EXACT TANGENT STIFFNESS FOR IMPERFECT BEAM-COLUMN MEMBERS[a]

Discussion by Lip H. Teh,[3] Kuo-Mo Hsiao,[4] Donald W. White,[5] and Ronald D. Ziemian[6]

The authors present a stability-function-based element claimed to provide an exact solution for an imperfect beam-column within the context of Timoshenko's beam-column theory. Unfortunately, there are a number of fundamental flaws within the authors' formulation. Furthermore, the authors make a number of incorrect statements regarding cubic beam-column elements as well as the application of nonlinear analysis in design. Recent advancements in analysis methods are providing the tools necessary for engineers to address two- and three-dimensional stability issues in frame design with ease and rigor. However, for these advancements to reach their full fruition, fundamental flaws and misconceptions in nonlinear frame analysis must be corrected. In this spirit, the discussers offer the following comments.

The authors state that, based on their experience, the cubic element should only be used when the axial force is small. They reference White et al. (1993) for a suggestion that the cubic element should not be used for axial loads larger than 40% of the Euler buckling load P_e (based on the length of the finite element). This criterion is based on the fact that the largest error in any of the terms of the basic, second-order elastic, planar, cubic-element stiffness versus the corresponding stiffness coefficients associated with the elastic stability functions exceeds 1% at P/P_e = 0.43. However, other terms within the cubic-element stiffness are less sensitive to the value of the axial force, and for nearly all practical purposes, two cubic elements per member are sufficient for accurate planar second-order elastic analysis when the element is properly formulated (Teh 2001). For members that are subject to relative sway between their ends, one cubic element per member is sufficient to obtain accurate second-order elastic solutions. The ''practical'' worst-case errors associated with the use of the elastic cubic element, for either buckling or for second-order load-deflection analysis, are exhibited within linear buckling solutions for a strut. Results for a full range of end conditions are shown in Table 1. These results can be easily verified by hand or using frame analysis programs that have a linear buckling analysis facility using cubic elements, such as the MASTAN2 educational software (Ziemian and McGuire 2000).

If the errors shown in Table 1 are acceptable, then one can conclude that at most only two cubic elements per member are required when there is substantial sidesway restraint, and that only one cubic element per member is necessary in typical sway frames. The authors' results with one cubic element per member in Fig. 4, and with two cubic elements per member in Figs. 6–8, support the aforementioned conclusion for second-order elastic load-deflection problems.

The authors imply in Fig. 3 that eight cubic elements are required for accurate load-deflection analysis of a sidesway-inhibited imperfect fixed-fixed column, and they state that two elements per member are insufficient. Fig. 9 shows an example solution of this problem with the geometrically imperfect cubic element developed by Hsiao and Hou (1987). This element assumes a cubic curve for the initial out-of-straightness, as opposed to the sinusoidal curve used in the authors' formulation. It can be observed that there is only a negligible difference in the analysis results between the models using two and ten cubic elements for the built-in strut having a transverse imperfection of 1% of the strut length.

The paper references Neuenhofer and Filippou (1998) for confirmation of their conclusions regarding the cubic element. Neuenhofer and Filipou (1998) presents three examples. Example 1 of this paper is a simply-supported column subjected to an eccentric axial load. No load-displacement solutions are shown, only internal moments at a particular level of loading. Both the internal moments and the load-displacement response of the cubic element are quite accurate with two elements per member in this problem. The third example is a snap-through buckling problem similar to the Williams Toggle. As shown by Teh and Clarke (1998a), a properly formulated cubic element is capable of solving this problem accurately with only one element per member.

The point of the discussers is not to discourage research on new approaches that may offer distinct advantages relative to the cubic element, such as the work conducted by the authors and by Neuenhofer and Filippou. The important issue is that new elements need to be tested comprehensively, compared with the best performing existing elements, and the qualities and limitations of the different elements need to be reported

TABLE 1. Errors in Prediction of Flexural Buckling Loads for Strut with Different End Conditions

Problem description	Sway	Elements	% error
Fixed-fixed strut	N	2	1.32
Fixed-pinned strut	N	2	2.57
Pinned-pinned strut	N	2	0.75
Fixed-fixed strut	Y	1	1.32
Fixed-pinned strut	Y	1	0.75
Cantilever, full fixity at one end	Y	1	0.75

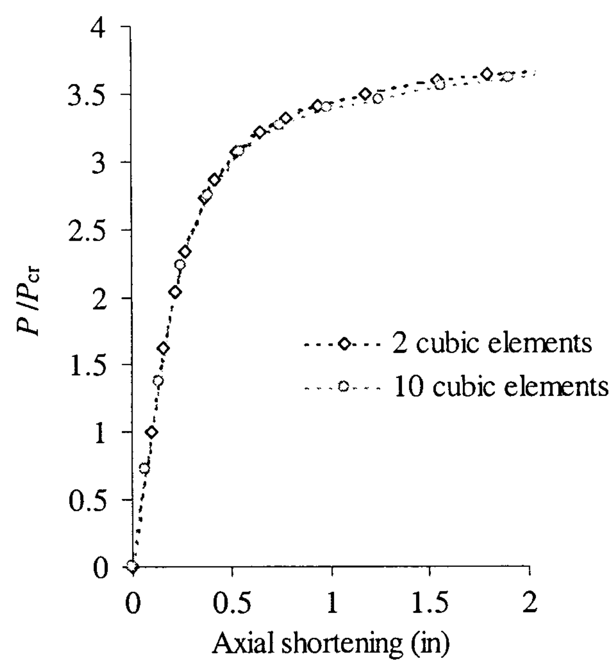

FIG. 9. Load-Deflection Graphs for Built-in Strut with Imperfection of 1%

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correctly and fairly. The discussers argue that the use of two elements per member does not present any significant difficulty for practical structural analysis of frameworks. Furthermore, this provides transverse deflection information directly at the midlength of members, which can be useful within a design context.

The authors state that the inclusion of member initial imperfections in the analysis is mandatory in codes and unavoidable in practice. In current practice by all design standards of which the discussers are aware (BSI 1990, CEN 1992, SAA 1998, CSA 1998, and AISC 1999), member out-of-straightness is addressed within the development of the column strength curve. Therefore, none of the present design standards actually require that out-of-straightness be modeled in the analysis. Nevertheless, the handling of initial out-of-straightness in the element formulation, as suggested by the authors and addressed in prior literature on cubic elements, can be useful. For many cases, however, where the response is dominated by sidesway stability and/or primary bending, the effect of the initial out-of-straightness is inconsequential (Lui 1992; Ziemian et al. 1992; Clarke and Bridge 1995).

The authors state that the initial out-of-straightness of the members can be established in the direction of the deflections caused by the external loads, apparently based on preliminary linear elastic analysis or the first iteration of the first increment of a nonlinear analysis. This is a useful approach for certain problems (Teh and Clarke 1998b), particularly since it can be automated within software for nonlinear analysis. However, this approach is not appropriate for structures that tend to fail by bifurcation onto a secondary equilibrium path. Fig. 10 depicts the lowest buckling mode of an experimental model tested by Kani and McConnel (1987), which is identical to the star dome analyzed in the paper except for the section properties and support condition. The bifurcation load of the star dome is significantly lower than the snap-through limit load predicted using the member imperfections that correspond to the primary deflection path. For a general imperfect structure of this type, the bifurcation problem becomes a load-deflection one.

The modeling of geometric imperfections in general (joint misalignment or out-of-plumbness as well as member out-of-straightness) as proposed by the authors may also be inappropriate for structures that involve significant reversal in the direction of displacements prior to reaching their maximum load. Such behavior is common for space domes. In such structures, the direction of the deflections due to the external loads can depend significantly on the load level.

The authors refer to their element as ''exact.'' However, the element is missing important terms involving coupling between torsion and flexure, as evidenced by the zeros within the fifth row and column of (36). This error appears to be due to the authors having developed their element based on a two-dimensional formulation, with the torsional effects subsequently included as per (26). State-of-the-art cubic finite elements have been published that include the torsional-flexural coupling terms missing in the authors' formulation, e.g., Nukala (1997), Hsiao and Lin (2000), and McGuire and Ziemian (2000), among others. The discussers are not aware of any stability-function-based formulation that has properly incorporated the coupling between flexure and torsion. This shortcoming is a major drawback that has been recognized by many authors (McGuire and Ziemian 1988; White and Hajjar 1991; Hancock 1994). Limit states involving torsional-flexural coupling are a practical reality (Trahair 1993; McGuire and Ziemian 2000; Teh et al. 2000).

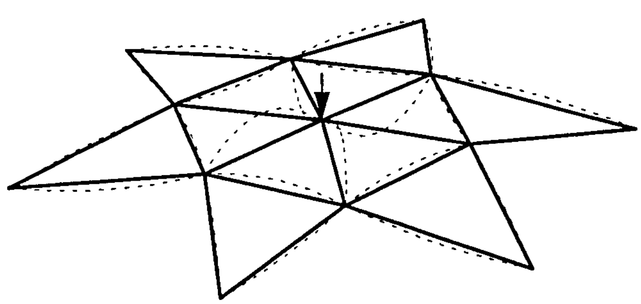

FIG. 10. Bifurcation Buckling of Star Dome (Kani and McConnel 1987)

In addition, the stability matrix [*N*] quoted from Ho and Chan (1991), who in turn refer to Meek and Tan (1984) for its form and derivation, does not properly account for the finite rotation kinematics or the behavior of nodal moments in three dimensions. This flaw results in the failure of the nonlinear analysis to detect flexural-torsional instability of framed structures (Teh and Clarke 1998a). The properly derived spatial cubic element is capable of detecting various modes of out-of-plane buckling, including flexural-torsional buckling of beam-columns (Hsiao and Lin 2000; McGuire and Ziemian 2000; Teh et al. 2000) and flexural buckling of torsion members (Trahair and Teh 2001).

Finally, the authors' two-dimensional element formulation is exact only for a linearly elastic prismatic member, assuming small displacements and rotations relative to the member chord. In any problems in which distributed yielding due to the presence of residual stresses, etc., is important, the author's element involves significant approximations of the member behavior. The discussers are not aware of any beam-column element that is capable of solving the general second-order inelastic problem accurately with one element per member. With respect to potential future applications of inelastic frame analysis for structural analysis/design of steel frames, one crucial advantage of the cubic element over the stability-function-based beam-column is that it is readily extended to material nonlinear analysis. While the cubic element has been successfully used for plastic-zone (Izzuddin and Smith 1996; Teh and Clarke 1999) and plastic-hinge (McGuire et al. 2000) analyses, the stability functions such as those described in the paper are valid only for a linearly elastic material.

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Closure by Siu-Lai Chan[7] and Jian-Xin Gu[8]

The writers are thankful for the discussers' interest and valuable comments on their paper.

First, the term ''exact'' refers to the formulation under the assumptions made in the paper. It is unfair to extend the application of the formulation outside the assumptions stated clearly by the writers—such as in the lateral-torsional buckling of beams mentioned by the discussers—since not a theory in the world can cover all cases without any prerequisite assumptions.

The ''Advanced Analysis'' in Appendix D of AS 4100 (1998) also requires the provision of full lateral restraint, and the proposed method is an efficient, robust, and practical technique for implementation of the method by the use of a single element per member, with the static design load limited to the first plastic hinge as a conventional design requirement in practice. Nevertheless, it is good to see the discussers agree that the element is exact under the assumptions of linear elastic and small rotations about the chord, which are also stated in the writers' paper. This represents a benchmark solution for this type of element that can be found to have a wide application in practical design of steel structures based on the first plastic hinge assumption.

In the writers' method, the elemental P-δ effect is automatically handled by the element formulation while the global structural imperfection or the P-Δ is modeled by insertion of notional forces or imperfect structural geometry at real joints between members. A model for a linear analysis can be adopted here, and artificial division of a member into two elements is not needed. Note that the sole replacement of the P-δ effect by notional forces may not be possible when a single element models a member. For example, the column imperfection in a simple nonsway frame with sway at all floor levels prevented by a shear wall cannot be simulated by insertion of notional force unless two elements are used to model a column, which is inconvenient in practical design. Also, web members in a plane or a space truss may face a similar difficulty.

The discussers' Table 1 shows the need of using two elements per member in the buckling analysis for such cases as the fix-fixed strut or the fix-pinned strut. Consider the case when we use the cubic element for analysis of moderate or large-sized practical frames such as the ones conducted by the writers (Chan 1999, 2001), in which there are thousands of members to be designed and analyzed nonlinearly. Some of them are connected to other, stiffer members and these columns are close to the boundary conditions of a fixed-fixed case. For the typical nonsway case of a column connected to a much stiffer beam or slab at top and at bottom, the use of a single cubic element is inadequate because the buckling mode is close to the fix-fix case.

The use of two elements per member will lead to a substantial increase in computer time in a nonlinear analysis, as in the case of analyzing a high-rise building of more than 16,000 members (Chan 2001). More importantly, the statement by the discussers that ''the transverse deflection information directly at the midlength of members . . . can be useful within a design context'' is not valid in many cases. For the common case of a column in a sway frame under lateral wind load, the column is bent under double curvatures, the midspan deflection is not maximum, and the P-δ effect at midheight is not critical. In another example, the writers agree that the use of eight elements is unnecessary, but a common practice in finite element is to use many elements for the *exact* solution.

Teh and Clarke (1998) showed that the cubic element is adequate for snap-through analysis of the William toggle frame. The reason is that they used a problem where the P-δ effect is unimportant and the P-Δ effect controls; this point has also been made by the writers in the example of the star-shaped frame. In this case, the axial force is small when compared with Euler's buckling load of the column. Here, the discussers are reminded that there are two principal and common sources of geometrically nonlinear effects, the P-Δ and the P-δ effects.

The ability to handle the snap-through buckling of shallow domes shows that the analysis method is able to consider the P-Δ effect accurately, but it does not mean that it can do the same for the P-δ effect. This point has been confused by many other researchers who consider their element or method to be sufficiently accurate when they can analyze the snap-through buckling of shallow toggle frame. Note that one can convert the P-δ effect to the P-Δ effect by using more elements, and this is another way of explaining the reason that two less accurate cubic elements are adequate for the buckling analysis of a simple column. Therefore, before the discussers can claim that their element is sufficiently accurate for buckling analysis using one element per member, they must first try problems with members under high axial force; otherwise their element is not properly tested.

The P-Δ effect is considered in the writers's paper by continuously updating the coordinates and the rotations using the corotational method for joint and member orientation. This approach can handle rotations larger than 15° (Oran 1973), which is believed to be more than sufficient for practical civil engineering structures.

While the element by the writers and Chan and Zhou (1995) converges to the cubic-element solution for columns with no axial force and is superior to the cubic element in the presence of axial force, the writers cannot see any reason to avoid its use. The discussers' assessment of a single cubic element per member for sway or nonsway cases is not needed when using a better element. The writers wonder if engineers will ever bother to assess its validity manually when deciding on

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whether one or two elements per member should be used, provided that a more reliable element is available. Also, the analyst needs to insert imperfection or notional forces at nodes between the two elements for simulation of member imperfection which, although possible by computer programming, involves more complicated effort than simply adopting the writers' element. This additional and unnecessary work makes the second-order analysis and design less attractive to practitioners.

The handling of initial imperfection proposed by the writers is to impose the imperfection in the same direction as the deflection for consistency with the design code. It has been demonstrated by the first writer, using the computer program NAF-Nida, that the program contains options of imposing imperfection in the eigen-buckling mode and also in the direction of deflections. The first consideration may be useful for an additional check after analysis by the second approach.

The discussers suggest the use of a column strength curve in place of member imperfection. They should be aware that the strength curves are often plotted from the formula for an imperfect column. For example, of the first code cited by the discussers, the Perry-Robertson formula is used to generate the column buckling curve, which assumes the initial imperfection as 0.1% of column length [see Appendix C, BSI (1990)]. The proposed element is capable of generating the same buckling strength curve (see Chan and Zhou 1998), which means that the P-δ effect is theoretically included in the curved element formulation. This feature allows the writers' element to produce the same design results as the code for cases where the effective length is obvious, while the proposed method can be extended to cases where the effective length is not obvious.

Most important of all, the disadvantage of using the strength curve in code instead of allowing the effect through element formulation is that the variation of member stiffness cannot be considered in the analysis and the forces in indeterminate frames cannot be computed accurately. For example, the compressive member in a simple cross bracing in Fig. 11 takes a much smaller load than the force in the tension member. To the extreme, it does not buckle before the tension member fails by yielding since the compression member near buckling does not take force due to its small stiffness (Chan and Zhou 1998). When we use strength curve to control failure, this variation of stiffness cannot be accurately modeled and the output by a single less accurate cubic element per member is therefore again incorrect. This is another reason to justify the use of an accurate element in a second-order analysis.

About the limitation of the writers' element in considering coupling between torsion and flexure, this point has been stated clearly in the writers' paper. Many researchers have attempted to develop an element that can consider all buckling modes, including those mentioned by the discussers as well as one not mentioned by the discussers, the local plate buckling for slender sections (Chan 1990). However, this objective has not been achieved and the writers believe the problem is more complicated than the discussers think, because the effective length of a beam cantilever can vary from 0.5 to 7.5 of its length, depending on the connection details and load height [see Table 10, BSI (1990)], which are not directly related to the currently used analysis parameters. If we include these details into the analysis program, in addition to the complication in formulation, the analysis will become much more complicated than the method proposed by the writers.

The [N] matrix by Ho and Chan (1991) is used in the tangent stiffness matrix. The accuracy is known to be unimportant in an incremental-iterative Newton-Raphson method type of numerical analysis, provided that numerical convergence can be achieved. For this purpose, the accuracy of [N] is adequate for the present analysis allowing for the second-order effect due to axial force only. For example, the accuracy of the tangent stiffness in the modified Newton-Raphson method is purposely abandoned for the sake of computational efficiency by reforming the tangent stiffness matrix only once in a load increment. Only the accuracy of the secant stiffness relation is important in the incremental-iterative analysis of path-independent problems.

The use of the refined element by Chan and Zhou (1995) in inelastic analysis is possible (see Liew et al. 1999). Use of a single element per member for inelastic buckling analysis is also feasible, and a paper describing the work is under preparation and should be published soon. Fig. 12 shows the buckling plot of such an analysis.

Finally, the writers are working to make the second-order-analysis-based design method practical. In doing so, the method must be simple to use so that modeling of a member by two elements is not needed. It must produce reliable results under practical conditions and be consistent with the design codes when their assumptions are the same. This will build confidence in adopting the advanced technique. The concept should be introduced to the engineers perhaps through an incremental-iterative manner, where more buckling and design factors are incorporated gradually into the method and with iteration for refinement in satisfying their needs and requirements. The computer program NAF-Nida (1996) has been used by a number of research groups and engineers for second-order analysis and design of virtual and real steel frames with satisfactory performance (see, for example, Peng et al. 1997).

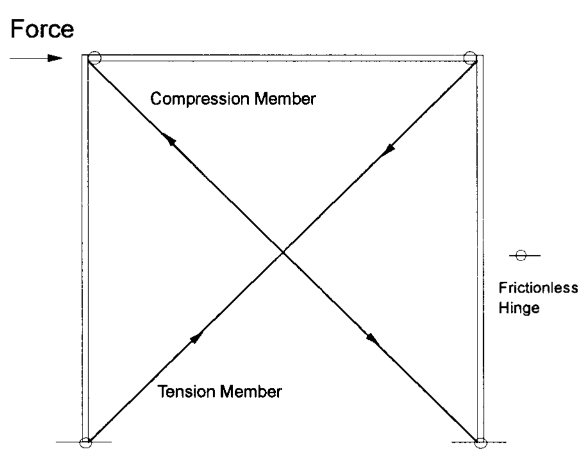

FIG. 11. Slender Cross Bracing with Stiffer Tension Members Taking Most Force

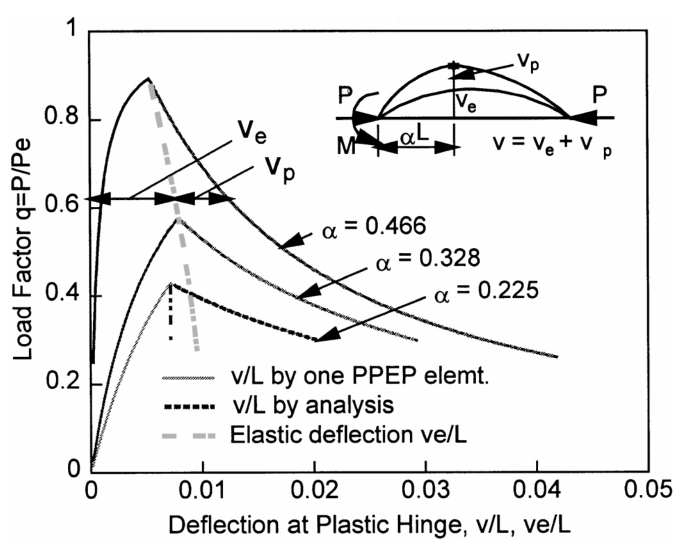

FIG. 12. Load versus Deflection at Plastic Hinge by 1 PPEP Element/ Member

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