# An Automatic Focusing Algorithm 

Sheng-Fuu Lin, You-Tasi Lin, Chien-Kun Su<br>Department of Electrical and Control Engineering, National Chiao Tung University, 1001 Ta Hsueh Road, Hsinchu, Taiwan 300, ROC

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#### Abstract

A simple and effective automatic focusing algorithm is proposed in this article. The principle of the proposed automatic focusing algorithm is based on that, for the radial test pattern, a best-focused image should have the smallest blurred region in the middle of the acquired image, and hence, should have the smallest equivalent radius. The circular Hough transform has became a common method in numerous image-processing applications for circle detection. Various modifications to the basic circular Hough transform have been suggested, such as: the inclusion of edge orientation, simultaneous consideration of a range of circle radii, the use of a complex accumulator array with the phase proportional to the log of the radius, or for filter operations. The purpose of this work is to show that a radius of a circular region extracted by a normalized circular Hough transform is a possible solution for determining the sharpness of images. To acquire high quality images with a given CCD camera, it is crucial that the camera be located exactly at the back length of the lens, i.e., the focus position of the lens. In the best conditions, the contours of the acquired images are of the sharpest, with none of the blurring effects associated with unfocused images. Acquiring such high quality images by these means is the main goal of the automatic focusing algorithm proposed in this article. © 2003 Wiley Periodicals, Inc. Int J Imaging Syst Technol, 12, 235-238, 2002; Published online in Wiley InterScience (www.interscience.wiley.com). DOI 10.1002/ima. 10029


## I. INTRODUCTION

One of the most important requirements of a technically good picture is well focused. A well-focused image looks sharp. In other words, the focusing process is to maximize the sharpness of images. There is a vast amount of literature on automatic focusing, but most of the automatic focusing mechanisms used in commercial cameras need a sophisticated optical and mechanical system (Goldberg, 1992). Some cameras rely on the method of maximizing the high frequency energy of the image transform to find an optimal focus (Luther, 1998). A simple and effective automatic focusing algorithm is proposed in this article. The proposed algorithm is especially suitable for those imaging systems with a fixed distance between the target and the lens. Figure 1 shows the configuration of such a

[^0]system. The principle of the proposed automatic focusing algorithm is based on that, for the radial test pattern, a best-focused image should have the smallest blurred region in the middle of the acquired image, and hence, should have the smallest equivalent radius. The circular Hough transform is used to determine the radius of the blurred region of an image, since it is reliable for detecting circles.

The circular Hough transform has became a common method for circle detection in numerous image processes (Antti and Nahum, 1994; Atherton and Kerbyson, 1999; Pei and Horng, 1994). Within an image, the transform is used to find circular formations of a given radius $R$. It can detect circles and circular arcs by choosing the center and the radius as parameters (Antti and Nahum, 1994). The location of a peak indicates the contour where a circle or an arc lies upon it. The number of computations in the circular Hough transform depends on the number of edge points in the image (Pei and Horng, 1994). Probabilistic Hough transforms are fast Hough algorithms that use polling instead of voting, i.e., a random selection of only a small part of the data set as input for the algorithm.

This paper is organized as follows: Section II describes the automatic focusing algorithm that is based on the circular Hough transform. The experimental results for automatic focusing are presented in Section III. Finally, in Section IV, conclusions are given.

## II. AUTOMATIC FOCUSING ALGORITHM

For circular Hough transform calculation, a separate circle filter can be used for each radius of circle to be detected. This forms the familiar three-dimensional parameter space, where two dimensions represent the position of the circle center $\left(c_{x} c_{y}\right)$ and the third is radius $R$, as

$$
\begin{equation*}
\left(x-c_{x}\right)^{2}+\left(y-c_{y}\right)^{2}=R^{2} . \tag{1}
\end{equation*}
$$

The mapping relation between $x, y$-plane and accumulation space is shown in Figure 2. The bold circle indicates a set of edge points, within the original image. Each edge point contributes a circle of radius $R$ to an output accumulator space, shown by the light circles. The output accumulator space has a peak where the contributing circles overlap at the center of the original circle. The circular Hough transform presents the transformation from the points on the circle in $x, y$-plane to the peak in the accumulation space.


Figure 1. A digital camera system with a constant distance between the target and the lens.

If the radius $R$ is given, the circular Hough transform can detect every circle of radius $R$ within an image (Atherton and Kerbyson, 1999). The operator is defined as
$O(u, v)= \begin{cases}1, & \text { if }(R-0.5)^{2}<(u-x)^{2}+(v-y)^{2}<(R+0.5)^{2}, \\ 0, & \text { otherwise, }\end{cases}$
where $(u, v)$ and $(x, y)$ are an accumulator coordinate in the accumulation space and an edge point coordinate in an image plane, respectively.

If we have a range of radii, then using phase coding results in a complex convolution operator that has a real and imaginary part for each operator coefficient (Atherton and Kerbyson, 1999). The real and imaginary parts of the phase coded annulus are defined as

$$
O(u, v)= \begin{cases}\exp \left(j \varphi_{u, v}\right), & \text { if } R_{\min }^{2}<(u-x)^{2}+(v-y)^{2}<R_{\max }^{2}  \tag{3}\\ 0, & \text { otherwise }\end{cases}
$$



Figure 2. The contribution of edge points to the accumulator space.


Figure 3. An example of circular Hough transform, (a) source image, (b) result of fixed radius, (c) result of phase coded annulus.
where $R_{\max }$ and $R_{\text {min }}$ are maximum and minimum radii of detected circle, respectively. The phase coding $\varphi_{u, v}$ is given by

$$
\begin{equation*}
\varphi_{u, v}=2 \pi\left(\frac{\sqrt{(u-x)^{2}+(v-y)^{2}}-R_{\min }}{R_{\max }-R_{\min }}\right) . \tag{4}
\end{equation*}
$$

Figure 4. Source image of automatic focusing.


Figure 5. The decomposition of the peak value of accumulation space.

An example of a circular Hough transform is illustrated in Figure 3. The source image containing a circle with a radius of 40 pixels is shown in Figure 3(a). The images in Figure 3(a-c) present the results for a circular Hough transform that has the given radius $R=$

40, and the magnitude of the phase coded annulus that has the given range of radius $R=\backslash 30,50]$, respectively. Hence, the coordinates of the peaks in Figures 3(b) and 3(c) indicate the center of the circle in Figure 3(a). Suppose that the phase component of the peak value in Figure 3(c) is $\varphi_{u, v}^{\prime}$, then the radius $R^{\prime}$ can be derived by

$$
\begin{equation*}
R^{\prime}=\frac{\varphi_{\mu, v}^{\prime}}{2 \pi}\left(R_{\max }-R_{\min }\right)+R_{\min } . \tag{5}
\end{equation*}
$$

The circular Hough transform can be formulated as a convolution whose mask coefficients are set on the circle boundary and are zero elsewhere (Atherton and Kerbyson, 1999). Then, the transform resolves the radius of the blurred intersection region.

The image with $1 \times 1 \mathrm{~cm}$ shown in Figure 4, is used as the source image for the automatic focusing since the blurred region at the image center is like a circle when the focus of the CCD camera is not properly set. The radius of this circle can be detected by the modified circular Hough transform with normalized phase coded annulus $O^{\prime}(u, v)$, which is defined as

$$
O^{\prime}(u, v)= \begin{cases}\exp \left(j \varphi_{u, v}\right) / \sqrt{(u-x)^{2}+(v-y)^{2}}, & \text { if } R_{\min }^{2}<(u-x)^{2}+(v-y)^{2}<R_{\max }^{2},  \tag{6}\\ 0, & \text { otherwise },\end{cases}
$$

where the phase coding $\varphi_{u, v}$ (Atherton and Kerbyson, 1999) is defined in (4).

Since the peak value in the accumulation space of the phase coded annulus is proportional to the radius of a detected circle, the effect of the different values of the radii on the peak value can be avoided by using (6), i.e., the accumulated values of each circle have the same magnitude at the center of the concentric circles.

Suppose that the concentric circles that have a maximum radius $r_{\text {max }}$ and a minimum radius $r_{\text {min }}$, then the mean radius is $r^{\prime}=$ $\left(r_{\text {max }}+r_{\text {min }}\right) / 2$. The decomposition of the peak value of the accumulation space in a complex number coordinate system is shown in Figure 5, where the dashed lines $v_{\text {max }}$ and $v_{\text {min }}$ present the accumulations of the edge points on the circles that have maximum and minimum radii, respectively, and the light line $v_{\text {peak }}$ denotes the composition of $v_{\text {max }}$ and $v_{\text {min }}$. The radii $r_{\text {max }}$ and $r_{\text {min }}$ are included in the phase components of $v_{\text {max }}$ and $v_{\text {min }}$, respectively, then the mean radius can be retrieved by the phase component of $v_{\text {peak. }}$. The circle with the mean radius within the central blurred region in

Figure 6 is detected by the proposed modified circular Hough transform. After this detection, a lookup table is established in which these radii and the corresponding elevations of the CCD camera are stored. As can be observed in Figure 7, image sharpness is in inverse proportion to the radius. Thus, the image with the smallest radius at the central blurred region is of relatively superior quality and the corresponding elevation is approximate to the back length of the lens.

## III. EXPERIMENTAL RESULTS

Since the absolute location of a CCD camera in experimental conditions cannot be known, the distance of its elevation can only be recorded relatively. The $60 \times 60$-pixel images taken at elevations ranging from 0 to 0.65 mm are illustrated in Figure 6. As is obvious, images (a), (b), (c), (j), (k), (l), (m), (n) suffer more serious blur than the others. The binary images $b(x, y)$, shown in Figure 7, are produced by the threshold method (Gonzalez and Woods, 1996) with the gray-level value $t=128$, which is defined as


Figure 6. Acquired images with relative elevation (a) 0 , (b) 0.05 mm , (c) 0.1 mm , (d) 0.15 mm , (e) 0.2 mm , (f) 0.25 mm , (g) 0.3 mm , (h) 0.35 mm , (i) 0.4 mm , (j) 0.45 mm , (k) 0.5 mm , (I) 0.55 mm , (m) 0.6 mm , (n) 0.65 mm .


Figure 7. The corresponding binary images of Fig. 6 after threshold.

Table I. The extracted radii using normalized phase coded annulus.

| Image index | a | b | c | d | e | f | g | h | i | j | k | l | m | n |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Relative elevation |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\quad(\mathrm{mm})$ | 0.00 | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 | 0.55 | 0.60 | 0.65 |
| Radius (pixel) | 8.6 | 8.5 | 8.3 | 7.7 | 7.0 | 6.5 | 6.2 | 6.9 | 7.8 | 7.9 | 8.0 | 8.1 | 8.1 | 8.2 |

$$
b(x, y)= \begin{cases}255, & \text { if } f(x, y)>t  \tag{7}\\ 0, & \text { otherwise }\end{cases}
$$

where $f(x, y)$ means the images shown in Figure 6. The radii at the central blurred regions are extracted by means of the circular Hough transform and the normalized phase coded annulus in (6) with $r_{\text {min }}=2$ pixel and $r_{\text {max }}=30$ pixel. The data for this are presented in Table I. When the relative elevation of the CCD camera is 0.3 mm , the dependent radius image (g) in Figure 6 has the sharpest contour of all the images, and a back length with a relative elevation of 0.3 mm is claimed.

## IV. CONCLUSIONS

With image processing, the fundamental problem is to filter out environment interference when attempting to acquire an image. By means of a modified circular Hough transform, automatic focusing can be used to determine the elevation of a CCD camera and to ensure that it is located at the back length of the lens.

In the article, an automatic focusing algorithm was successfully used to determine first the relative sharpness of acquired images and second the radius of the central region of the test pattern. From the
lookup table given in Section III, the elevation of the CCD camera to the focus of the lens can be accurately matched.

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[^0]:    *Correspondence to: Sheng-Fuu Lin, Department of Electrical and Control Engineering, National Chiao Tung University, 1001 Ta Hsueh Road, Hsinchu, Taiwan 300, ROC; E-mail: sflin@mail.nctu.edu.tw

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