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Spectral analysis on pounding probability of adjacent buildings

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Abstract

Presented herein is a spectral approach to evaluate the seismic pounding probability of two adjacent buildings simulated by multi-degree-of-freedom systems and separated by a minimum code-specified separation during a period of time. The analytical approach is based on random vibration theory and total probability theory. Numerical simulations of 36 cases are presented in this study. Results of this investigation reveal that the period ratio of the adjacent buildings plays a major role that affects the pounding risk of adjacent buildings. Also noted is that the effect of period ratio on pounding risk has not yet been taken into account in the seismic pounding related provisions of the Uniform Building Code.

Keywords: Pounding probability; Separation distance; Risk analysis

1. Introduction

Out-of-phase vibrations may be induced where adjacent buildings are subjected to earthquake loading and pounding may occur if the separation distance is inadequate.

For metropolitan cities located in regions of active seismicity, the pounding of adjacent buildings may pose a potentially serious problem since a maximum land use is often required due to high population density. Pounding damages were observed in the past earthquakes such as Thessaloniki, Greece (1978), Central Greece (1981), Guerrero-Michoacan, Mexico (1985), and Loma Prieta, Santa Cruz (1989), etc. [1–4].

In recent years, there were studies regarding the pounding behavior of buildings under the action of earthquakes [5–16], most of the investigations emphasized the deterministic aspect of the problem. Based on a literature survey conducted by the authors, there are no published results found on the seismic pounding probability of adjacent buildings.

Building codes providing a set of minimum technical rules and a legal basis for the practice of structural engineering are intended to ensure safety and play a transfer role of technology from research to practice. Despite significant advances in structural engineering design in recent years, uncertainty induced by structural loads and material strengths, however, gives rise to risk. The framers of building codes must address the questions: “How many chances of structural failure will occur in the future?” and “How safe is safe enough?”. Therefore, the most critical objective of design codes is to control risk to socially acceptable levels.

The need to investigate the level of seismic pounding risk of buildings is quite apparent in future code calibrations. In order to provide some clarity and insight into the pounding risk of adjacent buildings, this study develops a spectral method to estimate the seismic pounding risk of adjacent buildings separated by a minimum code-specified separation during a certain period of time.

2. Literature review

In recent years, valuable insights into the structural pounding and formulas for evaluating separations based on linear or equivalent linear procedures have been proposed.

Miller and Fatemi [5] investigated the pounding problem of adjacent buildings subjected to harmonic motions by vibroimpact concept. Anagnostopoulos [6] analyzed

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the effect of pounding for buildings under strong ground motions by simplified single-degree-of-freedom (SDOF) model. Anagnostopoulos and Spiliopoulos [7] investigated the response of mutual pounding between adjacent buildings in city blocks to several strong earthquakes. In the study, the buildings were idealized as lumped-mass, shear beam type, multi-degree-of-freedom (MDOF) systems. Westermo [8] applied links to adjacent buildings to reduce the pounding effect. Maison and Kasai [9] modeled the buildings as multiple-degree-of-freedom systems and analyzed the response of structural pounding with different types of idealizations. Papadrakakis et al. [10] studied the pounding response of two or more adjacent buildings based on the Lagrange multiplier approach by which the geometric compatibility conditions due to contact are enforced. A three-dimensional model developed for the simulation of the pounding response of adjacent buildings is presented by Papadrakakis et al. [11].

In evaluation of building separation, Jeng et al. [12] estimated the minimum separation distance required to avoid pounding of adjacent buildings by the spectral difference (SPD) method. Kasai et al. [13] extended Jeng's results and proposed a simplified rule to predict the inelastic vibration phase of buildings based on the numerical results of dynamic time history analyses. Penzien [14] proposed a formula for evaluating separations of two buildings, based on the procedures of equivalent linearization and the assumptions that the required minimum separation $S_{\text{req'd}}$ is controlled by the first-mode type of responses and the mode shape of responses is linear. The first writer [15] proposed a theoretical solution based on random vibration theory to predict the statistics of separations of adjacent buildings, assuming linear elastic responses. Hao and Zhang [16] investigated earthquake ground motion spatial variation effects on relative linear elastic response of adjacent building structures.

3. Code requirement regarding building separation

Based on the 1988 seismic provisions by the Structural Engineers Association of California (SEAOC) [17], the 1988 Uniform Building Code (UBC) [18] requires that all structures shall be separated from adjoining structures and provides detailed requirements for building separations. Calibrations of the related seismic pounding provisions of the building code have been made over the past decade. However, the literature survey conducted by the authors revealed that no published results on the seismic pounding risk of adjacent buildings were presented, although the concept and the philosophy of reliability-based design have been accepted for many years.

The need to investigate the level of seismic pounding risk of buildings is quite apparent in future code calibrations. In order to provide some clarity and insight into the risks of buildings, this paper presents a spectral method to estimate the pounding probability of buildings.

The 1994 Uniform Building Code [19] requires that all structures shall be separated from adjoining structures, and separations shall allow for $3(R_w/8)$ times the displacement due to seismic forces, where R_w is the system performance factor. In addition, it is also required that the story drift shall not exceed $0.04/R_w$ or 0.005 times the story height for structures having a fundamental period of less than 0.7 s and the story drift shall not exceed $0.03/R_w$ or 0.004 times the story height for structures having a fundamental period of 0.7 s or greater.

If the story drift ratios in each of the adjacent buildings are within the maximum value mentioned above, the minimum code-specified separation of two adjacent buildings having the same structural system can be expressed by

$$S_{\text{code}}=(3R_w/8)\left\{\sum_{i=1}^{na}(\theta_{ai}\cdot h_{ai})+\sum_{i=1}^{nb}(\theta_{bi}\cdot h_{bi})\right\}, \tag{1}$$

in which θ_{ai} is the ith story drift ratio of building A, h_{ai} is the ith story height of building A, and na is the story number of building A; θ_{bi} is the ith story drift ratio of building B, h_{bi} is the ith story height of building B, and nb is the story number of building B.

4. Proposed approach for seismic pounding risk analysis

4.1. Basic assumptions

For the analytical procedures presented herein, it is assumed that dynamic responses of buildings can well be simulated by dynamic responses of lumped-mass structure systems. Excitations can be considered as stationary Gaussian random processes with zero mean. It is noted that although earthquake motion is not stationary, during the time of strong motion a typical earthquake is approximately stationary, and good estimates of response can be made using random vibration theory [20]. Thus the building structures can be simplified by multi-degree-of-freedom models and the responses of the structures will be stationary Gaussian random processes with zero mean. For buildings having different heights, the pounding location of adjacent buildings is assumed to occur at the top of the shorter building.

4.2. Relative displacement at location of potential pounding

As shown in Fig. 1, if $y_{a,1}(t)$ and $y_{b,nb-na+1}(t)$ are the displacement time histories and $Z(t)$ is the relative dis-

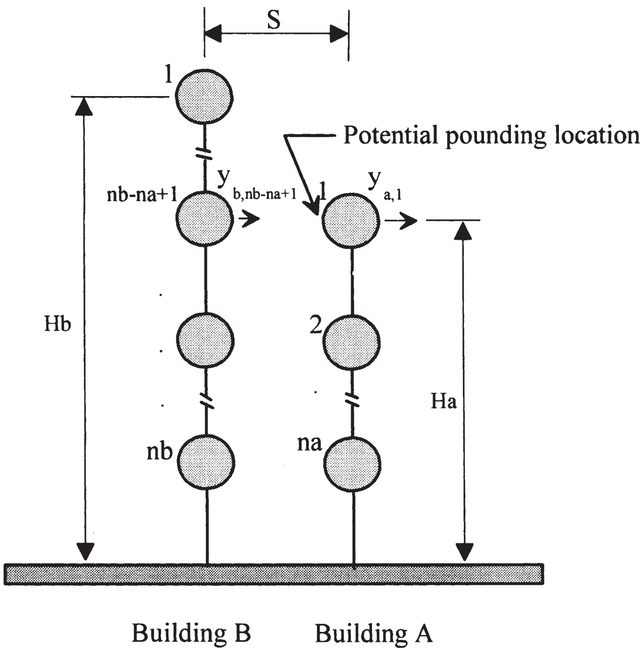

Fig. 1. Analytical model.

placement time history of two adjacent buildings A and B at the potential pounding position, then $Z(t)$ can be expressed as

$$Z(t)=y_{\mathrm{b,nb-na+1}}(t)-y_{\mathrm{a,1}}(t), \tag{2}$$

where na and nb are the number of degree of freedom of system A and system B, respectively. It is assumed that na is less than nb.

The minimum separation distance required to avoid pounding may be defined as

$$S_{\mathrm{req'd}}=\sup(Z(t)), \tag{3}$$

where "sup" implies the maximum value of the entire range of the relative displacement time history. The structural pounding may occur once the separation of adjacent buildings is less than $S_{\mathrm{req'd}}$. Thus, $Z(t)$ may be evaluated once the displacement time histories $y_{\mathrm{a,1}}(t)$ and $y_{\mathrm{b,nb-na+1}}(t)$ are determined.

For the linear structure systems, if the excitations are stationary Gaussian random processes with zero mean, the response processes of the structures will be stationary Gaussian random processes with zero mean, thus the relative displacement processes of adjacent buildings at the potential pounding location are stationary Gaussian random processes with zero mean and can be given by

$$Z(t)=\langle\varphi_{\mathrm{b}}(nb-na+1,1)\varphi_{\mathrm{b}}(nb-na+1,2)\ldots\varphi_{\mathrm{b}}(nb-na+1,nb)\rangle\begin{Bmatrix}Y_{\mathrm{b1}}(t)\\Y_{\mathrm{b2}}(t)\\\cdot\\\cdot\\Y_{\mathrm{bnb}}(t)\end{Bmatrix} \tag{4}$$

$$-\langle\varphi_{\mathrm{a}}(1,1)\varphi_{\mathrm{a}}(1,2)\ldots\varphi_{\mathrm{a}}(1,na)\rangle\begin{Bmatrix}Y_{\mathrm{a1}}(t)\\Y_{\mathrm{a2}}(t)\\\cdot\\\cdot\\Y_{\mathrm{ana}}(t)\end{Bmatrix},$$

where $\varphi_{\mathrm{a}}(1,i)$ and $Y_{\mathrm{a}i}(t)$ are the first component of the ith mode shape and the ith modal coordinate of building A, respectively. Similarly, $\varphi_{\mathrm{b}}(nb-na+1,i)$ and $Y_{\mathrm{b}i}(t)$ are the $nb-na+1$th component of the ith mode shape and the ith modal coordinate of building B, respectively.

The autocorrelation function $R_{ZZ}(\tau)$ for the relative displacement process of building A and building B at the potential pounding location is by definition,

$$R_{ZZ}(\tau)=E(Z(t)Z(t+\tau)) \tag{5}$$

Substituting Eq. (4) into Eq. (5), yields

$$R_{ZZ}(\tau)=R_{y_{\mathrm{b,nb-na+1}}y_{\mathrm{b,nb-na+1}}}(\tau)+R_{y_{\mathrm{a,1}}y_{\mathrm{a,1}}}(\tau)-R_{y_{\mathrm{a,1}}y_{\mathrm{b,nb-na+1}}}(\tau)-R_{y_{\mathrm{b,nb-na+1}}y_{\mathrm{a,1}}}(\tau) \tag{6}$$

where

$$R_{y_{\mathrm{b,nb-na+1}}y_{\mathrm{b,nb-na+1}}}(\tau)=\sum_{j=1}^{nb}\sum_{k=1}^{nb}\varphi_{\mathrm{b}}(nb-na+1,j)\varphi_{\mathrm{b}}(nb-na+1,k)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}E[G_{\mathrm{bj}}(t-\theta_1)G_{\mathrm{bk}}(t+\tau-\theta_2)]h_{\mathrm{bj}}(\theta_1)h_{\mathrm{bk}}(\theta_2)d\theta_1 d\theta_2, \tag{7}$$

$$R_{y_{\mathrm{a,1}}y_{\mathrm{a,1}}}(\tau)=\sum_{j=1}^{na}\sum_{k=1}^{na}\varphi_{\mathrm{a}}(1,j)\varphi_{\mathrm{a}}(1,k)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}E[G_{\mathrm{aj}}(t-\theta_1)G_{\mathrm{ak}}(t+\tau-\theta_2)]h_{\mathrm{aj}}(\theta_1)h_{\mathrm{ak}}(\theta_2)d\theta_1 d\theta_2, \tag{8}$$

$$R_{y_{\mathrm{b,nb-na+1}}y_{\mathrm{a,1}}}(\tau)=\sum_{j=1}^{nb}\sum_{k=1}^{na}\varphi_{\mathrm{b}}(nb-na+1,j)\varphi_{\mathrm{a}}(1,k)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}E[G_{\mathrm{bj}}(t-\theta_1)G_{\mathrm{ak}}(t+\tau-\theta_2)]h_{\mathrm{bj}}(\theta_1)h_{\mathrm{ak}}(\theta_2)d\theta_1 d\theta_2, \tag{9}$$

$$R_{y_{\mathrm{a,1}}y_{\mathrm{b,nb-na+1}}}(\tau)=\sum_{j=1}^{na}\sum_{k=1}^{nb}\varphi_{\mathrm{a}}(1,j)\varphi_{\mathrm{b}}(nb-na+1,k)\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}E[G_{\mathrm{aj}}(t-\theta_1)G_{\mathrm{bk}}(t+\tau-\theta_2)]h_{\mathrm{aj}}(\theta_1)h_{\mathrm{bk}}(\theta_2)d\theta_1 d\theta_2, \tag{10}$$

$$G_{bj}(t)=\frac{1}{M_{bj}}\langle\varphi_b(1,j)\varphi_b(2,j)\ldots\varphi_b(nb,j)\rangle\begin{Bmatrix}f_{b1}(t)\\f_{b2}(t)\\\cdot\\\cdot\\f_{bnb}(t)\end{Bmatrix} \quad (11)$$

$$G_{aj}(t)=\frac{1}{M_{aj}}\langle\varphi_a(1,j)\varphi_a(2,j)\ldots\varphi_a(na,j)\rangle\begin{Bmatrix}f_{a1,}(t)\\f_{a2}(t)\\\cdot\\\cdot\\f_{ana}(t)\end{Bmatrix}, \quad (12)$$

$$h_{bj}(\theta_1)=\frac{e^{-\xi_{bj}\omega_{bj}\theta_1}\sin(\omega_{bj}\sqrt{1-\xi_{bj}^2}\theta_1)}{\omega_{bj}\sqrt{1-\xi_{bj}^2}}, \quad (13)$$

$$h_{aj}(\theta_1)=\frac{e^{-\xi_{aj}\omega_{aj}\theta_1}\sin(\omega_{aj}\sqrt{1-\xi_{aj}^2}\theta_1)}{\omega_{aj}\sqrt{1-\xi_{aj}^2}}; \quad (14)$$

$f_{ai}(t)$, ω_{aj}, ξ_{aj}, and M_{aj} are the ith component of external force vector, the jth modal frequency, the jth modal damping, and the jth generalized mass of building A, respectively, and $f_{bi}(t)$, ω_{bj}, ξ_{bj} and M_{bj} are the ith component of external force vector, the jth modal frequency, the jth modal damping, and the jth generalized mass of building B, respectively.

In the frequency domain analysis, the spectral density function for the relative displacement process $Z(t)$ is by definition

$$S_{ZZ}(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}R_{ZZ}(\tau)e^{-i\omega\tau}d\tau. \quad (15)$$

Substituting R_{ZZ} of Eq. (6) into Eq. (15) gives

$$S_{ZZ}(\omega)=Re[S_{y_{b,nb-na+1}y_{b,nb-na+1}}(\omega)+S_{y_{a,1}y_{a,1}}(\omega)] -2Re]S_{y_{a,1}y_{b,nb-na+1}}(\omega)] \quad (16)$$

where

$$S_{y_{b,nb-na+1}y_{b,nb-na+1}}(\omega)=\sum_{j=1}^{nb}\sum_{k=1}^{nb}\varphi_b(nb-na+1,j)\varphi_b(nb-na+1,k)\frac{1}{M_{bj}M_{bk}}\left(\sum_{l=1}^{nb}\sum_{m=1}^{nb}\varphi_b(l,j)\varphi_b(m,k)S_{f_{bl}f_{bm}}(\omega)\right)H_{bj}(\omega)H_{bk}(-\omega), \quad (17)$$

$$S_{y_{a,1}y_{a,1}}(\omega)=\sum_{j=1}^{na}\sum_{k=1}^{na}\varphi_a(1,j)\varphi_a(1,k)\frac{1}{M_{aj}M_{ak}}\left(\sum_{l=1}^{na}\sum_{m=1}^{na}\varphi_a(l,j)\varphi_a(m,k)S_{f_{al}f_{am}}(\omega)\right) \quad (18)$$

$$H_{aj}(\omega)H_{ak}(-\omega),$$

$$Re[S_{y_{a,1}y_{b,nb-na+1}}(\omega)]=\sum_{j=1}^{nb}\sum_{k=1}^{nb}\varphi_a(1,j)\varphi_b(nb-na+1,k)\frac{1}{M_{aj}M_{bk}}\left(\sum_{l=1}^{na}\sum_{m=1}^{nb}\varphi_a(l,j)\varphi_b(m,k)S_{f_{al}f_{am}}(\omega)\right) \quad (19)$$

$$Re[H_{aj}(\omega)H_{bk}(-\omega)],$$

$$Re[H_{aj}(\omega)H_{bk}(-\omega)]=\frac{(\omega_{aj}^2-\omega^2)(\omega_{bk}^2-\omega^2)+(2\xi_{aj}\omega_{aj}\omega)(2\xi_{bk}\omega_{bk}\omega)}{((\omega_{aj}^2-\omega^2)^2+(2\xi_{aj}\omega_{aj}\omega)^2)((\omega_{bk}^2-\omega^2)^2+(2\xi_{bk}\omega_{bk}\omega)^2)}, \quad (20)$$

$$H_{bj}(\omega)=\frac{1}{\omega_{bj}^2+i2\xi_{bj}\omega_{bj}\omega-\omega^2}, \quad (21)$$

$$H_{aj}(\omega)=\frac{1}{\omega_{aj}^2+i2\xi_{aj}\omega_{aj}\omega-\omega^2}, \quad (22)$$

and $S_{f_{al}f_{am}}(\omega)$ and $S_{f_{bl}f_{bm}}(\omega)$ are the cross-spectral density function of the lth and the mth component of external force vector of building A and building B, respectively. Assuming that the lumped mass at the ith degree of freedom is m_i and $\ddot{v}_g(t)$ is the input ground motion, the external force vector can then be expressed by

$$\begin{Bmatrix}f_1(t)\\f_2(t)\\\cdot\\\cdot\\f_n(t)\end{Bmatrix}_{n\times1}=-\begin{bmatrix}m_1&&&\\&m_2&&\\&&\cdot&\\&&&\cdot\\&&&&m_n\end{bmatrix}_{n\times n}\begin{Bmatrix}1\\1\\\cdot\\\cdot\\1\end{Bmatrix}_{n\times1}\ddot{v}_g(t). \quad (23)$$

Hence the cross-spectral density function of the lth and the mth component of external force vector of building A and building B can be, respectively, given by

$$S_{f_{al}f_{am}}(\omega)=m_{al}m_{am}S_{aa}(\omega), \quad (24)$$

$$S_{f_{bl}f_{bm}}(\omega)=m_{bl}m_{bm}S_{aa}(\omega), \quad (25)$$

and

$$S_{f_{al}f_{bm}}(\omega)=m_{al}m_{bm}S_{aa}(\omega), \quad (26)$$

where $S_{aa}(\omega)$ is the two-sided power spectral density function of $\ddot{v}_g(t)$.

Substituting Eqs. (24)–(26) into Eq. (16), the spectral density function of the relative displacement process $Z(t)$ is then given by

$$S_{ZZ}(\omega)=S_{aa}(\omega)\left\{\sum_{j=1}^{nb}\sum_{k=1}^{nb}\varphi_b(nb-na+1,j)\varphi_b(nb-na+1,k)\frac{1}{M_{bj}M_{bk}}\left(\sum_{l=1}^{nb}\sum_{m=1}^{nb}m_{bl}m_{bm}\varphi_b(l,j)\varphi_b(m,k)\right)Re[H_{bj}(\omega)H_{bk}(-\omega)]+\sum_{j=1}^{na}\sum_{k=1}^{na}\phi_a(1,j)\varphi_a(1,k)\frac{1}{M_{aj}M_{ak}}\left(\sum_{l=1}^{na}\sum_{m=1}^{na}m_{al}m_{am}\right.\right. \quad (27)$$

$$\varphi_{\mathrm{a}}(l,j)\varphi_{\mathrm{a}}(m,k)\Bigg)Re[H_{\mathrm{aj}}(\omega)H_{\mathrm{ak}}(-\omega)]-2\sum_{j=1}^{na}\sum_{k=1}^{nb}\varphi_{\mathrm{a}}(1,j)$$

$$\varphi_{\mathrm{b}}(nb-na+1,k)\frac{1}{M_{\mathrm{aj}}M_{\mathrm{bk}}}\left(\sum_{l=1}^{na}\sum_{m=1}^{nb}m_{\mathrm{al}}m_{\mathrm{bm}}\varphi_{\mathrm{a}}(l,j)\varphi_{\mathrm{b}}(m,k)\right)$$

$$Re[H_{\mathrm{aj}}(\omega)H_{\mathrm{bk}}(-\omega)]\Bigg\}.$$

The ensemble mean squares of the relative displacement process $Z(t)$ and the relative velocity process $\dot{Z}(t)$ are related to the spectral density function $S_{ZZ}(\omega)$ by the equation

$$\sigma_{\mathrm{Z}}^2=\int_{-\infty}^{\infty}S_{\mathrm{ZZ}}(\omega)d\omega \tag{28}$$

and

$$\sigma_{\dot{\mathrm{Z}}}^2=\int_{-\infty}^{\infty}\omega^2S_{\mathrm{ZZ}}(\omega)d\omega, \tag{29}$$

respectively. Substituting Eq. (27) into Eqs. (28) and (29) gives

$$\sigma_{\mathrm{Z}}^2=\sum_{j=1}^{nb}\sum_{k=1}^{nb}\varphi_{\mathrm{b}}(nb-na+1,j)\varphi_{\mathrm{b}}(nb-na+1,k)\frac{1}{M_{\mathrm{bj}}M_{\mathrm{bk}}}$$

$$\left(\int_0^{\infty}S_{\mathrm{g}}(\omega)Re[H_{\mathrm{bj}}(\omega)H_{\mathrm{bk}}(-\omega)]d\omega\right)\sum_{l=1}^{nb}\sum_{m=1}^{nb}m_{\mathrm{bl}}m_{\mathrm{bm}}$$

$$\varphi_{\mathrm{b}}(l,j)\varphi_{\mathrm{b}}(m,k)+\sum_{j=1}^{na}\sum_{k=1}^{na}\varphi_{\mathrm{a}}(1,j)\varphi_{\mathrm{a}}(1,k)\frac{1}{M_{\mathrm{aj}}M_{\mathrm{ak}}}$$

$$\left(\int_0^{\infty}S_{\mathrm{g}}(\omega)Re[H_{\mathrm{aj}}(\omega)H_{\mathrm{ak}}(-\omega)]d\omega\right)\sum_{l=1}^{na}\sum_{m=1}^{na}m_{\mathrm{al}}m_{\mathrm{am}} \tag{30}$$

$$\varphi_{\mathrm{a}}(l,j)\varphi_{\mathrm{a}}(m,k)-2\sum_{j=1}^{na}\sum_{k=1}^{nb}\varphi_{\mathrm{a}}(1,j)\varphi_{\mathrm{b}}(nb-na+1,k)\frac{1}{M_{\mathrm{aj}}M_{\mathrm{bk}}}$$

$$\left(\int_0^{\infty}S_{\mathrm{g}}(\omega)Re[H_{\mathrm{aj}}(\omega)H_{\mathrm{bk}}(-\omega)d\omega\right)\sum_{l=1}^{na}\sum_{m=1}^{nb}m_{\mathrm{al}}m_{\mathrm{bm}}$$

$$\varphi_{\mathrm{a}}(l,j)\varphi_{\mathrm{b}}(m,k)$$

and

$$\sigma_{\dot{\mathrm{Z}}}^2=\sum_{j=1}^{nb}\sum_{k=1}^{nb}\varphi_{\mathrm{b}}(nb-na+1,j)\varphi_{\mathrm{b}}(nb-na+1,k)\frac{1}{M_{\mathrm{bj}}M_{\mathrm{bk}}}$$

$$\left(\int_0^{\infty}\omega^2S_{\mathrm{g}}(\omega)Re[H_{\mathrm{bj}}(\omega)H_{\mathrm{bk}}(-\omega)]d\omega\right)\sum_{l=1}^{nb}\sum_{m=1}^{nb}m_{\mathrm{bl}}m_{\mathrm{bm}}$$

$$\varphi_{\mathrm{b}}(l,j)\varphi_{\mathrm{b}}(m,k)+\sum_{j=1}^{na}\sum_{k=1}^{na}\varphi_{\mathrm{a}}(1,j)\varphi_{a}(1,k)\frac{1}{M_{\mathrm{aj}}M_{\mathrm{ak}}}$$

$$\left(\int_0^{\infty}\omega^2S_{\mathrm{g}}(\omega)Re[H_{\mathrm{aj}}(\omega)H_{\mathrm{ak}}(-\omega)]d\omega\right)\sum_{l=1}^{na}\sum_{m=1}^{na}m_{\mathrm{al}} \tag{31}$$

$$m_{\mathrm{am}}\varphi_{\mathrm{a}}(l,j)\varphi_{\mathrm{a}}(m,k)-2\sum_{j=1}^{na}\sum_{k=1}^{nb}\varphi_{\mathrm{a}}(1,j)\varphi_{\mathrm{b}}(nb-na+1,k)\frac{1}{M_{\mathrm{aj}}M_{\mathrm{bk}}}$$

$$\left(\int_0^{\infty}\omega^2S_{\mathrm{g}}(\omega)Re[H_{\mathrm{aj}}(\omega)H_{\mathrm{bk}}(-\omega)]d\omega\right)$$

$$\sum_{l=1}^{na}\sum_{m=1}^{nb}m_{\mathrm{al}}m_{\mathrm{bm}}\varphi_{\mathrm{a}}(l,j)\varphi_{\mathrm{b}}(m,k),$$

where $S_{\mathrm{g}}(\omega)$ is the one-sided earthquake power spectrum.

The simplified versions of Eqs. (30) and (31), based on the assumptions that all of the modal frequencies of buildings are well separated and all of the modal dampings of buildings are small, have been developed by the first writer [15]. It is noted that a real structure is much more complex than a simplified structure in the form of cantilever beam model. The use of the calculated results proposed in this study is subjected to errors resulting from the simplification of the analytical model. However, in order to establish a mathematical model, this study adopts the commonly used lumped-mass-cantilever beam model in structural dynamic analysis. The primary objective of the model is to illustrate the physical behavior and pounding probability of adjacent buildings subjected to earthquakes.

4.3. Building separation distance to avoid seismic pounding

For a zero-mean stationary Gaussian process $X(t)$, Davenport [21] has shown, relying in part on earlier work by Cartwright and Lonquet-Higgins [22], that the mean and the standard deviation of the extreme-values are given by the approximate relation

$$\bar{X}_{\mathrm{e}}\cong\left((2\ln(\nu T))^{0.5}+\frac{\gamma}{(2\ln(\nu T))^{0.5}}\right)\sigma_{\mathrm{X}} \tag{32}$$

and

$$\sigma_{X_{\mathrm{e}}}\cong\left(\frac{\pi}{\sqrt{6}(2\ln(\nu T))^{0.5}}\right)\sigma_{\mathrm{X}}, \tag{33}$$

respectively, where T is a time duration, γ is Euler's constant, equal to 0.5772, and

$$\nu = \frac{\sigma_{\dot{X}}}{\pi \sigma_X}. \tag{34}$$

Using Eqs. (32)–(34), the mean and the standard deviation of the separation distance of adjacent buildings to avoid pounding can, respectively, be expressed by the approximate relation

$$\bar{S}_{\text{req'd}} \cong \left((2\ln(\nu T))^{0.5} + \frac{0.5772}{2\ln(\nu T))^{0.5}} \right) \sigma_z \tag{35}$$

and

$$\sigma_{S_{\text{req'd}}} \cong \left(\frac{\pi}{\sqrt{6}(2\ln(\nu T))^{0.5}} \right) \sigma_z \tag{36}$$

in which

$$\nu = \frac{\sigma_{\dot{z}}}{\pi \sigma_z}. \tag{37}$$

4.4. Conditional pounding probability of adjacent buildings

For a stationary Gaussian random process, Z, the expected largest value (called the extreme value), $S_{\text{req'd}}$, is a random variable and the extreme value probability distribution of Z whose cumulative distribution function is of the exponential type asymptotically converge to the Type I extreme value probability distribution [23]. The type I asymptotic extreme value distribution can be given by the form

$$G(S_{\text{req'd}}) = \exp\{-\exp\{-\alpha_n(S_{\text{req'd}} - u_n)\}\}, \tag{38}$$

where

$$\alpha_n = \frac{\pi}{\sqrt{6}\sigma_{S_{\text{req'd}}}}, \tag{39}$$

and

$$u_n = \bar{S}_{\text{req'd}} - 0.577/\alpha_n, \tag{40}$$

in which $\bar{S}_{\text{req'd}}$ and $\sigma_{S_{\text{req'd}}}$ are the mean and the standard deviation of random variable $S_{\text{req'd}}$, respectively. Hence, the cumulative distribution function of $S_{\text{req'd}}$ given in Eq. (38) can be evaluated with the aid of Eqs. (39) and (40). The pounding probability of two adjacent buildings separated by a minimum code-specified separation and subjected to earthquakes with a "specified" peak ground acceleration (called the "conditional" pounding probability of adjacent buildings) can be evaluated, if the $\bar{S}_{\text{req'd}}$, $\sigma_{S_{\text{req'd}}}$, and S_{code} are determined. In other words, if the $\bar{S}_{\text{req'd}}$, $\sigma_{S_{\text{req'd}}}$, and S_{code} are known, the conditional pounding probability of adjacent buildings, $P_{p/a}$, can be evaluated by Eq. (41).

$$P_{p/a} = 1 - \exp\{-\exp\{-\alpha_n(S_{\text{code}} - u_n)\}\} \tag{41}$$

where α_n and u_n can be determined from Eqs. (39) and (40), respectively.

4.5. Overall pounding probability of adjacent buildings

The design earthquake ground motion by itself does not determine pounding risk of adjacent buildings; the pounding risk is also affected by the design rules and analysis procedures used in connection with the design ground motion. It is noted that the overall pounding probability of adjacent buildings, P_p, can be evaluated by total probability theory. In other words, the overall pounding probability of adjacent buildings during some period of time can be evaluated by combining the results of the seismic hazard analyses and the relations of PGA and the conditional pounding probability of adjacent buildings, which means the pounding probability of adjacent buildings subjected to earthquakes with a specified PGA.

If the ground motion intensity is characterized by the peak acceleration, a^*, then the seismic pounding risk evaluation proceeds as follows. For structural pounding to occur, two events must happen. First, a ground motion with intensity, a^*, must occur; secondly, this motion must cause pounding. All possible values of a^* must be considered. The overall probability that pounding will occur during some period of time, P_p, may be expressed as follows.

$$P_p = \int_a P_{p/a*} P_{a*} da^* = \int_a P_{p/a}{}^* \frac{d\gamma}{da^*} da^* \cong \sum_i (P_{p/a})_i (P_a \Delta a)_i \cong \sum_i (P_{p/a})_i (\Delta\gamma)_i. \tag{42}$$

in which $P_{p/a*}$ expresses the conditional pounding probability of adjacent buildings, which means the pounding probability of adjacent buildings subjected to earthquakes with a specified PGA, a^*; $P_{a*}da^*$ or $(d\gamma)/(da^*)da^*$ expresses the probability of occurrence of a ground motion with intensity between a^* and a^*+da^*. The values a_1, a_2, a_3, … provide a suitable discretization of the continuous intensity parameter. For convenience of numerical calculation, the function to be integrated has been evaluated at equal increments Δa. The numerical integration of Eq. (42) requires the evaluation of $(\Delta\gamma)_i$ and $(P_{p/a})_i$ of a ground motion with intensity between a_i and

$a_i+\Delta a$. In integration procedure, it is assumed that $P_{p/a*}$ remains constant between a_i and $a_i+\Delta a$ and can be evaluated from the relation curves of PGA and conditional pounding probability of adjacent buildings, expressed as $(P_{p/a})_i$. This assumption is available so long as a short enough Δa is used. Additionally, the value of $(\Delta\gamma)_i$ can be evaluated from the results of the seismic hazard analyses.

Fig. 2(a), which is based on information supplied by Algermissen and Perkins [24,25] from their study, is a result of the seismic hazard analyses at a given location and indicates the probabilities of not being exceeded in a 50 year interval if the levels of PGA were to be selected. A probability of not being exceeded can be translated into other quantities such as mean recurrence interval or average annual risk (Fig. 2(b)). A 90 percent probability of not being exceeded in a 50 year interval is equivalent to a mean recurrence interval of 475 years or an average annual risk of 0.002 events per year. As shown in Fig. 2, there is 90 percent probability that the PGA will not exceed 0.4g at this location. The value of $(P_a\Delta a)_i$ or $(\Delta\gamma)_i$, which is the occurrence probability of a ground motion with intensity between a_i and $a_i+\Delta a$ in a 50 year interval, can then be evaluated from this figure. The numerical summation process of Eq. (42) is depicted graphically in Fig. 3.

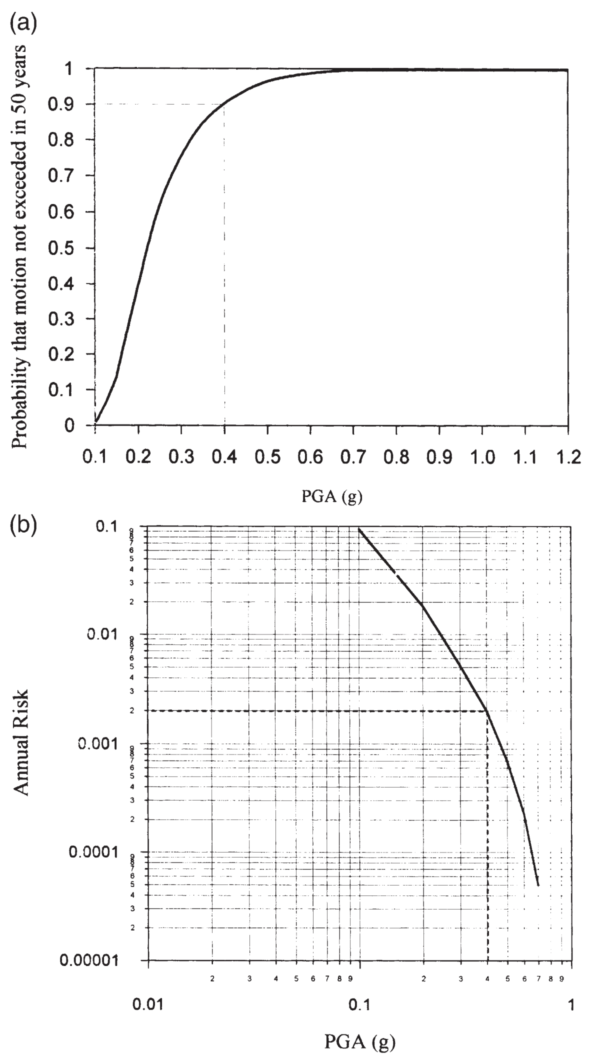

Fig. 2. Seismic hazard curve: (a) Probabilities of not being exceeded in 50 year life; (b) Annual probability of exceedance.

5. Earthquake power spectrum

The smooth earthquake power spectrum, $S_g(\omega)$, used in Eqs. (30) and (31) can be represented by the Kanai–Tajimi expression [26]

$$S_g(\omega)=\frac{\omega_g^4+4\xi_g^2\omega_g^2\omega^2}{(\omega_g^2-\omega^2)^2+4\xi_g^2\omega_g^2\omega^2}G_0 \tag{43}$$

where $S_g(\omega)$ reflects the frequency content during the most intense part of the ground motion and is a one-sided power spectrum, ω_g is the predominant ground frequency, ξ_g is the ground damping and G_0 is a measure of ground intensity.

The values $\omega_g=4\pi$ and $\xi_g=0.6$ have been suggested for firm ground sites and G_0 can be related to the peak ground acceleration, a, by [26]

$$G_0=\frac{0.141\xi_g}{\omega_g(1+4\xi_g^2)^{1/2}}a^2 \tag{44}$$

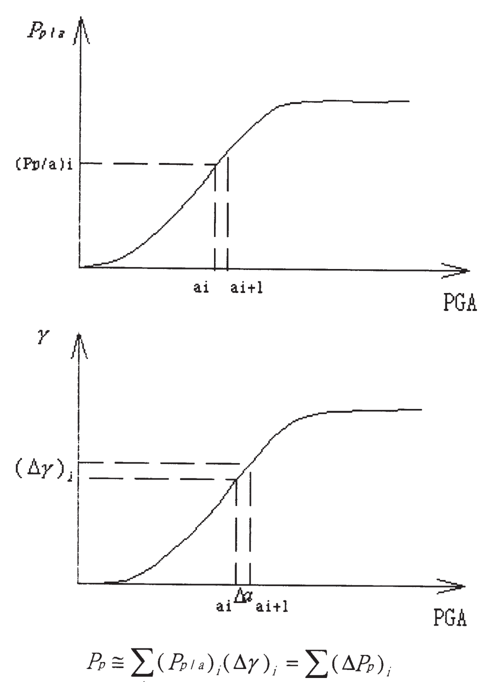

Fig. 3. Numerical summation process of overall seismic pounding probability.

6. Assessment of seismic risk at a site

The method of seismic risk analysis developed by Cornell [27] combines information about times of occurrence of earthquakes, areal distribution of seismicity, and attenuation of motion intensity, to yield probabilistic statements about the seismic threat at a given site.

On the basis of a statistical study of earthquake occurrence in time and space, Gutenberg and Richter [28] have found that an approximate linear relationship exits between N, the average number of earthquakes per year greater than or equal to M, and the magnitude M and can be expressed by the form

$$\log(N)=A-bM, \quad (45)$$

or

$$N=C\exp(-BM); \quad (46)$$

value of B close to 2 are found in most parts of the world [29].

Traditionally the peak ground acceleration (PGA) have been described as a function of magnitude and distance from the source. For example, using actual strong-motion records from firm ground sites in the western United States, Esteva and Rosenblueth [30] determined that the peak ground acceleration, a, is related to the magnitude, M, and the focal distance, R, in the following expression

$$a=\frac{2000\exp(0.8M)}{R^2} \quad (47)$$

Substituting Eq. (47) into Eq. (46) and considering all subsources, the average annual number, N_a, of earthquakes which cause peak ground accelerations greater than or equal to some fixed value a can be expressed by Eq. (48), if Esteva's results are used.

$$N_a=\sum_{all\ i}N_{i,a}=\tilde{G}\left(\frac{a}{2000}\right)^{-2.5} \quad (48)$$

where

$$\tilde{G}=\sum_{all\ i}C_iR_i^{-5} \quad (49)$$

and is a factor which depends on the geometry and relative activity of the various sources.

The mean return period T_a is simply the reciprocal of N_a.

$$T_a=\frac{1}{N_a} \quad (50)$$

The exceedance probability γ that a will be exceeded in the next 50 years can be expressed as

$$\gamma=1-(1-N_a)^{50} \quad (51)$$

The exceedance probability γ used in this study is shown in Fig. 2(a) which is based on information supplied by Algermissen and Perkins [24,25] from their study.

7. Analytical procedure and numerical examples

7.1. Procedure for seismic pounding risk analysis

The analytical procedure of pounding probability of two adjacent buildings during a period of time are briefly summarized in a step-by-step format as follows:

1. Determine mode shapes and frequencies of buildings by any available method.
2. Calculate the code-required separation distance of adjacent buildings from Eq. (1).
3. Determine earthquake power spectrum from Eq. (43), using G_0 in Eq. (44) for a specified peak ground acceleration.
4. Calculate ensemble mean squares of the relative displacement process, $Z(t)$, and the relative velocity process, $\dot{Z}(t)$, from Eqs. (30) and (31).
5. Calculate mean and standard deviation of the separation distance of adjacent buildings to avoid pounding from Eq. (35) to Eq. (37).
6. Calculate the conditional pounding probability of adjacent buildings, $P_{p/a}$, from Eq. (41), using α_n and u_n in Eqs. (39) and (40), respectively.
7. Repeat steps 3–6 with different peak ground accelerations until the relations between conditional pounding probability of adjacent buildings and peak ground acceleration are constructed.
8. Calculate the overall pounding probability of adjacent buildings from Eq. (42).

7.2. Numerical examples

A total of 36 cases of adjacent buildings are investigated, which include 4 cases for building A (story number of building A, na=6, 10, 14, 18) and 9 cases for building B (story number of building B, nb=4, 6, 8, 10, 12, 14, 16, 18, 20). The parameter values of buildings are given in Table 1. The earthquake intensity, expressed by the peak ground acceleration (PGA), is taken as 0.1 g, 0.2 g, 0.3 g, 0.4 g, 0.5 g, 0.6 g, 0.8 g, 1.0 g, and 1.2 g. Time duration, T, equals 30 s. The parameters of the Kanai–Tajimi model (Eq. (43)), namely, ω_g and ξ_g are taken as 4π and 0.6, respectively.

Table 1
Parameter values used in numerical examples

Degree of freedom, n	Fundamental period, T (s)	Stiffness, k (kg/cm)	Mass, m (kg*s^2/cm)	Damping ratio (%)
2	0.342	401594238		
4	0.575	449967041		
6	0.780	5077491455		
8	0.967	563400879		
10	1.144	613773193	454545.5	5
12	1.311	662145996		
14	1.472	706212158		
16	1.627	748586426		
18	1.777	789038086		
20	1.923	827490234		

7.3. Discussions

Fig. 4 shows the influence of the period of building B on the seismic pounding probability of adjacent buildings in 50 year life for Ta of 0.78 s, 1.144 s, 1.472 s and 1.777 s. As shown in Fig. 4, the pounding probabilities of adjacent buildings vary with the periods. It is noted that the probability of exceeding the design basis ground motion specified in the UBC'94 during a 50-year period is 10%. However, for the most critical case shown in Fig. 4, the pounding probability of adjacent buildings based on the seismic provisions of the UBC'94 is 17%. On the other hand, for the cases where the periods of adjacent buildings are well separated or extremely closed, the separation distance specified by the UBC'94 is too conservative due to improper treatment of the vibration phase of adjacent buildings. Based on the above observations, it is found that the method used in the UBC'94 provides poor estimates for the required separation distance and results in a non-uniform risk for studied cases. These observations are also detected in Fig. 5.

The relations between seismic pounding probability and period ratio of adjacent buildings are constructed and shown in Fig. 5. As shown in Fig. 5, the pounding probabilities of adjacent buildings vary significantly with period ratios of adjacent buildings. In other words, the period ratio of adjacent buildings plays a major role that affects the pounding risk of adjacent buildings. However, the effect of period ratio on pounding risk has not yet been taken into account in the seismic pounding related provisions of the Uniform Building Code.

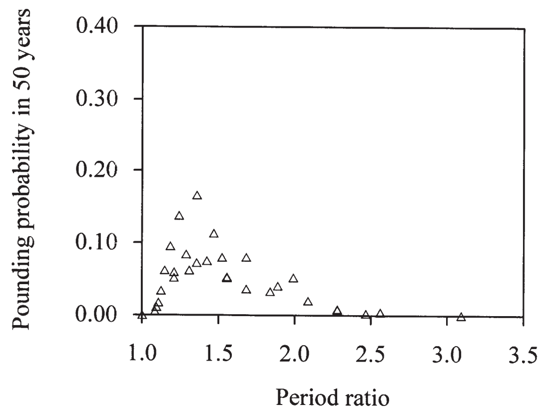

Fig. 5. Seismic pounding probability vs period ratio of adjacent buildings in 50 year life.

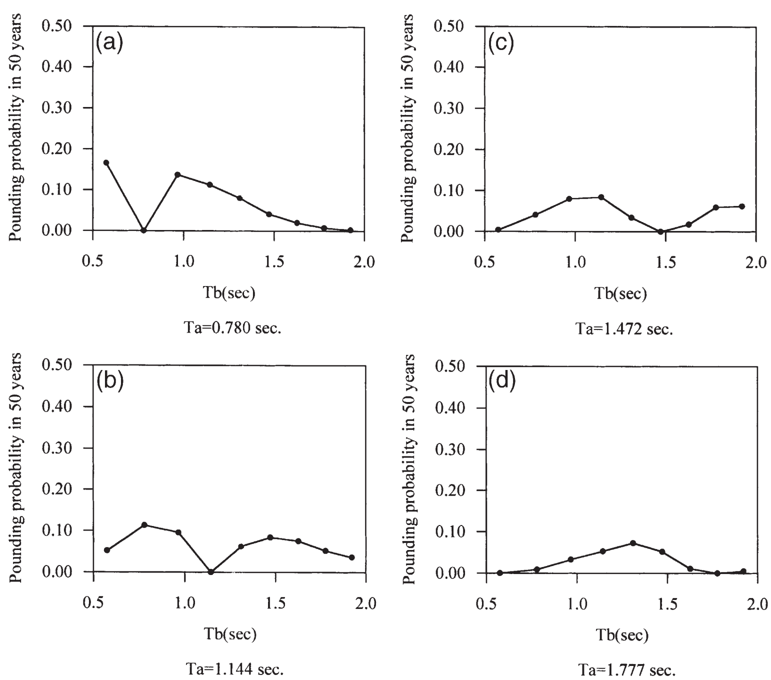

Fig. 4. Seismic pounding probability vs period of buildings in 50 year life: (a) Ta=0.780 s; (b) Ta=1.144 s; (c) Ta=1.472 s ; (d) Ta=1.777 s.

It is noted that the results of Figs. 4 and 5 may change with different buildings and earthquake models. However, the emphasis in this study is on the method and procedure of evaluating the seismic pounding risk of adjacent buildings.

It is also noted that investigating the seismic pounding risk of adjacent buildings may subject to some errors. A better accuracy, if desired, may be achieved by improving the precision of seismic hazard analysis, the confidence of the probability distribution of separations, and the accuracy of the parameters used to model the behaviors of the buildings.

8. Concluding remarks

Based on the assumptions of linear elastic structure responses, this study presents a spectral approach to evaluate the pounding probability of adjacent buildings separated by the minimum code-specified separation during their useful life of 50 years. Detailed procedures of the analytical method and some numerical examples are presented in this paper.

Results of this study reveal that the periods and the period ratio of the adjacent buildings are major parameters that affect the pounding risk of adjacent buildings. From the numerical examples presented in this study, it is found that significant differences of the pounding risk are observed for the adjacent buildings separated according to the minimum separation distance specified in the Uniform Building Code. It is therefore suggested that the future seismic provisions to include the effect of period ratio of adjacent buildings on the pounding risk of buildings.

It is noted that the proposed solutions are based on the assumption that floor elevations are the same for all buildings so that pounding occurs only at these elevations where the masses are lumped. In addition, the pounding location is assumed to occur at the top level of the shorter building. Therefore, for adjacent buildings with floor levels at different elevations, the proposed solutions cannot be used directly. On the other hand, the need, in the future research, to extend the current study from elastic to inelastic structural behavior is apparent since the responses of buildings subjected to major earthquakes may easily proceed to the inelastic range.

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