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Characterizing the Bit Permutation Networks Obtained from the Line Digraphs of Bit Permutation Networks

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A bit permutation network is an s-stage interconnection network composed of d^{n-1} $d \times d$ crossbar switches in each stage. This class of networks includes most of the multistage interconnection networks. Recently, Chang et al. [Networks 33 (1999), 261–267] showed that an s-stage d-nary bit permutation network N with d^n inputs (outputs) can be characterized by an $(s-1)$-vector $(k_1, \ldots, k_{s-1})$, where $k_t \in \{1, \ldots, n-1\}$. In this paper, we give a simple (but not trivial) formula to determine the characteristic vector of a new network $G(N)^+$, which is, approximately, the line digraph of N. We use this formula to obtain relations between some well-studied bit permutation networks. © 2001 John Wiley & Sons, Inc.

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1. INTRODUCTION

Chang et al. [1] proposed the notion of a *bit permutation network* which is an s-stage interconnection network composed of d^{n-1} $d \times d$ crossbar switches in each stage, where a crossbar switch, or just a crossbar, can connect any one-to-one mapping from inputs to outputs. This class of networks includes *the Beneš network*, *the Omega network*, *the banyan network*, *the baseline network*, and their *extra-stage* versions, namely, most of the multistage interconnection networks. Suppose that the d^{n-1} crossbars in a stage are each labeled by a distinct d-nary $(n-1)$-vector. They showed that an s-stage d-nary bit permutation network N with d^n inputs (outputs) can be characterized by a $(s-1)$-vector $(k_1, \ldots, k_{s-1})$, where $k_t = j \in \{1, \ldots, n-1\}$ means that N is topologically equivalent to a network whose linking pattern between stage t and $t+1$ consists of d^{n-2} disjoint complete bipartite graphs where each such graph connects all crossbars in stage t and $t+1$ having the same d-nary $(n-1)$-vectors except bit j. Fig 1 shows a bit permutation network with characteristic vector $(3, 1, 2)$ and is topologically equivalent to the network in Fig 2.

The line digraph $G(N)$ of a multistage crossbar network N is obtained by taking the links of N as nodes in $G(N)$, and an arc from node p to node q in $G(N)$ exists if link p is adjacent to and precedes link q in N. Note that nodes of the same stage in $G(N)$ are ordered by the starting points of their corresponding links in N (see Fig 3). Let $G(N)^+$ be obtained from $G(N)$ by adding d inlets (outlets) to each input (output) node. By interpreting nodes as crossbars, then $G(N)^+$ can also be viewed as a multistage crossbar network (see Fig 4). It is well known that being crosstalk-free (each crossbar carries at most one path) is an essential property for photonic switching, which uses optical fiber instead of electric wire as the transmission media. Lea [3] observed that if two paths are link-disjoint in N then their corresponding paths are node-disjoint in $G(N)$. Furthermore, Hwang and Lin [2] gave formulas relating the nonblocking properties of N to the crosstalk-free nonblocking properties of $G(N)^+$. Therefore, it is of interest to know that if N is a bit permutation network, what kind of network is $G(N)^+$.

In this paper, we will prove that if N is an s-stage d-nary bit permutation network with d^n inputs (outputs)

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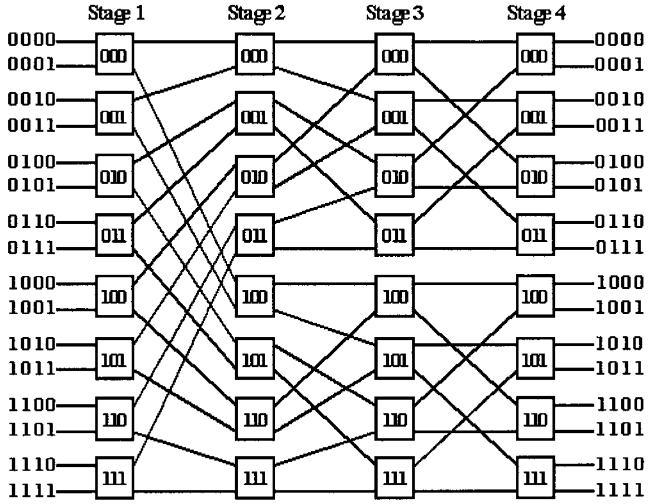

FIG. 1. A bit permutation network $N_2(4; u, v, f_1, f_2, f_3)$.

then $G(N)^+$ is an $(s+1)$-stage d-nary bit permutation network with d^{n+1} inputs (outputs). Furthermore, we give a simple (but not trivial) formula to determine the characteristic vector of $G(N)^+$ from that of N. Finally, we use this formula to obtain relations between some well-studied bit permutation networks.

2. BIT PERMUTATION NETWORKS

Consider a multistage interconnection network with d^n inputs (outputs) and s stages of d^{n-1} crossbars of size $d \times d$. Let the i^{th} crossbar in a stage be labeled by i in the d-nary $(n-1)$-vector. Define a bit-j group as the set of crossbars in a stage identical in their labels except the j^{th} bit. Such a group will also be labeled by a d-nary $(n-1)$-bit vector which is identical to any member in the group except that bit j is replaced by the symbol x_0, which stands for the set $\{0, 1, \ldots, d-1\}$. Chang et al. [1] called an s-stage d-nary interconnection network a bit permutation network if the links from stage t to $t+1$ are always from a bit-u_t group Z to a bit-v_t group Z', where Z' is a permutation of Z, for $t = 1, \ldots, s-1$. Those values u_t and v_t, $1 \le t \le s-1$, can be represented by two functions u and v from set $\{1, \ldots, s-1\}$ to set $\{1, \ldots, n-1\}$. For our purpose, we will restate their main results in a slightly different way (and provide proofs for justification).

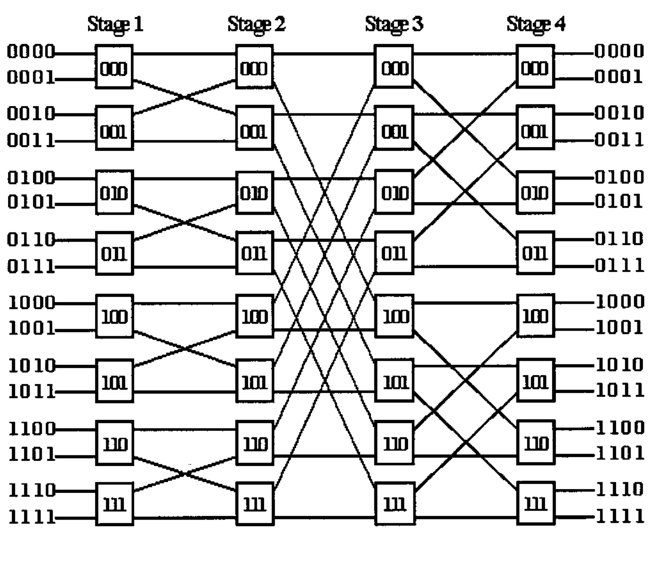

FIG. 2. A bit permutation network $N_2(4; I_3, I_1, I_2)$.

Assume that N is an s-stage d-nary bit permutation network with d^n inputs (outputs). Let f_t, $t = 1, \ldots, s-1$, denote the group linking function between stage t and $t+1$ of N. Then, N can be represented by $N_d(n; u, v, f_1, \ldots, f_{s-1})$. Note that f_t is a permutation of $\{1, \ldots, n-1\}$ and $(f_t)^{-1}(u_t) = v_t$.

The network in Figure 1 shows a bit permutation network with 16 inputs (outputs), in which crossbar i is represented by its binary 3-bit vector (x_1, x_2, x_3). Ignoring the inputs and outputs, then the network in Figure 1 can be viewed as a digraph whose nodes are those 32 crossbars labeled by $(x_1(t), x_2(t), x_3(t))$ (t is often omitted) and links are directed from left to right, where $1 \le t \le 4$ and $x_1, x_2, x_3 \in \{0, 1\}$. The links are from a bit-3 group (x_1, x_2, x_0) in stage 1 to a bit-1 group (x_0, x_1, x_2) in stage 2, from a bit-2 group (x_1, x_0, x_3) in stage 2 to a bit-3 group (x_1, x_3, x_0) in stage 3, and from a bit-2 group (x_1, x_0, x_3) in stage 3 to a bit-2 group (x_1, x_0, x_3) in stage 4, where $x_0 \in \{0, 1\}$. Thus,

$$u_1 = 3, v_1 = 1, f_1(1) = 3, f_1(2) = 1, f_1(3) = 2,$$

$$u_2 = 2, v_2 = 3, f_2(1) = 1, f_2(2) = 3, f_2(3) = 2,$$

$$u_3 = 2, v_3 = 2, f_3(1) = 1, f_3(2) = 2, f_3(3) = 3.$$

In this paper, we shall use the *cycle* notation for permutations, that is, the cycle $(i_1, i_2, \ldots, i_n)$ represents the permutation f with $f(i_1) = i_2, f(i_2) = i_3, \ldots, f(i_{n-1}) = i_n, f(i_n) = i_1$, and the cycle (j) represents f with $f(j) = j$. Then, f_1 can be represented by $(1, 3, 2)$; f_2, by $(1)(2, 3)$; and f_3, by $(1)(2)(3)$.

Theorem 1. *If there exist permutations $g_1, \ldots, g_s$ on $\{1, \ldots, n-1\}$ such that $u'_t = (g_t)^{-1}(u_t)$, $v'_t = (g_{t+1})^{-1}(v_t)$, and $f'_t = (g_t)^{-1} \circ f_t \circ g_{t+1}$ for $1 \le t \le s-1$, then two bit permutation networks $N_d(n; u, v, f_1, \ldots, f_{s-1})$ and $N_d(n; u', v', f'_1, \ldots, f'_{s-1})$ are equivalent.*

Proof. Consider the bijection g_t from the crossbar of $N_d(n; u, v, f_1, \ldots, f_{s-1})$ to the crossbar of $N_d(n; u', v', f'_1, \ldots, f'_{s-1})$ defined by

$$g_t((x_1, \ldots, x_{n-1})) = (x_{g_t(1)}, \ldots, x_{g_t(n-1)}) \text{ for } 1 \le t \le s.$$

In other words, $g_t((x_1, \ldots, x_{n-1})) = (x'_1, \ldots, x'_{n-1})$ whenever $x'_j = x_{g_t(j)}$ for $1 \le j \le n-1$.

To see that these two networks are equivalent, we only need to check that $g_1, \ldots, g_s$ are link-preserving.

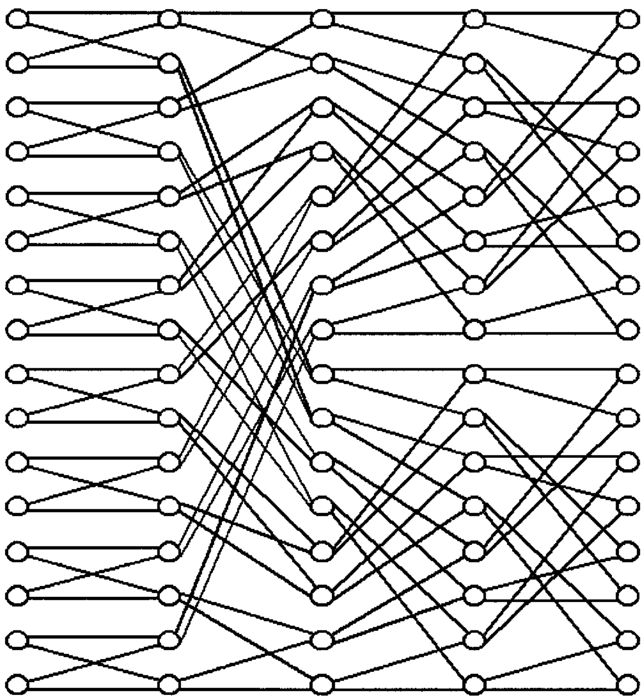

FIG. 3. The line digraph $G(N)$ obtained from the network in Figure 1. (links are directed from left to right)

Without loss of generality, suppose that the links between stage t and $t + 1$ of $N_d(n; u, v, f_1, \ldots, f_{s-1})$ are from a bit-u_t group $(x_1, \ldots, x_{u_t}, \ldots, x_{n-1})$ to a bit-v_t group $(y_1, \ldots, y_{v_t}, \ldots, y_{n-1})$, that is, $y_j = x_{f_t(j)}$ for $1 \le j \le n-1$. Let

$$g_t((x_1, \ldots, x_{u_t}, \ldots, x_{n-1})) = (x'_1, \ldots, x'_{(g_t)^{-1}(u_t)}, \ldots, x'_{n-1}),$$
$$\text{i.e., } x'_j = x_{g_t(j)} \text{ for } 1 \le j \le n-1,$$

and

$$g_{t+1}((y_1, \ldots, y_{v_t}, \ldots, y_{n-1})) = (y'_1, \ldots, y'_{(g_{t+1})^{-1}(v_t)}, \ldots, y'_{n-1}),$$
$$\text{i.e., } y'_j = y_{g_{t+1}(j)} \text{ for } 1 \le j \le n-1.$$

Then,

$$y'_j = y_{g_{t+1}(j)} = x_{f_t \circ g_{t+1}(j)} = x_{g_t \circ f'_t(j)} = x'_{f'_t(j)}$$

for

$$1 \le j \le n-1.$$

Thus, there exist links from a bit-$(g_t)^{-1}(u_t)$ group $(x'_1, \ldots, x'_{(g_t)^{-1}(u_t)}, \ldots, x'_{n-1})$ to a bit-$(g_{t+1})^{-1}(v_t)$ group $(x'_{f'_t(1)}, \ldots, x'_{f'_t((g_{t+1})^{-1}(v_t))}, \ldots, x'_{f'_t(n-1)})$ between stage t and $t + 1$ of $N_d(n; u', v', f'_1, \ldots, f'_{s-1})$. Conversely, the links in $N_d(n; u', v', f'_1, \ldots, f'_{s-1})$ also correspond to the links in $N_d(n; u, v, f_1, \ldots, f_{s-1})$. ∎

In Theorem 1, the permutations $g_1, \ldots, g_s$ change the labels of crossbars in $N_d(n; u, v, f_1, \ldots, f_{s-1})$, but preserve the linking pattern of $N_d(n; u', v', f'_1, \ldots, f'_{s-1})$.

Let I denote the identity permutation $(1)(2) \cdots (n-1)$ and $N_d(n; I_{k_1}, \ldots, I_{k_{s-1}})$ denote the bit permutation network $N_d(n; u, v, f_1, \ldots, f_{s-1})$ with $f_t = I$ and $u_t = v_t = k_t$ for all t. While [1] proved that $N_d(n; u, v, f_1, \ldots, f_{s-1})$ is equivalent to $N_d(n; I_{k_1}, \ldots, I_{k_{s-1}})$ for some $(k_1, \ldots, k_{s-1})$, we give an explicit formula to compute k_t for $1 \le t \le s-1$.

Theorem 2. *A bit permutation network $N_d(n;\ u,\ v,\ f_1, \ldots, f_{s-1})$ is equivalent to $N_d(n; I_{k_1}, \ldots, I_{k_{s-1}})$, where $k_1 = u_1$ and $k_t = (f_1 \circ \cdots \circ f_{t-1})(u_t)$ for $t = 2, \ldots, s-1$.*

Proof. Setting $g_t = I$ except $g_2 = (f_1)^{-1}$, from Theorem 1, $(g_1)^{-1}(u_1) = u_1$, $(g_2)^{-1}(v_1) = f_1((f_1)^{-1}(u_1)) = u_1$, and $(g_1)^{-1} \circ f_1 \circ g_2 = I$, we can verify that $N_d(n; u, v, f_1, \ldots, f_{s-1})$ is equivalent to $N_d(n; u', v', I_{k_1}, f'_2, \ldots, f'_{s-1})$, where $u'_2 = f_1(u_2)$, $v'_2 = v_2 = (f'_2)^{-1}(u'_2)$, $f'_2 = f_1 \circ f_2$, $u'_t = u_t$, $v'_t = v_t$, and $f'_t = f_t$ for $t = 3, \ldots, s-1$.

Assume the induction hypothesis that $N_d(n;\ u,\ v,\ f_1, \ldots, f_{s-1})$ is equivalent to $N_d(n;\ u',\ v',\ I_{k_1}, \ldots, I_{k_{j-1}},\ f'_j, \ldots, f'_{s-1})$, where $u'_j = (f_1 \circ \cdots \circ f_{j-1})(u_j)$, $v'_j = v_j = (f'_j)^{-1}(u'_j)$, $f'_j = f_1 \circ \cdots \circ f_j$, $u'_t = u_t$, $v'_t = v_t$, and $f'_t = f_t$ for $t = j+1, \ldots, s-1$. We prove that $N_d(n; u, v, f_1, \ldots, f_{s-1})$ is equivalent to $N_d(n; u'', v'', I_{k_1}, \ldots, I_{k_{j-1}}, I_{k_j}, f''_{j+1}, \ldots, f''_{s-1})$, where $u''_{j+1} = (f_1 \circ \cdots \circ f_j)(u_{j+1})$, $v''_{j+1} = v_{j+1} = (f''_{j+1})^{-1}(u''_{j+1})$, $f''_{j+1} = f_1 \circ \cdots \circ f_{j+1}$, $u''_t = u_t$, $v''_t = v_t$, and $f''_t = f_t$ for $t = j+2, \ldots, s-1$.

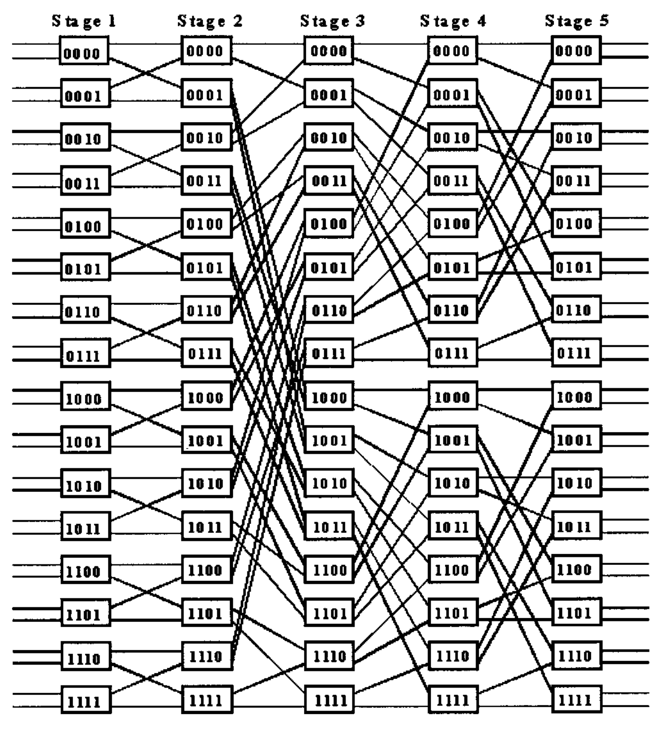

FIG. 4. The network $G(N)^+$ obtained from the network in Figure 1.

Again, by setting $g_t = I$ except $g_{j+1} = (f'_j)^{-1}$, from Theorem 1, $(g_j)^{-1}(u'_j) = (g_{j+1})^{-1}(v'_j) = u'_j = (f_1 \circ \cdots \circ f_{j-1})(u_j)$, and $(g_j)^{-1} \circ f'_j \circ g_{j+1} = I$, the network $N_d(n; u', v', I_{k_1}, \ldots, I_{k_{j-1}}, f'_j, \ldots, f'_{s-1})$ is equivalent to $N_d(n; u'', v'', I_{k_1}, \ldots, I_{k_{j-1}}, I_{k_j}, f''_{j+1}, \ldots, f''_{s-1})$. ∎

For convenience, we shall use $(k_1, \ldots, k_{s-1})$ as a short notation for the network $N_d(n; I_{k_1}, \ldots, I_{k_{s-1}})$. By Theorem 2, we say that a bit permutation network $N_d(n; u, v, f_1, \ldots, f_{s-1})$ can be characterized by a $(s-1)$-vector $(k_1, \ldots, k_{s-1})$.

Theorem 3. *If g is a permutation of* $\{1, \ldots, n-1\}$*, then* $N_d(n; I_{k_1}, \ldots, I_{k_{s-1}})$ *is equivalent to* $N_d(n; I_{g(k_1)}, \ldots, I_{g(k_{s-1})})$.

Proof. Choose all g_t as $(g)^{-1}$ in Theorem 1. Since $g \circ I_{k_t} \circ (g)^{-1} = I_{g(k_t)}$, the theorem is proved. ∎

3. MAIN RESULTS

Let N be an s-stage d-nary bit permutation network with d^n inputs (outputs). It is easily seen that $G(N)^+$ is an $(s+1)$-stage d-nary crossbar network with d^{n+1} inputs (outputs). We show that $G(N)^+$ is also a bit permutation network and how the group linking functions of N determine those of $G(N)^+$.

Theorem 4. *If a bit permutation network N is represented by* $N_d(n; u, v, f_1, \ldots, f_{s-1})$*, then* $G(N)^+$ *is a bit permutation network represented by* $N_d(n+1; u^*, v^*, h_1, \ldots, h_s)$*, where* $u_1^* = v_1^* = n$*,* h_1 *is the identity permutation* $(1)\cdots(n)$*,* $u_t^* = u_{t-1}$*,* $v_t^* = n$*, and* h_t *is the same as* f_{t-1} *except* $h_t(n) = u_{t-1}$ *and* $h_t(v_{t-1}) = n$ *for* $t = 2, \ldots, s$.

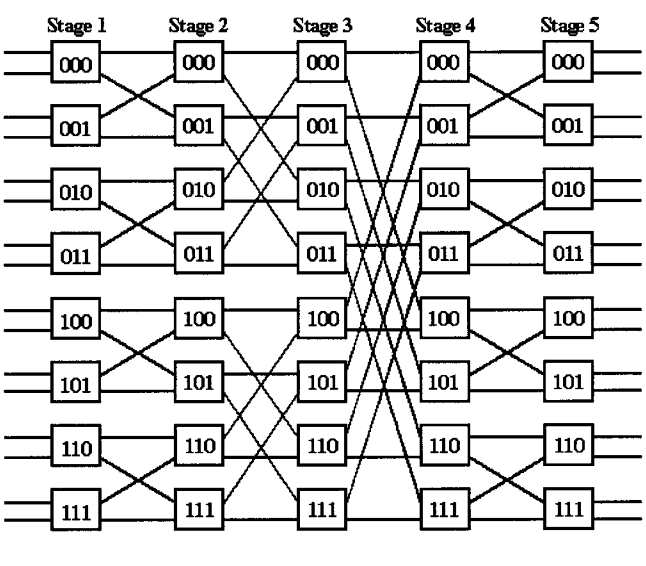

FIG. 5. The network $BY_F(1, 4)$.

Proof. Let the j^{th} link incident to the crossbars of each stage of N be labeled by j in the d-nary n-vector $(x_1, \ldots, x_n)$. Note that the links are ordered by the starting points of them. According to the construction rules of $G(N)^+$, the group linking function h_t between the crossbars of stage t and $t+1$ in $G(N)^+$ is equal to the relation between their corresponding links incident to the crossbars of stage t in N.

In stage 1 of N, since the links $(x_1, \ldots, x_{n-1}, x_0)$ are adjacent to and precede the links $(x_1, \ldots, x_{n-1}, x_0)$, where $x_0 \in \{0, 1, \ldots, d-1\}$, we know that $u_1^* = v_1^* = n$ and h_1 is equal to $(1)\cdots(n)$. For $t = 2, \ldots, s$, if the permutation f_{t-1} of N is from a bit-u_{t-1} group $(x_1, \ldots, x_{u_{t-1}-1}, x_0, x_{u_{t-1}+1}, \ldots, x_{n-1})$ to a bit-v_{t-1} group $(x_{f_{t-1}(1)}, \ldots, x_{f_{t-1}(v_{t-1}-1)}, x_0, x_{f_{t-1}(v_{t-1}+1)}, \ldots, x_{f_{t-1}(n-1)})$, then the links $(x_1, \ldots, x_{u_{t-1}-1}, x_0, x_{u_{t-1}+1}, \ldots, x_n)$ are adjacent to and precede the links $(x_{f_{t-1}(1)}, \ldots, x_{f_{t-1}(v_{t-1}-1)}, x_n, x_{f_{t-1}(v_{t-1}+1)}, \ldots, x_{f_{t-1}(n-1)}, x_0)$, where $x_0 \in \{0, 1, \ldots, d-1\}$ in stage t of N. Hence, $u_t^* = u_{t-1}$, $v_t^* = n$, and h_t is the same as f_{t-1} except that $h_t(n) = u_{t-1}$ and $h_t(v_{t-1}) = n$ for $t = 2, \ldots, s$. From the above, we also prove that $G(N)^+$ is a bit permutation network. ∎

Theorem 5. *Suppose that the characteristic vector of a bit permutation network* $N_d(n; u, v, f_1, \ldots, f_{s-1})$ *is* $(k_1, \ldots, k_{s-1})$*. Then, the characteristic vector of* $G(N_d(n; u, v, f_1, \ldots, f_{s-1}))^+$ *is* $(l_1, \ldots, l_s)$*, where* $l_1 = n$ *and* $l_t = k_{t-1}$ *if* $k_{t-1} \notin \{k_1, \ldots, k_{t-2}\}$ *or* $l_t = l_i$*, where* $i = max\{j \mid k_j = k_{t-1}, 1 \le j \le t-2\}$ *if* $k_{t-1} \in \{k_1, \ldots, k_{t-2}\}$ *for* $t = 2, \ldots, s$.

Proof. Since the characteristic vector of $N_d(n; u, v, f_1, \ldots, f_{s-1})$ is $(k_1, \ldots, k_{s-1})$, where $k_t \in \{1, \ldots, n-1\}$, by Theorems 2 and 4, we can prove that the characteristic vector of $G(N_d(n; I_{k_1}, \ldots, I_{k_{j-1}}, f_j, \ldots, f_{s-1}))^+$ equals that of $G(N_d(n; I_{k_1}, \ldots, I_{k_{j-1}}, I_{k_j}, f'_{j+1}, \ldots, f'_{s-1}))^+$ for $1 \le j \le s-1$. Hence, the characteristic vectors of $G(N_d(n; u, v, f_1, \ldots, f_{s-1}))^+$ and $G(N_d(n; I_{k_1}, \ldots, I_{k_{s-1}}))^+$ are the same.

By Theorem 4, $G(N_d(n; I_{k_1}, \ldots, I_{k_{s-1}}))^+$ is a bit permutation network represented by $N_d(n+1; u^*, v^*, h_1, \ldots, h_s)$, where $u_1^* = n$, $h_1 = (1)\cdots(n)$, $u_t^* = k_{t-1}$, and $h_t = (1)\cdots(k_{t-1}-1)(k_{t-1}+1)\cdots(n-1)(k_{t-1}, n)$ for $t = 2, \ldots, s$. Hence, $h_j(m) = m$ if $m \notin \{k_{j-1}, n\}$ for $m \in \{1, \ldots, n\}$ and $j = 1, \ldots, s$. By Theorem 2, the characteristic vector of $G(N_d(n; I_{k_1}, \ldots, I_{k_{s-1}}))^+$ is $(l_1, \ldots, l_s)$, where $l_1 = n$ and $l_t = (h_1 \circ \cdots \circ h_{t-1})(k_{t-1})$ for $t = 2, \ldots, s$. Thus, $l_t = k_{t-1}$ if $k_{t-1} \notin \{k_1, \ldots, k_{t-2}\}$. If $k_{t-1} \in \{k_1, \ldots, k_{t-2}\}$, then $i = max\{j \mid k_j = k_{t-1}, 1 \le j \le t-2\} \ge 1$ and $l_t = (h_1 \circ \cdots \circ h_{t-1})(k_{t-1}) = (h_1 \circ \cdots \circ h_{i+1})(k_{t-1}) = (h_1 \circ \cdots \circ h_{i+1})(k_i) = (h_1 \circ \cdots \circ h_i)(n) = (h_1 \circ \cdots \circ h_{i-1})(k_{i-1}) = l_i$. ∎

For example, if the characteristic vector of a bit permutation network N with d^4 inputs (outputs) is $(1, 3, 3, 2, 2, 3, 1, 3, 1, 1, 2, 3, 2, 2, 1)$, then the characteristic vector of $G(N)^+$ is $(4, 1, 3, 1, 2, 1, 3, 4, 1, 3, 1, 2, 4,$

1, 4, 3). Here, $l_1 = n = 4$, $l_2 = k_1 = 1$, $l_3 = k_2 = 3$, and $l_4 = l_2 = 1$ since $k_3 = 3 = k_2$. The formula obtained from Theorem 5 can be useful for some well-studied bit permutation networks.

Let us consider the network obtained by adding k extra stages to *the banyan network* with 2^n inputs (outputs) and by specifying that the extra k stages should be identical to the first k stages (denote this way of adding extra stages by F). Represent the above network by $BY_F(k,n)$. If the extra k stages are identical to the mirror image of the first k stages, then denote the network by $BY_{F^{-1}}(k,n)$. Fig 5 shows the network $BY_F(1,4)$.

Theorem 6. *The network* $G(BY_F(k,n))^+$, $0 \leq k \leq n$, *is equivalent to the network* $BY_F(k,n+1)$, *where* F *can be replaced by* F^{-1}.

Proof. Since $BY_F(k,n)$ is represented by $N_2(n; I_{n-1}, I_{n-2}, \ldots, I_1, I_{n-1}, I_{n-2}, \ldots, I_{n-k})$, from Theorem 5, the characteristic vector of $G(BY_F(k,n))^+$ is $(n, n-1, n-2, \ldots, 1, n, n-1, \ldots, n-k+1)$. This means that $G(BY_F(k,n))^+$ is equivalent to the network $N_2(n+1; I_n, I_{n-1}, I_{n-2}, \ldots, I_1, I_n, I_{n-1}, \ldots, I_{n-k+1})$. Hence, $G(BY_F(k,n))^+$ is equivalent to $BY_F(k, n+1)$. Similarly, we can obtain the result if F is replaced by F^{-1}. ∎

Let W^{-1} denote the inverse network of W, that is, the network obtained from W by reversing the order of the stages. It is easy to see that

Theorem 7. *The network* $G(BY_F^{-1}(k,n))^+$, $0 \leq k \leq n$, *is equivalent to the network* $BY_F^{-1}(k,n+1)$, *where* F *can be replaced by* F^{-1}.

Proof. Since $BY_F^{-1}(k,n)$ is represented by $N_2(n; I_1, I_2, \ldots, I_{n-1}, I_1, I_2, \ldots, I_k)$, from Theorem 5, the characteristic vector of $G(BY_F^{-1}(k,n))^+$ is $(n, 1, 2, \ldots, n-1, n, 1, \ldots, k-1)$. We can find the permutation $g = (1, 2, \ldots, n)$ such that $N_2(n+1; I_n, I_1, I_2, \ldots, I_{n-1}, I_n, I_1, \ldots, I_{k-1})$ is equivalent to $N_2(n+1; I_1, I_2, I_3, \ldots, I_n, I_1, I_2, \ldots, I_k)$ by Theorem 3. Hence, $G(BY_F^{-1}(k,n))^+$ is equivalent to $BY_F^{-1}(k, n+1)$. If F is replaced by F^{-1}, then we can also obtain the similar result. ∎

Theorem 7 was crucially used in [2] to prove the crosstalk-free property of $BY_{F^{-1}}^{-1}(k,n)$ essential to photonic switching.

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