Optik 113, No. 6 (2002) 267–271

http://www.urbanfischer.de/journals/optik

Optik Optics
International Journal for Light and Electron Optics

# Femtosecond soliton propagation in an optical fiber

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**Abstract:** An accurate wave equation beyond the slowly varying envelope approximation for femtosecond soliton propagation in an optical fiber is derived by the iterative method. The derived equation contains higher nonlinear terms than the generalized nonlinear Schrödinger equation obtained previously. For a silica-based weakly guiding single mode fiber, it is found that those more higher-order nonlinear terms, whose coefficients are proportional to the second-order dispersion parameter, are much smaller than the shock term. The 2.5-fs fundamental solitons is numerically simulated by using the generalized nonlinear Schrödinger equation and the full Maxwell's equations. Comparing these two results, we have found that the generalized nonlinear Schrödinger equation well describes the propagation of the pulse even containing a single optical cycle.

**Key words:** Femtosecond soliton slowly varying envelope approximation – nonlinear Schrödinger equation

## 1. Introduction

As the rapid development of the laser technology, ultrashort optical pulses containing only a few optical cycles and with pulsewidth less than 10 fs have been generated [1]. The propagation of an ultrashort pulse in a fiber is usually described by the generalized nonlinear Schrödinger equation based on the slowly varying envelope approximation (SVEA) [2]. The validity of this equation becomes questionable when the pulse contains only a few optical cycles. To resolve this problem, several approaches, which do not make the SVEA, have been employed [3–7]. The full-vector nonlinear Maxwell's equations have been solved by direct integration [3, 4], but this is very time consuming. By using an operator method and assuming that the nonlinearity is small [5, 6] or by representing the electric field as the superposition of monochromatic waves [7], modified wave equations can be derived. In addition, an iterative method has been used to derive a wave equation for ultrashort pulses [8].

In this paper, we will derive a wave equation using the electric field expansion of [7], the iterative method of [8] and the order of magnitude considerations of [9]. Through the first iteration, we obtain a wave equation, which has four more higher-order nonlinear terms than the equation obtained previously [5–8]. For a silica-based weakly guiding single mode fiber, we have found those more higher-order nonlinear terms, the coefficients of which are proportional to the second-order dispersion parameter, are much smaller than the shock term. We numerically investigate 2.5-fs fundamental solitons by using the generalized nonlinear Schrödinger equation and the full Maxwell's equations [3, 4]. After we compare these two results, we have found that the generalized nonlinear Schrödinger equation well describes the propagation of the pulse even containing a single optical cycle.

Received 25 January 2002; accepted 28 May 2002.

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## 2. Derivation of the wave equation

We derive the wave equation for a femtosecond pulse propagating in a single-mode fiber with a third-order nonlinearity. The electric field $E(x, y, z, t)$ which propagates in the fiber along the $z$-direction can be expressed by

$$E(x, y, z, t) = F(x, y) \cdot \phi(z, t) , \tag{1}$$

where $F(x, y)$ is the normalized linear eigenfunction of the mode excited in the fiber and $\phi(z, t)$ can be further represented as a superposition of monochromatic waves,

$$\phi(z, t) = \frac{1}{2\pi} \int \varphi(z, \omega) \exp\{\mathrm{i}[\beta(\omega) z - \omega t]\} \, \mathrm{d}\omega , \tag{2}$$

where $\beta(\omega) = n(\omega)\omega/c$ is the mode propagation constant at frequency $\omega$, $c$ is the velocity of light in vacuum, and $n(\omega)$ is effective refractive index. It is customary to express $\phi(z, t)$ by

$$\phi(z, t) = A(z, t) \exp[\mathrm{i}(\beta_0 z - \omega_0 t)] , \tag{3}$$

where $A(z, t)$is the field envelope, $\omega_0$ is the angular frequency of the carrier wave, and $\beta_0 = \beta(\omega_0)$. From the Maxwell's equations, we obtain the wave equation

$$\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = -\mu_0 \frac{\partial^2 P_L}{\partial t^2} - \mu_0 \frac{\partial^2 P_{NL}}{\partial t^2} , \tag{4}$$

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where $\mu_0$ is the permeability in vacuum, and the linear part $P_L$ and the nonlinear part $P_{NL}$ of the induced polarization are related to electric field $E(x,y,z,t)$through the following equations:

$$P_L(x,y,z,t) = \varepsilon_0 \int_{-\infty}^{\infty} \chi^{(1)}(t-t')\, E(x,y,z,t)\,\mathrm{d}t' , \quad (5)$$

$$P_{NL}(x,y,z,t) = \varepsilon_0 \iiint_{-\infty}^{\infty} \chi^{(3)}(t-t_1,\, t-t_2, t-t_3) \times E(x,y,z,t_1)\, E(x,y,z,t_2)\, E^*(x,y,z,t_3)\,\mathrm{d}t_1\,\mathrm{d}t_2\,\mathrm{d}t_3 , \quad (6)$$

where $\varepsilon_0$ is the vacuum permittivity, $\chi^{(1)}(t-t')$ is the linear susceptibility response function, and $\chi^{(3)}(t-t_1,\, t-t_2, t-t_3)$ is the third-order nonlinear susceptibility response function. Substituting eqs. (1), (2), (5), and (6) into eq. (4), we have

$$\frac{\partial\varphi(z,\omega)}{\partial z} = \frac{\mathrm{i}\varkappa\omega^2}{2c^2\beta(\omega)} \iint \mathrm{d}\omega'\,\mathrm{d}\omega''\,\varphi(z,\omega')\,\varphi(z,\omega'') \times \varphi^*(z,\omega'+\omega''-\omega)\,\chi^{(3)}(\omega-\omega')\cdot\exp(\mathrm{i}\,\Delta\beta\, z) + \frac{\mathrm{i}}{2\beta(\omega)}\cdot\frac{\partial^2\varphi(z,\omega)}{\partial z^2} , \quad (7)$$

where $\varkappa = \iint |F(x,y)|^4\,\mathrm{d}x\,\mathrm{d}y / \iint |F(x,y)|^2\,\mathrm{d}x\,\mathrm{d}y$, $\chi^{(3)}(\omega) = \int \chi^{(3)}(t)\exp(\mathrm{i}\omega t)\,\mathrm{d}t$ is the third-order susceptibility, and $\Delta\beta = \beta(\omega')+\beta(\omega'')-\beta(\omega'+\omega''-\omega)-\beta(\omega)$. We expand $\beta(\omega)$ around $\omega_0$ up to the fourth order, we have $\beta(\omega) = \beta_0 + \beta_1\,\Delta\omega + \frac{\beta_2}{2}\,\Delta\omega^2 + \frac{\beta_3}{6}\,\Delta\omega^3 + \frac{\beta_4}{24}\,\Delta\omega^4$, where $\Delta\omega = \omega-\omega_0$, $\beta_0 = \beta(\omega_0)$, $\beta_j = \partial^j\beta/\partial\omega^j|_{\omega=\omega_0}$ for $j=1$ to 4. $\beta_1$ is the reciprocal group velocity. $\beta_2$, $\beta_3$, and $\beta_4$ are the second-order, third-order, and fourth-order dispersion parameters, respectively. Substituting $\varphi(z,\omega) = \tilde{A}(z,\Delta\omega)\exp\{-\mathrm{i}[\beta(\omega)-\beta(\omega_0)]\,z\}$ into eq. (7) and taking the inverse Fourier transform $A(z,t) = \frac{1}{2\pi}\int \tilde{A}(z,\omega-\omega_0)\exp[-\mathrm{i}(\omega-\omega_0)\,t]\,\mathrm{d}\omega$, we have

$$\frac{\partial A}{\partial z} = H + \frac{\mathrm{i}}{2\beta_0}\, C_\beta A_h , \quad (8)$$

where

$$H = -\beta_1\frac{\partial A}{\partial t} - \frac{\mathrm{i}\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3} + \frac{\mathrm{i}\beta_4}{24}\frac{\partial^4 A}{\partial t^4} + \mathrm{i}\gamma\Bigg[1 + \mathrm{i}\left(\frac{2}{\omega_0} - \frac{\beta_1}{\beta_0}\right)\frac{\partial}{\partial t} - \left(\frac{1}{\omega_0^2} - \frac{2\beta_1}{\beta_0\omega_0} + \frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{2\beta_0}\right)\frac{\partial^2}{\partial t^2}\Bigg] NA , \quad (9)$$

$$C_\beta = 1 - \mathrm{i}\beta_1\frac{\partial}{\partial t} + \left(\frac{\beta_2}{2\beta_0} - \frac{\beta_1^2}{\beta_0^2}\right)\frac{\partial^2}{\partial t^2} , \quad (10)$$

$$A_h = \frac{\partial^2 A}{\partial^2 z} + \left(2\beta_1\frac{\partial}{\partial t} + \mathrm{i}\beta_2\frac{\partial^2}{\partial t^2} - \frac{\beta_3}{3}\frac{\partial^3}{\partial t^3}\right)\frac{\partial A}{\partial z} - \left[\mathrm{i}\beta_1\frac{\partial}{\partial t} - \frac{\beta_2}{2}\frac{\partial^2}{\partial t^2} - \frac{\mathrm{i}\beta_3}{6}\frac{\partial^3}{\partial t^3}\right]^2 A , \quad (11)$$

where $\gamma = \frac{n_2\omega_0}{cA_{\mathrm{eff}}}$, $n_2$ is the Kerr coefficient, $A_{\mathrm{eff}}$ is effective fiber cross section, $A_{\mathrm{eff}} = \frac{\iint |F(x,y)|^2\,\mathrm{d}x\,\mathrm{d}y}{\varkappa}$, and the higher order terms are neglected. The response $N(z,t)$ is described by [10]

$$N(z,t) = (1-\alpha)\,|A(z,t)|^2 + \alpha\int_{-\infty}^{t}\mathrm{d}t'\,f(t-t') \times |A(z,t')|^2 . \quad (12)$$

On the right-hand side of eq. (12), the first term represents Kerr nonresonant virtual electronic transitions in the order of about 1 fs or less [4], the second term represents delayed Raman response, $f(t)$ is the delayed response function, and $\alpha = 0.18$ parameterizes the relative strengths of Kerr and Raman interactions. In this paper, $f(t)$ models a single Lorentzian line centered on the optical phonon frequency $1/\tau_1$ and having a bandwidth of $1/\tau_2$ (the reciprocal phonon lifetime).

$$f(t) = \frac{\tau_1^2+\tau_2^2}{\tau_1\cdot\tau_2^2}\,\exp(-t/\tau_2)\cdot\sin(t/\tau_1) , \quad (13)$$

where $\tau_1 = 12.2fs$, $\tau_2 = 32fs$.

We now use order of magnitude considerations to simplify the calculation. The dispersion and the nonlinear terms in the nonlinear Schrödinger equation are of the same order of magnitude for the fundamental soliton, hence, we have

$$\left|\beta_2\,\frac{\partial^2 A}{\partial t^2}\right| \approx \left|\frac{\beta_2 A}{T_0^2}\right| \approx \left|\frac{\gamma|A|^2 A}{N_p^2}\right| , \quad (14)$$

where the parameter $N_p = [\gamma P_0 T_0^2/|\beta_2|]^{1/2}$ is the order of the soliton and $N_p = 1$ for the fundamental soliton, $T_0 = T_w/1.763$, $T_w$ is the pulse full width at half maximum, and $P_0$ is peak power of the incident pulse. In a silica-based weakly guiding single mode fiber, $\beta_0 \approx \frac{n_0\omega_0}{c}$, $\beta_1 \approx \frac{n_0}{c}$, $\beta_1^2 \gg \beta_0\beta_2$, and $\beta_1^3 \gg \beta_0\beta_3$. Defining $\sigma = \frac{1}{\omega_0 T_0}$, we obtain from eq. (14) for the fundamental soliton

$$\frac{|\beta_2|}{T_0^2} \approx \gamma|A|^2 \approx \omega_0^2|\beta_2|\,\sigma^2 . \quad (15)$$

By using the iterative technique and the order of magnitude considerations, we first neglect the second term on the right-hand side of eq. (8) and obtain in the zeroth order approximation

$$\frac{\partial A}{\partial z} = H . \quad (16)$$

From eq. (16), the first order approximation of $A_h$ is

$$A_h = -\gamma^2 N^2 A - 2\gamma\beta_2 A\left[(1-\alpha)\left|\frac{\partial A(z,t)}{\partial t}\right|^2 + \alpha\int_{-\infty}^{t}\mathrm{d}t'\,f(t-t')\cdot\left|\frac{\partial A(z,t')}{\partial t'}\right|^2\right] - 2\gamma\beta_2\,\frac{\partial N}{\partial t}\,\frac{\partial A}{\partial t} . \quad (17)$$

By substituting eq. (16) into eq. (8), the first order approximation of the wave equation is

$$\frac{\partial A}{\partial z} = \left(-\beta_1 \frac{\partial A}{\partial t} - \frac{i\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + \frac{\beta_3}{6}\frac{\partial^3 A}{\partial t^3} + \frac{i\beta_4}{24}\frac{\partial^4 A}{\partial t^4}\right) + i\gamma\left(NA + i\alpha_1 \frac{\partial}{\partial t} NA\right) - i\gamma\alpha_2 \frac{\partial^2 NA}{\partial t^2} - \frac{i\gamma\beta_2 A}{\beta_0} \times \left[(1-\alpha)\left|\frac{\partial A(z,t)}{\partial t}\right|^2 + \alpha \int_{-\infty}^{t} dt' f(t-t') \cdot \left|\frac{\partial A(z,t')}{\partial t'}\right|^2\right] - \frac{i\gamma\beta_2}{\beta_0}\frac{\partial N}{\partial t}\frac{\partial A}{\partial t} - \frac{i}{2\beta_0}\gamma^2 N^2 A\,, \tag{18}$$

where $\alpha_1 = \frac{2}{\omega_0} - \frac{\beta_1}{\beta_0} \approx \frac{1}{\omega_0}$, $\alpha_2 = \frac{1}{\omega_0^2} - \frac{2\beta_1}{\beta_0\omega_0} + \frac{\beta_1^2}{\beta_0^2} - \frac{\beta_2}{2\beta_0} \approx -\frac{\beta_2}{2\beta_0}$, and retaining all terms to the order of $\sigma^4$. If we make a second iteration, we find that the equation does not change up to the order of $\sigma^4$. On the right-hand side of eq. (18), the term with coefficient $\alpha_1$ is of order $\sigma^3$, and the last five terms representing nonlinear high-order terms of order $\sigma^4$ which are newly derived terms. Comparing the $\sigma^4$ term with coefficient $\alpha_2$ with the $\sigma^3$ term with coefficient $\alpha_1$, we have

$$r = \frac{\left|\alpha_2\left(\frac{\partial^2 NA}{\partial t^2}\right)\right|_{\max}}{\left|\alpha_1\left(\frac{\partial NA}{\partial t}\right)\right|_{\max}} \approx \frac{\frac{|\beta_2|}{\beta_0 T_0^2}}{\frac{1}{\omega_0 T_0}} \approx \frac{|\beta_2|\, c}{n_0 T_0}\,. \tag{19}$$

We consider a single cycle pulse. At the wavelength $\lambda = 1.55\ \mu m$, $\beta_2 = -20\ fs^2/mm$, pulsewidth $T_w \approx 5.17$ fs and we have $r = 1.4 \times 10^{-3}$. At $\lambda = 0.8\ \mu m$, $\beta_2 = 38.5\ fs^2/mm$, pulsewidth $T_w \approx 2.67$ fs and we have $r = 5.1 \times 10^{-3}$ [6]. Similarly we can show that other $\sigma^4$ terms are much smaller than the $\sigma^3$ term. Therefore, all $\sigma^4$ terms can be neglected for the single cycle pulse in the low loss window of the fiber.

The equation (18) is used to describe the propagation. In dimensionless soliton units, it can be rewritten as

$$\frac{\partial}{\partial \xi} u = \frac{i}{2}\frac{\partial^2 u}{\partial \tau^2} + \beta\frac{\partial^3 u}{\partial \tau^3} + \frac{i\beta_4}{24|\beta_2|\, T_0^2}\frac{\partial^4 u}{\partial \tau^4} + i\bar{N}u - \frac{1}{\omega_0 T_0}\frac{\partial}{\partial \tau} \times \bar{N}u - \frac{i\beta_2}{\beta_0 T_0^2}\left\{\frac{1}{2}\frac{\partial^2 \bar{N}u}{\partial \tau^2} + \left[(1-\alpha)\left|\frac{\partial u}{\partial \tau}\right|^2 + \alpha\int_{-\infty}^{\tau} d\tau' \cdot f(\tau-\tau') \cdot \left|\frac{\partial u}{\partial \tau'}\right|^2\right] + \frac{\partial \bar{N}}{\partial \tau}\frac{\partial u}{\partial \tau} + \frac{1}{2}\bar{N}^2 u\right\}, \tag{20}$$

where $\xi = \frac{z}{L_D}$, $\tau = \frac{t - \beta_1 z}{T_0}$, $u = \frac{N_P A}{\sqrt{P_0}}$, $\beta \equiv \frac{\beta_3}{6|\beta_2|\, T_0}$, $L_D = \frac{T_0^2}{|\beta_2|}$ is dispersion length, and $\bar{N} = \frac{N_P^2 N}{P_0}$. When the terms of order $\sigma^4$ are neglected, eq. (20) reduces to

$$\frac{\partial}{\partial \xi} u = \frac{i}{2}\frac{\partial^2 u}{\partial \tau^2} + \beta\frac{\partial^3 u}{\partial \tau^3} + \frac{i\beta_4}{24|\beta_2|\, T_0^2}\frac{\partial^4 u}{\partial \tau^4} + i\bar{N}u - \frac{1}{\omega_0 T_0}\frac{\partial}{\partial \tau}\bar{N}u\,. \tag{21}$$

## 3. Full Maxwell's equations model

The Maxwell's equations for the optical pulse linearly polarized in $x$-direction propagation in $z$-direction are written as

$$\frac{\partial H_y}{\partial t} = \frac{1}{\mu_0}\frac{\partial E_x}{\partial z}\,, \tag{22a}$$

$$\frac{\partial D_x}{\partial t} = \frac{\partial H_y}{\partial t}\,, \tag{22b}$$

$$D_x = \varepsilon_0 \varepsilon_r E_x + P_x\,. \tag{22c}$$

Here $\mu_0$ and $\varepsilon_0$ are the permeability and permittivity coefficients in free space, $\varepsilon_r$ is the relative permittivity, $D_x$ is the electric filed displacement and $P_x$ is the electric polaritation.

$P_x$ consists of linear part $P_x^L$ and nonlinear part $P_x^{NL}$, $P_x = P_x^L + P_x^{NL}$. The linear polarization $P_z^L$ is given by convolution of $E_z(x,t)$ and first-order susceptibility function $\chi^{(1)}(t)$.

$$P_z^L(x,t) = \varepsilon_0 \int_{-\infty}^{\infty} \chi^{(1)}(t-\bar{t})\, E_z(x,t)\, d\bar{t}\,, \tag{23}$$

where

$$\chi^{(1)}(t) = \frac{\omega_r^2(\varepsilon_s - \varepsilon_r)}{\sqrt{\omega_r^2 - \frac{\delta^2}{4}}} \exp(-\delta t/2) \sin\left(\sqrt{\omega_r^2 - \frac{\delta^2}{4}}\, t\right).$$

$\omega_r$ is dipole resonant frequency $\delta$ is damping constant. $P_z^{NL}$ is given by convolution of $E_z(x,t)$ and third-order susceptibility $\chi^{(3)}(t)$

$$P_z^{NL}(x,t) = \varepsilon_0 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \chi^{(3)}(t-\bar{t}_1, t-\bar{t}_2, t-\bar{t}_3) \times E_z(x,\bar{t}_1)\, E_z(x,\bar{t}_2)\, E_z(x,\bar{t}_3)\, d\bar{t}_1\, d\bar{t}_2\, d\bar{t}_3\,. \tag{24}$$

We consider the nonlinear polarization with single time convolution

$$P_z^{NL}(x,t) = \varepsilon_0 \chi^{(3)} E_z(x,t) \int_{-\infty}^{\infty} g(t-\bar{t})\, E_z^2(x,\bar{t})\, d\bar{t}\,, \tag{25}$$

where $\chi^{(3)}$ is the nonlinear coefficient. The response is given by phonon interaction $f(t)$ and nonresonant electric effects $\delta(t)$.

$$g(t) = (1-\alpha)\,\delta(t) + \alpha \cdot f(t)\,, \tag{26}$$

where $f(t)$ is given in eq. (13).

## 4. Numerical results

In an attempt to verify the validity of the eq. (21) for describing the propagating of ultrashort pulse, we simulate the 2.5-fs fundamental soliton propagation by eqs. (20), (21) and the full Maxwell's equations. The fiber parameters are: soliton wavelength $\lambda = 1.55$ μm, $\omega_r = 8 \times 10^{13}$ rad/s, $\varepsilon_\infty = 2.25$, $\varepsilon_s = 5.25$, $\delta = 1.0 \times 10^9\ \text{s}^{-1}$, and the nonlinearity $\gamma = 2 \times 10^{-6}\ \text{W}^{-1}\ \text{mm}^{-1}$. From the permittivity function $\chi^{(1)}(t)$, we obtain $\beta_1 = 5.01 \times 10^3$ fs/mm, $\beta_2 = -24.56\ \text{fs}^2$/mm, $\beta_3 = 61.97\ \text{fs}^3$/mm, and $\beta_4 = -209.64\ \text{fs}^3$/mm. eqs. (20) and (21) are solved by the split-step Fourier method. eq. (22) is directly and iteratively computed by following the algorithm of the FD-TD method [4]. Figs. 1a and 1b are the pulse shapes of the 2.5-fs fundamental soliton in $5L_D$ simulated by using the generalized nonlinear Schrödinger equation, eq. (21), and the full Maxwell's equations, eq. (22), respectively. Using the moving frame relation $\left(\tau = \dfrac{t - \beta_1 z}{T_0}\right)$, we transform Fig. 1b into temporal distribution. After the transformation, one can see that Fig. 1a and Fig. 1b are almost the same. From Fig. 1, the soliton phenomenon is induced by third-order dispersion [11, 12], self-steepening effect [13] and delayed Raman response [14, 15]. The angular frequency of the pulse is 200 THz, which is 10-times to the spectrum of Raman gain spectrum. It can be seen from Figs. 1a and b that the pulse is with oscillation structure and dispersive wave in tailing edge. The dominant effect by third-order dispersion is shown. We can find the pulse shape in Figs. 1a is consistent with that in Fig. 1b. The same propagation also simulated by using eq. (20), and it is found that two numerical results by using eq. (20) and (21) differ less than 0.5%. It is demonstrated that eq. (21) could well describe the propagation of the 2.5-fs fundamental soliton in fiber. On the other hand, the coefficient of $\sigma^4$ order is found to be negligible.

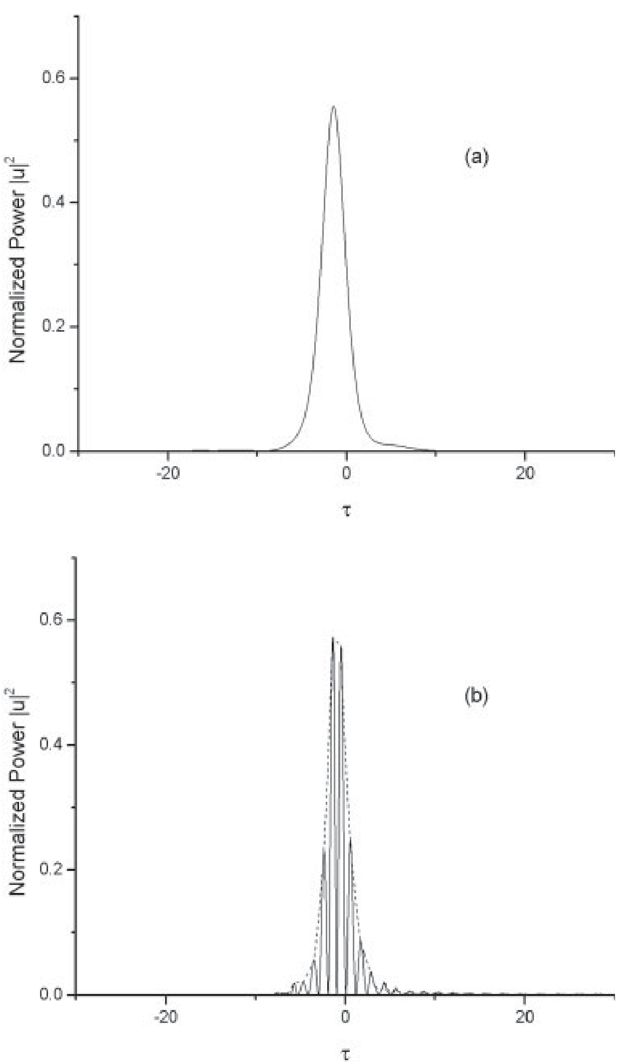


**Fig. 1. Pulse shapes of the 2.5-fs fundamental soliton in 5 $L_D$ simulated by using a) the generalized nonlinear Schrödinger equation and b) the full Maxwell's equations.**

## 5. Conclusion

In conclusion, we have used iterative method to derive a wave equation for femtosecond soliton propagation in an optical fiber. The derived equation contains higher nonlinear terms than the equation obtained previously. It is found that those more higher-order nonlinear terms, the coefficients of which are proportional to the second-order dispersion parameter, are much smaller than the shock term in a silica-based weakly guiding single mode fiber. The propagations of 2.5-fs fundamental soliton by using the generalized nonlinear Schrödinger equation and the full Maxwell's equations are numerically simulated. Comparing these two results, we found that the generalized nonlinear Schrödinger equation well describes the propagation of the pulse even containing a single optical cycle.

**Acknowledgement.** This work was supported in part by the National Science Council, Taiwan, R.O.C. under Contract NSC 89-2215-E-009-112, the Academic excellence program of R.O.C. Ministry of Education under Contract 90-E-FA06-1-4-90X023 and National Center for High-Performance Computing.

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