Minimal Energy Decentralized Estimation via Exploiting the Statistical Knowledge of Sensor Noise Variance

Jwo-Yuh Wu, Qian-Zhi Huang, and Ta-Sung Lee

***Abstract*—We study the problem of minimal-energy decentralized estimation via sensor networks with the best-linear-unbiased-estimator fusion rule. While most of the existing solutions require the knowledge of instantaneous noise variances for energy allocation, the proposed approach instead relies on an associated statistical model. The minimization of total energy is subject to a performance constraint in terms of the reciprocal of mean square errors averaged over the considered distribution. A closed-form formula for such a mean distortion metric, as well as an associated tractable lower bound, is derived. By imposing a target distortion constraint in terms of this bound and further through feasible set relaxation, the problem can be reformulated in the form of convex optimization and is then analytically solved. The proposed method shares several attractive features of the existing designs via instantaneous noise variances. Through simulations it is seen to significantly improve the energy efficiency against the uniform allocation scheme.**

***Index Terms*—Convex optimization, decentralized estimation, energy minimization, quantization, sensor networks.**

I. Introduction

Decentralized estimation has been one key issue in signal processing research for sensor networks [9], [10]. Subject to severe energy and bandwidth limitations, each sensor in this scenario is typically allowed to transmit only a quantized version of its raw measurement to the fusion center (FC) to generate a final parameter estimate. While quantized messages with longer bit length yield improved data fidelity, the consumed transmission energy is however proportional to the bit loads [3], [8]. To avoid quick energy drainage and prolong the network lifetime, several energy-efficient decentralized estimation schemes, formulated via an optimal bit-loading setup, have been recently reported in [3], [7], and [8]. One key feature common to these works is that the energy (or bits) allocated to each sensor must be determined via *instantaneous* local sensor noise characteristics, e.g., the noise variance if the fusion rule follows the best-linear-unbiased-estimator (BLUE) principle [1]. Since timely knowledge of the instantaneous noise profile calls for frequent training/update and would be too costly to acquire, a plausible alternative is to instead exploit the partial (or long-term) information of the noise characteristics [8]. Such related solutions, however, remain yet to be developed.

This paper attempts to provide a solution to minimal-energy decentralized estimation (under BLUE fusion rule) by exploiting long-term noise variance information. We focus on a commonly used statistical model for noise variance, and the estimation performance is assessed through the reciprocal of the mean square error (MSE) averaged with respect to the considered distribution. A closed-form expression of the averaged distortion measure is obtained, which is seen to be highly nonlinear in the sensor bit loads. To facilitate analysis an associated tractable lower bound of the averaged distortion measure is derived. By instead imposing the performance requirement in terms of this bound and further through feasible set relaxation, the energy-minimization problem can be reformulated in the form of convex optimization and is then analytically solved based on the standard Lagrange techniques. The proposed scheme possesses several appealing features pertaining to the existing solutions based on the instantaneous noise variance information: sensors with bad channel quality (specified via the path distance to FC) are shut off to conserve energy, and for those active nodes the allocated energy is proportional to the individual channel gain. Simulation results show that the proposed optimal solution yields significant energy saving against the equal-bit allocation scheme.

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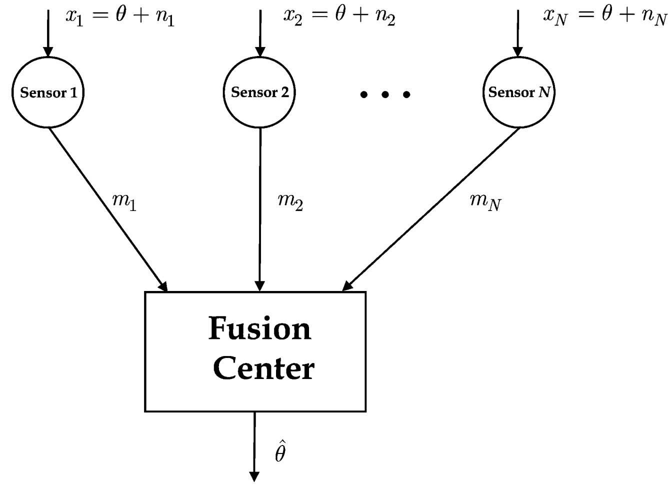

Fig. 1. Sensor network system diagram.

The rest of this paper is organized as follows. Section II is the preliminary. Section III presents the main results. Section IV shows the numerical simulation. Finally, Section V is the conclusion.

II. Preliminary

Consider a wireless sensor network as depicted in Fig. 1, in which N spatially deployed sensors cooperate with an FC for estimating an unknown deterministic parameter θ. The local observation at the ith node is

$$x_i = \theta + n_i, \quad 1 \le i \le N \tag{2.1}$$

where n_i is a zero-mean measurement noise with variance σ_i^2. Due to bandwidth and power limitations each sensor quantizes its observation into a b_i-bit message, and then transmits this locally compressed data to the FC to generate a final estimate of θ. In this paper the uniform quantization scheme with nearest-rounding [4], [6], is adopted; the quantized message at the ith sensor can thus be modeled as[1]

$$m_i = x_i + q_i, \quad 1 \le i \le N \tag{2.2}$$

where q_i is the quantization error which is uniformly distributed with zero mean and variance $\sigma_{q_i}^2 = R^2/(12 \cdot 4^{b_i})$ [4], where $[-R/2, R/2]$ is

[1]Such a quantizer model and error assumption, though being valid only when the number of quantization bits is sufficiently large and the signal amplitude tends to span over all the quantization intervals [4], are widely used in the literature due to its analytical tractability.

the available signal amplitude range common to all sensors. With (2.1) and (2.2), the received data from all sensor outputs can be expressed in a vector form as[2]

$$[m_1 \cdots m_N]^T = [1 \cdots 1]^T \theta + \underbrace{[n_1 \cdots n_N]^T}_{:=\mathbf{n}} + \underbrace{[q_1 \cdots q_N]^T}_{:=\mathbf{q}} \quad (2.3)$$

where $(\cdot)^T$ denotes the transpose. This paper focuses on linear fusion rules for parameter recovery. More specifically, by assuming that the noise components $\{\mathbf{n}, \mathbf{q}\}$ in (2.3) are mutually independent and the respective samples n_i's and q_i's are also independent across sensors, the parameter θ is retrieved via the BLUE [1] estimator via

$$\hat{\theta} = \left(\sum_{i=1}^{N} \frac{m_i}{\sigma_i^2 + R^2 4^{-b_i}/12}\right)\left(\sum_{i=1}^{N} \frac{1}{\sigma_i^2 + R^2 4^{-b_i}/12}\right)^{-1} \quad (2.4)$$

and the incurred MSE is thus [1]

$$E|\hat{\theta} - \theta|^2 = \left(\sum_{i=1}^{N} \frac{1}{\sigma_i^2 + R^2 4^{-b_i}/12}\right)^{-1}. \quad (2.5)$$

A commonly used statistical description for sensor noise variance is [3], [8]

$$\sigma_i^2 = \delta + \alpha z_i, \quad 1 \le i \le N \quad (2.6)$$

where δ models the network-wide noise variance threshold, α controls the underlying variation from the nominal minimum, and $z_i \sim \chi_1^2$ is a central chi-square distributed random variable with degrees-of-freedom equal to one [2, p. 24]. In the sequel, we will exploit the noise variance model (2.6) for minimal- energy decentralized estimation.

III. Main Results

This section presents the proposed minimal energy scheduling scheme. Section III-A first introduces the mathematical formulation. Then we will show in Section III-B the adopted approach toward solving the problem. The optimal solution is derived in Section III-C, and the associated key features are discussed in Section III-D.

A. Problem Formulation

We assume as in [3] that the consumed energy for transmitting the message m_i at the ith sensor is proportional to the number of bits b_i in m_i, that is

$$E_i = w_i b_i \text{ for some } w_i, \quad 1 \le i \le N \quad (3.1)$$

where the energy density factor w_i is defined as [3]

$$w_i := \eta d_i^{\kappa} \cdot \frac{(2^s - 1)}{s} \cdot \ln\left(\frac{4(1 - 2^{-s})}{sP_b}\right) \quad (3.2)$$

in which η is a constant depending on the noise profile, d_i is the distance between the ith node and the FC, κ is the path loss exponent common to all sensor-to-FC links, s is the number of bits per QAM/PSK symbol, and P_b is the target bit error rate. With (3.1), the energy allocated to the ith sensor is thus completely determined by the number of quantization bits b_i. For a fixed set of noise variances σ_i^2's, the energy minimization problem subject to an allowable parameter distortion level γ (in terms of MSE) can be formulated as

$$\text{Minimize } \sum_{i=1}^{N} w_i b_i,$$
$$\text{subject to } \left(\sum_{i=1}^{N} \frac{1}{\sigma_i^2 + (R^2/12)4^{-b_i}}\right)^{-1} \le \gamma \text{ and}$$
$$b_i \text{ nonnegative integer, } 1 \le i \le N \quad (3.3)$$

or equivalently

$$\text{Minimize } \sum_{i=1}^{N} w_i b_i, \text{ subject to } \sum_{i=1}^{N} \frac{1}{\sigma_i^2 + (R^2/12)4^{-b_i}} \ge \gamma^{-1}$$
$$\text{and } b_i \text{ nonnegative integer, } 1 \le i \le N. \quad (3.4)$$

To obtain a universal solution irrespective of instantaneous measurement noise conditions, we will consider the following optimization problem, in which the equivalent MSE performance metric in (3.4) is instead averaged with respect to the noise variance statistic characterized in (2.6):

$$\text{Minimize } \sum_{i=1}^{N} w_i b_i,$$
$$\text{subject to } \int_{\mathbf{z}} \sum_{i=1}^{N} \frac{1}{\delta + \alpha z_i + (R^2/12)4^{-b_i}} p(\mathbf{z}) d\mathbf{z} \ge \gamma^{-1}$$
$$\text{and } b_i \ge 0, \ 1 \le i \le N, \quad (3.5)$$

where $\mathbf{z} := [z_1 \cdots z_N]^T$ with $p(\mathbf{z})$ denoting the associated distribution. In (3.5), the constraint that all b_i are nonnegative integers are relaxed to be $b_i \ge 0$ so as to render the problem tractable; once the optimal (real valued) b_i's are computed, the associated bit loads can be obtained through upper integer rounding, as in [3] and [8]. The solution to problem (3.5) is discussed next.

B. Proposed Approach

To solve (3.5), a crucial step is to derive an analytic expression of the average MSE performance measure. For this we first note that, since $z_i \sim \chi_1^2$ is i.i.d. and [2, p. 24]

$$p_{\chi_1^2}(z) = \frac{1}{\sqrt{2\pi z}} \exp(-z/2) u(z) \quad (3.6)$$

where $u(z)$ denotes the unit-step function, we have

$$\int_{\mathbf{z}} \sum_{i=1}^{N} \frac{1}{\delta + \alpha z_i + (R^2/12)4^{-b_i}} p(\mathbf{z}) d\mathbf{z}$$
$$= \sum_{i=1}^{N} \int_0^{\infty} \frac{1}{\alpha z_i + \underbrace{[\delta + R^2 4^{-b_i}/12]}_{:=\beta_i}} \cdot \frac{e^{-z_i/2}}{\sqrt{2\pi z_i}} dz_i$$
$$= \frac{1}{\sqrt{2\pi}} \sum_{i=1}^{N} \int_0^{\infty} \frac{e^{-z_i/2}}{(\alpha z_i + \beta_i)\sqrt{z_i}} dz_i. \quad (3.7)$$

The following lemma, with proof given in Appendix A, provides a closed-form expression of the integral involved in the summation in (3.7).

[2]We assume perfect reception of all the messages m_i' at the FC, and the resultant MSE thus serves as a yardstick performance. When the transmission link is modeled as a binary symmetric channel, by following the procedures as in [8] it can be shown that, if the bit-error-rate is below a certain threshold, the incurred MSE is at most a constant factor away from the benchmark measure. It is such a channel-aware capability that makes the proposed design paradigm meaningful; a similar approach is also adopted in [3].

Lemma 3.1: With $\alpha > 0$ and $\beta_i > 0$ as defined in (3.7), we have

$$\int_0^\infty \frac{e^{-z_i/2}}{(\alpha z_i + \beta_i)\sqrt{z_i}} dz_i = \frac{2\pi \cdot e^{\beta_i/2\alpha} \cdot Q(\sqrt{\beta_i/\alpha})}{\sqrt{\alpha\beta_i}} \tag{3.8}$$

where $Q(x) := \int_x^\infty (e^{-t^2/2}/\sqrt{2\pi})dt$ is the Gaussian tail function. □

With (3.7) and (3.8), the optimization problem (3.5) can be equivalently rewritten as

$$\text{Minimize} \sum_{i=1}^N w_i b_i, \text{ subject to } \sqrt{\frac{2\pi}{\alpha}} \times \sum_{i=1}^N \frac{e^{\beta_i/2\alpha} \cdot Q(\sqrt{\beta_i/\alpha})}{\sqrt{\beta_i}} \geq \gamma^{-1}, \; b_i \geq 0, \; 1 \leq i \leq N. \tag{3.9}$$

Exact solutions to problem (3.9) appear intractable since the design constraint, in particular, the one accounting for the target MSE, is highly nonlinear in b_i. We will thus seek for suboptimal alternatives which can otherwise admit simple analytic expressions. The underlying approach toward this end is to derive an easy-to-tackle lower bound on the target distortion metric, and then replace the distortion constraint in (3.9) by one which forces the lower bound to be above γ^{-1}: such a procedure will considerably simplify the analysis without incurring any loss in the desired estimation performance. This is done with the aid of the next lemma (see Appendix B for a proof).

Lemma 3.2: The following inequality holds:

$$\sqrt{\frac{2\pi}{\alpha}} \sum_{i=1}^N \frac{e^{\beta_i/2\alpha} \cdot Q(\sqrt{\beta_i/\alpha})}{\sqrt{\beta_i}} \geq cNQ\left(\frac{1}{N}\sum_{i=1}^N (\sqrt{\delta/\alpha} + R2^{-b_i}/\sqrt{12\alpha})\right) \tag{3.10}$$

where c is a constant defined by $c := \sqrt{2\pi/\alpha} \cdot (e^{\delta/2\alpha}/\sqrt{\delta + R^2/12})$. □

Inequality (3.10) suggests that we can replace the performance constraint in (3.9) by the following one without incurring any penalty in the target distortion:

$$cNQ\left(\frac{1}{N}\sum_{i=1}^N (\sqrt{\delta/\alpha} + R2^{-b_i}/\sqrt{12\alpha})\right) \geq \gamma^{-1} \tag{3.11}$$

or equivalently

$$\frac{1}{N}\sum_{i=1}^N (\sqrt{\delta/\alpha} + R2^{-b_i}/\sqrt{12\alpha}) \leq Q^{-1}\left(\frac{1}{cN\gamma}\right) \tag{3.12}$$

since $Q(\cdot)$ is one-to-one and monotone decreasing. We will thus instead focus on the optimization problem with a modified MSE performance constraint:

$$\text{Minimize} \sum_{i=1}^N w_i b_i, \text{ subject to } \frac{R}{\sqrt{12\alpha}N}\sum_{i=1}^N 2^{-b_i} \leq Q^{-1}\left(\frac{1}{cN\gamma}\right) - \sqrt{\frac{\delta}{\alpha}}, \; b_i \geq 0, \; 1 \leq i \leq N. \tag{3.13}$$

The main advantage of the alternative design formulation is that, in (3.13), the cost function is linear and the constraints are convex; it is thus a convex optimization problem and will moreover lead to a simple closed-form solution as shown hereafter.

C. Optimal Solution

To solve problem (3.13), let us form the Lagrangian as

$$\begin{aligned} L(b_1, \cdots, b_N, \lambda, \mu_1, \cdots \mu_N) &= \sum_{i=1}^N w_i b_i + \lambda\left(\frac{R}{\sqrt{12\alpha}N}\sum_{i=1}^N 2^{-b_i} - Q^{-1}\left(\frac{1}{cN\gamma}\right) + \sqrt{\frac{\delta}{\alpha}}\right) - \sum_{i=1}^N \mu_i b_i; \end{aligned} \tag{3.14}$$

the associated set of KKT conditions then reads

$$w_i + \lambda \cdot \frac{(-\ln 2)R2^{-b_i}}{\sqrt{12\alpha}N} - \mu_i = 0, \quad 1 \leq i \leq N \tag{3.15}$$

$$\lambda\left(\frac{R}{\sqrt{12\alpha}N}\sum_{i=1}^N 2^{-b_i} - Q^{-1}\left(\frac{1}{cN\gamma}\right) + \sqrt{\frac{\delta}{\alpha}}\right) = 0 \tag{3.16}$$

$$\lambda \geq 0, \quad \mu_i \geq 0, \quad \mu_i b_i = 0, \quad b_i \geq 0, \quad 1 \leq i \leq N. \tag{3.17}$$

We first observe that, if $\lambda = 0$, (3.15) implies $\mu_i = w_i > 0$ for all $1 \leq i \leq N$, and hence $b_i = 0$, $1 \leq i \leq N$: this case should be precluded since otherwise all the sensors will remain silent. Accordingly, we must have $\lambda > 0$, meaning that the MSE constraint in (3.13) is active so that

$$\frac{R}{\sqrt{12\alpha}N}\sum_{i=1}^N 2^{-b_i} = Q^{-1}\left(\frac{1}{cN\gamma}\right) - \sqrt{\frac{\delta}{\alpha}}. \tag{3.18}$$

Solving (3.15) and (3.18) leads to

$$b_i = \log_2\left\{\frac{R\overline{\lambda}}{\sqrt{12\alpha}N(w_i - \mu_i)}\right\} \tag{3.19}$$

where

$$\overline{\lambda} := \lambda \ln 2 = \left[Q^{-1}(c^{-1}N^{-1}\gamma^{-1}) - \sqrt{\delta/\alpha}\right]^{-1} \sum_{i=1}^N (w_i - \mu_i). \tag{3.20}$$

By taking into account the constraint $b_i \geq 0$, the optimal pair $(b_i^{\text{opt}}, \overline{\lambda}^{\text{opt}})$ is given by the next theorem (see Appendix C for detailed derivation).

Theorem 3.3: Assume $w_1 \geq w_2 \geq \cdots \geq w_N$ without loss of generality, and define the function

$$f(K) := (Nw_K)^{-1} \sum_{j=N-K+1}^N w_j, \quad 1 \leq K \leq N. \tag{3.21}$$

Let $1 \leq K_1 \leq N$ be such that $f(K_1 - 1) < 1$ and $f(K_1) \geq 1$. Then we have

$$b_i^{\text{opt}} = \begin{cases} 0, & 1 \leq i \leq N - K_1 \\ \log_2\left\{\frac{R\overline{\lambda}^{\text{opt}}}{\sqrt{12\alpha}Nw_i}\right\}, & N - K_1 + 1 \leq i \leq N \end{cases} \tag{3.22}$$

where

$$\overline{\lambda}^{\text{opt}} = \left[Q^{-1}(c^{-1}N^{-1}\gamma^{-1}) - \sqrt{\delta/\alpha}\right]^{-1} \sum_{j=N-K_1+1}^N w_j. \tag{3.23}$$

□

D. Discussions

1) We note that the target distortion level γ cannot be set unlimitedly small: it is otherwise lower bounded by the MSE attained by the benchmark estimate based on unquantized raw sensor measurements (i.e., the case when $b_i = \infty$, $1 \le i \le N$). Indeed, by setting $b_i = \infty$ in the mean distortion specified in (3.9), the minimal allowable γ can be immediately determined as

$$\gamma \ge \underbrace{\left[N e^{\delta/2\alpha} Q(\sqrt{\delta/\alpha}) \sqrt{\frac{2\pi}{\alpha\delta}} \right]^{-1}}_{:=\gamma_0}. \tag{3.24}$$

2) Since $0 \le b_i \le \infty$, a necessary condition for validating the MSE constraint in (3.13) is therefore

$$Q^{-1}\left(\frac{1}{cN\gamma}\right) - \sqrt{\frac{\delta}{\alpha}} \ge 0, \text{ or } \frac{1}{cN\gamma} \le Q\left(\sqrt{\frac{\delta}{\alpha}}\right) \tag{3.25}$$

because $Q(\cdot)$ is one-to-one and monotone decreasing. By definition of the constant c in Lemma 3.2 and with (3.25), the distortion level attainable by the proposed method is lower bounded by

$$\gamma \ge \left[N e^{\delta/2\alpha} Q(\sqrt{\delta/\alpha}) \sqrt{\frac{2\pi}{\alpha(\delta + R^2/12)}} \right]^{-1}. \tag{3.26}$$

We note that the lower bound (3.26) is indeed larger than the minimal threshold γ_0 defined in (3.24).

3) Recall from (3.2) that the energy density factor w_i is proportional to the path loss gain d_i^κ (if the same bit error rate is assumed throughout all the links). Large values of w_i, in particular, correspond to sensors deployed far away from the FC (with large d_i), usually with poor background channel gains. In light of this point, the proposed optimal solution (3.22) is intuitively attractive: sensors associated with the $(N - K_1)$th largest w_i's are turned off to conserve energy. We note that a similar energy conservation strategy via shutting off sensors along poor channel links is also found in [8], which instead exploits the knowledge of the instantaneous noise variances for parameter estimation.
4) We further note from (3.22) that, for those active nodes, the assigned message length is inversely proportional to w_i: this is intuitively reasonable since sensors with better link conditions should be allocated with more bits (energy) toward MSE reduction and energy conservation.
5) Based on the inequality constraint for MSE in (3.13), the equal-bit scheme for maintaining the target MSE can be obtained by solving

$$\frac{R 2^{-\tilde{b}}}{\sqrt{12\alpha}} = Q^{-1}\left(\frac{1}{cN\gamma}\right) - \sqrt{\frac{\delta}{\alpha}} \tag{3.27}$$

leading to

$$\tilde{b} = \log_2 \left\{ \frac{R}{\sqrt{12\alpha}\left[Q^{-1}(c^{-1}N^{-1}\gamma^{-1}) - \sqrt{\delta/\alpha}\right]} \right\}. \tag{3.28}$$

Simulation results in the next section show that the proposed optimal scheme (3.22) yields significant energy saving when compared with (3.28). □

IV. Numerical Simulation

This section illustrates through numerical simulation the energy saving efficiency of the proposed solution b_i^{opt} in (3.22) over the uniform allocation scheme $\tilde{b}$ in (3.28). For a fixed set of energy density factors w_i's, $1 \le i \le N$, the performance is measured via the percentage of energy saving (PES) [3], [8]:

$$\text{PES} := \frac{\tilde{b} \sum_{i=1}^{N} w_i - \sum_{i=1}^{N} w_i b_i^{\text{opt}}}{\tilde{b} \sum_{i=1}^{N} w_i} \times 100. \tag{4.1}$$

The total number of sensors is $N = 1500$, the target mean MSE is $\gamma = 0.005$, and the link channel gain is set to be $w_i = d_i^\kappa$. For a positive random variable V we define the associated normalized deviation as $\rho(V) := \sqrt{\text{var}(V)}/E(V)$, which measures the tendency of heterogeneity of V [3], [8] (the larger such a ratio is, the more heterogeneous the random variable V will be).

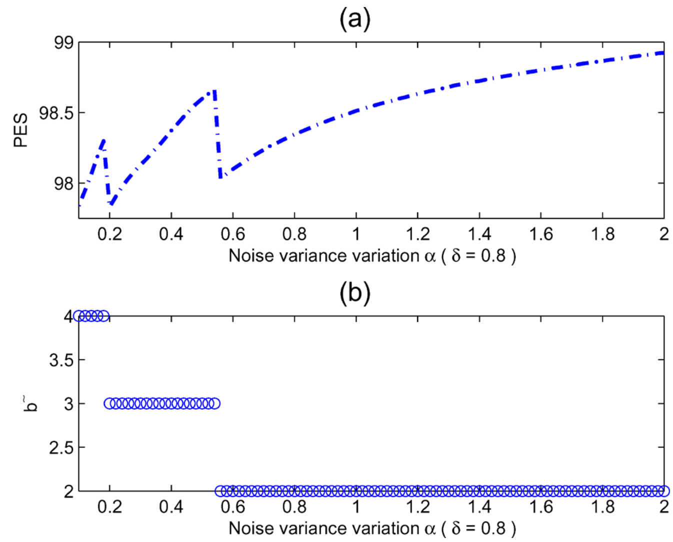

Fig. 2. (a) Percentage of energy saving. (b) Bits of equal-energy scheme ($\delta = 0.8$).

A. Impact Due to Heterogeneity of Sensor Noise Variance

This simulation illustrates the impact due to the heterogeneity of sensor noise variance on the energy saving performance. The path loss factor is set to be $\kappa = 3.5$, and the link distances follow the model $d_i = 10 + 10Z_i$, with $Z_i \sim \chi_1^2(z)$ being i.i.d. The normalized deviation factor of the sensor noise variance in (2.6) is verified to be

$$\rho\left(\sigma_i^2\right) = \frac{\sqrt{2}\alpha}{\alpha + \delta} = \frac{\sqrt{2}}{1 + (\delta/\alpha)}. \tag{4.2}$$

It is easy to see from (4.2) that $\rho(\sigma_i^2)$ increases either when α is enlarged or δ is reduced. With fixed $\delta = 0.8$, Fig. 2(a) shows the PES for $0.1 \le \alpha \le 1.6$ (corresponding to $0.157 \le \rho(\sigma_i^2) \le 0.943$), and Fig. 2(b) depicts the computed $\tilde{b}$ in (3.28). We first observe that the PES exhibits two "jumps": this accounts for the two level changes of $\tilde{b}$ as α varies. Also, within each duration of constant $\tilde{b}$, energy efficiency of the solution (3.22) improves as α increases (or $\rho(\sigma_i^2)$ is enlarged). We repeat the experiment by fixing $\alpha = 0.4$ and increasing δ from zero to four (thus yielding $1.414 \ge \rho(\sigma_i^2) \ge 0.129$); the results are shown in Fig. 3. The figure shows that, for each duration of constant $\tilde{b}$, the PES is nonetheless reduced as δ increases (or $\rho(\sigma_i^2)$ is lowered). A very rough interpretation of this tendency is that, since large δ's incur severe noise corruption in *all* sensor measurements, more sensor nodes should be turned on (thus potentially more energy consumption) for fulfilling the target MSE requirement.

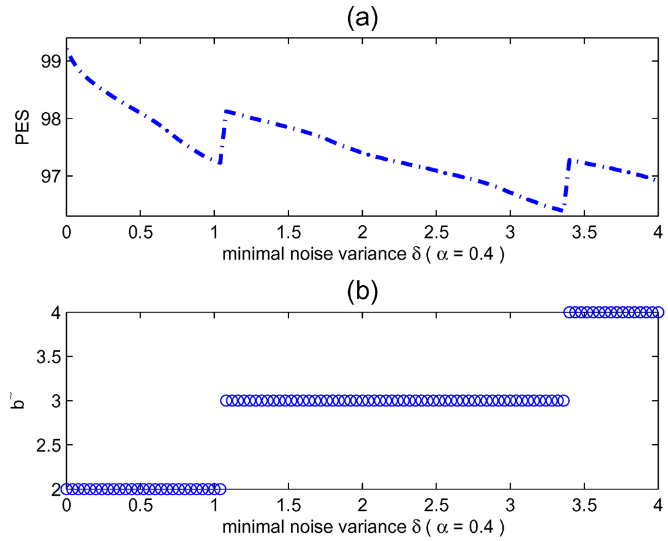

Fig. 3. (a) Percentage of energy saving. (b) Bits of equal-energy scheme $(\alpha = 0.4)$.

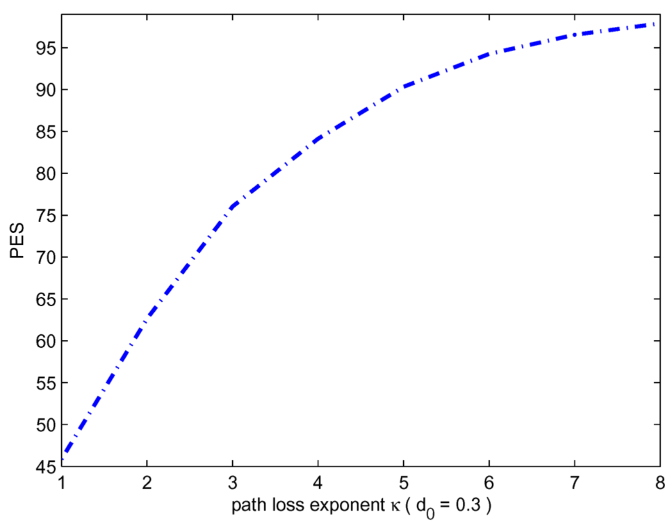

Fig. 4. Percentage of energy saving w.r.t. different κ.

B. Impact Due to Heterogeneity of Energy Density Factors

This simulation further investigates the PES performance with respect to the heterogeneity of the energy density factors w_i's. We set $\delta = 0.8$ and $\alpha = 0.2$ in the sensor noise variance model (2.6); as such the uniform allocation scheme (3.28) yields $\tilde{b} = 3$. Toward tractable derivation of the normalized deviation $\rho(w_i)$, we assume as in [3] that the sensor nodes are uniformly deployed inside the unity disk whose center is the FC. The link distance d_i is specifically characterized as a uniform random variable drawn from the interval $[d_0, 1]$, where $0 < d_0 < 1$ models the distance threshold. With this assumption direct manipulations show

$$\rho(w_i) = \sqrt{\frac{(1+\kappa)^2 \left(1 - d_0^{1+2\kappa}\right)(1-d_0)}{(1+2\kappa)\left(1-d_0^{1+\alpha}\right)^2} - 1}. \tag{4.3}$$

With (4.3) it can be checked that $\rho(w_i)$ increases either when κ is enlarged or d_0 is decreased. With fixed $d_0 = 0.3$ Fig. 4 shows the PES curve for $1 \le \kappa \le 8$ (yielding $0.311 \le \rho(w_i) \le 1.528$); with fixed

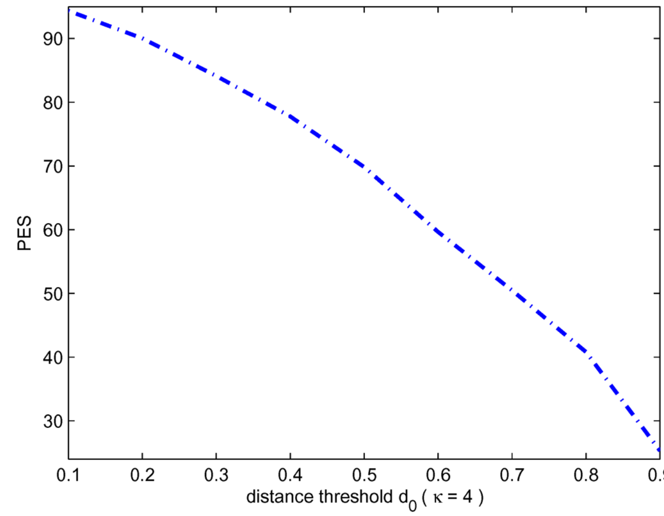

Fig. 5. Percentage of energy saving w.r.t. different d_0.

$\kappa = 4$ Fig. 5 depicts the PES for $0.1 \le d_0 \le 0.9$ (corresponding to $1.225 \ge \rho(w_i) \ge 0.121$). As we can see from both figures, PES improves when $\rho(w_i)$ is large. This is intuitively reasonable since, as w_i's get more heterogeneous, the proposed scheme (3.22) via channel gain (or node location) selection can avoid severe energy consumption along the transmission links with poor channel quality.

Based on the above numerical experiments, we conclude that large energy savings can be achieved as the sensing environment becomes more heterogeneous. A similar phenomenon has also been observed in [3] and [8], in which parameter estimation is done based on the knowledge of instantaneous sensor noise variances. Since the proposed solution (3.22) (via exploiting the statistical noise variance description) accounts for the long-term characteristics of the schemes in [3] and [8], this consistency is thus expected.

V. Conclusion

This paper provides a solution to the minimal-energy decentralized estimation problem by exploiting a statistical noise variance model. Based on a closed-form expression of the reciprocal of MSE averaged over the noise variance distribution and by leveraging an associated tractable lower bound, energy minimization is reformulated as a convex optimization problem. The analytic nature of the resultant solution reveals the underlying energy saving policy: simply allocate energies to sensors with large channel gains, and shut off those suffering from poor link quality. Numerical simulation shows that the proposed optimal solution is capable of reducing more than 90% energy consumption when compared with the uniform-allocation scheme; the energy saving efficiency is significant particularly when as the sensing environment gets more heterogeneous. In the future work we will generalize the results to the vector parameter case.

Appendix A
Proof of Lemma 3.1

By change of variable $u = \alpha z_i + \beta_i$, and hence $z_i = (u - \beta_i)/\alpha$, we have

$$\int_0^\infty \frac{e^{-z_i/2}}{(\alpha z_i + \beta_i)\sqrt{z_i}} dz_i = \int_{\beta_i}^\infty \frac{e^{(-u+\beta_i)/2\alpha}}{u\sqrt{(u-\beta_i)/\alpha}} \cdot \frac{1}{\alpha} du$$

$$= \frac{e^{\beta_i/2\alpha}}{\sqrt{\alpha}} \int_{\beta_i}^\infty \frac{e^{-u/2\alpha}}{u\sqrt{u-\beta_i}} du. \tag{A.1}$$

It thus suffices to check

$$\int_{\beta_i}^{\infty} \frac{e^{-u/2\alpha}}{u\sqrt{u-\beta_i}} du = \frac{2\pi}{\sqrt{\beta_i}} Q(\sqrt{\beta_i/\alpha}). \quad \text{(A.2)}$$

Let us define $u = \beta_i \csc^2\theta$, and hence $du = -2\beta_i \csc^2\theta \cot\theta d\theta$. We then have

$$\int_{\beta_i}^{\infty} \frac{e^{-u/2\alpha}}{u\sqrt{u-\beta_i}} du = \int_{\pi/2}^{0} \frac{e^{-\beta_i \csc^2\theta/2\alpha}}{\beta_i \csc^2\theta \cdot \sqrt{\beta_i}\cot\theta} (-2\beta_i \csc^2\theta \cot\theta) d\theta$$
$$= \frac{2}{\sqrt{\beta_i}} \int_0^{\pi/2} e^{-\beta_i/2\alpha \sin^2\theta} d\theta. \quad \text{(A.3)}$$

We note that the $Q(\cdot)$ function admits the following alternative expression [5, p. 71]

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} e^{-x^2/2\sin^2\theta} d\theta. \quad \text{(A.4)}$$

The assertion (A.2) follows immediately from (A.3) and (A.4). □

Appendix B
Proof of Lemma 3.2

We first observe that, since $\beta_i = \delta + R^2 4^{-b_i}/12$ and $0 \le b_i < \infty$, we have $e^{\beta_i/2\alpha} \ge e^{\delta/2\alpha}$ and $\sqrt{\beta_i} \le \sqrt{\delta + R^2/12}$, leading to

$$\sqrt{\frac{2\pi}{\alpha}} \cdot \frac{e^{\beta_i/2\alpha} \cdot Q(\sqrt{\beta_i/\alpha})}{\sqrt{\beta_i}} \ge \underbrace{\sqrt{\frac{2\pi}{\alpha}} \cdot \frac{e^{\delta/2\alpha}}{\sqrt{\delta + R^2/12}}}_{=c} \cdot Q(\sqrt{\beta_i/\alpha}),\ 1 \le i \le N. \quad \text{(B.1)}$$

Also, as $\sqrt{\beta_i/\alpha} = \sqrt{(\delta + R^2 4^{-b_i}/12)/\alpha} \le \sqrt{\delta/\alpha} + \sqrt{R^2 4^{-b_i}/12\alpha} = \sqrt{\delta/\alpha} + R2^{-b_i}/\sqrt{12\alpha}$ and $Q(\cdot)$ is one-to-one and monotone decreasing, we have

$$Q(\sqrt{\beta_i/\alpha}) \ge Q(\sqrt{\delta/\alpha} + R2^{-b_i}/\sqrt{12\alpha}). \quad \text{(B.2)}$$

Inequalities (B.1) and (B.2) then imply

$$\sqrt{\frac{2\pi}{\alpha}} \sum_{i=1}^{N} \frac{e^{\beta_i/2\alpha} \cdot Q(\sqrt{\beta_i/\alpha})}{\sqrt{\beta_i}} \ge c \sum_{i=1}^{N} Q(\sqrt{\delta/\alpha} + R2^{-b_i}/\sqrt{12\alpha}). \quad \text{(B.3)}$$

Further, since $Q(t)$ is convex for $t > 0$, it follows

$$\frac{1}{N} \sum_{i=1}^{N} Q(\sqrt{\delta/\alpha} + R2^{-b_i}/\sqrt{12\alpha}) \ge Q\left(\frac{1}{N} \sum_{i=1}^{N} (\sqrt{\delta/\alpha} + R2^{-b_i}/\sqrt{12\alpha})\right) \quad \text{(B.4)}$$

and hence

$$c \sum_{i=1}^{N} Q(\sqrt{\delta/\alpha} + R2^{-b_i}/\sqrt{12\alpha}) \ge cNQ\left(\frac{1}{N} \sum_{i=1}^{N} (\sqrt{\delta/\alpha} + R2^{-b_i}/\sqrt{12\alpha})\right) \quad \text{(B.5)}$$

and the result thus follows. □

Appendix C
Derivation of Optimal Solution (3.22)

By substituting $\overline{\lambda}$ into (3.20) into (3.19), it is straightforward to see that the constraint $b_i \ge 0$ is equivalent to

$$R\left(\sum_{i=1}^{N} w_i - \mu_i\right)\left[\sqrt{12a}N \times \left(Q^{-1}(c^{-1}N^{-1}\gamma^{-1}) - \sqrt{\delta/\alpha}\right)(w_i - \mu_i)\right]^{-1} \ge 1 \quad \text{(C.1)}$$

hence $\mu_i \ge 0$ must be properly chosen to simultaneously meet (C.1) and the equality constraint (3.18), which based on (3.19) can be equivalently rewritten as

$$\overline{\lambda}^{-1} \sum_{i=1}^{N} (w_i - \mu_i) = Q^{-1}(c^{-1}N^{-1}\gamma^{-1}) - \sqrt{\delta/\alpha}. \quad \text{(C.2)}$$

For this we first observe from (3.19) that the constraint $b_i \ge 0$ also implies $\overline{\lambda}^{-1}(w_i - \mu_i) \le R/(\sqrt{12\alpha}N)$, which along with (C.2) requires $Q^{-1}(c^{-1}N^{-1}\gamma^{-1}) - \sqrt{\delta/\alpha} \le R/\sqrt{12\alpha}$, or

$$R\left(\sqrt{12a}\left(Q^{-1}(c^{-1}N^{-1}\gamma^{-1}) - \sqrt{\delta/\alpha}\right)\right)^{-1} \ge 1. \quad \text{(C.3)}$$

Note that constraint (C.3) is equivalent to $\gamma \le (Ne^{\delta/2\alpha}Q(\sqrt{\delta/\alpha} + (R/\sqrt{12\alpha})\sqrt{2\pi/\alpha(\delta + R^2/12)})^{-1}$; sine this upper bound is feasible (in that it is larger than the minimal threshold (3.26)), we may without loss of generality choose γ to be within this range so that (C.3) holds. If the integer K_1 exists, then based on (3.21) and (C.3) it is straightforward to show that $\mu_i = w_i$ for $1 \le i \le N - K_1$ and $\mu_i = 0$ for $N - K_1 + 1 \le i \le N$ fulfill (C.1): the solutions $\overline{\lambda}^{\text{opt}}$ in (3.23) and b_i^{opt} in (3.22) then follows, respectively, from (C.2) and (3.19). The existence of K_1 is indeed guaranteed by the construction of $f(K)$ in (3.21): $f(1) = w_N/(Nw_1) \le 1/N$, $f(K)$ is monotone increasing with K, and $f(N) = (\sum_{i=1}^{N} w_i)/(Nw_N) \ge N/N = 1$. □

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