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An integrated inventory model for a single vendor and multiple buyers with ordering cost reduction

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Abstract

Forming strategic alliances and utilizing modern information technologies have been the two most important ways for firms to gain such competitive advantages as lower logistics costs and secure customers' loyalty. In this paper, we consider an integrated inventory system where a single vendor purchases and processes raw materials in order to deliver finished items to multiple buyers. The vendor and all buyers are willing to invest in reducing the ordering cost (e.g., establishing an electronic data interchange based inventory control system) in order to decrease their joint total cost. The amount of investment determines the planned ordering cost and hence affects their replenishment decisions. One major managerial implication from this ordering cost reduction is that the efforts to streamline and speedup transactions via the application of information technologies may result in a higher degree of coordination and automation among allied trading parties. An analytical model is developed to derive the optimal investment amount and replenishment decisions for both vendor and buyers. The exponential ordering cost function is then applied to our general model, and a numerical analysis is performed to provide interesting insights of the model. Numerical results show that the vendor and all the buyers can benefit directly from substantial cost savings by this ordering cost reduction investment. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

It has been a trend for firms to establish inter-organizational information systems with their buyers in order to gain such competitive advantages as lower logistics costs and securer customers' loyalty. One of the most contemporary information systems is to apply the electronic data interchange (EDI) technology to not only link but also automate the ordering, shipping, inquiring, and payment activities between vendor and

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buyers. An important advantage of using EDI to connect a vendor with his/her buyers is that the products consumption information from each buyer can be automatically and instantly transmitted to the vendor. Based on the information, the vendor can decide when and how many items to deliver to which buyer so that the overall system cost is optimized and the performance of inventory control can be significantly improved.

EDI systems can also result in better vendor–buyer integration and thus streamline the supply chain of traditional goods. This point is well illustrated in the channel partnership established between Levi Strauss and its retailers. Levi Strauss, an apparel manufacturer, operates the LeviLink, an EDI system linking it with its retailers, to speedup the processing of orders and to respond quickly to the customers' changing tastes [1]. The functions of LeviLink include management of inventory, management and reconciliation of purchase orders, tracking of purchase orders, processing and payment of invoices, capturing of point of sales information, and analysis of market trends. Many other successful cases of utilizing the modern information technology in operations management have also been reported [2].

In recent years, many studies have focused on the benefit from ordering cost reduction in the inventory systems but only from the single party's viewpoint (for examples, see [3–6]). However, considering the dyadic relationship between the vendor and buyer is necessary for implementing an EDI-based ordering system since the implementation needs both the trading partners to interchange transaction documents, to standardize transaction procedures, and to integrate related applications [7]. To address the vendor–buyer integration of EDI, Banerjee and Banerjee [8] consider an EDI-based vendor-managed inventory (VMI) system in which the vendor makes all replenishment decisions for his/her buyers to improve the joint inventory cost. Their work focuses solely on the inventory policy by assuming that an EDI system has already been operated between vendor and buyers and hence no ordering cost will incur for both parties.

In order to streamline the supply chain, any vendor is expected to synchronize his/her production cycles with buyers' ordering cycles as well as raw material procurement cycles so that the total inventory cost for the entire chain can be reduced. The cooperation between vendor and buyers for improving the performance of inventory control has thus received a great deal of attention from researchers. Several authors have studied the integrated inventory models in which the vendor and the buyer coordinate their production and ordering policies, in order to lower the joint inventory costs (for examples, see [9–12]). Other research works of integrated inventory models have been summarized in the related review articles [13,14]. Most previous work on integrated vendor–buyer inventory systems does not incorporate raw material procurement decisions into consideration. However, some researchers have taken the procurement of raw materials into account for developing their inventory models in which the manufacturer unifies procurement and production policies to minimize his/her own total inventory cost (for examples, see [15–19]).

In this paper, we investigate an integrated inventory system where a single vendor purchases and processes raw materials in order to deliver finished items to multiple buyers. Our work extends Banerjee and Banerjee's modes [8] to incorporate ordering cost reduction and raw material procurement into the integrated inventory decisions. Our study is motivated by the fact that more and more firms in practice have devoted tremendous efforts to reduce ordering times and costs with their trading partners but few formal models are available in the literature to evaluate those efforts. Therefore, our model serves as a pioneering work on investigating the effects of ordering cost reduction on the integrated inventory system. One major managerial implication of this ordering cost reduction is that the efforts to streamline and speedup transactions via the application of information technologies may result in a higher degree of coordination and automation between allied trading parties. Assumptions and notations used in our work are presented in the next section. In Section 3, we derive an analytical model in which the planned ordering cost is a general function of the expenditure to operate the ordering system, and also develop a solution procedure to find the optimal decisions. Then, in Section 4, we apply the special case of exponential cost function to our general model and perform a numerical analysis to gain some interesting insights. Finally, conclusions are summarized in Section 5.

2. Assumptions and notations

In our model, we assume that the vendor purchases raw materials outside to produce finished items. The procurement lot size of raw materials is assumed to be an integral multiple of the usage of each production batch. This policy has also been considered in some previous works [15–17], which are more general than the lot-for-lot procurement policy adopted in other work [18,19]. The finished items are then delivered to multiple buyers at a common cycle and backorders are allowed for all buyers (see Fig. 1). Following the assumption in Banerjee and Banerjee [8], all buyers adopt the VMI policy, that is, the vendor makes replenishment decisions for all buyers to optimize the joint total cost. Now, the vendor and his/her buyers plan to establish new ordering systems (e.g., EDI-based ordering systems) between them to reduce the ordering cost. It is expected that the system will result in lower joint total cost and, more importantly, safer vendor–buyer relationship because the buyers can benefit directly from this cost saving. Our model is useful, particularly, in inventory systems where a vendor and his/her buyers form strategic alliances for profit sharing. Other assumptions for our model are:

1. Shortages are not allowed for the vendor.
2. The information of buyers' replenishment decision parameters is available to the vendor.
3. The planned ordering cost for each buyer depends on the expenditure incurred per unit time to operate the new ordering system. This expenditure could be the leasing cost of equipment and the operating cost to keep the system working effectively.

The problem for the vendor is therefore to simultaneously determine the buyers' backlogging quantities, the cycle times for both finished items delivery and raw materials procurement, and the operating cost for new ordering systems so that the joint total cost can be minimized.

The notations used in our model are shown as follows:

D_i demand rate for buyer i, which is a known constant

P production rate for vendor, which is a known constant and $P \geqslant \sum_{i=1}^{m} D_i$, where m is the number of buyers

n integral number of production batches per raw materials procurement cycle, which is a decision variable and $n \geqslant 1$

K expenditure per unit time to operate the planned ordering system between vendor and all buyers, which is a decision variable

C common cycle time for buyers, which is a decision variable

f_i fraction of backlogging time in a cycle for buyer i, which is a decision variable

M usage rate of raw materials for producing each finished item

A ordering cost per raw materials order for vendor

S setup cost per production run for vendor

T_{0i} original ordering cost per buyer i's order

K_0 expenditure required per unit time for reducing the largest ordering cost among buyers to zero

$T_i(K)$ planned ordering cost per buyer i's order, which is a strictly decreasing function of K with $T_i(0) = T_{0i}$ and $T_i(K_0) = 0$

H_{vm} carrying cost per unit of raw materials held per unit time for vendor

H_{vp} carrying cost per finished item held per unit time for vendor

H_{bi} carrying cost per unit held per unit time for buyer i

L_i backlogging cost per unit backlogged per unit time for buyer i

HC_{vm} vendor's carrying cost per procurement cycle for raw materials

TC_{vm} vendor's total cost per unit time for raw materials

TC_{vp} vendor's total cost per unit time for finished items

TC_v vendor's total inventory cost per unit time
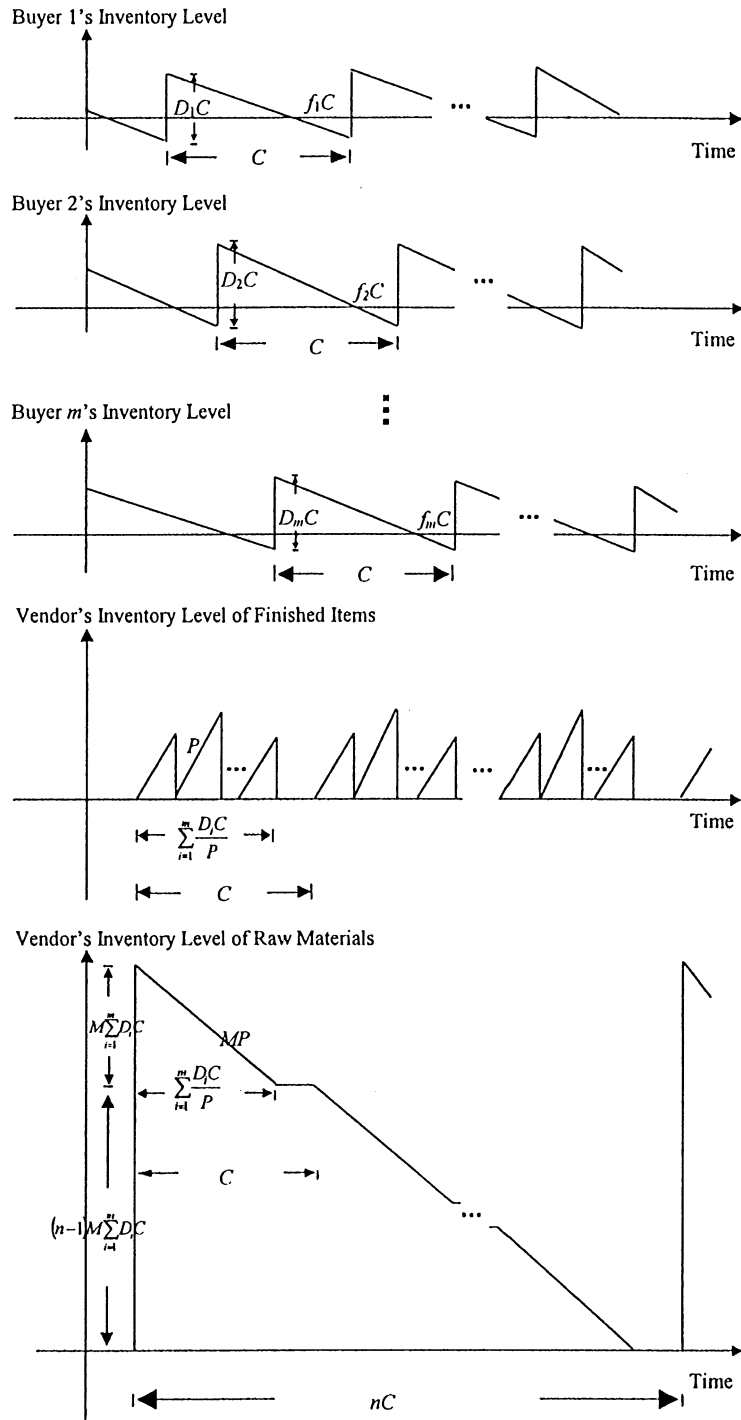
Buyer 1's Inventory Level

Buyer 2's Inventory Level

Buyer *m*'s Inventory Level

Vendor's Inventory Level of Finished Items

Vendor's Inventory Level of Raw Materials

Fig. 1. The inventory levels for vendor and buyers.

TC_{bi} buyers i's total inventory cost per unit time

JTC   joint total cost per unit time for vendor and all buyers

3. The general model

The behavior of inventory levels for the vendor and all buyers is illustrated in Fig. 1. The arrival of each raw materials procurement will coincide with the start of a new production run. The procurement lot size of raw materials is equal to n multiples of the usage of each production batch and hence each procurement cycle equals nC time units. The stock of raw materials will be consumed continuously during each production run period $\sum_{i=1}^{m} D_i C/P$, and then be held until the next production run starts. Therefore, the vendor's raw materials stock per procurement cycle consists of n triangles and $(n-1)$ rectangles and the carrying cost for raw materials per cycle is

$$\text{HC}_{vm} = H_{vm}\left[\frac{n}{2}\left(\frac{M\sum_{i=1}^{m} D_i C}{MP}\right)\left(M\sum_{i=1}^{m} D_i C\right) + \sum_{j=1}^{n-1} j\left(M\sum_{i=1}^{m} D_i C^2\right)\right]$$

$$= \frac{nMH_{vm}}{2P}\left(\sum_{i=1}^{m} D_i C\right)^2 + \frac{n(n-1)MH_{vm}}{2}\sum_{i=1}^{m} D_i C^2.$$

Thus, the vendor's total inventory cost for raw materials per unit time is

$$\text{TC}_{vm} = \frac{1}{nC}(A + \text{HC}_{vm}) = \frac{A}{nC} + \frac{CMH_{vm}}{2}\sum_{i=1}^{m} D_i\left(n - 1 + \frac{\sum_{i=1}^{m} D_i}{P}\right)$$

and his/her total inventory cost for finished items per unit time is

$$\text{TC}_{vp} = \frac{1}{C}\left(S + H_{vp}\sum_{i=1}^{m}\frac{D_i^2 C^2}{2P}\right) = \frac{S}{C} + \frac{CH_{vp}}{2P}\sum_{i=1}^{m} D_i^2.$$

Then, the vendor's total inventory cost per unit time is

$$\text{TC}_v = \text{TC}_{vm} + \text{TC}_{vp}, = \frac{1}{C}\left(\frac{A}{n} + S\right) + \frac{C}{2}\left[MH_{vm}\sum_{i=1}^{m} D_i\left(n - 1 + \frac{\sum_{i=1}^{m} D_i}{P}\right) + \frac{H_{vp}}{P}\sum_{i=1}^{m} D_i^2\right].$$

The buyer i's total inventory cost per unit time is given as

$$\text{TC}_{bi} = \frac{1}{C}\left[T_i(K) + \frac{H_{bi}(1-f_i)^2 D_i^2 C^2}{2D_i} + \frac{L_i f_i^2 D_i^2 C^2}{2D_i}\right]$$

$$= \frac{T_i(K)}{C} + \frac{C}{2}[H_{bi}(1-f_i)^2 D_i + L_i f_i^2 D_i].$$

Therefore, the joint total cost for the vendor and all the buyers per unit time is

$$\text{JTC} = K + TC_v + \sum_{i=1}^{m} TC_{bi}$$

$$= K + \frac{1}{C}\left[\frac{A}{n} + S + \sum_{i=1}^{m} T_i(K)\right] + \frac{C}{2}\left[MH_{vm}\sum_{i=1}^{m} D_i\left(n - 1 + \frac{\sum_{i=1}^{m} D_i}{P}\right)\right.$$

$$\left. + \frac{H_{vp}}{P}\sum_{i=1}^{m} D_i^2 + \sum_{i=1}^{m} H_{bi}(1-f_i)^2 D_i + \sum_{i=1}^{m} L_i f_i^2 D_i\right]. \tag{1}$$

The decision is to minimize JTC with the optimal values of f_i, C, K, and n.

From (1), since

$$\frac{\partial^2 \text{JTC}}{\partial f_i^2} = CD_i(H_{bi} + L_i) > 0,$$

where JTC is convex of f_i for any given C, K, and n; and the optimal value of f_i can easily be obtained as

$$f_i^* = \frac{H_{bi}}{H_{bi} + L_i}. \tag{2}$$

Note that the optimal fraction of backlogging time in a cycle expressed in (2) for buyer i is identical to that of the economic order quantity model (EOQ) with backorders, which to the optimal fraction of carrying stock time is equal to the ratio of unit carrying cost to unit backlogging cost. By substituting (2) into (1) and rearranging the result, we can have

$$\text{JTC} = K + \frac{1}{C}\left[\frac{A}{n} + S + \sum_{i=1}^{m} T_i(K)\right] + \frac{C}{2}\left[MH_{vm}\sum_{i=1}^{m} D_i\left(n - 1 + \frac{\sum_{i=1}^{m} D_i}{P}\right)\right.$$

$$\left. + \frac{H_{vp}}{P}\sum_{i=1}^{m} D_i^2 + \sum_{i=1}^{m} \frac{H_{bi}L_iD_i}{H_{bi} + L_i}\right]. \tag{3}$$

Since the second derivative of (3) with respect to C is

$$\frac{\partial^2 \text{JTC}}{\partial C^2} = \frac{2}{C^3}\left[\frac{A}{n} + S + \sum_{i=1}^{m} T_i(K)\right] > 0,$$

JTC in (3) is convex of C for any given K and n. Thus, the optimal value of C for (3) is

$$C^* = \sqrt{\frac{2\left[A/n + S + \sum_{i=1}^{m} T_i(K)\right]}{H\sum_{i=1}^{m} D_i}}, \tag{4}$$

where

$$H = MH_{vm}\left(n - 1 + \frac{\sum_{i=1}^{m} D_i}{P}\right) + \frac{1}{\sum_{i=1}^{m} D_i}\left(\frac{H_{vp}}{P}\sum_{i=1}^{m} D_i^2 + \sum_{i=1}^{m} \frac{H_{bi}L_iD_i}{H_{bi} + L_i}\right).$$

The term H is composed of carrying cost items only, and represents an integrated unit carrying cost for both vendor and buyers. Also note that the optimal common cycle time for buyers expressed in (4) is in the same form as that of the EOQ model in terms of overall demand rate and relevant cost components. Substituting (4) into (3), the joint total cost for any given K and n becomes

$$\text{JTC}(n, K) = K + \sqrt{2H\sum_{i=1}^{m} D_i\left[\frac{A}{n} + S + \sum_{i=1}^{m} T_i(K)\right]}. \tag{5}$$

Now, for any fixed K, the optimal value of n for (5) can be determined as (see Appendix A for proof)

$$n^*(K) = \begin{cases} n(n-1) < \dfrac{AG}{MH_{vm}\left[S + \sum_{i=1}^{m} T_i(K)\right]} \leqslant n(n+1) & \text{if } G > 0, \\ 1 & \text{otherwise,} \end{cases} \tag{6}$$

where

$$G = \frac{1}{\sum_{i=1}^{m} D_i} \left(\frac{H_{vp}}{P} \sum_{i=1}^{m} D_i^2 + \sum_{i=1}^{m} \frac{H_{bi} L_i D_i}{H_{bi} + L_i} \right) - M H_{vm} \left(1 - \frac{\sum_{i=1}^{m} D_i}{P} \right).$$

The term G implies a trade-off between the carrying cost of each finished item for both vendor and buyers represented by the former term and the carrying cost of idled raw materials for each finished item represented by the latter term. If the unit carrying cost for finished items is greater than that for idled raw materials, i.e., $G > 0$, then the optimal integral number of production batches for each raw material procurement will increase as the difference increases. Otherwise, it is best for the system to apply a lot-for-lot policy to its raw materials procurements. In addition, since $T_i(K)$ is strictly decreasing with K, the optimal integral number n in (6) is non-decreasing with K and consequently, from (4), the values of C^* and hence $f_i^* C^*$ are also strictly decreasing with K. This implies that the greater the expenditure in ordering cost reduction, the smaller the buyers' order quantities and backorders. It is intuitively appealing and also the main reason why the integrated inventory system is willing to invest to reduce the ordering cost. Since the optimal n is non-decreasing with K, it is apparent from (6) that n is bounded by the cases when $K = 0$ and $K = K_0$. Furthermore, for any given n, a necessary condition for K to minimize (5) is

$$\frac{\partial \text{JTC}(n, K)}{\partial K} = 1 + \sqrt{\frac{H \sum_{i=1}^{m} D_i}{2 \left[(A/n) + S + \sum_{i=1}^{m} T_i(K) \right]}} \sum_{i=1}^{m} \frac{\mathrm{d} T_i(K)}{\mathrm{d} K} = 0. \tag{7}$$

Therefore, for each n, $n = n^*(0), n^*(0) + 1, \ldots, n^*(K_0)$, the optimal values of K and hence the joint total cost can be determined by first obtaining the solutions of (7), namely K_7's, and then applying the following rules (see Appendix B for proof):

$$\text{JTC}(n) = \begin{cases} \text{Min}[\text{JTC}(n, K_7)] & \text{if } K_7 \text{ lies within } (0, K_0) \text{ and } \sum_{i=1}^{m} \frac{\mathrm{d}^2 T_i(K_7)}{\mathrm{d} K_7^2} > \frac{1}{H \sum_{i=1}^{m} D_i}, \\ \text{Min}[\text{JTC}(n, 0), \text{JTC}(n, K_0)] & \text{otherwise.} \end{cases}$$

Note that the condition for the first term is to identify those $\text{JTC}(n, K_7)$'s that are valid local minima. Then, the optimal values of n and joint total cost can be derived as

$$\text{JTC}^* = \begin{cases} \text{Min}[\text{JTC}(n) | n = n^*(0), n^*(0) + 1, \ldots, n^*(K_0)] & \text{if } G > 0, \\ \text{JTC}(1) & \text{otherwise.} \end{cases}$$

Finally, the optimal values of C and f_i can then be decided from (4) and (2), respectively.

4. The exponential cost function

To further investigate the behavior of our model, we first apply a special case of exponential ordering cost function to the general model and then perform a numerical analysis to gain some insights.

4.1. Derivation of the optimal solutions

We now assume that the buyer i's ordering cost $T_i(K)$ is an exponential function of expenditure per unit time K and defined as

$$T_i(K) = T_{0i} \mathrm{e}^{-rk}, \quad r > 0 \quad \text{and} \quad i = 1, 2, \ldots, m. \tag{8}$$

Note that there is no upper limit for the expenditure, i.e., K_0 is infinite in this case. By following the procedure described in the previous section, we have $n^*(0)$ and $n^*(K_0)$ from (6) for the cases when $T_i(K) = T_{0i}$ and $T_i(K) = 0$, respectively. Now, for each n, $n = n^*(0)$, $n^*(0) + 1$, ..., $n^*(K_0)$, substitute (8) into (7) and a unique K_7 can be derived as

$$K_7 = \frac{1}{r}\ln\frac{Hr^2\sum_{i=1}^{m}D_i\sum_{i=1}^{m}T_{0i}}{1 + \sqrt{1 + 2Hr^2\sum_{i=1}^{m}D_i((A/n) + S)}}. \tag{9}$$

Note that $K_7 > 0$ if and only if $n > N$, where (see Appendix C for derivation)

$$N = \frac{2(S + \sum_{i=1}^{m}T_{0i}) - Gr^2\sum_{i=1}^{m}D_i\left(\sum_{i=1}^{m}T_{0i}\right)^2}{2MH_{vm}r^2\sum_{i=1}^{m}D_i\left(\sum_{i=1}^{m}T_{0i}\right)^2}$$

$$+ \frac{\sqrt{\left[2(S + \sum_{i=1}^{m}T_{0i}) - Gr^2\sum_{i=1}^{m}D_i\left(\sum_{i=1}^{m}T_{0i}\right)^2\right]^2 + 8AMH_{vm}r^2\sum_{i=1}^{m}D^i\left(\sum_{i=1}^{m}T_{0i}\right)^2}}{2MH_{vm}r^2\sum_{i=1}^{m}D_i\left(\sum_{i=1}^{m}T_{0i}\right)^2}.$$

Since

$$\sum_{i=1}^{m}\frac{\mathrm{d}^2 T_i(K_7)}{\mathrm{d}K_7^2} = \frac{1 + \sqrt{1 + 2Hr^2\sum_{i=1}^{m}D_i((A/n) + S)}}{H\sum_{i=1}^{m}D_i} > \frac{1}{H\sum_{i=1}^{m}D_i}$$

and K_0 is infinite in this case, we have

$$\mathrm{JTC}(n) = \begin{cases} \mathrm{JTC}(n, K_7) & \text{if } n > N, \\ \mathrm{JTC}(n, 0) & \text{otherwise.} \end{cases}$$

Then, the rest of the optimal solutions can also be determined successively by the same procedure as described in the previous section.

4.2. The numerical analysis

We now illustrate the application of our model by a numerical example. The optimal decisions along with the sensitivity analysis for all parameters are presented in Table 1 and several interesting findings can be summarized as follows:

1. Overall, as mentioned in the previous section for the general model, the greater the investment in ordering cost reduction, the shorter the common cycle time and hence the smaller the buyers' order quantities and backorders. Furthermore, the vendor and all buyers can share substantial cost savings (more than 20% in most cases) from this ordering cost reduction investment.
2. When each buyer i's demand rate D_i, original ordering cost T_{0i}, unit carrying cost H_{bi}, or unit backlogging cost L_i, increases, the ordering cost reduction investment K^* and the improvement rate of joint total cost, namely $\Delta\mathrm{JTC}$, increase. A larger D_i implies a higher inventory consumption rate and will result in a higher-order frequency; therefore, a larger investment in buyers' ordering costs reduction is preferred. Each buyer i's inventory cost will increase as T_{0i} or H_{bi} increases, which will desire for a smaller order quantity and, hence, a shorter delivery cycle and a larger reduction investment. L_i has exactly the same effects on the model as H_{bi} does but in the scale of H_{bi}/L_i.
3. When the vendor's production rate P decreases or unit carrying cost for finished items H_{vp} increases, K^* and $\Delta\mathrm{JTC}$ increase. A smaller P or a H_{vp} implies a larger carrying cost for the system, which will encourage a smaller production lot size and, thus, a shorter production cycle and a larger expenditure in ordering cost reduction.

Table 1
Sensitivity analysis for the exponential function case[a]

		n^*	K^*	$T_i(K^*)$	C^*	JTC*	ΔJTC[b] (%)	n_0[c]	C_0
Base example		2	417	1.6	0.047	13,512	23.3	1	0.080
D^i	5000	2	377	2.3	0.069	9,248	21.1	1	0.119
	20,000	2	460	1.0	0.030	20,503	25.6	1	0.051
P	30,000	2	425	1.4	0.043	14,627	25.0	1	0.072
	120,000	2	412	1.6	0.049	12,916	22.1	1	0.084
A	100	1	405	1.8	0.053	12,031	26.2	1	0.074
	400	2	402	1.8	0.054	15,507	21.9	2	0.071
S	100	2	437	1.3	0.038	11,147	31.6	1	0.074
	400	1	370	2.5	0.074	16,771	16.0	1	0.090
T_{0i}	50	2	347	1.6	0.047	13,442	13.9	1	0.070
	200	2	486	1.5	0.047	13,581	35.5	1	0.095
H_{vm}	1	2	408	1.7	0.051	12,419	26.3	2	0.071
	4	1	397	1.9	0.057	14,680	21.8	1	0.075
H_{vp}	2	2	415	1.6	0.047	13,277	22.8	1	0.081
	8	2	420	1.6	0.044	13,969	24.0	1	0.076
H_{bi}	4	1	371	2.5	0.074	11,426	21.2	1	0.097
	16	2	431	1.3	0.040	15,565	26.1	1	0.066
L_i	10	4	381	2.2	0.066	12,592	21.4	1	0.087
	40	2	422	1.5	0.044	14,160	24.3	1	0.075
r	0.005	2	693	3.1	0.047	13,889	21.1	1	0.080
	0.02	2	243	0.8	0.046	13,288	24.5	1	0.080

[a] Base example: $D_i(i = 1, 2, 3) = 10,000$ units/year, $P = 60,000$ units/year, $M = 1$, $A = \$200$/order, $S = \$200$/setup, $T_{0i}(i = 1, 2, 3) =$ $\$100$/order, $H_{vm} = \$2$/unit/year, $H_{vp} = \$4$/unit/year, $H_{bi}(i = 1, 2, 3) = \$8$/unit/year, $L_i(i = 1, 2, 3) = \$20$/unit/year, $r = 0.01$.

[b] ΔJTC = [(JTC$_0$ − JTC*)/JTC$_0$] × 100, where JTC$_0$ is the minimal joint total cost with no ordering cost reduction.

[c] n_0 and C_0 are the optimal solutions with no ordering cost reduction.

4. When the vendor's production setup cost S decreases, K^* and ΔJTC increase. A larger S implies a smaller fraction of the buyers' ordering costs to the overall ordering and setup costs, which will discourage ordering cost reduction expenditure because the return of such investment will be less worthwhile. In addition, this result implies that the investment in ordering cost reduction will be beneficial especially to inventory systems where the vendor has a low production setup cost and can therefore provide just-in-time (JIT) deliveries.

5. The procurement cycle of raw materials n^*C^* will decrease but the improvement rate of joint total cost ΔJTC will increase as the vendor's ordering cost for raw materials A decreases or carrying cost of raw materials for each finished item MH_{vm} increases. Notice from (1) that the usage rate of raw materials per finished item M has exactly the same effects on the model as the unit carrying cost of raw materials H_{vm} does.

6. The increase of the parameter of exponential cost function r will result in a smaller ordering cost reduction investment K^* but a higher cost improvement rate ΔJTC because r represents the ordering cost improvement rate of the related investment.

7. The investment decision and the improvement rate of joint total cost are more sensitive to the vendor's production setup cost S and each buyers' i's original ordering cost T_{0i}, than the other parameters. The inventory decisions are however insensitive to T_{0i} and r because they affect mainly the ordering cost reduction decision.

Our general model is applicable to and our solution procedure can also be applied to find the optimal inventory policy for all other types of ordering cost functions in addition to the exponential one. The solution

procedure can easily be implemented via a simple computer program in any common language (FORTRAN in our case). Furthermore, since the searching range for the optimal integral number of production batches in each raw material procurement cycle is relatively narrow, it will take only about a second to find each set of optimal solutions for the exponential cost function case. Our general model can also be quite efficient for some other cases because the solutions of (7) can easily be obtained for well-behaved ordering cost functions such as linear and stepwise ones.

5. Conclusions

Reducing inventory levels of raw materials, work-in-process, and finished items simultaneously in different stages has become the major focus for supply chain management. In recent years, there has been a growing trend in both research work and practical applications of VMI policy for various industries. An important ingredient for a successful VMI policy is a closer coordination and cooperation among trading parties. Therefore, the application of modern information technologies to reduce communication and order processing times has been inevitable for firms to improve their inventory management. In this paper, we investigate an integrated inventory system where a single vendor purchases and processes raw materials in order to deliver finished items to multiple buyers. The vendor and all the buyers are willing to invest in reducing the ordering cost in order to decrease their joint total cost. The amount of investment determines the planned ordering cost and hence affects their replenishment decisions. Our work is useful particularly for inventory systems where the vendor and his/her buyers form a strategic alliance for profit sharing.

We first derive an analytical model in which the planned ordering cost is a general function of the expenditure to operate the new ordering system, and also develop a solution procedure to find the optimal investment and replenishment decisions for both vendor and buyers. As one might expect, our model proves that the greater the investment in ordering cost reduction, the shorter the common cycle time and hence smaller the buyers' order quantities and backorders. A numerical analysis is then performed for the special case of exponential ordering cost function. Numerical results show that the vendor and all buyers can share substantial cost savings from this ordering cost reduction investment and also provide the following findings about our model:

1. The investment in ordering cost reduction and the improvement rate of joint total cost will increase as each buyer's demand, original ordering cost, carrying cost, backordering cost, or the vendor's carrying cost for finished items increases but as the vendor's production rate or setup cost decreases.
2. The ordering cost reduction investment and the cost improvement rate are more sensitive to the vendor's production setup cost and each buyer's original ordering cost than to other system parameters. In other words, the ordering cost reduction investment followed by JIT deliveres will be beneficial especially to inventory systems where the vendor has a low setup cost and/or each buyer has a high ordering cost originally.

Appendix A

For any fixed K, the minimization of JTC(n, K) in (5) can be achieved by minimizing the following function:

$$Z(n) = \frac{X}{n} + Yn,$$

where

$$X = AG, \quad Y = MH_{vm}\left[S + \sum_{i=1}^{m} T_i(K)\right].$$

Since

$$Z(n + 1) - Z(n) = -\frac{X}{n(n + 1)} + Y$$

and

$$[Z(n + 1) - Z(n)] - [Z(n) - Z(n - 1)] = \frac{2X}{n(n^2 - 1)},$$

the optimal value of n can be determined as follows:

1. When $X > 0$, $Z(n)$ is a strictly convex function and the sufficient and necessary condition for $Z(n)$ to be minimal at n is therefore

$$Z(n + 1) - Z(n) \geqslant 0 \quad \text{and} \quad Z(n - 1) - Z(n) > 0,$$

   that is,

$$n(n - 1) < \frac{X}{Y} \leqslant n(n + 1).$$

   The equality sign is on the right-hand side because a smaller n is preferred in order to lessen the vendor's inventory stock of raw materials.

2. When $X \leqslant 0$, $Z(n)$ is strictly increasing with n and the optimal n is therefore equal to 1.

Appendix B

For any given n, the second derivative of $\mathrm{JTC}(n, K)$ in (5) with respect to K can be derived as

$$\frac{\partial^2 \mathrm{JTC}(n, K)}{\partial K^2} = \sqrt{\frac{H\sum_{i=1}^{m} D_i}{2[(A/n) + S + \sum_{i=1}^{m} T_i(K)]}} \left\{ \sum_{i=1}^{m} \frac{\mathrm{d}^2 T_i(K)}{\mathrm{d}K^2} - \frac{\sum_{i=1}^{m}[\mathrm{d}T_i(K)/\mathrm{d}K]^2}{2[(A/n) + S + \sum_{i=1}^{m} T_i(K)]} \right\}.$$

Since, from (7)

$$\sum_{i=1}^{m} \frac{\mathrm{d}T_i(K_7)}{\mathrm{d}K_7} = -\sqrt{\frac{2[(A/n) + S + \sum_{i=1}^{m} T_i(K_7)]}{H\sum_{i=1}^{m} D_i}}$$

we have

$$\frac{\partial^2 \mathrm{JTC}(n, K_7)}{\partial K_7^2} = \sqrt{\frac{H\sum_{i=1}^{m} D_i}{2[(A/n) + S + \sum_{i=1}^{m} T_i(K_7)]}} \left[\sum_{i=1}^{m} \frac{\mathrm{d}^2 T_i(K_7)}{\mathrm{d}K_7^2} - \frac{1}{H\sum_{i=1}^{m} D_i} \right].$$

Therefore, for any given n, the optimal values of K and hence joint total cost can be determined as follows:

1. If there is no K_7 lying within the interval of $(0, K_0)$, then it is apparent that

$$\mathrm{JTC}(n) = \mathrm{Min}[\mathrm{JTC}(n, 0), \mathrm{JTC}(n, K_0)].$$

2. If $\sum_{i=1}^{m} \mathrm{d}^2 T_i(K_7)/\mathrm{d}K_7^2 > 1/H\sum_{i=1}^{m} D_i$ for some K_7's lying within the interval of $(0, K_0)$, then each $\mathrm{JTC}(n, K_7)$ for those K_7's is a local minimum and hence

$$\mathrm{JTC}(n) = \mathrm{Min}[\mathrm{JTC}(n, K_7)].$$

3. If $\sum_{i=1}^{m} \mathrm{d}^2 T_i(K_7)/\mathrm{d}K_7^2 \leqslant 1/H\sum_{i=1}^{m} D_i$ for all K_7's lying within the interval of $(0, K_0)$, then each $\mathrm{JTC}(n, K_7)$ for those K_7's is either a local maximum or an inflection point and hence

$$\mathrm{JTC}(n) = \mathrm{Min}[\mathrm{JTC}(n, 0), \mathrm{JTC}(n, K_0)].$$

Appendix C

From (9), $K_7 > 0$ if and only if

$$Hr^2 \sum_{i=1}^{m} D_i \sum_{i=1}^{m} T_{0i} > 1 + \sqrt{1 + 2Hr^2 \sum_{i=1}^{m} D_i((A/n) + S)}.$$

By rearranging the inequality and taking square for both sides, we can have

$$\left(Hr^2 \sum_{i=1}^{m} D_i \sum_{i=1}^{m} T_{0i} - 1\right)^2 > 1 + 2Hr^2 \sum_{i=1}^{m} D_i((A/n) + S),$$

which can be further rearranged as

$$n^2 M H_{vm} r^2 \sum_{i=1}^{m} D_i \left(\sum_{i=1}^{m} T_{0i}\right)^2 - n\left[2\left(S + \sum_{i=1}^{m} T_{0i}\right) - Gr^2 \sum_{i=1}^{m} D_i \left(\sum_{i=1}^{m} T_{0i}\right)^2\right] - 2A > 0,$$

where $G = H - nMH_{vm}$. Therefore, $K_7 > 0$ if and only if

$$n >$$

$$\sqrt{\frac{2\left(S + \sum_{i=1}^{m} T_{0i}\right) - Gr^2 \sum_{i=1}^{m} D_i \left(\sum_{i=1}^{m} T_{0i}\right)^2 + \sqrt{\left[2\left(S + \sum_{i=1}^{m} T_{0i}\right) - Gr^2 \sum_{i=1}^{m} D_i \left(\sum_{i=1}^{m} T_{0i}\right)^2\right]^2 + 8AMH_{vm} r^2 \sum_{i=1}^{m} D_i \left(\sum_{i=1}^{m} T_{0i}\right)^2}}{2MH_{vm} r^2 \sum_{i=1}^{m} D_i \left(\sum_{i=1}^{m} T_{0i}\right)^2}}.$$

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