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International Journal of Production Research
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tprs20

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Published online: 14 Nov 2010.

To cite this article: K. S. Chen , M. L. Huang & R. K. Li (2001) Process capability analysis for an entire product, International Journal of Production Research, 39:17, 4077-4087, DOI: 10.1080/00207540110073082

To link to this article: http://dx.doi.org/10.1080/00207540110073082

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INT. J. PROD. RES., 2001, VOL. 39, NO. 17, 4077–4087

Process capability analysis for an entire product

K. S. CHEN†*, M. L. HUANG‡ and R. K. LI‡

Process capability indices (PCIs) are powerful means of studying the process ability for manufacturing a product that meets specifications. Several capability indices including C_p, C_{pu}, C_{pl} and C_{pk} have been widely used in manufacturing industry to provide common quantitative measures on process potential and performance. The formulas for these indices are easily understood and can be straightforwardly applied. However, those process capability indices are inappropriate for asymmetric tolerances and could not be applied to evaluate multi-process products. Based on C_p, C_{pu}, C_{pl} and C_{pk}, this research aims to develop one process capability analysis chart (PCAC) for precisely measuring an entire product composed of symmetric tolerances, asymmetric tolerances, larger-the-better and smaller-the-better characteristics. The process capability analysis chart evaluates the capabilities of multi-process products and provides chances for continuous improvement on the manufacturing process.

1. Introduction

Process yield, process expected loss and process capability indices (PCIs) are three basic means that have been widely applied in measuring product potential and performance. Of the three, process capability indices are easily understood and can be straightforwardly applied to the manufacturing industry. The larger process capability index implies the higher process yield, and the larger process capability index also indicates the lower process expected loss. Therefore, the process capability index can be viewed as an effective and excellent means of measuring product quality and performance. Many engineering designers and shop floor controllers use process capability indices as communication indicators to evaluate and elevate the manufacturing process. For example, process capability indices can assist in solving manufacturing problems when engineering designers negotiate with shop floor supervisors on manufacturing problems. In addition, business sales and customers can communicate with each other about product characteristics via process capability indices. Customers normally preset product specifications and a mutually agreed quality level is necessary to establish communication between customers and manufacturers through process capability indices.

Process capability indices have been widely used to measure product qualities that meet specifications in automated, semiconductor and IC assembly manufacturing industries. Numerous statisticians and quality engineers—such as Kane (1986), Chan *et al.* (1988), Choi and Owen (1990), Boyles (1991), Pearn *et al.* (1992), Kotz

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Revision received April 2001.

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International Journal of Production Research ISSN 0020–7543 print/ISSN 1366–588X online © 2001 Taylor & Francis Ltd
http://www.tandf.co.uk/journals
DOI: 10.1080/00207540110073082

and Johnson. (1993), Boyles (1994), and Spiring (1997)—have emphasized research into process capability indices to propose more effective methods of evaluating process potential and performance. The two well-known process capability indices C_{pu} and C_{pl}, proposed by Kane (1986), which measure smaller-the-better and larger-the-better process capabilities are (see section 6 for nomenclature):

$$C_{pu} = \frac{USL - \mu}{3\sigma},$$

$$C_{pl} = \frac{\mu - LSL}{3\sigma},$$

where USL and LSL are, respectively, the upper specification limit and the lower specification limit, μ is process mean and σ is process deviation. However, these approaches consider processes with single quality characteristics only and restrict the application to multi-process products.

Although Chen (1994), Boyles (1996) and others have presented multivariate capability indices, those indices were appropriate for products with numerous unilateral specifications or products with bilateral specifications exclusively. A Multi-process Performance Analysis Chart (MPPAC), proposed by Singhal (1991), evaluates the performance of a multi-process product with symmetric bilateral specifications. As noted by Davis (1992), most multi-process products are composed of numerous unilateral specifications and bilateral specifications, and customers are satisfied when all quality characteristics of an entire product meet preset specifications. Obviously, neither univariate process capability indices nor multivariate process capability indices can fulfil the above requirements.

According to Boyles (1994), C_p and C_{pk} are capability indices with respect to process yield, and are irrelevant to process target (T). The one-to-one mathematical relation between C_p and process yield is $[p = 2\Phi(3C_p) - 1]$ when $\mu = (USL + LSL)/2$, and the one-to-one mathematical relation does not exist when $\mu \neq (USL + LSL)/2$. Boyles showed that the mathematical relation between C_{pk} and process yield is $p \geq 2\Phi(3C_{pk}) - 1$. $C_{pk} = 1$ ensures the process yield exceeds 99.73%. Obviously, C_{pk} is a better index than C_p. However, C_{pk} is still an inadequate measure of process centring. The important problem is that the above process capability indices cannot be applied to assess bilateral products with asymmetric tolerances. Pearn and Chen (1998) proposed a process capability index C_{pa} for a process with asymmetric tolerances to solve the restriction:

$$C_{pa} = \frac{D^* - A^*}{3\sigma},$$

where $A^* = \max\{d^*(\mu - T)/D_u,\ d^*(T - \mu)/D_l\}$, $D_u = USL - T$, $D_l = T - LSL$ and $d^* = \{\min D_u, D_l\}$. Obviously, $A^* = |\mu - T|$ when $T = m$ (symmetric case), and C_{pa} is reduced to the original index C_{pk}. The factor A^* ensures that the new generalization C_{pa} obtains its maximal value at $\mu = T$ (process is on-target) regardless of whether the tolerances are symmetric ($T = m$) or asymmetric ($T \neq m$). $C_{pa} = 0$ can be verified when the process mean is on the specification limit ($\mu = LSL$, or $\mu = USL$). On the other hand, $C_{pa} > 0$ when $LSL < \mu < USL$. For a fixed σ, the value of C_{pa} decreases when μ shifts away from T. In reality, the value of C_{pa} decreases faster when μ moves away from T to the closer specification limit, and decreases slower when μ moves away from T to the farther specification limit.

Combing Singhal's MPPAC with asymmetric process capability index C_{pa}, plus considering unilateral characteristics, we aim to propose a Process Capability Analysis Chart (PCAC) to evaluate process potential and performance for an entire product composed of smaller-the-better specifications, larger-the-better specifications, symmetric specifications and asymmetric specifications.

2. Product capability analysis chart

The Multi-process Performance Analysis Chart (MPPAC), proposed by Singhal (1991), evaluates the performance of a multi-process product with symmetric bilateral specifications. C_{pu} and C_{pl} represent the X-axis and Y-axis, respectively in MPPAC, whereas C_p is the average of C_{pu} and C_{pl}, namely, $C_p = 1/2(C_{pu} + C_{pl})$ and C_{pk} is the minimum value of the X- and Y-axes, namely, $C_{pk} = \min\{C_{pu}, C_{pl}\}$. Based on MPPAC, first, this chart is revised to evaluate a multi-process product with both symmetric and asymmetric bilateral specifications. Secondly, the vertical and horizontal axes of the chart are to evaluate larger-the-better and smaller-the-better characteristics. The third step is based on the critical values of individual process capabilities for the multi-process product, marking the process capability zone with bold lines. Finally, the values of all individual process capability indices of the entire product are located on PCAC. The process capabilities are capable when individual process capability indices are located within the process capability zone. Conversely, processes must be upgraded when some of the process capability indices are located outside the capability zone. It is easy to distinguish process performance with respect to the locations of process capability indices on PCAC. Hence, our Product Capability Analysis Chart (PCAC) not only distinguishes process capabilities, but also reveals the degree of quality accuracy for multi-process products. Therefore, PCAC is an effective and efficient means of evaluating multi-process products.

Based on the description in section 1, C_{pa} reasonably represents the process situation when the process shifts away from the target for asymmetric tolerances. The definition of C_{pa} can be rewritten as $C_{pa} = \min\{C_{du}, C_{dl}\}$, where $C_{du} = (d^*/D_u)$ C_{pu} and $C_{dl} = (d^*/D_l)$ C_{pl}. In the two-dimensional space, the X-axis simultaneously represents C_{du} for the nominal-the-best process and represents C_{pu} for the smaller-the-better process. Similarly, the Y-axis simultaneously represents C_{dl} for the nominal-the-best process and represents C_{pl} for the larger-the-better process. Axes X and Y construct the Process Capability Analysis Chart (PCAC) as shown in figure 1. PCAC characterizes not only multi-process capabilities with symmetric and asymmetric tolerances on the dimensional space, but also multi-process capabilities with smaller-the-better and larger-the-better types on the X and Y axes, respectively. Because the X-axis represents distinct indices, C_{du} and C_{pu}, and Y-axis represents distinct indices, C_{dl} and C_{pl}, it is better to build a table for PCAC to apply easily. The use of the table will be discussed in section 4.

Considering the loss function stated in the Taguchi method, the closer the process mean to the process target implies the better quality and fewer process losses. Conversely, the farther the process mean from the process target implies the worse process capabilities. Likewise, keeping the process on-target is crucial. A few subsidiary lines of C_a can be added on PCAC for precisely controlling the process shifts, where $C_a = 1 - [\max\{(\mu - T)/D_u, (T - \mu)/D_l\}]$. Obviously, C_a measures the relative distance of the shift from process mean to preset target. The definition of relative distance is $(\mu - T)/D_u$ or $(T - \mu)/D_l$. Equal relative distances result in the same values of C_a. For example, the specifications for two nominal-the-best processes A

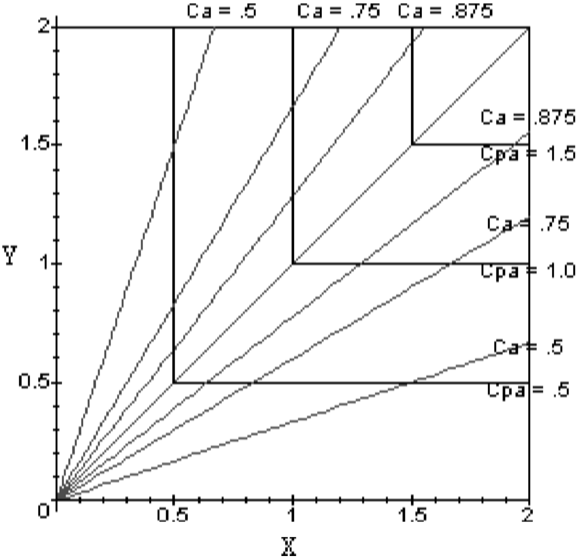

Figure 1. Product capability analysis chart.

and B are the same as $(LSL, T, USL) = (12, 16, 18)$. If the process mean for A is 14 and process mean for B is 17, then both relative distances of shifts are 1/2 and both C_a are 1/2. Off-diagonal subsidiary lines are plotted when C_a are 0.5, 0.75 and 0.875 in figure 1. $C_a < 0.875$ denotes that the process is not accurate; actions to shift the process mean closer to the process target are required. Namely, $C_a \geq 0.875$ indicates a process with good accuracy. In general, C_a cannot be too small since a smaller C_a implies the process mean shifts farther away from the process target and results in much process loss. Let $C_a = (1 - 1/a)$, then the values of μ are $[T + (1/a) \times D_u]$ and $[T - (1/a) \times D_l]$ for each C_a. The slope of the corresponding subsidiary line is $(a + 1)/(a - 1)$ when the process mean is greater than the process target, and the slope of the corresponding subsidiary line is $(a - 1)/(a + 1)$ when the process mean is smaller than the process target. Table 1 briefly displays the values of C_a and the corresponding μ.

Values of C_a	Values of μ	A	Slope
1.000	T	∞	1.0000
0.875	$T + 0.125\,D_u$	8	1.2857
	$T - 0.125\,D_l$	8	0.7778
0.750	$T + 0.250\,D_u$	4	1.6667
	$T - 0.250\,D_l$	4	0.6000
0.500	$T + 0.500\,D_u$	2	3.0000
	$T - 0.500\,D_l$	2	0.3333
0.250	$T + 0.750\,D_u$	1.33	7.0000
	$T - 0.750\,D_l$	1.33	0.1429
0.000	USL	1	∞
	LSL	1	0

Table 1. Values of C_a and the corresponding μ.

3. Application of process capability analysis chart

C_{pa}, C_{pu} and C_{pl} are three indices to evaluate the process capabilities on a Process Capability Analysis Chart (PCAC). For a multi-process product, assume there are n_a nominal-the-best processes evaluated by C_{paj} $(j = 1, 2, \ldots, n_a)$, n_u smaller-the-better processes evaluated by C_{puj} $(j = 1, 2, \ldots, n_u)$, and n_l larger-the-better processes evaluated by C_{plj} $(j = 1, 2, \ldots, n_l)$. The unilateral specifications C_{puj} and C_{plj} hold the one-to-one mathematical relation to the process yield under normal assumptions. Let X be the random number of process means, and the formula for the smaller-the-better process can be described as:

$$p_{ij} = P(X < USL) = P\left(\frac{X-\mu}{\sigma} < \frac{USL-\mu}{\sigma}\right) = P(Z < 3C_{pij}) = \Phi(3C_{pij}).$$

Similarly, the formula for the larger-the-better process can be described as:

$$p_{ij} = P(x > LSL) = \Phi(3C_{pij}).$$

Thus, the general form for unilateral characteristics can be written as: $p_{ij} = \Phi(3C_{pij})$, where Φ denotes the standard normal cumulative distribution function, $i \in \{u, l\}, j = 1, 2, \ldots, n_i$.

Whereas C_{pa} is a valid index for bilateral specification with asymmetric tolerance, Pearn and Chen (1998) derived the formula regarding C_{pa} and process yield p_{ij} in the following:

$$p_{ij} \geq 2\Phi(3C_{paj}) - 1, \qquad j = 1, 2, \ldots, n_a.$$

Combining the above equation with $\Phi(x) \geq 2\Phi(x) - 1$, we attain that the better the process capability, the higher the process yield. And the relation between process yield p_{ij} and process capability index is:

$$p_{ij} \geq 2\Phi(3C_{pij}) - 1, \quad i \in (S = \{u, l, a\}, \quad j = 1, 2, \ldots, n_i.$$

Based on the above analysis, the process yield is evaluated in terms of an integrated process capability index C_T in the following:

$$C_T = \left(\frac{1}{3}\right)\Phi^{-1}\left(\left[\left(\prod_{i\in S}\prod_{j=1}^{n_i}[2\Phi(3C_{pij}) - 1]\right) + 1\right]\Big/2\right).$$

In particular, when $C_T = c$, $\prod_{i\in S}\prod_{j=1}^{n_i}(2\Phi(3C_{pij}) - 1) = 2\Phi(3c) - 1$.

As far as the integrated process capability for a multi-process product is concerned, the integrated process capability is definitely lower than any individual process capability. Assume the individual process yields are independent; the entire process yield p can be described as:

$$p = \prod_{i\in S}\prod_{j=1}^{n_i}(p_{ij}).$$

Because $p_{ij} \geq 2\Phi(3C_{pij}) - 1$, the formula for process yield is $p \geq 2\Phi(3c) - 1$. Obviously, a larger process capability implies a higher process yield. For instance, $C_T = 1.0$ ensures the process yield exceeds 99.73%.

Mostly, the entire process yield of a multi-process product is lower than any individual process yield ($p \leq p_{ij}$). Similarly, when the entire process yield (or entire product capability) is preset to satisfy the required level, the individual process yield

Quality condition	Values of C_0
Inadequate	$C_0 < 1.00$
Capable	$1.00 \leq C_0 < 1.33$
Satisfactory	$1.33 \leq C_0 < 1.50$
Excellent	$1.50 \leq C_0 < 2.00$
Super	$2.00 \leq C_0$

Table 2. The five quality conditions

(or individual process capability) should exceed the preset standard for the entire product. For instance, if $C_T \geq c$, then

$$C_T = \left(\frac{1}{3}\right)\Phi^{-1}\left(\left[\left(\prod_{i\in S}\prod_{j=1}^{n_i}[2\Phi(3C_{pij})-1]\right)+1\right]\Big/2\right) \geq c.$$

Specifically, when the preset minimum values of process capabilities for individual characteristics are equal, we have:

$$C_T = \left(\frac{1}{3}\right)\Phi^{-1}(((2\Phi(3C_{pij})-1)^n+1)/2) \geq c,$$

where $n = n_a + n_u + n_l$. The critical value $C_0(C_{pij} \geq C_0)$ for individual process capability can be attained by solving the previous inequality when the integrated process yield exceeds $c(C_T \geq c)$, where

$$C_0 = \left(\frac{1}{3}\right)\Phi^{-1}\left(\frac{\sqrt[n]{2\Phi(3c)-1}+1}{2}\right).$$

As noted by Pearn and Chen (1997), a process is ‘inadequate’ if the process capability index is less than 1.00; it indicates that the process is not adequate to

	Entire process capability index $C_T \geq c$			
N	$c = 1.0$	$c = 1.33$	$c = 1.5$	$c = 2.0$
1	1.000	1.330	1.500	2.000
2	1.068	1.384	1.548	2.037
3	1.107	1.414	1.576	2.059
4	1.133	1.436	1.595	2.074
5	1.153	1.452	1.610	2.085
6	1.170	1.465	1.622	2.095
7	1.183	1.477	1.632	2.103
8	1.195	1.486	1.641	2.110
9	1.205	1.495	1.649	2.116
10	1.214	1.502	1.656	2.121
11	1.222	1.509	1.662	2.216
12	1.230	1.515	1.667	2.130
13	1.236	1.520	1.673	2.135
14	1.243	1.526	1.677	2.138
15	1.248	1.530	1.682	2.142

When $n > 15$, $C_0 = \Phi^{-1}[(\sqrt[n]{2\Phi(3c)-1}+1)/2]/3)$.

Table 3. Values of individual process capability C_0.

the preset manufacturing specifications. A process is called ‘capable’ if the process capability index ranges from 1.00 and 1.33; it indicates that some process control is needed. A process is called ‘satisfactory’ if the process capability index ranges from 1.33 and 1.50; it indicates that process quality is satisfactory. A process is called ’excellent’ if the process capability index is between 1.50 and 2.00; finally, a process is ’super’ when the process capability index exceeds 2.00. Table 2 displays the five conditions and the corresponding values of C_0.

Based on the above, table 3 lists the minimum values of the process capability indices C_0 for individual process characteristics when the entire process capability indices C_T are preset to 1.00, 1.33, 1.50 and 2.00 versus n individual process characteristics (when $n > 15$, $C_0 = \Phi^{-1}[(\sqrt[n]{2\Phi(3c) - 1} + 1)/2]/3$. Namely, C_0 is specified when C_T and the number of individual process characteristics n are selected. In addition, if the process loss is considered, C_a is set to fit the specific requirement of the real situation. The process capability zone is marked on the Product Capability Analysis Chart (PCAC) according to the minimum individual process capability C_0 and the maximum process shift C_a.

4. Example

One example will be given for the purpose of illustration. Consider one product composed of five nominal-the-best ($n_a = 5$), two larger-the-better ($n_l = 2$), and two smaller-the-better ($n_u = 2$) process characteristics. The process capability indices on the dimensional space $(X, Y) = (C_{du}, C_{d_l}) = ((d_u)\ C_{pu}, (d_l)C_{pl})$, where $d_u = d^*/D_u$, and $d_l = d^*/D_l$, are located on PCAC for nominal-the-best characteristics. In addition, the X-axis characterizes process capabilities $(X, Y) = (C_{pu}, 0)$ for smaller-the-better characteristics, while the Y-axis characterizes process capabilities $(X, Y) = (0, C_{pl})$ for larger-the-better characteristics. Table 4 displays process specifications and capability indices for nine process characteristics. Assume the entire process capability is preset to exceed one ($C_T \geq c = 1$), the minimum individual process capability C_0 is 1.205 when $n = 9$ from table 3, and C_0 could also be verified by $C_0 = \Phi^{-1}[(\sqrt[9]{2\Phi(3) - 1} + 1)/3]/3 = 1.205$. In addition, when the process loss is considered, e.g. $C_a \geq 0.875$, the process capability zone according to C_0 and C_a is marked as shown in figure 2.

The product capabilities for nine process characteristics are discussed as below.

(1) Processes $N1 - N5$ are nominal-the-best type with bilateral specifications:

  (i) Process $N1$ is not located within the process capability zone and C_a is less than 0.875; it indicates that the process is not capable both on precision and accuracy, either to reduce the process variation or shift the process mean closer to the process target in order to upgrade the process capability.

  (ii) The process capability index for process $N2$ is not located within the process capability zone and the process capability is classified as inadequate. Actions must be taken to improve the process quality. Process engineers should monitor the process to figure out all assignable causes, and reduce the process variation to elevate the process capability.

  (iii) The process capability index for $N3$ is 1.50 and C_a is 0.90; it indicates that the process capability is satisfactory and no stringent quality control is required.

Process	type	LSL	T	USL	μ	σ	C_a	X-axis	Y-axis	C_{pa}
$N1$	Nominal-the-best	580	600	620	595	5.0	0.75	1.67	1.00	1.00
$N2$	Nominal-the-best	590	600	620	600	5.0	0	0.67	0.67	0.67
$N3$	Nominal-the-best	580	600	620	602	4.0	0.90	1.50	1.83	1.50
$N4$	Nominal-the-best	56	58	60	57.8	0.4	0.90	1.83	1.50	1.50
$N5$	Nominal-the-best	56	57	60	58	0.4	0.50	0.56	1.67	0.56
$L1$	Larger-the-better	15			20	1.1			1.52	1.52
$L2$	Larger-the-better	15			18	1.1			0.91	0.91
$S1$	Smaller-the-better			100	82	6.0		1.00		1.00
$S2$	Smaller-the-better			100	77	6.0		1.28		1.28

Table 4. Specifications and capability indices for nine process characteristics.

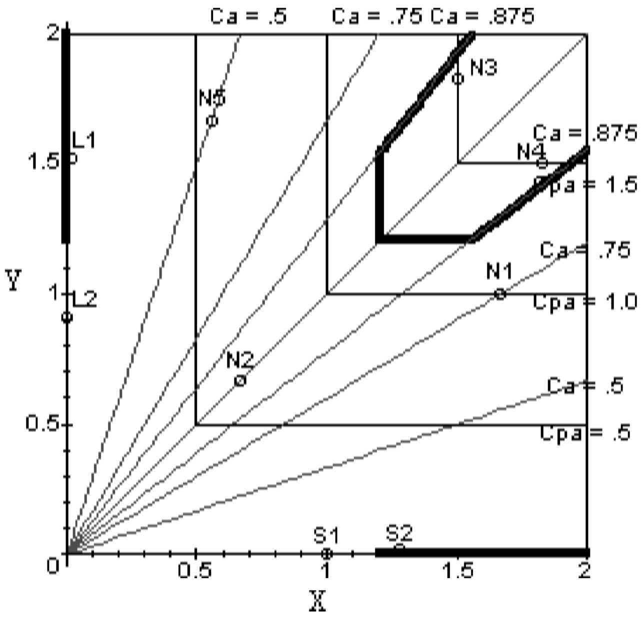

Figure 2. Process capability zone.

(iv) The process capability index for process $N4$ is 1.50 and C_a is 0.90; it means that the process capability is excellent.

(v) The process capability index for process $N5$ is 0.56 and C_a is 0.50; the process capability is classified as inadequate with too much process loss. Similar to process $N2$, all assignable causes should be removed and the process variation needs to be reduced. In addition, process engineers should bring the process mean back to the closer process target to avoid a huge process loss.

(2) Processes $L1$ and $L2$ are unilateral specifications, which are larger-the-better type with lower specification limits only. The process capability index of $L1$ is located within the process capability zone and the process capability is considered as satisfactory, while the process capability index of $L2$ is not located within the process capability zone and the process capability is classified as inadequate. To enhance the process quality of $L2$, either reduce the process variation or shift the process mean farther away from the lower specification limit.

(3) Processes $S1$ and $S2$ are unilateral specifications, which are smaller-the-better types with upper specification limits only. The process capability index of $S1$ is not located within the process capability zone and actions must be taken to reinforce the process capability. When the process capability index of $S2$ is located within the process capability zone, the process capability is capable.

On the whole, of the nine process characteristics, there are five process capability indices, $N1$, $N2$, $N5$, $L2$ and $S1$, not located within the process capability zone. Actions to bring all indices back within the process capability zone to enhance the process capabilities may be different but necessary. Under cost considerations, all indices are located within the process capability zone, and engineers can continue to reinforce the quality level by shifting the process mean to target and reducing the process variation in order to locate indices near the diagonal for a nominal-the-best characteristic process; and can continue to improve processes to attain larger process capability indices for processes with unilateral specifications.

5. Conclusions

Although process capability indices have been widely applied in manufacturing, the conventional indices restrict the application to multi-process products. In this paper, we construct an effective and efficient method via a Product Capability Analysis Chart (PCAC) to evaluate the process capability of an entire product composed of multiple process characteristics. First, Singhal's Multi-process Performance Analysis Chart (MPPAC) is modified to analyse processes with symmetric and asymmetric specifications. Secondly, smaller-the-better and larger-the-better characteristics are evaluated via the revised axes X and Y, respectively. The process capability zone is then marked according to a specified individual process capability C_0 and a preset process shift C_a. The Product Capability Analysis Chart interprets multi-process capabilities and distinguishes the process precision and accuracy with respect to the locations of the process capability indices on the Product Capability Analysis Chart. Finally, quality improvement actions are taken with respect to unsatisfactory processes to enhance the entire process capability. Thus, the revised Product Capability Analysis Chart is encouraged for the application of statistical process control in the factory.

6. Nomenclature

$$A^* = \max\{d^*(\mu - T)/D_u, d^*(T - \mu)/D_l\},$$
$$C_{du} = (d^*/D_u)C_{pu},$$
$$C_{dl} = (d^*/D_l)C_{pl},$$
$$C_p = 1/2(C_{pu} + C_{pl}),$$
$$C_{pa} = (d^* - A)/3\sigma,$$
$$C_{pu} = (USL - \mu)/3\sigma,$$
$$C_{pl} = (\mu - LSL)/3\sigma,$$
$$C_{pk} = \min\{C_{pu}, C_{pl}\},$$
$$d^* = \min\{D_u, D_l\},$$
$$D_u = USL - T,$$
$$D_l = T - LSL,$$

C_0	critical value,
C_T	integrated process capability index,
$MPPAC$	Multi-process Performance Analysis Chart,
p	process yield
$PCAC$	Process Capability Analysis Chart,
μ	process mean,
σ	process standard deviation,

T process target,
USL the upper specification limit
LSL the lower specification limit,
$\Phi(\cdot)$ the cumulative function of standard normal distribution,
Φ^{-1} the inverse cumulative function of standard normal distribution,
Z standard normal distribution.

Acknowledgements

The authors would like to thank the National Science Council of the Republic of China for financially supporting this research under Contract No. NSC-89-2213-E-167-004.

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