example, the chromosome in Fig. 2 represents a network design that selects only three wells, and the well numbers are 3, 10, and 30.

Step 1: Evaluate Total Cost and Fitness Value for Each Chromosome

The chromosome described in step 0 can be represented mathematically in the form $I_k = x_1, x_2, \ldots, x_M$ where I_k denotes a chromosome in the population and M is the number of total candidate wells. Since each element x_l has a binary value of 1 or 0, the number of wells in this chromosome can be calculated as follows:

$$N_{\text{well}} = \sum_{l=1}^{M} x_l \tag{9}$$

When the number and locations of pumping wells are determined, the fixed costs are readily calculated and the problem then involves evaluating the optimal operating costs for the network design. According to Eqs. (4)–(8), when a network alternative is selected, the discrete and inseparable nature of the problem is eliminated and the optimization model can then be rewritten as follows:

$$\min_{\substack{l \in \Omega \\ u_t^i, i \in I, l=1,\ldots,T}} J(I) = \sum_{i \in I} \left\{ \sum_{t=1}^{T} c_2 u_t^i(I)[L_*^i(I) - h_{t+1}^i(I)] \right\} + C_{\text{fix}} \tag{10}$$

subject to Eqs. (5), (6), (7), and (8) where C_{fix}=a constant representing the fixed costs and does not affect the operating costs. Therefore, the CDDP can be used to evaluate the optimal operating costs for the selected network design.

The CDDP algorithm exceeds conventional dynamic programming (DP) and mathematical programming algorithms in computational efficiency (Murray and Yakowitz 1979; Jones et al. 1987) and does not require discretization of the state and control vector. Accordingly, CDDP overcomes the "curse of dimensionality," a serious limitation for conventional DP (Bellman and Dreyfus 1962). The CDDP enables a significant reduction in the "working" dimensionality of the algorithm over that of mathematical programming algorithms by taking advantage of the dynamic nature of groundwater hydraulic or water quality optimization problems through stagewise decomposition. On the contrary, mathematical programming algorithms do not exploit the sequential time structure of these problems (Jones et al. 1987).

0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0

Fig. 2. Chromosome representation (I)

J. Water Resour. Plann. Manage. 2002.128:57-65.

The CDDP used herein is a procedure suggested by Murray and Yakowitz (1979) and is a successive approximation technique for solving optimal control problems, iteratively determining the optimal solution to the problem stated in Eq. (10) subject to Eqs. (5)–(8). The CDDP algorithm requires a quadratic approximation of the original problem. By substituting Eq. (5) into Eq. (10), the objective function becomes a function of control and state variables with identical time index (t). The second-order Taylor's expansion then approximates the objective function on the nominal policy. Since Eq. (5) is linear, in this study the Hessian matrix of the approximated objection function is positive definite. The approximated quadratic objective function and other linear constraints, Eqs. (6)–(8), create a convex quadratic problem at every time step.

Quadratic programming in the backward and forward sweep is employed to resolve the series of quadratic problems. In the backward sweep, the state variables are considered as unknown parameters, and the optimal control laws, which are a function of the unknown parameters, are computed. While in the forward sweep, using the initial value of state variables and the transfer function, Eq. (5), the value of the state variables can be specified at each time step. The quadratic programming is reapplied to solve the problem, which is moving forward, and the optimal policy is revealed. Notably, the computed optimal policy becomes the nominal policy for the next iteration. Since quadratic problems are only an approximation of the original problem, iterations are required. A detailed discussion of the CDDP algorithm is provided in Murray and Yakowitz (1979), Chang (1986), Jones et al. (1987), and Chang et al. (1992).

After the optimal total cost $J(I)$ for each chromosome I is calculated, the fitness for each chromosome can be evaluated as

$$f(I)=C_{\max}-J(I) \tag{11}$$

where $C_{\max}$ denotes an arbitrarily large number. The procedure is repeated for all chromosomes in each generation. Therefore, the CDDP is embedded in the GA to calculate the optimal operating costs as also indicated in Fig. 1.

Several computational issues related to the application of GA are worth mentioning. First, the capacity limitation in Eq. (8) requires that the number of wells for each chromosome must be within the maximum and minimum number of wells; otherwise, no feasible solutions will be available, and the CDDP algorithm is not executed. The maximum and minimum number of wells can be determined as follows:

$$\text{maximum number of wells}=\max(d_t)/u_{\min} \tag{12}$$

$$\text{minimum number of wells}=\max(d_t)/u_{\max} \tag{13}$$

If the number of wells in a chromosome does not satisfy the constraints, then the fitness of this chromosome is assigned a small value.

Second, since the CDDP algorithm requires nominal policies for initialization, a systematic procedure must be developed. The nominal policies for each chromosome (that is, network design) are calculated by solving a series of quadratic problems forward in time. The problems are defined as follows:

For $t=1,...,T$

$$\min_{u_t(l),i\subset\Omega} D_t^I(I)s_t(I) \tag{14}$$

subject to

$$h_{t+1}=Fh_t+G(I)u_t(I)+z,\;\; h_1=\overline{h_1},\;\; I\cup\Omega \tag{15}$$

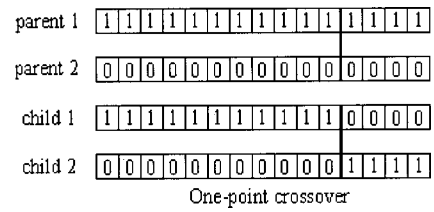

Fig. 3. Crossover operator

$$h_{t+1}\geqslant h_{\min} \tag{16}$$

$$\sum_{i\in I} u_t^i \geqslant d, \tag{17}$$

$$u_{\min}^i \leqslant u_t^i(I) \leqslant u_{\max}^i, I\subset\Omega \tag{18}$$

where $\overline{h_1}$ = initial hydraulic heads; $s_t(I)=h_{t+1}(I)-L_*(l)$ = vector of drawdowns. Incorporating Eq. (15) into Eq. (14) produces a quadratic form in u_i. Because Eqs. (16)–(18) are linear constraints, Eqs. (14)–(18) define a convex quadratic problem at each time step. Therefore, a standard quadratic programming can be used to obtain the decision variable vector. As stated previously, the quadratic problems are solved independently in the forward direction for each time step, t. A series of decision vectors, which is a nominal policy, can then be obtained. Eq. (14) implies that the computed policy minimizes the drawdown and satisfies the constraints, Eqs. (15)–(18), for all stages (time steps). Furthermore, the policy also satisfies the constraints (5)–(8) of the original problem. Therefore, if the network is designed adequately, the algorithm will produce a feasible solution. Otherwise, the fitness of this chromosome is assigned a small value, and the CDDP procedure is omitted. The quadratic programming technique used herein is a heuristic procedure for determining the nominal policies.

Step 2: Reproduce Best Strings

Reproduction is implemented in this study by using the roulette wheel approach. In roulette wheel reproduction, each chromosome has the probability $p_j(I)$ of being selected.

$$p_j(I)=\frac{f_j(I)}{\sum_{j=1}^{\text{pop}} f_j(I)} \tag{19}$$

where *pop* denotes the population size. This operation simulates natural selection, where a higher fitness value of a chromosome implies a higher probability that the chromosomes will survive. Therefore, the algorithm can converge to a set of chromosomes with high fitness values.

Step 3: Perform Crossover

Crossover involves random coupling of the newly reproduced strings, and each pair of strings partially exchanges information.

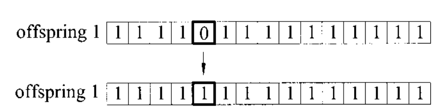

Fig. 4. Mutation operator

J. Water Resour. Plann. Manage. 2002.128:57-65.

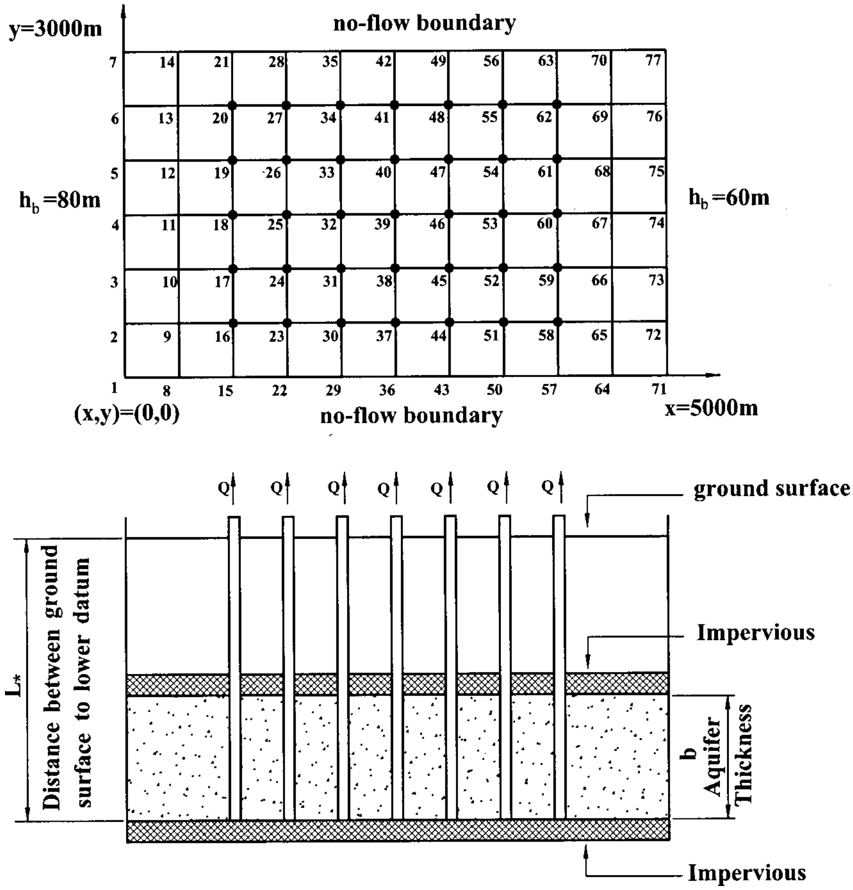

Fig. 5. Aquifer for water supply examples

Crossover aims to exchange gene information so as to produce new offspring strings that preserve the best material from two parent strings. In general, the crossover occurs at a certain probability (p_{cross}) so that it is performed on a majority of the population. In this study, we select one point crossover as shown in Fig. 3, where p_{cross} ranges from 0.5 to 1.0.

Step 4: Implement Mutation

Mutation restores lost or unexplored genetic material to the population to prevent the GA from prematurely converging to a local minimum. A mutation probability (p_{mutat}) is specified so that random mutations can be applied to individual genes. DeJong (1975) originally suggested that a mutation probability inversely proportional to the population size would prevent the search from locking onto a local optimum. This study follows DeJong's suggestion. Before implementing a mutation, a random number with uniform distribution is generated. If this number is smaller than the mutation probability, then mutation is performed; otherwise, mutation is disregarded. Notably, according to the specific probability, mutation changes a specific gene ($0 \rightarrow 1$ or $1 \rightarrow 0$) in the offspring strings produced by the crossover operation. An example of mutation is shown in Fig. 4 in the selected bit shown as the block is changed from 0 in the old string to 1 in the new string.

Table 1. Aquifer Properties of Example Application

Parameter	Value
Hydraulic conductivity	4.31×10^{-4} m/s
Storage coefficient	0.001
Porosity	0.2
Aquifer thickness (b)	50 m
L_*	120 m

Step 5: Perform Termination

After completion of steps 1–4, a new population is formed that requires evaluating the total cost as in step 1, which is used to assess the stopping criterion, which in turn is based on the change

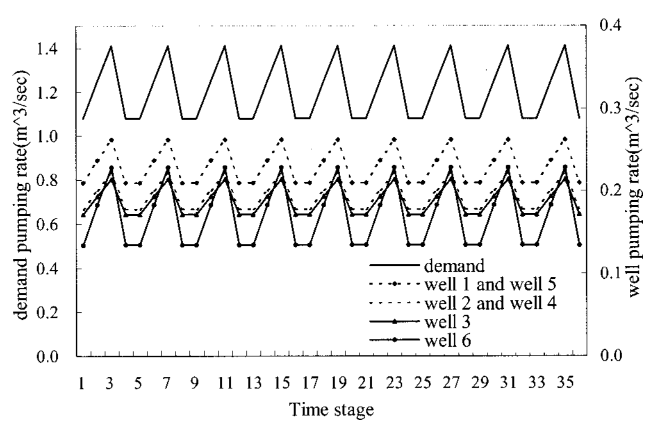

Fig. 6. Water demand for examples and optimal pumping rates for section 4.2

J. Water Resour. Plann. Manage. 2002.128:57-65.

Table 2. Optimal Solutions for Different Limits on Minimum Pumping Rate

Limits on minimum well pumping rates (m^3/s)	Optimal operating cost million	Calculated minimum pumping rate at first stage (m^3/s)	Optimal number of wells	Generations
0	$2.44278	0.000082	35	37
0.01	$2.44281	0.01	14	40

of either the objective function value (total cost) or the optimized parameters. If the user-defined stopping criterion is satisfied or when the maximum allowed number of generations is achieved, the procedure terminates; otherwise, return to step 1 to perform another cycle. The success and performance of the GA depend on several parameters: the population size, number of generations, and probabilities of crossover and mutation (McKinney and Lin 1994). Goldberg (1989) has suggested that a good GA performance requires the choice of high-crossover and low-mutation probabilities and a moderate population size. Therefore, solutions from a GA cannot be guaranteed to be optimal. However, GAs are robust and easy to hybridize with other optimization methods or simulation models.

Numerical Results

Several example problems are presented to demonstrate the effectiveness of the methodology integrating a GA and CDDP. All the examples are based on the same hypothetical system as depicted in Fig. 5, adapted from Chang et al. (1992). Several issues related to fixed costs and constraints on individual pumping wells are considered in this demonstration. The aquifer is assumed to be homogeneous, isotropic, and confined. The $3,000 \times 5,000$ m site is described with 77 finite-element nodes and 35 potential well locations with constant head and no-flow boundaries to circumvent the flow domain. Initial conditions on hydraulic head distribution prior to pumping are assumed to be in steady state with the aquifer properties listed in Table 1. In the management model, the planning horizon is divided into 36 stages over nine years. The total pumpage at each stage must satisfy the demand as shown in Fig. 6, with maximum and minimum well capacities of 0.5 and 0 or 0.01 m^3/s and minimum hydraulic head of 50 m.

Three examples are examined: no fixed costs; constant unit fixed costs, and varying unit fixed costs according to geological conditions. The value of coefficient c_3 in these examples is set at 0.045. The performance of all examples relies on proper setting of the crossover probability (p_{cross}), population size, and mutation probability (p_{mutat}). Numerical experiments indicate that, for p_{cross} in the range 0.5–1.0, population size between 50 and 100, and $p_{\text{mutat}} = 1/\text{population}$, the computation is likely to converge to an optimal or near-optimal solution within 22 to 43 generations. The solutions to the following examples are obtained using $p_{\text{cross}} = 0.8$ and a population size of 80 chromosomes.

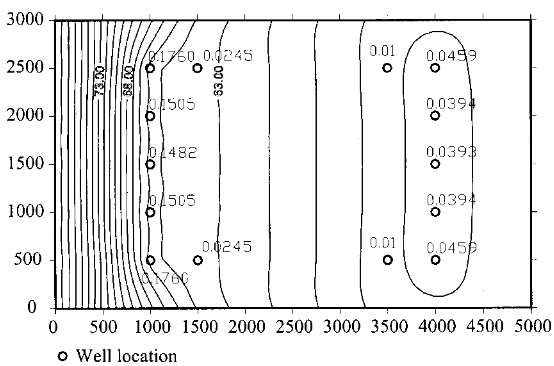

Fig. 7. Optimal head distribution and pumping rates at first time step for case of 0.01 minimum pumping rates (14 selected wells)

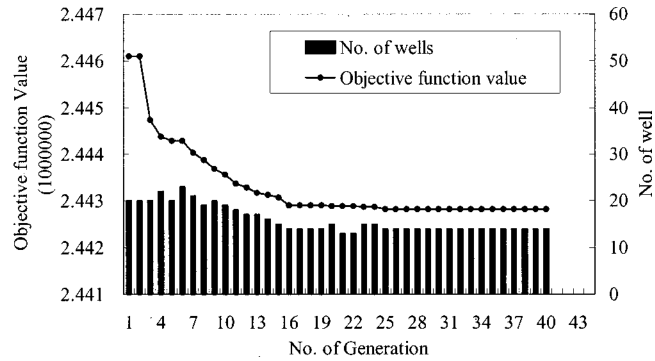

Fig. 8. Objective function values and number of wells versus number of generations

Effect of Omitting Fixed Costs

The first two cases investigate the consequences of omitting fixed costs in the planning of a groundwater supply system. The value of c_1 is zero, and two different minimum pumping constraints on individual wells of 0 and 0.01 m^3/s, respectively, are examined, with results summarized in Table 2. For the case of no fixed costs with a minimum pumping constraints of 0 m^3/s, the algorithm selects all 35 candidate wells, with several wells pumping at a small rate. Since small pumping rates are infeasible for practical applications, this finding corresponds to the statement of McKinney and Lin (1995) in their study on optimal aquifer remediation.

When the minimum pumping constraint is set at 0.01 m^3/s, the optimal number of well installations is reduced to 14. These results demonstrate that when omitting the fixed costs, the lower bound on pumpage strongly impacts the number of wells selected. However, since there are few physical or economic references, it is difficult to estimate suitable lower bounds. Fig. 7 illustrates the optimal hydraulic head distribution and pumping rates at the first time step for the case of 0.01 minimum pumping rate. Fig 8 summarizes the change of the value of the objective function and the number of wells for the optimal chromosomes in each GA generation. This simulation result indicates that solution converges after the 25th generation.

Table 3. Cost Comparison for Variation of Coefficient c_1

Coefficient c_1	$60.0 m^{-1}	$120.0 m^{-1}	$180.0 m^{-1}	$240.0 m^{-1}
Number of wells	8	7	6	5
Fixed cost ($)	57,600	100,800	129,600	144,000
Operating cost ($)	2,449,611	2,456,985	2,472,055	2,497,860
Total cost ($)	2,507,211	2,557,785	2,601,655	2,641,860
CPU time (s)	13,894	14,333	19,373	12,101

J. Water Resour. Plann. Manage. 2002.128:57-65.

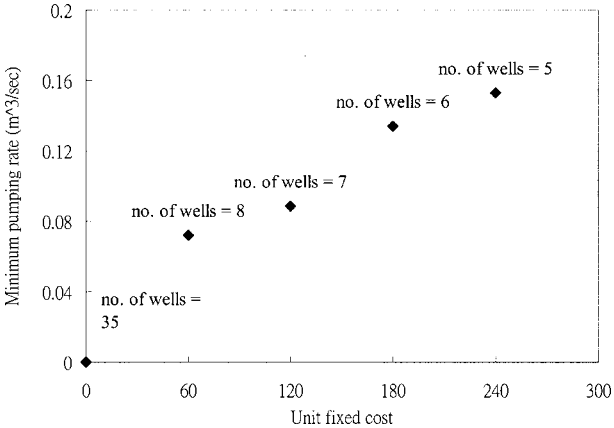

Fig. 9. Comparison of value of coefficient C_1 and minimum pumping rate

Effect of Uniform Fixed Cost on Number of Well Setup

This study also considers the impact of fixed costs given a range of values of the unit fixed cost. Within each case, the unit fixed cost of each well is assumed to be the same. Constraints on the individual minimum pumping rates were relaxed (that is, $u_{\min} = 0.0\,m^3/s$) for all cases. Other constraints and the system configuration are identical to those described previously. Total fixed cost can be estimated by multiplying well depth by the unit fixed cost. The optimal value of the objective function is the optimal total cost with both the fixed and operating costs considered. Table 3 summarizes the optimal total cost, optimal number of wells, and required CPU time with respect to distinct unit fixed cost c_1. These examples are calculated on a PC with an Intel Pentium II 300 MHz CPU.

According to Table 3, increasing the unit fixed cost increases the total fixed costs, operating costs, and total cost, but decreases the total number of wells. The relationship between the optimal number of wells and the unit fixed cost resembles that between the optimal number of wells and constraints on individual minimum pumping rate described previously. Fig 9 plots the relationship between unit fixed cost, the optimal number of wells, and the minimum pumping rates at the end of the first time step associated with each optimal solution. As expected, this figure reveals that increasing the unit fixed cost increases the calculated minimum pumping rates and, expectedly, reduces the number of wells.

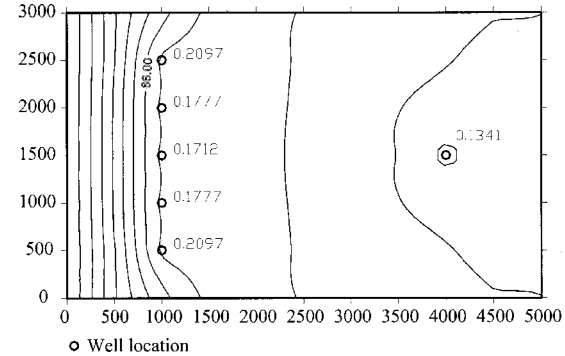

Fig. 10. Head distribution calculated using optimal well locations, optimal number of wells, and optimal stage 1 pumping rate for each well

The optimal head distribution at the first time step and the number and location of wells, assuming the unit fixed cost equals \$180.0 m^{-1}, are shown in Fig. 10. The number of wells of the optimal network is six, and the selected wells concentrate on the west side. The distribution of the location of wells is reasonable since hydraulic head is higher in the west region, thus requiring less pumping cost. Time-varying pumping rates for the six wells are shown in Fig. 6. Comparing the case of omitting fixed costs, the optimal number of wells is reduced from 14 to 6, the wells are repositioned, and the distribution of hydraulic head is altered.

Comparison of Total Cost with These Cases

The merits of considering both the fixed and operating costs are revealed by comparing the true total cost of the designed network for all cases considered. For the cases omitting fixed costs, the optimal network was determined based on only the operating costs; the operating costs are nearly the same, although the number of wells differs significantly (Table 2). The total cost can be estimated by adding the calculated operating costs to the fixed costs. Table 4 summarizes the total cost of the two networks given

Table 4. Total Costs for Varying Unit Fixed Cost (c_1) in Example 1

Unit fixed cost (c_1)	\$0 m^{-1}	\$60 m^{-1}	\$120 m^{-1}	\$180 m^{-1}	\$240 m^{-1}
Lower limits (0.0 m^3/s) (35 wells)	2.44 (M)	2.69 (M)	2.95 (M)	3.20 (M)	3.45 (M)
Lower limits (0.01 m^3/s) (14 wells)	2.45 (M)	2.55 (M)	2.65 (M)	2.75 (M)	2.85 (M)
Difference of total cost	−0.01 (M)	0.14 (M)	0.30 (M)	0.45 (M)	0.60 (M)

Note: M=million dollars.

Table 5. Comparison of Total Cost with and without Consideration of Fixed Costs in Optimization Model

Coefficient c_1	\$60 m^{-1}	\$120 m^{-1}	\$180 m^{-1}	\$240 m^{-1}
Total cost and well numbers considering fixed cost	2,507,211 (8 wells)	2,557,785 (7 wells)	2,601,655 (6 wells)	2,641,860 (5 wells)
Total cost and well numbers considering no fixed cost	2,694,780 (35 wells)	2,946,780 (35 wells)	3,198,780 (35 wells)	3,450,780 (35 wells)
Percentage of difference (%)	7.48	15.21	22.95	30.62

J. Water Resour. Plann. Manage. 2002.128:57-65.

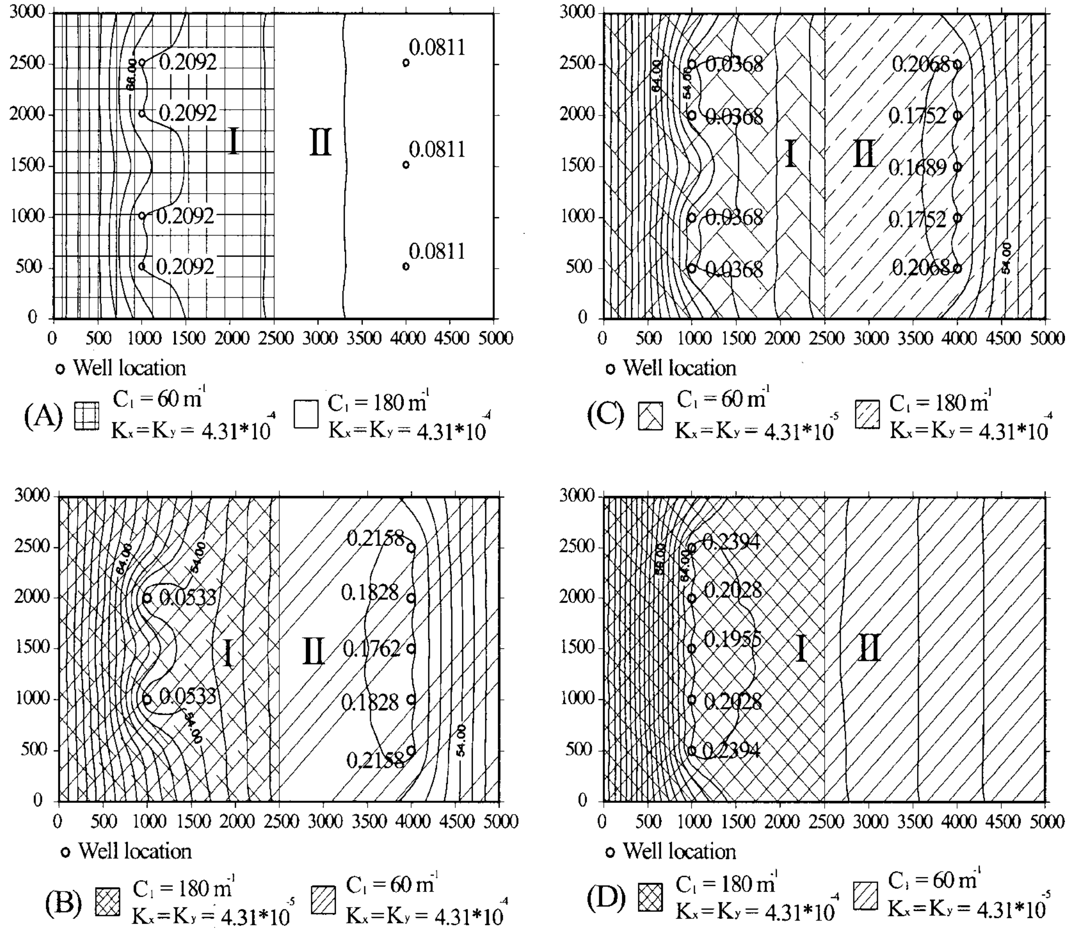

Fig. 11. Head distribution resulting from optimal network design and pumping scheme for first time step

a range of unit fixed costs. Contrary to the operating costs, the two cases differ from each other significantly when the unit fixed cost is high. Table 5 compares only the total cost of the network designation.

Table 5 reveals that for the value coefficient c_1 as \$180.0 m^{-1}, the total cost of the network for the case that does not consider fixed costs is 22.95% more than that which considers the fixed costs. This finding demonstrates the importance of considering fixed costs when the unit fixed cost is high.

Varying Unit Fixed Cost (c_1) According to Geological Conditions

The last example demonstrates the capability of this procedure to solve a problem with the unit fixed cost and hydraulic conductivity varying in space. Previously, the unit fixed cost and hydraulic conductivity were assumed to be the same in the study area; however, this is unlikely to be true due to typical heterogeneous geological conditions. Therefore, this example spatially varies the value of c_1 and hydraulic conductivity to simulate the consequence of geological heterogeneity. To simplify the analysis, the unit fixed costs (c_1) selected are \$60.0 and \$180.0 m^{-1}, and hydraulic conductivity is 4.31×10^{-5} and 4.31×10^{-4} m/s.

Figure 11 presents two geological zones and the corresponding unit fixed costs and also shows the optimal head distribution, number, and location of wells. Comparing Figs. 11(b) and 11(d) reveals that when unit fixed costs remain the same in the flow domain, the heterogeneity affects the number and location of wells. In addition, according to Figs. 11(a and c) the permeability in zone II is higher than zone I, and the optimal well number increases by two. This finding indicates that a high permeability zone is desired when placing a well. More interestingly, according to Fig. 11(d), when unit fixed costs do not significantly differ from permeability, the latter dominates the behavior of the well setup more than the former. Table 6 lists the fixed costs, operating costs, and total costs for the four cases.

Conclusion

This study presents a novel scheme that integrates a GA with CDDP to determine the optimal solution of a groundwater management problem while simultaneously considering fixed costs and time-varying operating costs. The decision variables involve the number and location of wells as well as the time-varying

Table 6. Fixed Costs, Operating Costs, and Total Costs for Examples of Varying Unit Fixed Cost (c_1) According to Geological Conditions

Case	Fixed cost (\$)	Operating cost (\$)	Total Cost (\$)
A	93,600	2,484,603	2,578,203
B	79,200	3,208,020	3,287,220
C	136,800	3,181,356	3,318,156
D	108,000	2,500,937	2,608,937

J. Water Resour. Plann. Manage. 2002.128:57-65.

pumping rates. The number and location of wells together form a discrete optimal combinatorial problem, and the time-varying pumping rates increase the computational complexity. The proposed model can incorporate binary variables simply into the optimization problem.

Simulation results demonstrate that the solution of the problem while omitting the fixed costs may be far from the true optimal network if the fixed costs are high. The fixed costs can reduce the number of wells in the network. This can also be achieved by assigning minimum pumping constraints on each well, but the appropriate pumping constraints are difficult to estimate in practice without economic or physical references. Therefore, the inclusion of fixed costs is important in the groundwater management problem, and the proposed algorithm provides a valuable reference for decision makers.

Although this study considers only groundwater supply, the proposed algorithm can be further extended to groundwater remediation planning. The computational loading required for the solution of groundwater remediation models increases with the complexity of the problem. Using the property of homogeneity of a GA can attain the speedup of convergence. That is, the chromosome does not require further calculation when it has been calculated in a previous generation of the GA. In addition, parallel implementations of the GA and the simulation model are likely to be required.

Acknowledgments

The writers would like to thank the National Science Council of the Republic of China for financially supporting this research under Contract No. NSC 88-2611-E-009-004. The writers would also like to express thanks for the useful comments of Dr. John W. Labadie, the editor, and anonymous reviewers.

References

Ahlfeld, D. P., Mulvey, J. M., and Pinder, G. F. (1988b). ‘‘Contaminated groundwater remediation design using simulation, optimization, and sensitivity theory, 2: Analysis of a field site.’’ *Water Resour. Res.*, 24(5), 443–452.

Ahlfeld, D. P., Mulvey, J. M., Pinder, G. F., and Wood, E. F. (1988a). ‘‘Contaminated groundwater remediation design using simulation, optimization, and sensitivity theory. 1: Model development.’’ *Water Resour. Res.*, 24(5), 431–441.

Aguado, E., and Remson, I. (1974). ‘‘Ground-water hydraulics in aquifer management.’’ *J. Hydraul. Div.*, 100(1), 103–118.

Basagaoglu, H., and Marino, M. A. (1999). ‘‘Joint management of surface and ground water supplies.’’ *Ground Water,* 37(2), 214–222.

Bellman, R. E., and Dreyfus, S. E. (1962). *Applied dynamic programming*, Princeton University Press, Princeton, N.J.

Chang, S. C. (1986). ‘‘A hierarchical, temporal decomposition approach to long horizon optimal control problems,’’ PhD dissertation, Univ. of Connecticut, Storrs.

Chang, L. C., Shoemaker, C. A., and Liu, P. L. F. (1992). ‘‘Optimal time-varying pumping rates for groundwater remediation: Application of a constrained optimal control algorithm.’’ *Water Resour. Res.*, 28(12), 3157–3171.

Cieniawski, S. E., Eheart, J. W., and Ranjithan, S. (1995). ‘‘Using genetic algorithm to solve a multiobjective groundwater monitoring problem.’’ *Water Resour. Res.*, 31(2), 399–409.

Culver, T. B., and Shoemaker, C. A. (1992). ‘‘Dynamic optimal control for groundwater remediation with flexible management periods.’’ *Water Resour. Res.*, 28(3), 629–641.

Culver, T. B., and Shoemaker, C. A. (1997). ‘‘Dynamic optimal groundwater reclamation with treatment capital costs.’’ *J. Water Resour. Plan. Manage.*, 123(1), 23–29.

DeJong, K. A. (1975). ‘‘An analysis of the behavior of a class of genetic adaptive systems.’’ PhD dissertation, Univ. of Michigan, Ann Arbor.

Goldberg, D. E. (1989). *Genetic algorithm in search, optimization, and machine learning*, Addison-Wesley, Reading, Mass.

Gorelick, S. M. (1983). ‘‘A review of distributed parameter groundwater management modeling methods.’’ *Water Resour. Res.*, 19(2), 305–319.

Gorelick, S. M., Voss, C. I., Gill, P. E., Murray, W., Saunders, M. A., and Wright, M. H. (1984). ‘‘Aquifer reclamation design: The use of contaminant transport simulation combined with nonlinear programming.’’ *Water Resour. Res.*, 20(4), 415–427.

Jones, L., Willis, R., and Yeh, W. W.-G. (1987). ‘‘Optimal control of nonlinear groundwater hydraulics using differential dynamic programming.’’ *Water Resour. Res.*, 23(11), 2097–2106.

Lin, X., and Yang, Y. (1991). ‘‘The optimization of groundwater supply system in SHI JIAZ-HUANG City, China.’’ *Water Sci. Technol.*, 24(11), 71–76.

McKinney, D. C., and Lin, M. D. (1994). ‘‘Genetic algorithm solution of groundwater management models.’’ *Water Resour. Res.*, 30(6), 1897–1906.

McKinney, D. C., and Lin, M. D. (1995). ‘‘Approximate mixed-integer nonlinear programming methods for optimal aquifer remediation design.’’ *Water Resour. Res.*, 31(3), 731–740.

Molz, F. J., and Bell, L. C. (1977). ‘‘Head gradient control in aquifers used for fluid storage.’’ *Water Resour. Res.*, 13(14), 795–798.

Morshed, J., and Kaluarachchi, J. J. (2000). ‘‘Enhancements to genetic algorithm for optimal ground-water management.’’ *J. Hydrologic Eng.*, 5(1), 67–73.

Murray, D. M., and Yakowitz, S. J. (1979). ‘‘Constrained differential dynamic programming and its application to multireservoir control.’’ *Water Resour. Res.*, 15(5), 1017–1027.

Murtagh, B. A., and Saunders, M. A., (1982). ‘‘A projected Lagrangian algorithm and its implementation for sparse nonlinear constraints.’’ *Mathematical programming study 16*, North-Holland, Amsterdam, The Netherlands.

Pezeshk, S., Helweg, O. J., and Oliver, K. E. (1994). ‘‘Optimal operation of ground-water supply distribution systems,’’ *J. Water Resour. Plng. and Mgmt.*, 120(5), 573–586.

Philbrick, C. R., and Kitanidis, P. K. (1998). ‘‘Optimal conjunctive-use operations and plans,’’ *Water Resour. Res.*, 34(5), 1307–1316.

Pinder, G. F. (1978). ‘‘Galerkin finite element models for aquifer simulation.’’ *Rep. 78-WR-5*, Dept. of Civ. Eng., Princeton Univ., Princeton, N.J.

Rosenwald, G. W., and Green, D. W. (1974). ‘‘A method for determining the optimum location of wells in a reservoir using mixed-integer programming.’’ *Soc. Pet. Eng. J.*, 14(1), 44–54.

Taghavi, S. A., Howitt, R. E., and Mariño, M. A. (1994). ‘‘Optimal control of ground-water quality management: Nonlinear programming approach,’’ *J. Water Resour. Plan. Manage.*, 120(6), 962–982.

Takahashi, S., and Peralta, R. C. (1995). ‘‘Optimal perennial yield planning for complex nonlinear aquifers: Methods and examples,’’ *Adv. Water Resour.*, 18(1), 49–62.

Wang, M., and Zheng, C. (1998). ‘‘Ground water management optimization using genetic algorithms and simulated annealing: Formulation and comparison.’’ *J. Am. Water Resour. Ass.*, 34(3), 519–530.

Wardlaw, R., and Sharif, M. (1999). ‘‘Evaluation of Genetic Algorithms for Optimal Reservoir System Operation.’’ *J. Water Resour. Plan. Manage.*, 125(1), 25–33.

Yeh, W. W.-G. (1992). ‘‘Systems analysis in ground-water planning and management.’’ *J. Water Resour. Plan. Manage.*, 118(3), 224–237.

Zheng, C., and Wang, P. P. (1999). ‘‘An integrated global and local optimization approach for remediation system design.’’ *Water Resour. Res.*, 35(1), 137–148.

J. Water Resour. Plann. Manage. 2002.128:57-65.