Formation of optical vortex lattices in solid-state microchip lasers: Spontaneous transverse mode locking

Y. F. Chen*
Department of Electrophysics, National Chiao Tung University, Hsinchu, Taiwan, Republic of China

Y. P. Lan
Institute of Electro-Optical Engineering, National Chiao Tung University, Hsinchu, Taiwan, Republic of China
(Received 2 April 2001; published 14 November 2001)

We experimentally investigate pattern formation in a solid-state microchip laser with a large Fresnel number. Controlling the reflectivity of the output coupler can generate the stable square pattern of optical vortex lattices. The formation of square vortex lattices is found to be a spontaneous process of transverse mode locking within almost-degenerated mode families. The frequency of self-induced oscillation in square vortex lattices agrees well with the numerical calculation. The chaotic relaxation oscillation is found in square vortex lattices due to the multi-longitudinal-mode operation.

DOI: 10.1103/PhysRevA.64.063807 PACS number(s): 42.60.−v, 42.65.−k, 42.50.−p

The study of pattern formation in laser systems has become one of the most active fields of research in recent years because there are some interesting similarities in behavior between optical and hydrodynamic systems [1]. Theoretical models used to describe lasers with a large Fresnel number were usually analyzed with partial differential equations derived from the Maxwell-Bloch equations [2–5]. These models predicted the formation of periodic structure such as square optical vortex lattices (SVLs) [3,6] and of localized structures such as spatial soliton [7]. Theoretical results also predicted that class-*A* lasers with a large Fresnel number emit stationary patterns of SVLs near the laser threshold [8]. However, the dynamic characteristics of class-*B* lasers such as CO_2 or solid-state lasers are those of oscillations with an inertial nonlinearity. In the case of such oscillators the perturbations exhibit oscillatory relaxation. Theoretical analysis puts in evidence the qualitative differences of transverse dynamics for class-*A* and -*B* lasers with a large Fresnel number. Even so, not much has been done so far to observe the difference.

Previous works carried out with CO_2 lasers have shown evidence of spatiotemporal complexity [5,9] but the pattern dynamics near the laser threshold differed from those predicted by the models. Experimental works in CO_2 lasers usually used long cavities in which the longitudinal-mode spacing is of the same order of magnitude as the transverse-mode spacing. The presence of several longitudinal modes probably constituted the main reason for the discrepancies between theoretical predictions and experimental observations.

The recent rapid progress of diode-pumped microchip lasers has driven a renaissance of solid-state laser-physics research and led to novel phenomena [10]. The diode-pumped microchip laser can be easily operated in single longitudinal mode more than the ten times above threshold before the second longitudinal mode reaches threshold because the microchip gain medium has a short absorption depth that reduces the spatial hole burning effect. In this paper, we present the first experimental results of pattern formation in a fiber-coupled diode end-pumped microchip laser with a large Fresnel number. A fiber-coupled laser diode is used to maintain the cylindrical symmetry in the laser cavity. It is found that pattern formation strongly depends on the reflectivity of the output coupler. When the output reflectivity is not high enough, the transverse-mode pattern near the pump threshold is usually a high-order Laguerre-Gaussian (LG) $TEM_{0,l}$ mode with the distribution $\cos^2 l\phi$ (or $\sin^2 l\phi$) in azimuthal angle, having $2l$ nodes in azimuth. Slightly above the pump threshold, the laser emits a pair of transverse LG $TEM_{0,l}$ cosine and sine modes with chaotic dynamics. On the other hand, a high Fresnel number microchip laser can emit the pattern of the SVLs when the output reflectivity is high enough. Especially, the dynamics of the SVLs are not the results of multimode operation but exhibit single-frequency characteristics. The SVL pattern can be described as transverse mode locking within almost-degenerated mode families. It is also found that the SVL pattern may display chaotic relaxation oscillations through the onset of multi-longitudinal-mode operation.

The experimental cavity we use is analogous to the one described in Ref. [11]. We set up the resonator length to be as short as possible for reaching a large Fresnel number. The total resonator length is ∼2.0 mm. The output coupler is a concave mirror with the radius of curvature of 50 mm. In the present resonator, the frequency spacing between consecutive longitudinal modes is about 60 GHz, while the frequency difference between consecutive transverse modes is ∼5 GHz. Since the longitudinal-mode spacing is greater by one order of magnitude than the transverse-mode spacing, the present laser can be easily operated in single longitudinal mode to study the pattern formation. The pump source is a 1-W fiber-coupled laser diode (Coherent, F-81-800C-100) with a 0.1 mm of core diameter. The Fresnel number for an end-pumped solid-state laser can be expressed as Fr $=\omega_p^2/(\lambda L)$, where ω_p is the pump size, λ is the lasing wavelength, and L is the resonator length. To have a high Fresnel

*Corresponding author. FAX: (886-35) 729134; email address: yfchen@cc.nctu.edu.tw

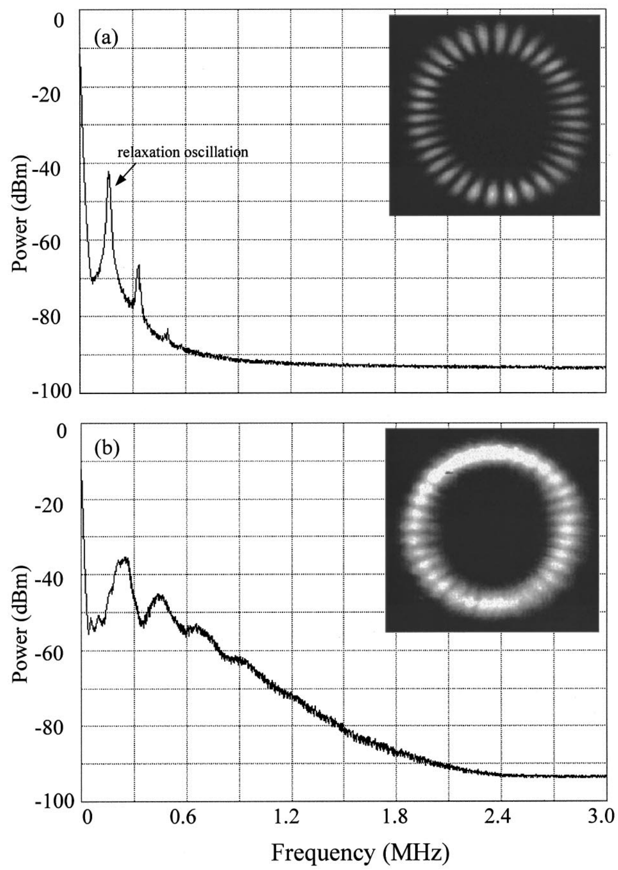

FIG. 1. Power-intensity spectra of laser emission for LG $\mathrm{TEM}_{0,17}$ mode: (a) near lasing threshold, (b) 1.25 times above threshold. Beam profiles are shown in the insets.

number, the pump size should be as large as possible. The pump size on the laser crystal can be easily adjusted by defocusing the pump source. In the present experiment, the pump size was adjusted within 0.5–1.5 mm. The maximum pump size depends on the lasing threshold. Using $\lambda = 1.064\ \mu\mathrm{m}$ and $L = 2$ mm, the Fresnel number in the experiment can be varied from 100 to 1000.

First we used an output coupler with the reflectivity of 97% in the laser resonator. Near lasing threshold, the laser emits a pure high-order LG $\mathrm{TEM}_{0,l}$ mode with the distribution $\cos^2 l\phi$ (or $\sin^2 l\phi$) in azimuthal angle, having $2l$ nodes in azimuth. The laser oscillating on a single high-order LG mode comes from the fact that the pump profile is similar to a doughnut-type distribution. As shown in Fig. 1(a), the free-running single-transverse-mode class-*B* laser displays relaxation oscillations that play an important role in the dynamics of multitransverse-mode class-*B* lasers. Slightly above lasing threshold, the present laser usually emits a pair of transverse LG $\mathrm{TEM}_{0,l}$ cosine and sine modes with chaotic dynamics, as shown in Fig. 1(b). The appearance of dynamic chaos is believed to arise from the interaction of the relaxation frequency and the astigmatism-induced frequency difference between two similar LG $\mathrm{TEM}_{0,l}$ modes. Even though the geometry is cylindrical symmetry, there is still certain astigmatism in the present cavity because of thermal lensing effect and anisotropic properties of the gain medium. Astigmatism-induced splitting of the two similar mode frequencies has a significant influence on laser dynamics. A nonlinear system of the Maxwell-Bloch equations [12] was used to investigate the dynamics of two similar LG $\mathrm{TEM}_{0,l}$ modes in a class-*B* laser. It is found that there is a chaotic set of solutions when the astigmatism-induced frequency difference is close to the relaxation frequency. Note that the system of equations for the dynamics of a class-*B* laser operating in two similar LG $\mathrm{TEM}_{0,l}$ modes is similar to the system describing generation of counterpropagating wave in a bidirectional-ring class-*B* laser, as discussed in Refs. [13], [14]. Therefore, the condition for chaotic emission is also predicted in a bidirectional-ring class-*B* laser [13].

The patterns observed so far can be interpreted as the simultaneous excitation of two similar LG $\mathrm{TEM}_{0,l}$ modes. This implies that the patterns were still linear because all their properties were determined by the boundary conditions, not by the nonlinearity of the gain medium. "Essentially nonlinear" pattern formation of lasers as it is predicted by the Maxwell-Bloch equations, requires not only a large Fresnel number of the resonator but also a continuum of transverse modes [6,8]. In order to excite a continuum of transverse modes simultaneously, we increased the reflectivity of the output coupler to 99%. With an output coupler with the reflectivity of 99% we have observed a succession of spatially well-organized SVL patterns whose complexity increases with the Fresnel number. These SVL patterns, as shown in Fig. 2, are very different from the monomode structures associated with Laguerre or Hermite-Gauss functions in an empty cavity. Surprisingly, the measurement of the optical spectrum indicated that the SVL pattern was a single-mode emission rather than a combination of lasing modes. In other words, the formation of SVL patterns can be interpreted as a spontaneous process of transverse mode locking of nearly degenerate modes, assisted by the laser nonlinearity. The nonlinearity is due to the dynamics of the saturation process. Although similar transverse locking in the generation of optical vortex crystals was demonstrated in broad-area VCSELs [15], optical systems so far have not generated such a large number of vortices in a single-mode emission. In addition, the present experiment provides the first observation of the transition from the linear to the essentially nonlinear pattern formation in a solid-state microchip laser. The highly regular crystal-like vortex patterns have also been demonstrated in CO_2 laser system [9]. However, the pattern generated in CO_2 laser system is a multiwavelength pattern not a coherent pattern. Therefore, the dynamics of the present SVL pattern is entirely different from that of the previous pattern found in CO_2 laser. Increasing the pumping power to the maximum pump power that is 5–8 times above lasing threshold, depending on the Fresnel number, the present SVL patterns were always preserved. Moreover, the SVL patterns were also preserved in free-space propagation.

The power spectrum was also measured to study the dynamics of the SVL patterns. Figures 3(a) and 3(b) show, respectively, the results of the power spectrum just near and seven times above lasing threshold for the SVL pattern with $\mathrm{Fr} \approx 500$. The fact that the power spectrum is almost independent of the region of the laser mode detected confirms the

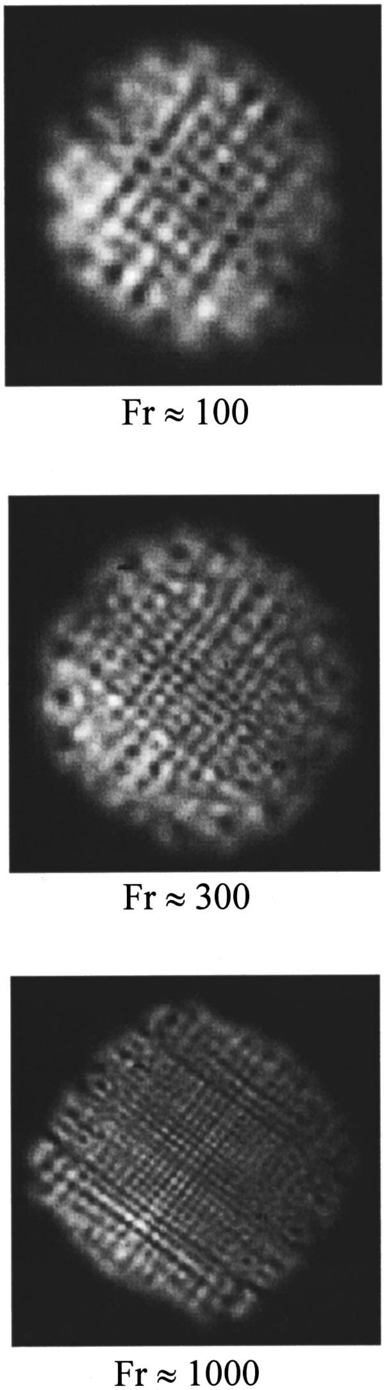

FIG. 2. Beam profiles of laser emission for SVL patterns, measured with the CCD camera, for three-different Fresnel numbers.

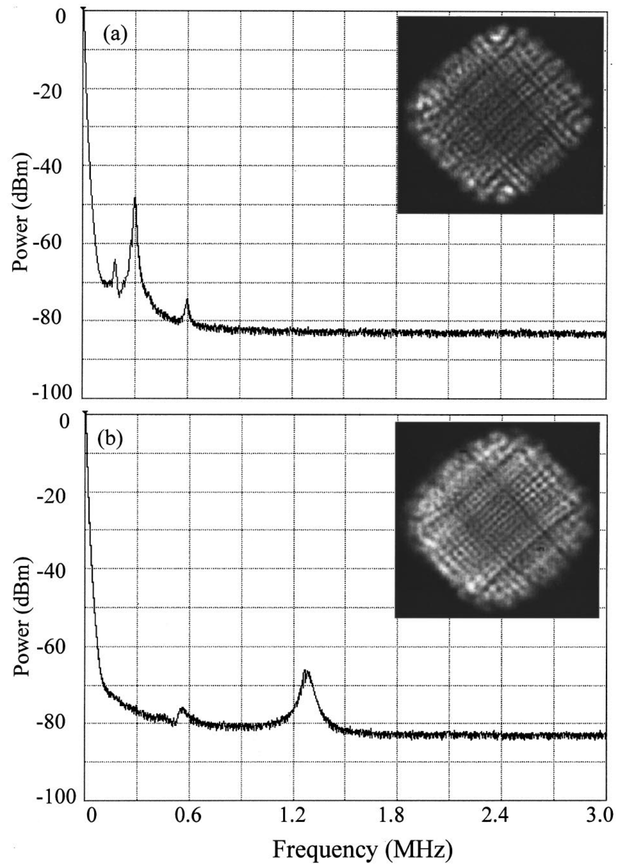

FIG. 3. Power-intensity spectra of laser emission for SVL pattern: (a) near lasing threshold, (b) seven times above threshold. Beam profiles are shown in the insets.

present SVL pattern to be a nearly pure single-mode emission. The temporal behavior was found to be similar to the dynamics of single transverse mode in class-*B* lasers except that there is an additional oscillation component with frequency lower than the relaxation frequency. We believe that the additional oscillation mode can be interpreted as the "acoustic" oscillation mode. The theoretical analysis [6] show that there are two pure vibrational modes of the self-induced dynamics of vortex lattices: (1) "acoustic" oscillation mode, where the neighboring vortices along a diagonal oscillation in phase; (2) "optical" oscillation mode, where the neighboring vortices along a diagonal oscillation in antiphase. The acoustic mode is the oscillation that only the transverse modes from the same degenerate family are involved, whereas the optical oscillation mode occurs when the transverse modes from two-different families are simultaneously excited. Since the present SVL emanates from a high level of degeneracy of transverse-mode families, the self-induced oscillation should belong to the acoustic mode. The numerical calculation [6] shows that the frequency of the acoustic oscillation is smaller by a factor of 2.8 ± 0.1 than the relaxation oscillation frequency. As shown in Fig. 3, the experimental result agrees very well with the theoretical prediction. The good agreement supports the affirmation that self-induced oscillation is the acoustic mode that resembles the oscillation of atoms in alkali-halide-type crystal when an acoustic phonon is excited.

The studies so far have been restricted to the dynamics of the SVL patterns belonging to a single longitudinal mode. To see the influence of multilongitudinal mode on the dynamics of the SVL patterns, we used an output coupler with the reflectivity of 99.8% in the resonator. The lasing threshold in the present case is less than 50 mW due to the low round-trip losses. The laser can be operated in multilongitudinal by increasing pump power more than ten times above threshold. Near lasing threshold, the SVL pattern and its power spec-

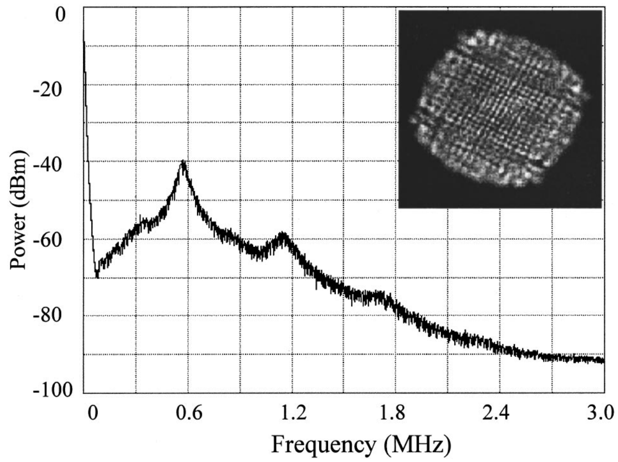

FIG. 4. Power spectrum for the self-chaotic oscillations of SVL pattern in the multi-longitudinal-mode operation. Beam profiles are shown in the insets.

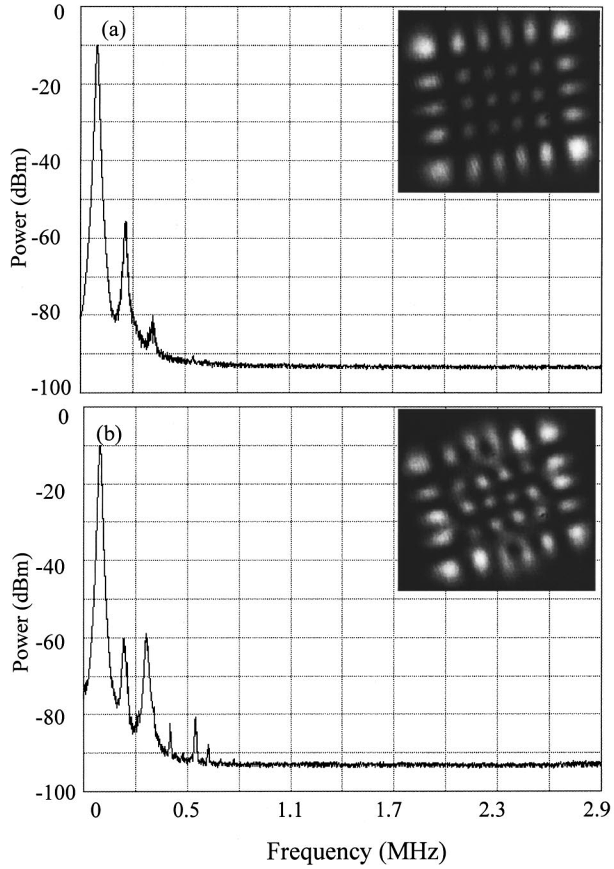

FIG. 5. Power-intensity spectra of laser emission for HG $TEM_{5,4}$ mode: (a) near lasing threshold, (b) 1.1 times above threshold. Beam profiles are shown in the insets.

trum is similar to the results shown in Fig. 3(a). When the pump power was increased to lead to multi-longitudinal-mode operation, chaotic relaxation oscillations were observed. As shown in Fig. 4, although the SVL pattern was nearly preserved in the multi-longitudinal-mode operation, the power-spectrum broadened. The broadening of the power spectrum indicates that the appearance of chaotic oscillations in free-running class-*B* lasers without external periodic perturbations is of considerable interest. The two-frequency route to chaotic relaxation oscillations has been observed in a diode-pumped microchip laser with the TEM_{00} mode output in two-longitudinal-mode oscillation regime [16]. The weak cross-gain coupling among two-longitudinal modes has been proposed to explain the relaxation oscillation instabilities. The instability of the SVL pattern in the multi-longitudinal-mode operation may originate from the same nonlinear gain mechanism.

Finally, it is worthwhile to mention that the mode profiles shown in Figs. 2–4 are rather different from higher-order mode of Hermite-Gaussian (HG) type in two dimensions. The distinctions can be found not only from the pattern profiles but also from the dynamics. To show the differences, we generated a higher-order two-dimensional (2D) HG mode by using a top-hat pump profile and a cavity in which the output coupler is a concave mirror with the radius of curvature of 50 mm and the reflectivity of 97.5%. Figure 5(a) shows the transverse pattern and power-intensity spectrum for HG $TEM_{5,4}$ mode near lasing threshold. The difference between the mode pattern shown in Fig. 2 and the 2D HG mode shown in Fig. 5(a) can be distinctly found. Experimental results reveal that the single 2D HG mode pattern certainly changes to a multimode pattern by slightly increasing the pump power, as shown in Fig. 5(b). Conversely, the transverse-mode-locking SVL patterns, as shown in Fig. 3, are insensitive to the pump power.

In summary, we demonstrated the generation of SVL patterns in solid-state microchip lasers with a large Fresnel number. The SVL pattern was found to emanate from a spontaneous process of transverse mode locking of nearly degenerate modes, assisted by the nonlinearity of gain medium. The dynamics of the SVL pattern agrees very well with the theoretical prediction. The ability to spontaneously generate optical vortex lattices in a microchip laser is both of a fundamental interest and could have applications in a variety of areas such as dynamic optical storage and processing.

The authors thank the National Science Council of the Republic of China for financially supporting this research under Contract No. NSC-89-2112-M-009-059.

[1] M. C. Cross and P. C. Hohenberg, Rev. Mod. Phys. **65**, 851 (1993).
[2] R. Lefever, L. A. Lugiato, W. Kaige, N. B. Abraham, and P. Mandel, Phys. Lett. A **135**, 354 (1989).
[3] P. Coullet, L. Gil, and F. Rocca, Opt. Commun. **73**, 403 (1989).
[4] G. L. Oppo, G. D'Alessandro, and W. J. Firth, Phys. Rev. A **44**, 4712 (1991).
[5] G. Huyet, C. M. Martinoni, J. R. Tredicce, and S. Rica, Phys.

Rev. Lett. **75**, 4027 (1995).
[6] K. Staliunas and C. O. Weiss, J. Opt. Soc. Am. B **12**, 1142 (1995).
[7] M. Brambilla, L. A. Lugiato, F. Prati, L. Spinell, and W. J. Firth, Phys. Rev. Lett. **79**, 2042 (1997).
[8] K. Staliunas and C. O. Weiss, Physica D **81**, 79 (1995).
[9] D. Dangoisse, D. Hennequin, C. Lepers, E. Louvergneaux, and P. Glorieux, Phys. Rev. A **46**, 5955 (1992).
[10] K. Otsuka, P. Mandel, and E. A. Viktorov, Phys. Rev. A **56**, 3226 (1997).
[11] Y. F. Chen, Y. P. Lan, and S. C. Wang, Appl. Phys. B: Lasers Opt. **72**, 167 (2000).
[12] D. V. Skryabin, A. G. Vladimirov, and A. M. Radin, Quantum Electron. **27**, 892 (1997).
[13] N. V. Kravtsov and E. G. Lariontsev, Quantum Electron. **24**, 841 (1994).
[14] A. G. Vladimirov, Opt. Commun. **149**, 67 (1998).
[15] J. Scheuer and M. Orenstein, Science **285**, 230 (1999).
[16] K. Otsuka, S. L. Hwong, and B. A. Nguyen, Phys. Rev. A **61**, 3815 (2000).