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Computer-aided manufacturing of spiral bevel and hypoid gears by applying optimization techniques

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Abstract

A mathematical model of an ideal spiral bevel and hypoid gear-tooth surfaces based on the Gleason hypoid gear generator mechanism is proposed. Using the proposed mathematical model, the tooth surface sensitivity matrix to the variations in machine–tool settings is investigated. Surface deviations of a real cut pinion and gear with respect to the theoretical tooth surfaces are also investigated. An optimization procedure for finding corrective machine–tool settings is then proposed to minimize surface deviations of real cut pinion and gear-tooth surfaces. The results reveal that surface deviations of real cut gear-tooth surfaces with respect to the ideal ones can be reduced to only a few microns. Therefore, the proposed method for obtaining corrective machine–tool settings can improve the conventional development process and can also be applied to different manufacturing machines and methods for spiral bevel and hypoid gear generation. An example is presented to demonstrate the application of the proposed optimization model. © 2001 Elsevier Science B.V. All rights reserved.

Keywords: Spiral bevel; Hypoid gear; Machine–tool settings; Optimization; Surface deviation

1. Introduction

Manufacturing spiral bevel and hypoid gears requires state-of-the-art machinery and techniques because such gears have complex tooth-surface geometries. Many analytical efforts such as tooth contact analysis (TCA), loaded tooth contact analysis (LTCA), stress analysis, undercut checking, kinematic optimization, among others, are successfully applied to the design of spiral bevel and hypoid gears to obtain optimal tooth surfaces with permissible contact pattern position, length, bias, smoothness of motion, and adjustability of assembly. Therefore, it is significant to develop a methodology that minimizes, within acceptable tolerances, the surface deviation of real cut spiral bevel and hypoid gear-tooth surfaces with respect to theoretical ones.

Recent technology development on CNC machinery makes it possible to manufacture and inspect spiral bevel and hypoid gears using full quantitative and qualitative controls. Several computer-aided inspection systems and closed-loop manufacturing systems that combine CNC coordinate measuring machines with theoretical gear-tooth-surface data, have been developed by Gleason Works [1], M&M Precision Systems [2], Soehne [3], and Lemanski [4] in the past few years. Theoretical gear-tooth-surface data can be obtained from mathematical models of bevel and hypoid gears. Krenzer [5] proposed computer-aided corrective machine settings for manufacturing bevel and hypoid gear sets using first-order and second-order sensitivity matrices. Litvin et al. [6,7] and Zhang and Litvin [8] proposed a series of methodologies to minimize deviations in real cut gear-tooth surfaces and to analyze the meshing and contact of real cut gear-tooth surfaces. However, all these studies investigated minimization of surface deviations by means of the so-called linear regression method. Since the tooth-surface geometry of spiral bevel and hypoid gears is quite complex and sensitive to machine–tool settings, gears with different characteristics should be generated using different manufacturing machines and methods. Therefore, it is desirable to build up a methodology that has the following characteristics: (a) gear-set mathematical models represented in terms of machine–tool settings and machine constants; (b) calculated corrective machine–tool settings in terms of actual machine–tool settings; (c) better numerical efficiency, reliability, and robustness than the linear regression method.

In this paper, a methodology for simulating manufacture of theoretically correct tooth surfaces of Gleason spiral

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bevel and hypoid gears is proposed. The sensitivity of tooth surfaces to variations of machine–tool settings is investigated using the sensitivity analysis technique. Therefore, the characteristics of the gear generator can thus be obtained and controlled. Using the proposed gear-set mathematical model, theoretical tooth-surface data can be determined and then down-loaded to CNC coordinate measuring machines. Using CNC coordinate measuring machines to measure sampling points on tooth surfaces, real cut gear-tooth-surface data can be obtained. The measured data can then be compared with the theoretical data to calculate gear-tooth-surface deviations. Using the measured surface deviations and the sensitivity matrix, corrective machine settings that minimize the surface deviations to within tolerances can be obtained by means of a quadratic optimization algorithm. Using this optimization technique to calculate corrective machine–tool settings is shown to be more efficient and successful than using the linear regression method.

In the optimization procedure, the maximum gear-tooth-surface deviations is chosen as the objective function, instead of the least-squares sum used by the linear regression method [7], and perturbations of the machine–tool settings are chosen as the design variables to be updated automatically by sequential quadratic programming (SQP). The tooth-surface characteristics vary with cutting machines and methods, and surface characteristics such as tilted root angle, tooth thickness, backlash, etc., should be considered in the optimization development procedure. In practice, the corrective machine–tool settings are bounded to a permissible range to match the practical machine–tool relationship. In this study, a prototype optimization software program: multifunctional optimization system tool (MOST) [9], based on the SQP method, is used as an optimization tool because of its accuracy, reliability, and efficiency [10]. An improved procedure for development of spiral bevel and hypoid gears based on the proposed approach is suggested. An example is presented to demonstrate the optimization technique and its applications.

2. Mathematical model of the Gleason spiral bevel and hypoid gears

In practice, spiral bevel and hypoid gears can be cut using the Gleason hypoid gear generator. The Gleason hypoid gear generator mechanism can be divided into four major parts: face-mill cutter; cradle assembly; feed and drive mechanisms; work-head assembly. Detailed description of the mechanism as presented by Fong and Tsay [11], Litvin et al. [12], and the Gleason Works [13–15] is omitted here. Cross-section *a–a* of the head cutter can be considered to be two straight lines, as shown in Fig. 1, and it can be expressed in coordinate system $S_1(x_1, y_1, z_1)$ as follows:

$$x_1 = [r_m \pm (\tfrac{1}{2}W + u_j \sin\varphi_j)]\sin\beta,$$
$$y_1 = [r_m \pm (\tfrac{1}{2}W + u_j \sin\varphi_j)]\cos\beta,$$
$$z_1 = -u_j \cos\varphi_j, \quad (1)$$

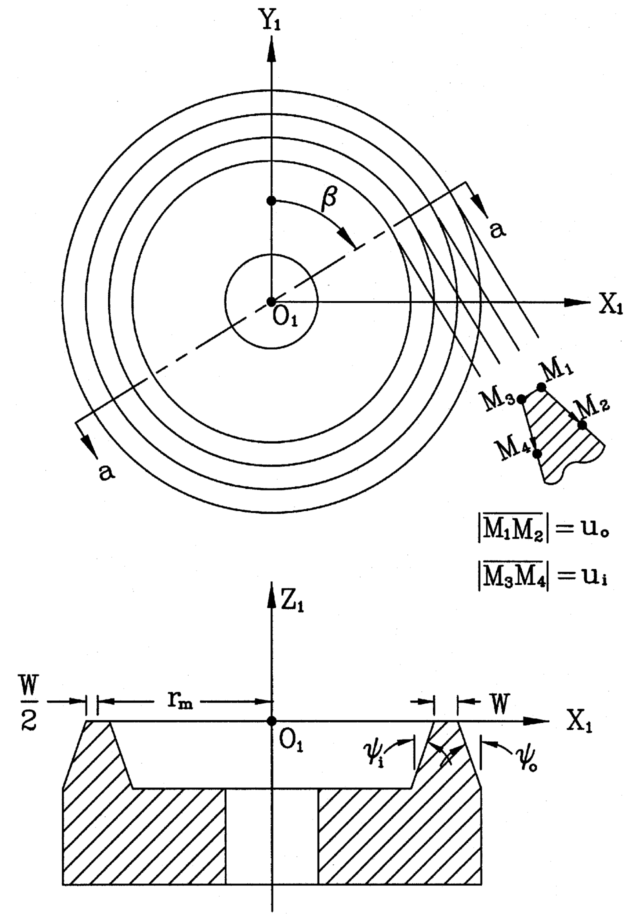

Fig. 1. Relationship between the coordinate system S_1 and the face mill cutter.

where j = i and o, and parameters u_i, β_i, u_o, β_o, are the head-cutter surface coordinates of the inside and outside blades, respectively. Subscript "i" indicates the inside blade, and "o" represents the outside blade; the "±" sign should be considered a "+" sign for the outside blade (j = o), and a "−" sign for the inside blade (j = i).

The Gleason spiral bevel and hypoid gears generating mechanism coordinate systems are shown in Fig. 2. The position vectors of the spiral bevel and hypoid gear-tooth surfaces $\boldsymbol{R}_i$ and their surface unit normals $\boldsymbol{n}_i$ were developed by Fong and Tsay [16].

3. Sensitivity analysis

The sensitivity of spiral bevel and hypoid gear-tooth surfaces to machine–tool settings was studied by Fong and Tsay [17], Krenzer [5], and Huston et al. [18]. Using the sensitivity analysis technique, the machine–tool settings characteristics of each spiral bevel and hypoid gear generator were obtained. Based on the repeatability of the same gear generator in the manufacturing process, the surface deviation of real cut gear-tooth surfaces can be minimized by choosing optimal corrective machine–tool settings.

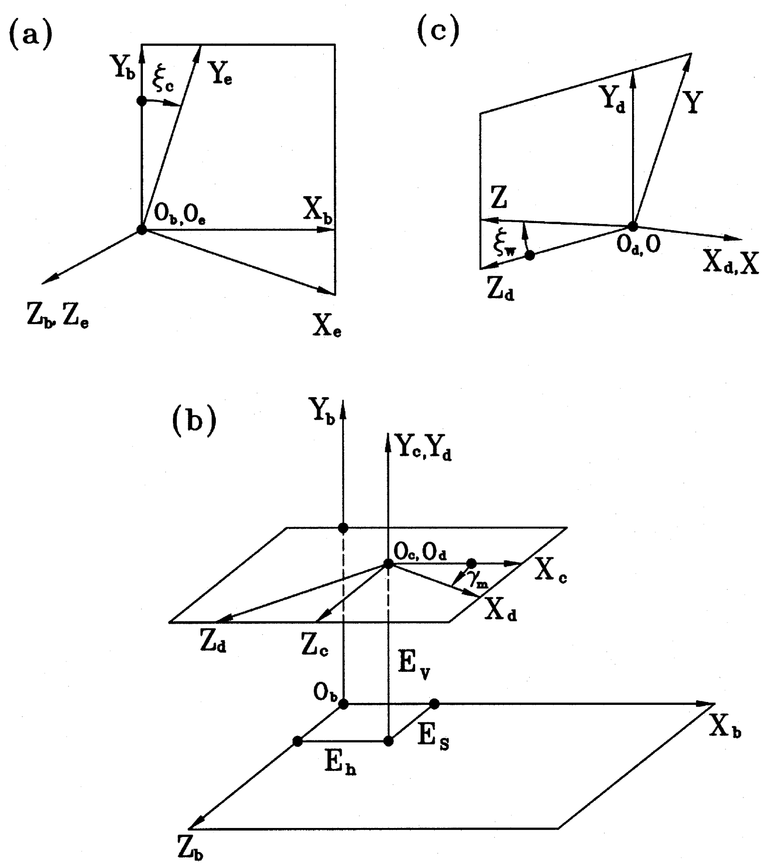

Fig. 2. Coordinate systems for the generating mechanism.

Detailed descriptions of the gear-tooth surface sensitivity to variations in machine–tool settings were discussed by Fong and Tsay [17], and are omitted here. The machine–tool settings chosen for sensitivity analysis include the cradle angle ϕ_c, eccentric angle ϕ_e, cutter spindle rotation angle ϕ_t, swivel angle ϕ_s, sliding base setting E_s, increment of machine center to back D_x, vertical offset E_v, machine root angle setting γ_m, ratio of Helical motion change gears η_h, and ratio of roll change gears η_r. The first variation in gear-tooth surfaces due to variations in machine–tool settings is defined as

$$\delta \underset{\sim}{\boldsymbol{R}}_i = \sum \frac{\partial \underset{\sim}{\boldsymbol{R}}_i}{\partial \varsigma_j} \delta \varsigma_j, \tag{2}$$

where $\delta \underset{\sim}{\boldsymbol{R}}_i$ represents the variation in the tooth surface position vector and parameter $\delta \varsigma_j$ indicates the perturbation increment of the machine–tool settings.

The perturbation increment of each machine–tool setting should be chosen according to the precision limitation of the Gleason spiral bevel and hypoid gears generator: 0.01 mm for linear displacement; 1 min for angular displacement; 0.0001 for the ratio of change gears. In sensitivity analysis of spiral bevel and hypoid gear-tooth surfaces, $m \times n$ discrete sampling points have been chosen to represent the tooth-surface geometric characteristics, as shown in Fig. 3. The values of m and n depend on tooth-surface geometry, sampling accuracy, machine precision as well as product requirements. The surface sampling points are numbered ascendant from the root to top, and the heel to toe. The sensitivity coefficient S_{ij} is defined as the displacement variation along the normal direction of each tooth-surface point due to the perturbation of each machine–tool setting $\delta \varsigma_j$. Therefore, Eq. (2) can be rewritten as

$$\{\delta \underset{\sim}{\boldsymbol{R}}_i\} = [S_{ij}]\{\delta \varsigma_j\},$$

$$S_{ij} = \frac{\partial \underset{\sim}{\boldsymbol{R}}_i}{\partial \varsigma_j} \quad (i = 1, \ldots, p \text{ and } j = 1, \ldots, q), \tag{3}$$

where $p = m \times n$ is the number of sampling points, q the number of machine–tool settings. The sensitivity matrix $[S_{ij}]$ can be applied to calculate corrective machine settings in the manufacturing development process for spiral bevel and hypoid gears.

4. Corrective machine–tool settings for gear set manufacturing

Deviations between the theoretical gear-tooth surfaces and real cut gear-tooth surfaces may exist for a number of reasons, such as machine–tool setting inaccuracies, machine constant error, machine flexibility, among others. Whatever the reason, the corrective machine–tool settings are required

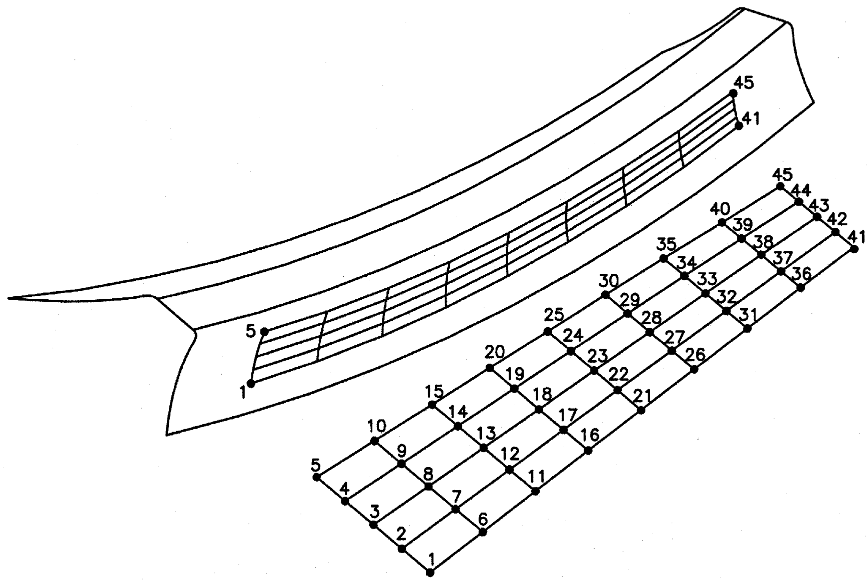

Fig. 3. Surface sampling points on the spiral bevel gear convex side.

to minimize tooth surface deviations to within permissible levels. Conventionally, the rolling test development was used to obtain the corrective machine–tool settings and compensate for tooth surface deviations [13–15]. However, it is a time-consuming and inefficient process for manufacturing development of spiral bevel and hypoid gears.

In this section, a quadratic optimization procedure is applied to reduce shop time during the development stage. Using the proposed gear-set mathematical model, theoretical tooth surfaces can be represented by the meshed sampling points, as shown in Fig. 3. The coordinates of the sampling points $\underset{\sim}{\boldsymbol{R}}_i$ are down-loaded to the CNC coordinate measuring machine, which then measures the corresponding points on the real cut gear-tooth surfaces and records the coordinates $\underset{\sim}{\boldsymbol{R}}_i^*$ on a data diskette. The measurement data are then compared with the theoretical

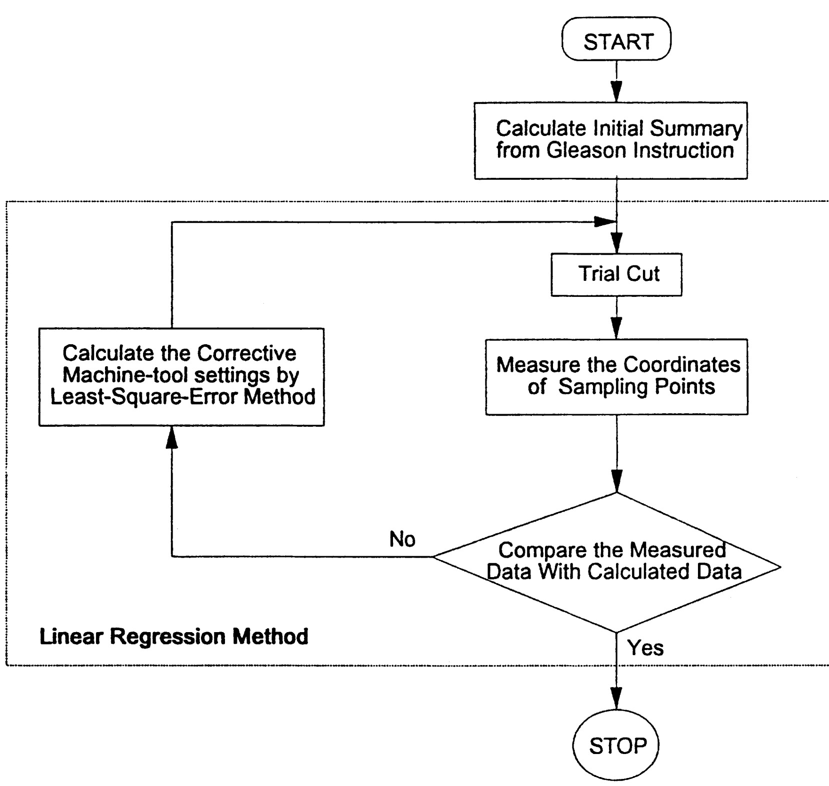

Fig. 4. Spiral bevel and hypoid gear development flow chart using the linear regression method.

data, and any deviations in the normal direction determined according to

$$\Delta R_i = (\underset{\sim}{\boldsymbol{R}}_i - \underset{\sim}{\boldsymbol{R}}_i^*) \cdot \underset{\sim}{\boldsymbol{n}}_i, \tag{4}$$

where subscript i designates the number of sampling points, and $\underset{\sim}{\boldsymbol{R}}_i$ and $\underset{\sim}{\boldsymbol{n}}_i$ represent the theoretical position vector and unit normal vector of the sampling points on the tooth surface, respectively.

Based on the calculated surface deviations ΔR_i and the sensitivity matrix $[S_{ij}]$ of the generator, the governing equation used to minimize deviations in real cut gear-tooth surfaces can be written as follows:

$$\begin{Bmatrix} \Delta R_1 \\ \Delta R_2 \\ \cdots \\ \cdots \\ \Delta R_p \end{Bmatrix} = \begin{Bmatrix} \frac{\partial R_1}{\partial \varsigma_1} & \cdots & \frac{\partial R_1}{\partial \varsigma_q} \\ \frac{\partial R_2}{\partial \varsigma_1} & \cdots & \frac{\partial R_2}{\partial \varsigma_q} \\ \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots \\ \frac{\partial R_p}{\partial \varsigma_1} & \cdots & \frac{\partial R_p}{\partial \varsigma_q} \end{Bmatrix} \begin{Bmatrix} \Delta \varsigma_1 \\ \Delta \varsigma_2 \\ \cdots \\ \cdots \\ \Delta \varsigma_q \end{Bmatrix}, \tag{5}$$

or

$$\{\Delta R_i\} = [S_{ij}]\{\Delta \varsigma_j\}, \tag{6}$$

where $\{\Delta R_i\}$ represents the surface deviation of sampling points, $[S_{ij}]$ the sensitivity matrix of partial derivatives $\partial \underset{\sim}{\boldsymbol{R}}_i / \partial \varsigma_j$, and $\{\Delta \varsigma_j\}$ the corrective machine–tool settings.

The system in Eq. (5) is over-determined, since $p \gg q$. Generally, an over-determined system of equations will not have a solution, so the linear regression method can be adopted to solve it. A schematic flow chart for spiral bevel and hypoid gear development using the linear regression method is shown in Fig. 4. However, from a practical point of view the linear regression method is not adequate for solving this problem because the corrective machine–tool settings obtained by the linear regression method cannot be constrained to a permissible range that matches the physical machine–tool relationship. In some situations, the obtained solutions may be meaningless because the corrective machine–tool settings are yielded out-of-range. In addition, Eq. (6) becomes linearly dependent when the cradle angle

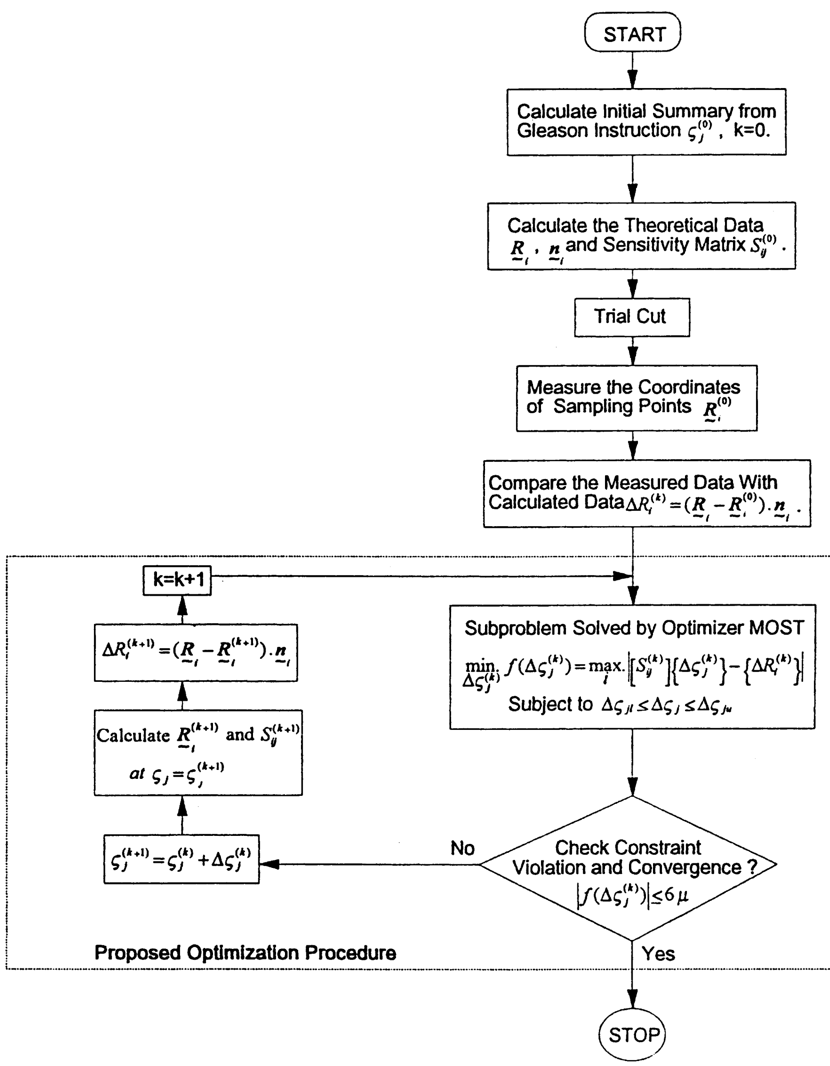

Fig. 5. Spiral bevel and hypoid gear development flow chart using the proposed optimization method.

and sliding base setting perturbations exist simultaneously during Helical motion. This results in a numerical divergence instability problem.

For the above-mentioned reasons, a development procedure using the proposed optimization procedure was adopted to overcome the weakness of the linear regression method. The flow chart for spiral bevel and hypoid gear development procedures using the optimization technique is proposed and is shown in Fig. 5. In this study, corrective machine–tool settings were regarded as problem constraints to be constrained to reasonable and desirable ranges. All machine–tool setting perturbations were chosen as design variables and updated automatically. Unlike the least-squares-error method used in linear regression analysis, the maximum deviation of real cut gear-tooth surfaces was chosen as the objective function. In the proposed development procedure, the sensitivity matrix is also updated automatically to obtain the most accurate corrective machine–tool settings. Therefore, the problem of how to obtain the corrective machine–tool settings is transformed into an optimization subproblem expressed as follows:

$$\min_{\Delta\varsigma_j^{(k)}} f(\Delta\varsigma_j^{(k)}) = \max_i |[S_{ij}^{(k)}]\{\Delta\varsigma_j^{(k)}\} - \{\Delta R_i^{(k)}\}|$$
$$(i = 1, \ldots, p; j = 1, \ldots, q \text{ and } k = \text{iteration counter}), \quad (7)$$

that is subject to the constraints: (a) the tooth thickness is constant; (b) the root angle is kept in the range to obtain an admissible gear clearance; (c) $\Delta\varsigma_{jl} \le \Delta\varsigma_j \le \Delta\varsigma_{ju}$; where $\Delta\varsigma_j$ $(j = 1, \ldots, q)$ are the design variables including the perturbation increments of cradle angle ϕ_c, eccentric angle ϕ_e, cutter spindle rotation angle ϕ_t, swivel angle ϕ_s, sliding base setting E_s, increment of machine center to back D_x, vertical offset E_v, machine root angle setting γ_m, ratio of Helical motion change gears η_h, and ratio of roll change gears η_r. All these perturbations are treated as design variables and can be represented as follows:

$$\Delta\varsigma_j = \{\Delta\phi_c, \Delta\phi_e, \Delta\phi_t, \Delta\phi_s, \Delta E_s, \Delta D_x, \Delta E_v, \Delta\gamma_m, \Delta\eta_h, \Delta\eta_r\}. \quad (8)$$

Constraint (a) indicates that the gear and pinion tooth thicknesses are held constant, and therefore the backlash is also kept within the design range. Constraint (b) means that the root angle of the gear generated by the corrective machine–tool settings is limited to the desired range to maintain clearance and to avoid interference between mating gears. Regarding constraint (c), the corrective machine–tool settings $\Delta\varsigma_i$ must be bounded to within a reasonable range between $\Delta\varsigma_{il}$ and $\Delta\varsigma_{iu}$ to match the practical machine–tool relationship. The values of $\Delta\varsigma_{il}$ and $\Delta\varsigma_{iu}$ are specified by

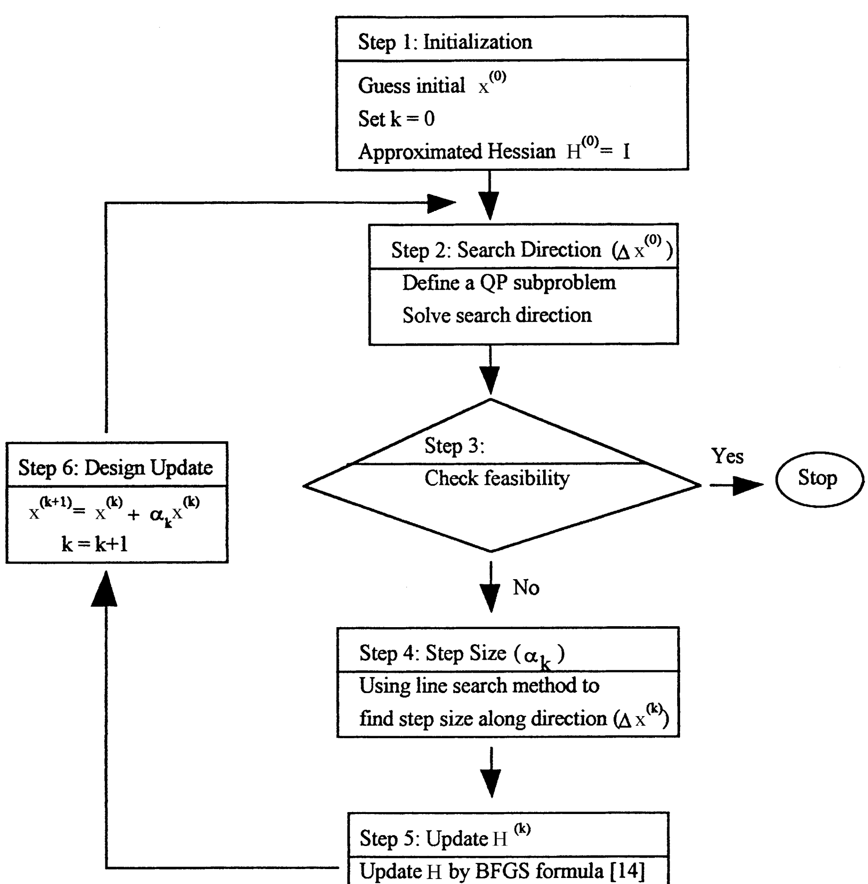

Fig. 6. Conceptual flow chart of the SQP algorithm.

Table 1
Gear blank dimensions, cutter specifications, and machine–tool settings (Gleason No. 106 hypoid generator settings)

Items	Pinion	Gear
Blank dimensions		
Number of teeth	22	22
Face width	24.000 mm	24.000 mm
Pitch angle	45°0′	45°0′
Outside diameter	93.311 mm	93.311 mm
Pitch apex to crown	41.344 mm	41.344 mm
Cutter specifications		
Mean cutter diameter	125.000 mm	125.000 mm
Inside blade angle	26°45′	26°45′
Outside blade angle	13°15′	13°15′
Point width	1.900 mm	1.900 mm
Tip fillet	0.500 mm	0.500 mm
Initial machine–tool settings		
Eccentric angle	40°5′	40°37′
Cutter spindle rotation angle	13°35′	18°11′
Swivel angle	189°42′	265°55′
Cradle angle	149°23′	355°12′
Machine root angle	35°24′	41°9′
Machine center to back	MD[a],−0.829 mm	MD[a],−0.404 mm
Blank offset	0.884 mm	0.000 mm
Sliding base	1.455 mm	5.081 mm
Nc/50 ratio gears	43/63×57/64	48/66×56/66
Helical motion gears	67/45×73/46	–
Helical motion position	No. 2	–

[a] Mounting distance.

the characteristics of the Gleason spiral bevel and hypoid gears generators.

The above optimization subproblem was solved by using MOST [9], a prototype optimization software program, in which the SQP method is adopted as an optimizer because of its accuracy, reliability, and efficiency [10]. The SQP algorithm is a generalized gradient-descent optimization method, and subsequently converges to a local rather than a global optimum. A conceptual flow chart of the SQP algorithm is shown in Fig. 6, which reflects the characteristics of the direct iterative optimization

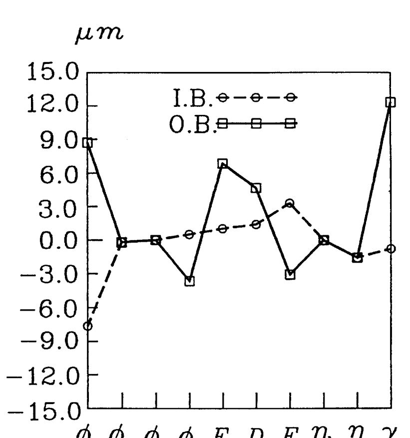

Fig. 7. Gear surface perturbations due to machine–tool setting variations.

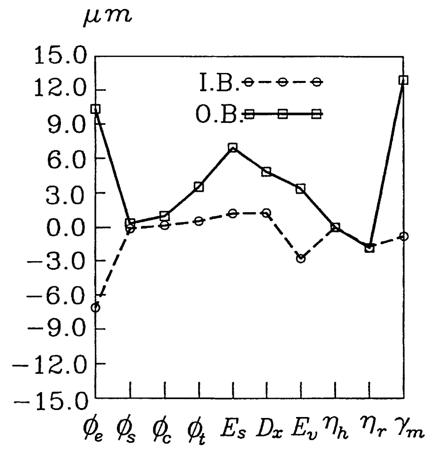

Fig. 8. Pinion surface perturbations due to machine–tool setting variations.

method. The steps of the algorithm are briefly summarized as follows:

Step 1. Initialization: set $k = 0$ and estimate $x^{(k)}$. Select two small numbers for allowable maximum constant violation (ε_1) and convergence parameter (ε_2), respectively. Let approximate Hessian matrix $H^{(0)}$ be an identity matrix.

Step 2. Search direction: linearize the objective and constraint functions about the current $x^{(k)}$ and a quadratic step size constraint is imposed for the linearized subproblem. This problem can be defined as a quadratic programming (QP) subproblem. Thus, solution of the QP problem yields a direction vector $\Delta x^{(k)}$.

Table 2
Machine–tool setting changes

Items	Pinion	Gear
Using linear regression method		
Eccentric angle	0°4′	0°2′
Cutter spindle rotation angle	0°32′	0°39′
Swivel angle	−61°51′	−10°37′
Cradle angle	−11107°15′	−587337°32′
Machine root angle	0°15′	0°41′
Machine center to back	−0.37 mm	−0.04 mm
Blank offset	−0.43 mm	−0.11 mm
Sliding Base	96.25 mm	−0.79 mm
Helical motion gears ratio	0.13	−0.20
Nc/50 ratio gears	0.01	0.01
Using SQP method		
Eccentric angle	0°−8′	0.00
Cutter spindle rotation angle	0°−7′	0°−7′
Swivel angle	0.00	0.00
Cradle angle	0.00	0.00
Machine root angle	0°1′	0°−1′
Machine center to back	0.00	−0.06 mm
Blank offset	−0.03 mm	−0.05 mm
Sliding base	−0.03 mm	−0.15 mm
Helical motion gears ratio	0.00	0.00
Nc/50 ratio gears	0.00	0.00

Step 3. Convergence criteria: if the maximum constraint violation and convergence parameter are less than the given accuracy ε_1 and ε_2, then stop the iteration and exit.
Step 4. Line search: a step size (α_k) along the direction is estimated based on a suitable line search method using descent functions.
Step 5. Update *H*: the BFGS (Broyden–Fletch–Goldfard–Shanno) updated formula [20] is selected to guarantee a positive definite updated Hessian $H^{(k)}$.
Step 6. Design updated: the new design is updated with step size (α_k) and search direction $\Delta x^{(k)}$ as iterative formula $X^{(k+1)} = x^{(k)} + \alpha_k \Delta x^{(k)}$.

5. Examples

In this section, a spiral bevel gear set generated by the Gleason No. 106 hypoid generator using the Duplex–Helical method is used as an example to demonstrate the proposed optimization procedure. This example investigates the sensitivity of surface characteristics to machine–tool setting variations, and illustrates the optimization procedure that calculates corrective machine–tool settings for minimizing deviations of real cut gear-tooth surfaces. The gear blank dimensions, cutter specifications, and machine–tool settings are listed in Table 1.

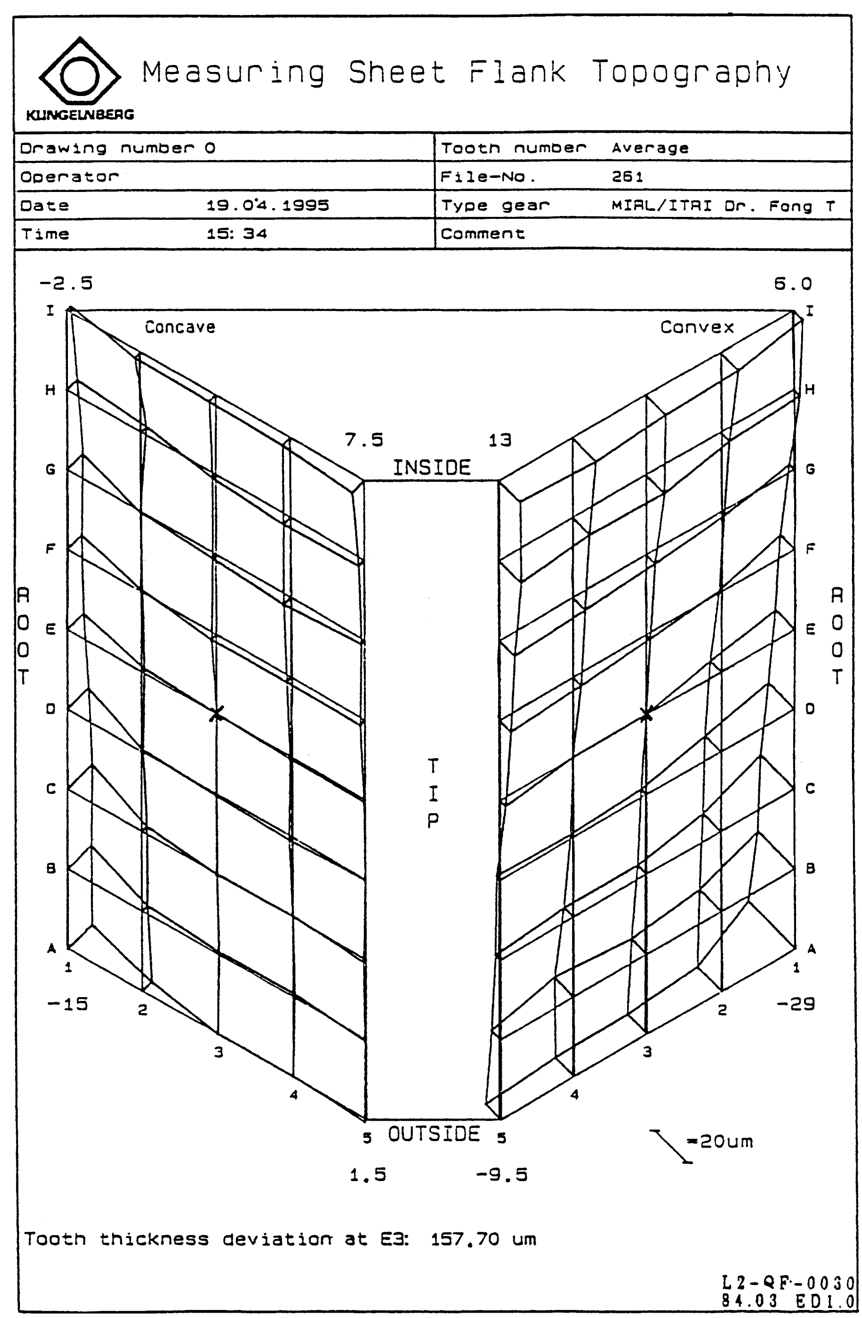

Fig. 9. Surface deviations of gear cut using primary machine settings.

The maximum displacement perturbations in each column of the sensitivity matrix $\{S_{ij}\}$ for the gear and pinion tooth surfaces are calculated according to Eq. (3), and shown in Figs. 7 and 8. The abbreviation I.B. denotes the convex side of the tooth surface, which is cut by the inside blade of the face-mill cutter, while O.B. denotes the concave side of the tooth surface. It was found that the maximum perturbation displacement is very sensitive to variations in the eccentric angle ϕ_e, sliding base setting E_s, vertical offset E_v, and machine root angle γ_m. Therefore, when making these machine–tool settings, care should be taken in measuring because they greatly affect real cut tooth-surface geometry. For this example, a spiral bevel gear set generated by the Duplex–Helical method was used. It is usually difficult to improve the surface characteristics of gears generated by the Duplex method because, as shown in Figs. 7 and 8, most of the parameters have conflicting effects on the displacement perturbations from side to side. Thus, improving one side on the tooth may result in a corresponding degradation of the other side. However, it is more

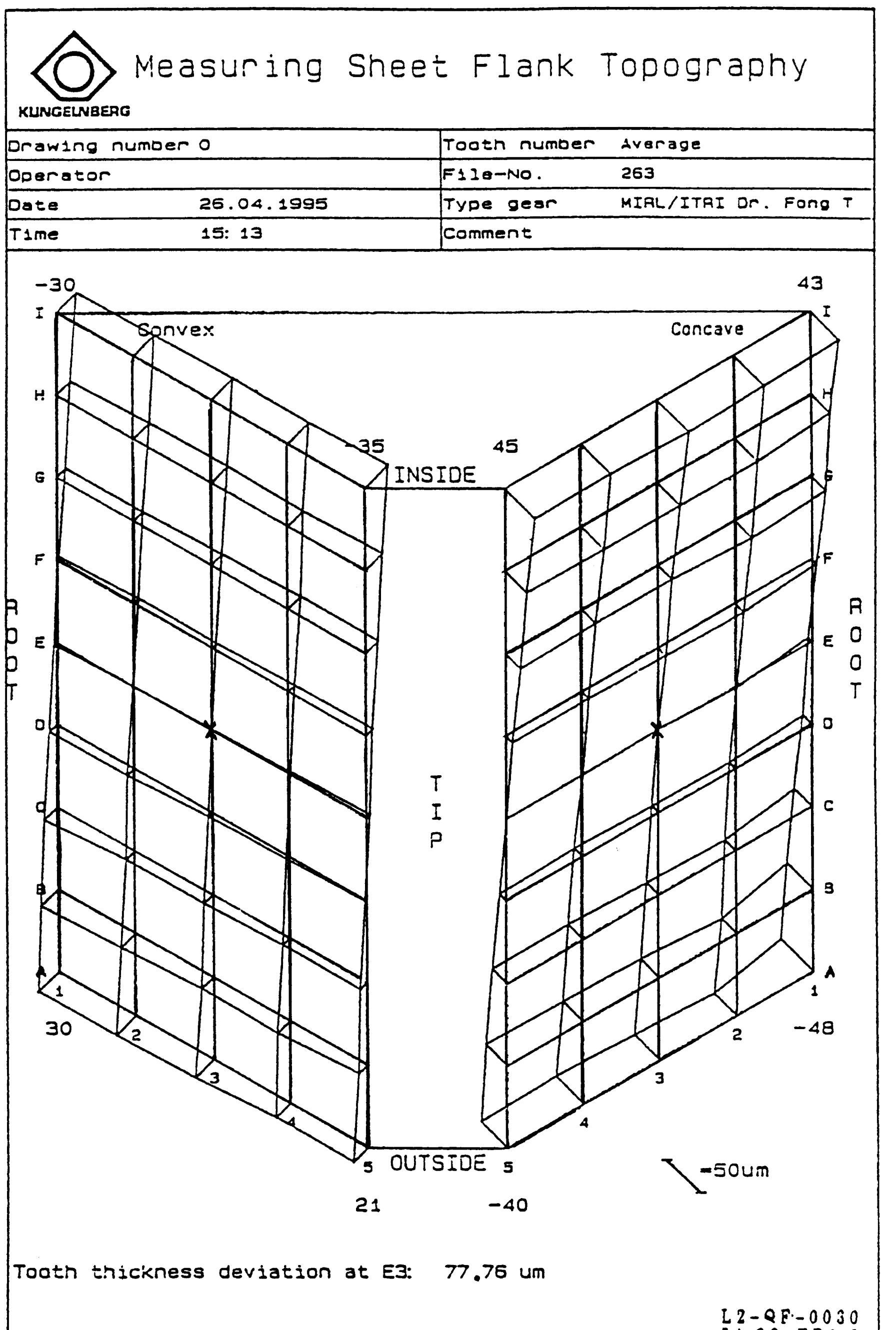

Fig. 10. Surface deviations of pinion cut using primary machine settings.

convenient and efficient to use the proposed optimization technique to minimize surface deviations than to use the linear regression method.

The sample gear was cut using the primary machine–tool settings shown in Table 1 and the coordinates of the surface sampling points on the real cut gear-tooth surfaces were measured using the CNC coordinate measuring machine. The measured data were then compared with the theoretical data obtained from the proposed gear-tooth mathematical model. For considerations of precision and minimization of run-out errors, four actual teeth were measured and the average measurement values were taken as actual surface data. Surface deviations at the sampling points on the real cut gear surface are shown in Fig. 9. The maximum deviation on the real cut gear-tooth surfaces was 0.029 mm, and occurred at sampling point A1 on the convex side; the tooth

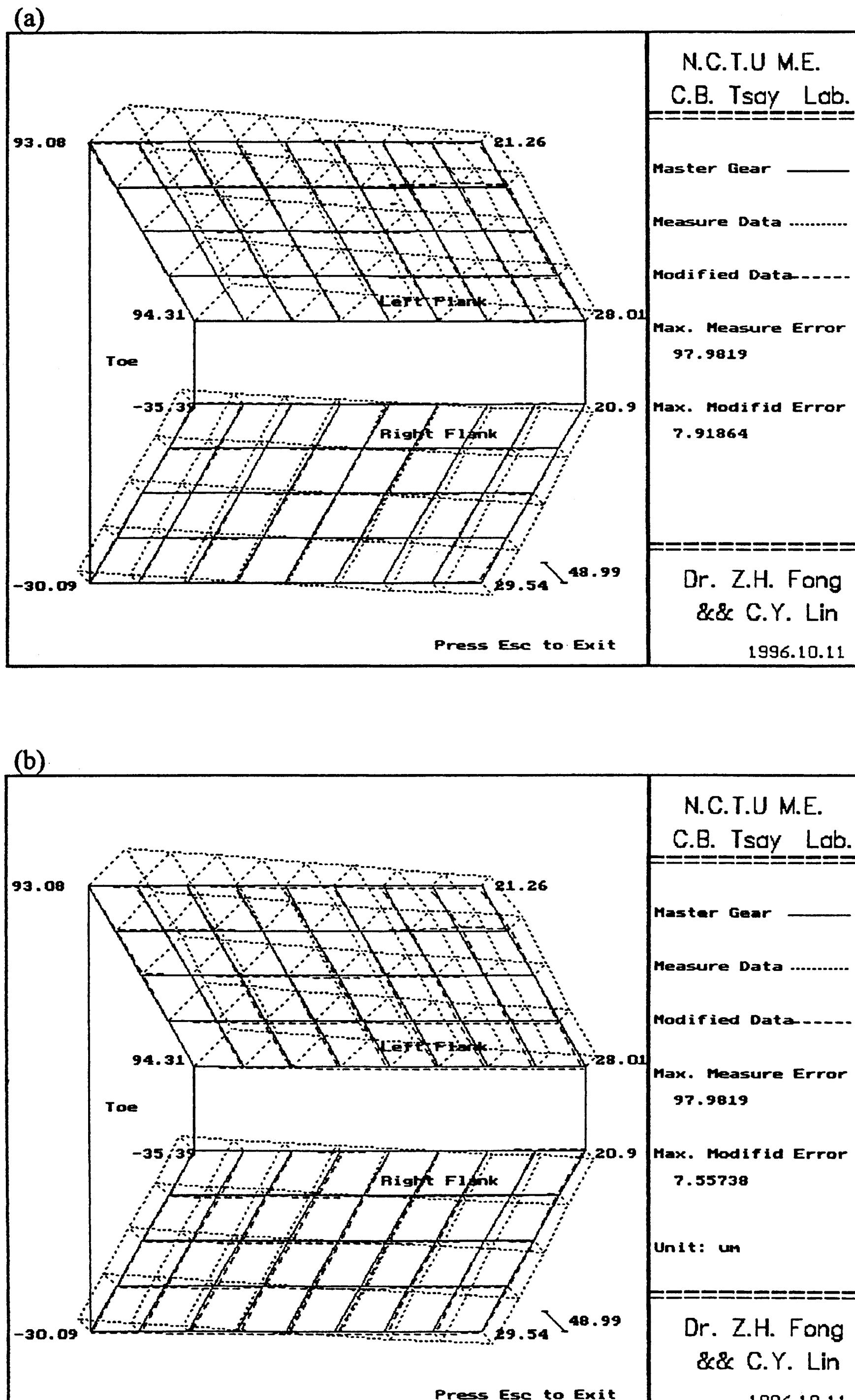

Fig. 11. Computer simulation of gear surface deviations using: (a) the linear regression method; (b) the SQP method.

thickness deviation at the basic reference point E3 was 0.158 mm. On the other hand, the pinion was cut according to the primary machine–tool settings shown in Table 1 and surface coordinates of the sampling points were measured. Surface deviations at the sampling points on the real cut pinion surface are shown in Fig. 10. The maximum deviation on the real cut pinion-tooth surface was 0.048 mm, and occurred at reference point A1 on the concave side. The tooth thickness deviation at the basic reference point E3 was 0.078 mm. Using the proposed development procedure and the developed computer simulation programs, modifications or changes in machine–tool settings were calculated by using linear regression method and SQP method, and are listed in Table 2, respectively. The computer simulation results are also shown in Figs. 11 and 12: in these figures the residual surface deviations are almost the same by using

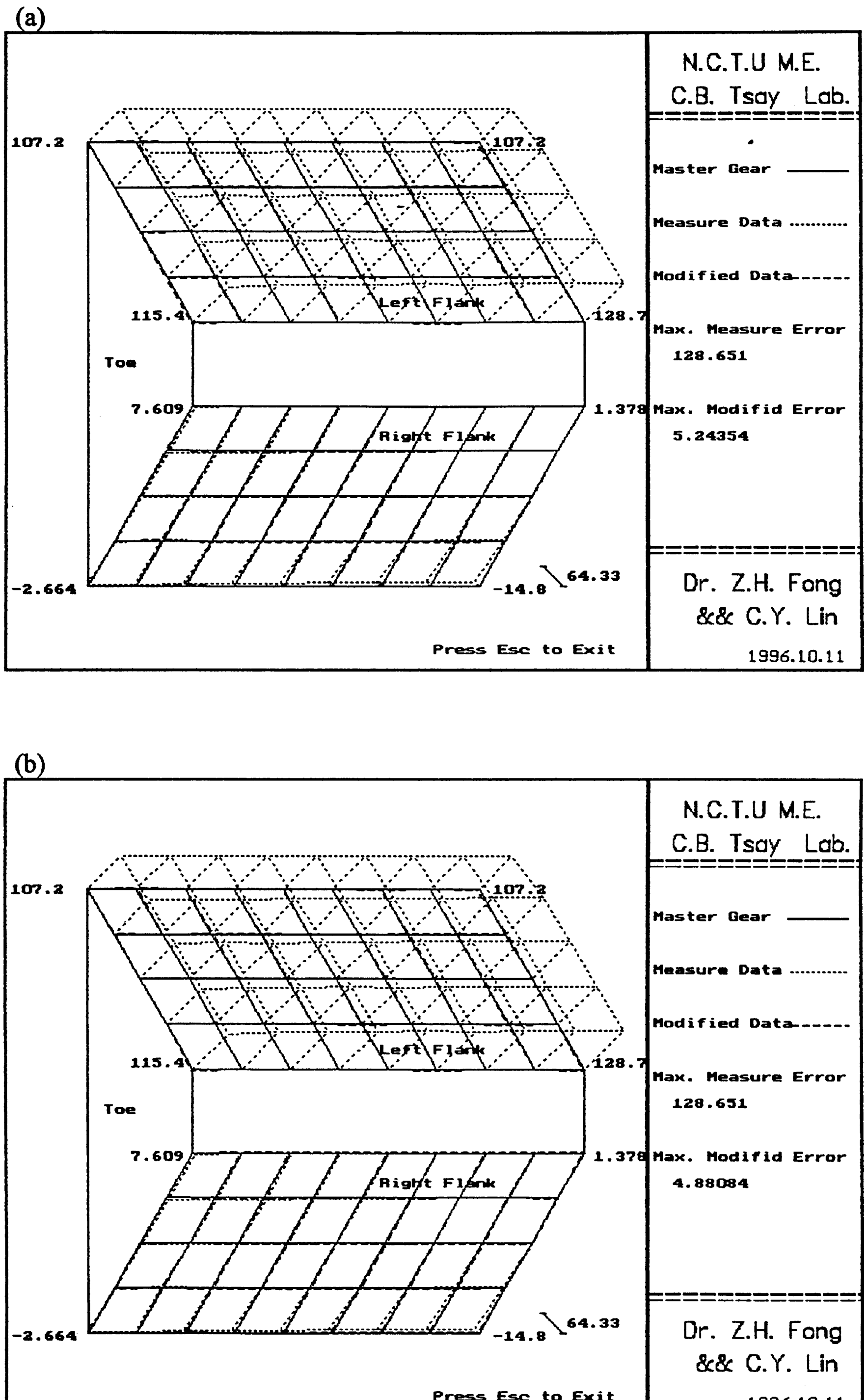

Fig. 12. Computer simulation of pinion surface deviations using: (a) the linear regression method; (b) the SQP method.

linear regression method and SQP method. However, since the linear regression method solves this problem dealing with unconstraint problem, the results of corrective machine–tool setting such as cradle angle and swivel angle setting shown in Table 2 were out-of-range and meaningless. Besides, the pinion was generated by Helical motion method. This also induced the linearly dependent problem of cradle angle and sliding base setting changes. In this study, the changes in machine–tool settings calculated by means of linear regression method using the singular value decomposition (SVD) [19] algorithm were typical during the course of trial-and-error. All of above-mentioned cases illustrated that the linear regression method existed weakness to solve this problem. This is why the SQP method is adopted in this problem.

Using the proposed optimization technique and the developed computer simulation programs, optimum changes in machine–tool settings were calculated, and are listed in Table 2. Based on these corrective machine–tool settings, a spiral bevel gear was cut using a Gleason No. 106 hypoid generator. The surface deviations at sampling points on the real cut gear-tooth surfaces are shown in Fig. 13. The tooth thickness deviation at the basic reference point E3 was reduced to 0.025 mm, and the maximum surface deviation

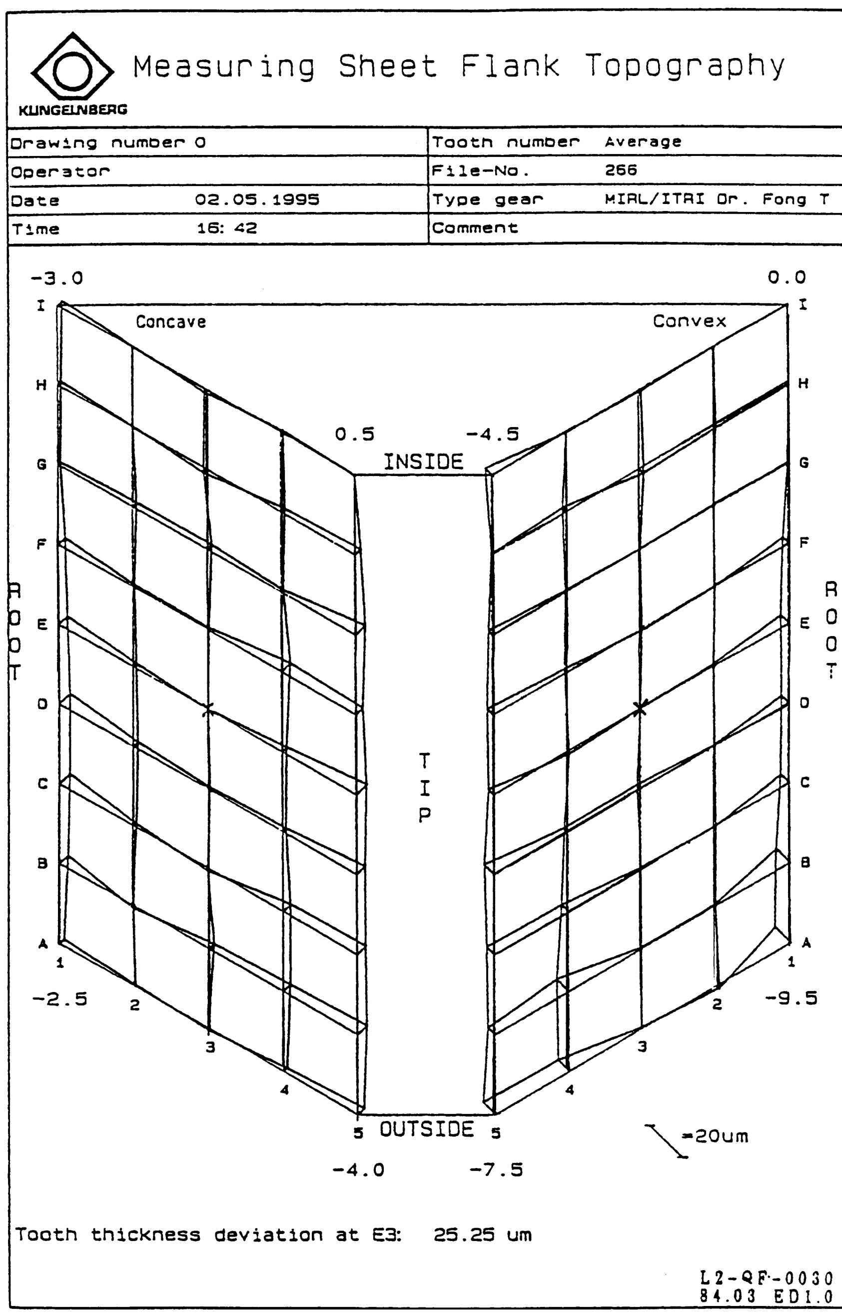

Fig. 13. Surface deviations of a gear cut using corrective machine–tool settings.

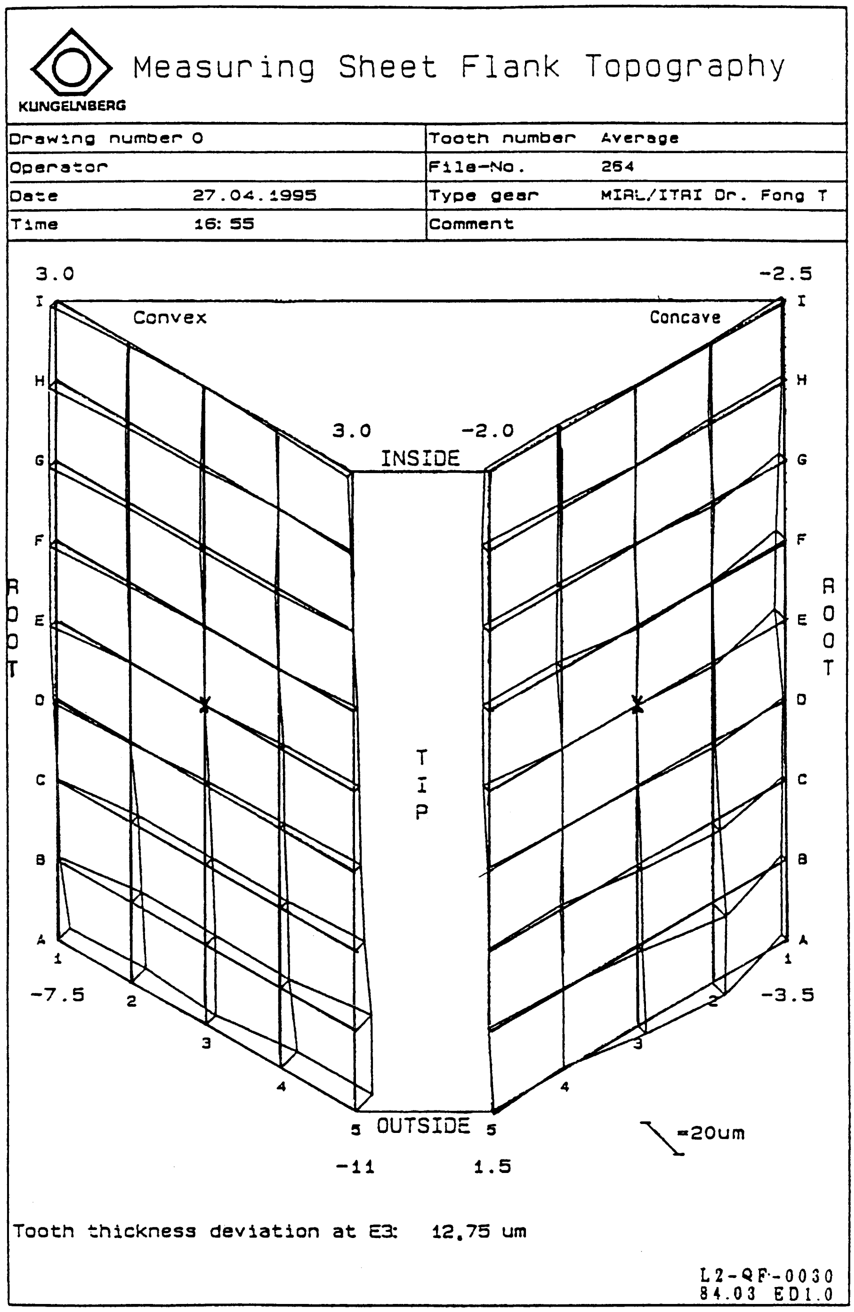

Fig. 14. Surface deviations of a pinion cut using corrective machine–tool settings.

of about 0.01 mm occurred at sampling point A1 on the convex side. These surface deviations are within tolerance and no further gear development is required. In addition, using the proposed optimum machine–tool setting changes shown in Table 2, a pinion was cut and its surface sampling point deviations are shown in Fig. 14. The tooth thickness deviation at the basic reference point E3 was reduced to 0.013 mm and the maximum surface deviation of 0.011 mm occurred at sampling point A5 on the convex side, which is acceptable. Therefore, the proposed method for obtaining corrective machine–tool settings to minimize surface deviations of real cut pinions and gears proved to be very useful. This indicates that the sensitivity analysis and optimization techniques were successfully applied in the proposed methodology.

6. Conclusion

Using a proposed mathematical model of Gleason spiral bevel and hypoid gears and a CNC measuring machine, sensitivity analysis of generated pinion and gear-tooth surfaces due to machine–tool settings have been investigated. Since the proposed mathematical model was derived directly in terms of the machine settings and machine constants, it is very easy to implement the mathematical model and to establish a closed-loop manufacturing system for the spiral bevel and hypoid gears.

Based on the sensitivity analysis and using a CNC coordinate measuring machine, an optimization procedure for corrective machine–tool setting calculation that minimizes surface deviations on real cut pinion and gear-tooth surfaces to within the permissible tolerances has been developed. The optimization method, which uses the SQP technique instead of the conventional linear regression method, has also been successfully applied to find corrective machine–tool settings within reasonable setting constraints. The developed optimization procedure is applicable to improving quality and quantity controls during manufacture of spiral bevel and hypoid gear sets generated by the Gleason–Duplex method, the Helical–Duplex method, the fixed setting method, and the modified roll method.

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