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Quantum regime of Cooper pair tunneling in small-area $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ mesas in magnetic fields

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Abstract

We show that application of an *c*-axis magnetic field decreases the effective interlayer Josephson coupling in small-area $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}$ (Bi-2212) mesas at low temperatures. As magnetic field increases, the crossover from the classical regime of Cooper pair tunneling to the quantum regime leads to an additional drastic suppression of the Josephson coupling due to the Coulomb blockade. As result, the stack of many intrinsic junctions is nonsuperconducting at high fields. The quantum regime of superconducting tunneling may be reached in mesas with N junctions and area $0.3 \mu\text{m}^2$ made of intercalated Bi-2212 crystals by applying *c*-axis magnetic fields $B > 4/N^2$ T at $N \gg 1$. © 2001 Published by Elsevier Science B.V.

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1. Introduction

Highly anisotropic Bi- and Tl-based cuprate superconductors may be considered as a stack of superconducting atomically thin CuO_2 layers coupled by the Josephson interaction. For the most studied system Bi-2212 the basic parameters of the intrinsic Josephson junctions between double CuO_2 layers separated by $s = 15.6 \text{ \AA}$ are well known. The Josephson critical current density J_0 was found to be in the interval 500–1200 A/cm² (depending on doping) from measurements of the IV characteristics [1,2]. Dissipation in these junctions controlled by the quasiparticle tunneling

between *d*-wave CuO_2 superconducting layers is characterized by the quasiparticle conductivity, σ_q , and $\sigma_q \approx 1.5\text{--}2 \text{ k}\Omega^{-1}\text{cm}^{-1}$ at very low temperatures [2]. Significantly lower values of J_0 and σ_q were obtained by the intercalation of Bi-2212 single crystals. In the Bi-2212 mesas with S in the interval 10–600 μm^2 , made of single crystals intercalated with HgBr_2 critical current density as low as $\approx 50 \text{ A/cm}^2$ was observed [3,4].

Hence intrinsic Josephson junctions are strongly underdamped with the “quality factor” $Q = \omega_p R_q C = \omega_p \epsilon_c / 4\pi \sigma_q$ about 200. Here $C = \epsilon_c S / 4\pi s$ is the capacitance of a single junction, $R_q = s / \sigma_q S$ is its resistance and S is the junction area. The Josephson energy of a single junction is $E_J = \Phi_0 J_0 S / 2\pi c$, while the charge energy for Cooper pair tunneling is given as $E_c = (2e)^2 / 2C$.

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The total energy of a stack of such intrinsic junctions is a sum of individual junctions and their coupling energy. In small-area junctions the latter is determined by weak electrostatic energy of charges due to the electric field outside of the junctions (negligible at $s \ll S^{1/2}$) and due to variations of the chemical potential in the layers caused by charge variations in atomically thin layers and possible deviations from equilibrium [5–8]. This coupling will be neglected in the following.

Recently mesa-like samples were fabricated with the area S as small as $0.3 \mu\text{m}^2$ [9], with the goal of reaching the quantum regime for Cooper pair tunneling and of using intrinsic Josephson junctions as basic elements for quantum computing. In the small mesas studied in Ref. [2] the critical current density at 4.2 K drops by about an order of magnitude as compared with that in mesas with area $S \geq 4 \mu\text{m}^2$. As S decreases the crossover to the quantum regime should indeed lead to a drop in the critical current due to Coulomb blockade of Cooper pair tunneling. However, fabrication of small-area samples may result in damage of superconductivity inside CuO_2 layers near edges and corresponding suppression of J_0 . Therefore, it is not clear whether drop of the critical current is caused by damage of superconductivity inside layers or by the crossover to the quantum regime.

In the following we study conditions on the temperature and on S to observe quantum effects in Bi-2212 samples with typical junction parameters mentioned above. We show that, for typical current density, mesas with much smaller area are needed to reach the quantum regime. Next we show that quantum regime may be achieved by applying a magnetic field along the c axis to suppress the Josephson coupling.

2. Conditions for quantum regime

The charge and the phase in the layer n we denote by $2eq_n$ and ϕ_n respectively, the variables q_n and ϕ_n are conjugated. We assume that the system is neutral, $\sum_n q_n = 0$. The phase difference and the electric field between layers $n-1$ and n are $\varphi_n = \phi_n - \phi_{n-1}$ and $(8e\pi/\epsilon_c S)P_n$ respectively. Here

$P_n = \sum_i^{n-1} q_i$. The variables P_n and φ_n are conjugated. The Hamiltonian in terms of these variables is the energy of the electric field between layers and the Josephson energy:

$$\mathcal{H} = \sum_n \mathcal{H}_n, \quad \mathcal{H}_n = E_c P_n^2 + E_J(1 - \cos \varphi_n), \quad (1)$$

where $P_n = \partial/i\partial\varphi_n$. For quantum effects to be important E_c should be at least comparable with the Josephson energy E_J , temperature T , and damping rate $\hbar/R_q C$ [10,11]. The condition $E_c \geq \hbar/R_q C$ reads as $2e^2 s/\hbar\sigma_q S \geq 1$. It is fulfilled in mesas with area $S \leq 1 \mu\text{m}^2$ which are available now. The condition $E_c \geq T$ is fulfilled in mesas with area $S \leq 1 \mu\text{m}^2$ at low temperatures $T < 0.1$ K. The inequality $E_c/E_J = (e^2/\hbar c)(16\pi e c s/\epsilon_c J_0 S^2) \geq 1$ turns out to be the most severe condition on the upper limit for S . For a single junction we define the crossover critical current density $J_{\text{cr}}(1) = 16e^2\pi^2 c s/\Phi_0 \epsilon_c S^2$ at which $E_c = E_J$. For smallest area available now, $S = 0.3 \mu\text{m}^2$, we get $J_{\text{cr}}(1) \approx 3$ A/cm². This value is about an order of magnitude below the critical current density found for small-area mesas studied in Ref. [9].

3. Field dependence of the critical current in the classical regime

At low temperatures magnetic fields above ≈ 0.04 T induce the vortex glass phase, with c -axis uncorrelated pancake vortices. These misaligned pancake vortices cause fluctuations of the gauge-invariant phase difference $\varphi_n(\mathbf{r})$ between the layers n and $n+1$. As result, the average maximum interlayer current, given as [12]:

$$I_{\text{max}}(B) = J_0 \left| \int d\mathbf{r} \exp[i\varphi_n(\mathbf{r})] \right| \quad (2)$$

drops with the concentration of vortices. Here $\langle \dots \rangle$ means averaging over disorder (pinning), which determines positions of vortices, $\varphi_n(\mathbf{r})$ is determined by the fixed positions of pancakes. For mesas with $S < \lambda_J^2$ and in magnetic fields $B \gg \Phi_0/S$ the phase difference is

$$\varphi_n(\mathbf{r}) = \sum_i [\phi_v(\mathbf{r} - \mathbf{r}_{ni}) - \phi_v(\mathbf{r} - \mathbf{r}_{n+1,i})]. \quad (3)$$

Here $\phi_v(\mathbf{r})$ is the polar angle of the point $\mathbf{r} = (x, y)$, inside layers and \mathbf{r}_{ni} is the coordinate of pancake i in the layer n . Coordinates \mathbf{r}_{ni} are random and uncorrelated for different n in the vortex glass phase. Hence, in a mesoscopic small-area mesa, suppression of the critical current in the vortex glass phase is random due and maximum Josephson current is characterized by a distribution function. Eqs. (2) and (3) leads to the estimate

$$\begin{aligned} \langle J_{\max}^2(B) \rangle &= J_0^2 \int d\mathbf{r} d\mathbf{r}' \langle \exp[i\varphi_n(\mathbf{r}) - i\varphi_n(\mathbf{r}')] \rangle \\ &\approx J_0^2 a^2 S, \end{aligned} \quad (4)$$

where $a = (\Phi_0/B)^{1/2}$ is the intervortex spacing. We used here the fact that for a vortex glass, $\langle \exp[i\varphi_n(\mathbf{r}) - i\varphi_n(0)] \rangle$, drops with $|\mathbf{r}|$ on the scale a at $a \ll S^{1/2}$. At $a^2 \ll S \ll \lambda_J^2$ the only scale for the distribution function of the critical current is $J_0 a S^{1/2}$, but its shape, $P(I/J_0 a S^{1/2})$, is unknown yet. This distribution is broad, dispersion of the maximum current, $J_0 a S^{1/2}$, is of order of the average maximum current. From Eq. (4) we obtain that the average maximum current density drops as $B^{-1/2}$ with B , i.e. $\langle J_{\max}(B) \rangle \leq (\langle J_{\max}^2 \rangle)^{1/2} \approx J_0 (\Phi_0/B S)^{1/2}$.

4. Quantum regime

For a stack of N intrinsic Josephson junctions the effect of quantum fluctuations is stronger than in a single junction due to one-dimensional coupling of junctions. To characterize the effect of fluctuations we calculate the response of the system to an external magnetic field (diamagnetic moment). For that we consider N junctions forming a ring. The magnetic flux through the ring is we denote by Φ . Now the Hamiltonian, Eq. (1), with the condition $\sum_n \varphi_n = 2\pi\Phi/\Phi_0$, can be written as

$$\begin{aligned} \mathcal{H} &= \sum_{n=1}^{N-1} \mathcal{H}_n + E_c \left(\sum_{n=1}^{N-1} P_n \right)^2 \\ &+ E_J \left[1 - \cos \left(\frac{2\pi\Phi}{\Phi_0} - \sum_{n=1}^{N-1} \varphi_n \right) \right]. \end{aligned} \quad (5)$$

We calculate the ground state energy $\mathcal{E}_0(\Phi)$ and then define the diamagnetic moment (which is proportional to the maximum Josephson current) as $M = -\partial\mathcal{E}_0/\partial\Phi$.

In the limit of strong Josephson coupling, $\alpha = E_J/E_c \gg 1$, we account for weak fluctuations of phase φ_n and use a variational approach taking the variational wave function of the system in the Gaussian form. At $N \gg 1$ we obtain the diamagnetic moment

$$M(\Phi) \propto -J_0 N \exp[-N/2\sqrt{\alpha}] \sin(2\pi\Phi/\Phi_0). \quad (6)$$

Hence, the maximum Josephson current drops exponentially with N and in the limit $N \rightarrow \infty$ the system is nonsuperconducting. However, at $N \ll 2\sqrt{\alpha}$ the effect of quantum fluctuations is negligible. For $N \gg 1$ it becomes significant at the crossover current density $J_{cr}(N) = J_{cr}(1)N^2/4$.

In the limit of weak Josephson coupling, $E_J \ll E_c$, we use perturbation theory with respect to Josephson energy. At $E_J = 0$ the ground state is when all $P_n = 0$. Excited states with nonzero P_n are separated by gaps of order E_c . The operator $\cos \varphi_n$ changes P_n by ± 1 . In Eq. (5) only the last term depends on Φ and it connects the ground state with the state where all $P_n = 1$. The dependence of \mathcal{E}_0 on Φ comes from terms of N th order in E_J in the perturbation series, which contain the product $[1 - \cos(2\pi\Phi/\Phi_0 - \sum_{n=1}^{N-1} \varphi_n)] \prod_{n=1}^{N-1} (1 - \cos \varphi_n)$. Therefore, we obtain again the exponential drop of maximum critical current with N :

$$M(\Phi) \propto -J_0 N (E_J/E_c)^{N-1} \sin(2\pi\Phi/\Phi_0). \quad (7)$$

The magnetic field suppresses the Josephson energy and thus the maximum Josephson current, i.e. the function $P(I)$ shifts to smaller I . This change for a single junction is strong for magnetic fields $B > B_{cr}(1)$, where $B_{cr}(1)$ is the crossover field defined by $\langle J_{cl}(B) \rangle = J_{cr}(1)$:

$$B_{cr}(1) = \Phi_0^3 S^3 \left(\frac{J_0 \epsilon_c}{16\pi^2 e^2 c s} \right)^2 = \frac{\Phi_0}{S} \left[\frac{J_0}{J_{cr}(1)} \right]^2. \quad (8)$$

For $J_0 \approx 50$ A/cm², $s = 21$ Å, and $S = 0.3$ μm² we obtain $B_{cr} \approx 1$ T. For N junctions we obtain $B_{cr}(N) \approx 4B_{cr}(1)/N^2$. As magnetic field increases, the crossover to the quantum regime and then to the nonsuperconducting state may be seen as a fast

drop of the maximum currents, it is faster here, than $B^{-1/2}$. Note, that in a stack with N junctions, due to distribution of the maximum current in the presence of vortices, some junctions will reach the quantum regime at magnetic fields smaller than B_{cr} .

5. Discussion

The experimental data on the behavior of the critical current in c -axis magnetic fields obtained by Morozov et al. [13] have confirmed at least qualitatively our theoretical results for the effect of the magnetic field on the critical currents in the classical regime. In Bi-2212 mesas with $S = 4 \mu\text{m}^2$ the average critical current density dropped significantly with B and a broadening of the distribution of the critical currents was observed. Typical variations of the critical currents were similar to average critical current.

From our estimates of $B_{cr}(1)$ we see that one needs quite low initial critical current density J_0 and small S to observe strong effect of the magnetic field and crossover to the quantum regime.

We note that crossover to the quantum regime and then transition to an insulating state in a one-dimensional system of Josephson junctions driven by the magnetic field was observed by Haviland et al. [14]. They used a magnetic field parallel to the junctions to suppress the Josephson coupling, and the junctions were coupled strongly due to small stray capacitance of each electrode.

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