TECHNICAL NOTE

Method for measuring the retardation of a wave plate

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An electro-optic modulator applied to a carrier frequency is used to measure the retardation of a wave plate. This method is not only suitable for any wave plate but also can be operated in real time.

Wave plates are often used in optical metrology, for example, ellipsometry[1–3] and phase-shifting interferometry.[4,5] The retardation error of a wave plate will significantly influence the experimental results.[6] Several papers on measuring the retardation error of a wave plate have been published. Some of them are overly complicated[7–10]; others can be used only to measure the quarter-wave plate[11] or the wave plate for the wavelength of a commercial He–Ne Zeeman laser.[12] A novel method for measuring the retardation of a wave plate, based on the general concept of converting an optical phase difference into an electrical phase difference,[13,14] is presented in this Technical Note. An electro-optic modulator is used to mix a carrier frequency to the reference signal and the test signal. Then, when the phase difference between two signals is measured, the retardation of the wave plate is measured.

A schematic diagram of the optical system is shown in Fig. 1. The linearly polarized light passing through an electro-optic modulator EO is incident on a beam splitter BS and is divided into two parts: the reference beam and the test beam. The reference beam reflected from BS passes through an analyzer AN_r, then enters the photodetector D_r, whereas the test beam transmitted through BS passes through a tested wave plate W and an analyzer AN_t and is detected by another photdetector D_t. For convenience the $+z$ axis is in the propagation direction and the y axis in the vertical direction. Let the incident light be linearly polarized in the horizontal direction. The fast axis of EO under an applied electric field and the fast axis of the wave plate are both at 45° to the x axis. Both the transmission axes of two analyzers AN_r and AN_t are along the y axis.

Let the amplitude of the reference beam be normalized to 1; then the Jones vector[15] of the reference beam is

$$\begin{bmatrix} E_{rx} \\ E_{ry} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \exp(i\phi_x) & 0 \\ 0 & \exp(-i\phi_y) \end{bmatrix} \times \begin{bmatrix} \cos\frac{\Gamma}{2} & -i\sin\frac{\Gamma}{2} \\ -i\sin\frac{\Gamma}{2} & \cos\frac{\Gamma}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -i\exp(i\phi_y)\sin\frac{\Gamma}{2} \end{bmatrix}, \quad (1)$$

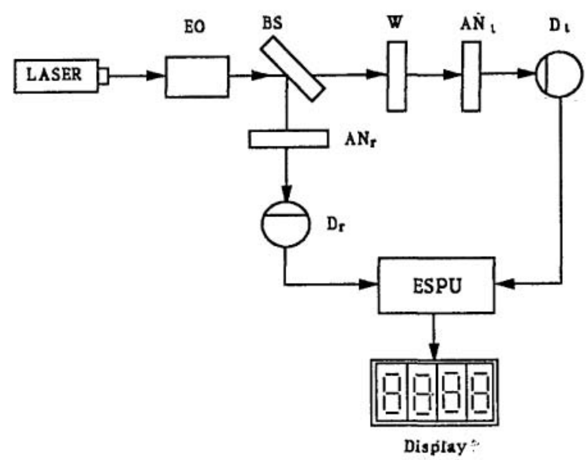

Fig. 1. Schematic diagram for measuring the retardation of a wave plate: EO, electro-optic modulator; BS, beam splitter; W, tested wave plate; AN, analyzer; D, photodetector; ESPU, electronic signal processing unit.

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where Γ is the phase introduced by the electro-optic modulator and ϕ_x and ϕ_y are the phase shifts for the x component (p polarization) and y component (s polarization) of the reflection beam from BS, respectively. The Jones vector of the test beam is

$$\begin{bmatrix} E_{rx} \\ E_{ry} \end{bmatrix} = a \exp(i\alpha) \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\frac{\delta}{2} & -i\sin\frac{\delta}{2} \\ -i\sin\frac{\delta}{2} & \cos\frac{\delta}{2} \end{bmatrix} \times \begin{bmatrix} \cos\frac{\Gamma}{2} & -i\sin\frac{\Gamma}{2} \\ -i\sin\frac{\Gamma}{2} & \cos\frac{\Gamma}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = a \exp(i\alpha) \begin{bmatrix} 0 \\ -i\sin\frac{\Gamma+\delta}{2} \end{bmatrix}, \quad (2)$$

where a is the amplitude of the test beam relative to the reference beam, α is the constant phase difference produced by the optical path difference between two beams, and δ is the retardation to be measured. Hence the intensities of the reference signal and the test signal are

$$I_r = \tfrac{1}{2}(1 - \cos\Gamma), \quad (3)$$

$$I_t = \tfrac{1}{2}a^2[1 - \cos(\Gamma + \delta)], \quad (4)$$

respectively. From Eq. (3) it is obvious that the intensity of the reference signal is independent to the phase shifts corresponding to the reflection from BS. If Γ is constant, i.e., without the electro-optic modulator or with the electro-optic modulator off, both I_r and I_t are unchanged, and it is difficult to evaluate the value of δ. If one applies a sawtooth signal to the electro-optic modulator with an amplitude that is sufficient to cause a total phase change of 2π during one cycle, then $\Gamma = \omega t$, where ω is the angular frequency of the sawtooth. Thus the detector output signals become sinusoidal with a phase difference of δ. Moreover these two sinusoidal signals are sent to the electronic signal processing unit (ESPU) as shown in Fig. 1. Finally the phase difference between these two signals is measured, and the retardation can be obtained with high accuracy.

To show the feasibility of this technique, a quarter-wave plate (WPQ-6328-4M) manufactured by Japan Sigma Koki Ltd. was tested. A He–Ne laser with a 632.8-nm wavelength and an electro-optic modulator (PC 100/2), manufactured by Electro-Optics Developments Ltd. with a half-wave voltage of 250 V, were used in this test. For easier observation these signals were monitored by an oscilloscope and are shown in Figs. 2 and 3. The upper parts of these two figures represent the external modulated sawtooth signal, which is applied to a electro-optic modulator with a frequency of 200 Hz. The middle and lower parts represent the wave forms of reference signals and test signals, respectively. From Fig. 2 one can see that the reference and test signals are inphase before the tested wave plate is located in the setup. In Fig. 3 it is clear that they have a phase difference after the introduction of the tested wave plate. The phase difference is easily read out with a digital phase meter. The results of 10 points across that quarter-wave plate are shown in Fig. 4; the average retardation is 94.7°, and the rms deviation is 0.46°.

In this Technical Note we presented an easy method for measuring the retardation of a wave plate. It is not only suitable for any wave plate but also has great accuracy and can be operated in real time. If a wave plate for any other wavelength is to be tested, the

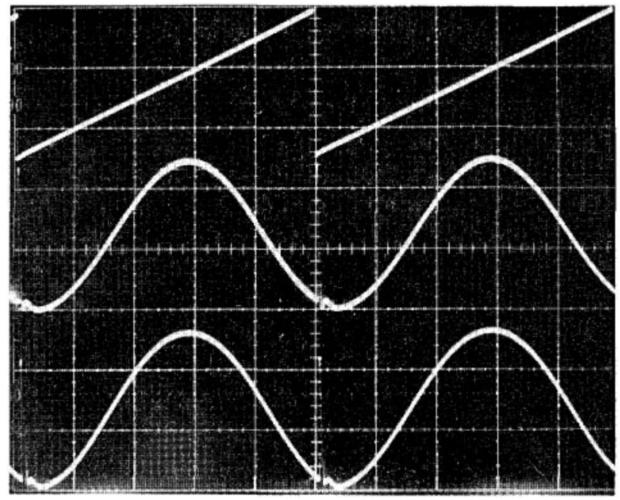

Fig. 2. Recorded signals without the tested quarter-wave plate: upper trace, external modulated signal; middle trace, reference signal; lower trace, test signal.

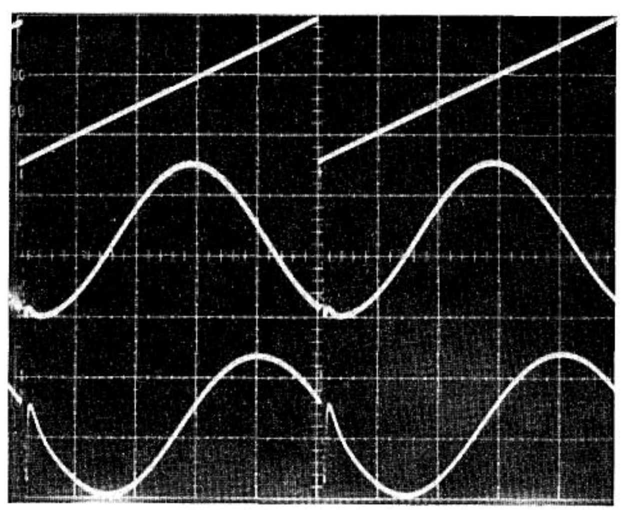

Fig. 3. Recorded signals with the tested quarter-wave plate: upper trace, external modulated signal; middle trace, reference signal; lower trace, test signal.

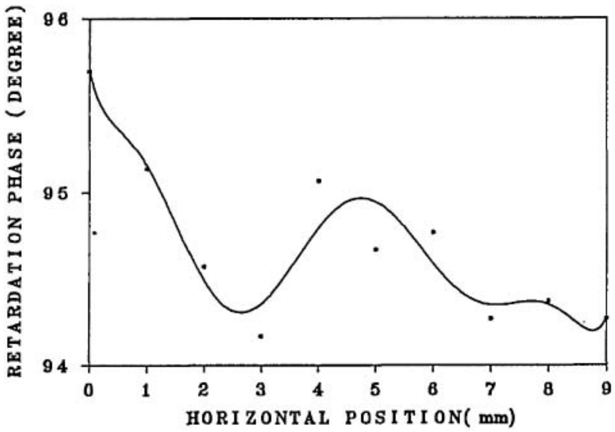

Fig. 4. Results of 10 points across the tested quarter-wave plate.

measurement processes are the same as above except that the light source should be converted to the corresponding wavelength.

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