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# The electric field effect in $g_3$ model of the layer superconductors

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**Abstract**

High-$T_c$ superconductor is considered as alternately stacked metal and insulating layers. The electric field is applied along the stacking direction and investigated in the frame of Ginzburg–Landau theory with the coupling $g_3$ between metallic layers for the transition temperature $T_c$. Then the system of equations for $T_c$ can be solved by the transfer matrix method. The shift of the transition temperature is analytically derived and we find the criterion for the enhancement of $T_c$ while the electric field is applied. 

*Keywords:* Electric field effect; Transition temperature; Transfer matrix method

## 1. Introduction

Since high-$T_c$ cuprate is found, shifts of the transition temperature $T_c$ of up to 30 K have been observed in high-$T_c$ cuprate films by an applied electric field [1]. Just last year, Schön et al. [2] made a field-effect transistor for $C_{60}$ and found that the induced superconductivity at a $T_c$ of 10 K in the positive applied voltage to be an electron-doped $C_{60}$. Later, Schön et al. [3] find that a $T_c$ of 52 K for the negative applied voltage is found to be hole-doped $C_{60}$. This new finding has raised a new method to get high-$T_c$ superconductivity. Also, a superconductor to insulator switch is carried out. Almost two years ago, Tao et al. [4] found that an electric field can drive high-$T_c$ superconductors (HTSC) particles together to form a round ball. This effect may be explained by increasing the surface tension of HTSC particles due to decrease the interlayer Josephson coupling while the electric field is applied. Hence, it is very interested in studying futher the electric field effect of HTSC, especially relating the electric field effect to the superconductivity mechanism.

The mechanism causing the large electric field effect for $T_c$ shifts of HTSC is most accepted to be the charge modulation model [5]. The usual approach is to use BCS weak coupling model, Poisson equation and Thomas–Fermi (TF) model for a weaker anisotropic HTSC. Sakai [6] used TF model and regarded HTSC as an alternately stacked superconductivity and insulating layers. Here, we use TF model, $g_3$ interlayer coupling model [7] and Plasmon BCS-like mechanism [8] to simulate the field effect of the shift of $T_c$ in HTSC. In deriving the shift of $T_c$, we propose to use the transfer matrix method. The condition of the $T_c$ shift of HTSC due to the electric field is gotten and discussed.

Our approach is briefly described in Section 2. The results and discussion are presented in Section 3. Finally, a brief conclusion is made.

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## 2. Theory

Consider a system of a MIS-like structure [6] shown in Fig. 1. The order of the stacked materials from the top metallic electrode ($z = 0$) to the bottom metallic (the positive $z$-axis is downward perpendicular to the surface of HTSC) electrode is insulating layer $I_o$ with thickness S, and then the layer HTSC. HTSC is modeled as stacked metal ($CuO_2$ layer) and insulating layer (with thickness $d$). When a constant voltage $V$ is applied along $c$-direction of HTSC, the charge density and Fermi level will be changed accordingly.

In deriving the shift of $T_c$ due to the uniform applied electric field, some assumptions are made: (1) there is an induced positive charge density $Q$ on the 1st metallic plate, (2) another metallic electrode contacts with the last insulating layer of HTSC, (3) there are $N$ layers of superconducting $CuO_2$ planes, (4) the electric field penetrating HTSC is treated by Poisson equation and TF model.

The induced charge density $Q$ at the gate electrode is $Q = (V - \phi_1)(\varepsilon_i\varepsilon_b)/(\varepsilon_i s + \varepsilon_b L)$, where $\phi_1$ is the electrostatic potential of the topmost $CuO_2$ layer of HTSC, $\varepsilon_i$ and $\varepsilon_b$ are the dielectric constant of the gate insulator and the insulator layer $I_o$, respectively. $V$ is a gate voltage. In TF model, the induced charge density on the $p$th $CuO_2$ plane is $\delta n_p = e\rho_F\phi(r_\parallel, z_p)$, where $\rho_F$ is density of state at the Fermi energy and $z_p = L + S + (p-1)d$. The electrostatic potential $\phi(r_\parallel, z)$ at the position $z$ may be determined by the Poisson's equation [9]

$$\frac{d^2\phi}{dz^2} = -\frac{Q}{\varepsilon_i}\delta(z) + \left\{\left[\varepsilon_s Q - \varepsilon_b(Q - e^2\rho_F\phi(z))\right]/\varepsilon_b\varepsilon_s\right\} \times \delta(z - z_1) + \frac{e^2\rho_F}{\varepsilon_s}\sum_{p=2}^{N}\phi(z)\delta(z - z_p) + \frac{\phi_N(z_N)}{d}\delta(z - z_{N+1})$$

where $\phi_N$ is the potential between $N$th and $(N+1)$th $CuO_2$ layer. $\varepsilon_s$ is the dielectric constant of the insulator inside HTSC. While $z \neq 0$ and $z \neq z_p$, the Poisson's equation becomes Laplace equation. The solution of this equation is $\phi_p(z) = A_p(z - z'_p) + B_p$, where $z'_p$ is a reference coordinate and may be taken to be $z'_p = z_p + d/2$. By using Gauss's law and the continuity of the electrostatic potential $\phi_p(z_{p+1}) = \phi_{p+1}(z_{p+1}) = \phi(z_{p+1})$, then we have the recurrence relation

$$\begin{pmatrix} A_p \\ B_p \end{pmatrix} = M^{p-1}\begin{pmatrix} A_1 \\ B_1 \end{pmatrix}$$

where

$$M^{p-1} = \frac{1}{2}\begin{pmatrix} +\lambda_+^{p-1} + \lambda_-^{p-1} & \lambda_+^{p-1} - \lambda_-^{p-1} \\ \lambda_+^{p-1} - \lambda_-^{p-1} & \lambda_+^{p-1} + \lambda_-^{p-1} \end{pmatrix}\begin{pmatrix} A_1 \\ B_1 \end{pmatrix}$$

and

$$\lambda_\pm = \left(1 + \frac{q_s d}{2}\right) \pm \sqrt{\left(1 + \frac{d q_s}{2}\right)^2 - 1},$$

$$q_s = e^2\rho_F/\varepsilon_s$$

To get the transition temperature $T_c$ of HTSC under the applied electric field, we use the Ginzburg–Landau theory approach. According to the $g_3$ model proposed by Schneider and his collaborators [7], the free energy $F$ of HTSC near $T_c$ is written as

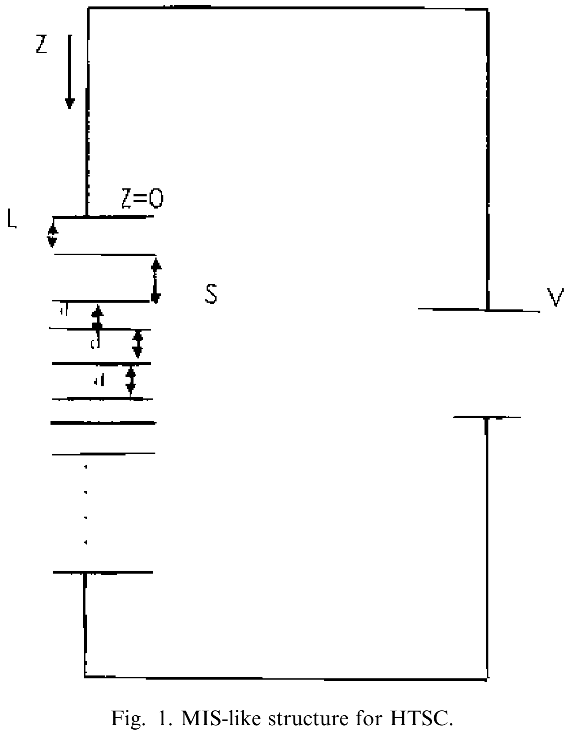


Fig. 1. MIS-like structure for HTSC.

$$F = \sum_p \int d\vec{r} \Big\{ \alpha(T - T_{cp})|\psi_p(\vec{r})|^2 - \frac{g^3}{2}\left[\psi_p^*(\vec{r})(\psi_{p+1} + \psi_{p-1}) + \text{c.c.}\right] \Big\}$$

where

$$T_{cp} = 1.134 E_{Fp}\left(-\frac{1}{\lambda_p}\right)$$

and

$$E_{Fp} = E_F^o + e\phi_p$$

$\lambda_p$ is a coupling contant on the $p$th $CuO_2$ layer. $\psi_P$ is the order parameter on the $P$th $CuO_2$ layer. $g_3$ is interlayer coupling constant and assumed to be independent of the electric field. From minimizing F with respect to $\psi_P^*$, we get the linear system equations, but we may cast this linear system equations into a transfer matrix form and then solve it easily. Here, we write out the results.

$$\begin{pmatrix} \psi_p \\ \psi_{p+1} \end{pmatrix} = M_p M_{p-2} \ldots M_1 \begin{pmatrix} o \\ \psi_{1'} \end{pmatrix} \tag{1}$$

where

$$M_i = \begin{pmatrix} 0 & 1 \\ -1 & \alpha(T_c - T_{ci})/g_3 \end{pmatrix} \tag{2}$$

Then $T_c$ can be determined by Eqs. (1) and (2). For instance, only one $CuO_2$ layer, i.e. $p = 1$, then $\psi_2 = 0$, and $T_c = T_{c1}$. For two $CuO_2$ layers, we have $-1 + \alpha^2(T_c - T_{c1})(T_c - T_{c2})/g_3^2 = 0$. In general, we need to find the eigenvalues of transfer matrix $M_i$. The eigenvalues of $M_i$ are

$$\beta_i = \frac{\alpha(T_c - T_{ci})}{2g_3} \pm \sqrt{\left(\frac{\alpha(T_c - T_{ci})}{2g_3}\right)^2 - 1}$$

Set $\beta_{i+}$ for the upper sign of $\beta_i$ and $\beta_{i-}$ for the lower sign of $\beta_i$. The corresponding eigenvectors of $M_i$ are

$$\frac{1}{\sqrt{\beta_{i+}^2 + 1}}\begin{pmatrix} 1 \\ \beta_{i+} \end{pmatrix}$$

and

$$\frac{1}{\sqrt{\beta_{i-}^2 + 1}}\begin{pmatrix} 1 \\ \beta_{i-} \end{pmatrix}$$

According to the results of Sakai [6], the electric field effect is limited within three superconductivity $CuO_2$ layers. That is, beyond three $CuO_2$ layers, $T_c$ would become a bulk value. Hence $\beta_{i\pm}$ $(i \geqslant 4)$ will become complex value. This is equivalent to require $\alpha(T_c - T_{ci}) < 2|g_3|$. Therefore, the electric field effect is enhanced when the condition $\alpha(T_c - T_{ci}) > 2|g_3|$ must be met for $i \leqslant 3$. That is, $T_c - (2|g_3|/\alpha) > T_{ci} = 1.134 E_{Fi} e^{-1/\lambda_i}$, where the applied voltage $V$ involves essentially in $E_{Fi}$. It is obvious that the limited value of the applied voltage strongly depends on the interlayer coupling $g_3$.

## 3. Results and discussion

The transfer matrix method is proposed to get $T_c$ of layered HTSC under the uniform applied electric field. The change of $T_c$ can be determined through Eqs. (1) and (2). The condition of the $T_c$ shift due to the applied voltage is derived. There is a limitation of the applied voltage to have the electric field effect on $T_c$ shift. Such a result is to be tested by the experimental results. The change of $T_c$ relates to the applied voltage is obvious in the expression of $T_{ci}$. For one and two $CuO_2$ layers, the $T_c$ change has been discussed in our previous work [10].

## 4. Conclusion

$T_c$ of HTSC in an applied electric field is derived on the basis of plasmon mechanism in the framework of TF model and $g_3$ interlayer coupling model. The electric field effect on HTSC is essentially limited within the Debye screening length. The change of $T_c$ due to electric field is shown above how to get it. The limitation of the applied voltage to have $T_c$ change is gotten, except the dielectric breakdown field. It is interesting in testing this result. Our approach may be extended to other mechanism.

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