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Vortex lattice in type II superconductors under magnetic field in the presence of inhomogeneities

B. Rosenstein [a,*], B.Ya. Shapiro [b], V. Zhuravlev [a]

[a] National Center for Theoretical Studies and Electrophysics Department, National Chiao Tung University, Hsinchu 30050, Taiwan, ROC
[b] Department of Physics, Bar Ilan University, Ramat Gan 52100, Israel

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Abstract

We investigate the structure, elastic and dynamical properties of the vortex matter in the presence of artificially created or intrinsic gradients of the critical temperature in the framework of the Ginzburg–Landau theory. The region of parameters in which vortex cores are not well separated is treated perturbatively in $1 - H_{c2}(T)/H_{c2}(0)$. Critical current for periodic pinning potential is obtained and general expressions for elastic moduli at long wavelength are derived. We show that it is impossible to restrict the system to lowest Landau level. We use it to provide a theory of the discontinuous peak effect in critical current which appears near $H_{c2}(T)$ line in low T_c strongly type II superconductors. Influence of thermal fluctuations is also considered and we find softening of the shear modulus in the vicinity of vortex lattice melting line.

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In type II, superconductors for which the penetration depth λ exceeds the correlation length ξ the magnetic field penetrates the sample in a form of Abrikosov vortices, which strongly interact thereby creating an elastic "vortex matter". Impurities, always present in a sample, lead to inhomogeneities, which greatly affect the thermodynamic and especially dynamic properties of the vortex matter. Recently various experimental techniques were developed, which allow to artificially create "pinning" on the scale of up to tens of nm in an controllable way. When the inhomogeneity is strong enough, it pins the vortex matter, resulting in dissipationless persistent current, thereby recovering an original defining property of superconductor. The pinning can be overcome if the current exceeds the critical current J_c or if thermal fluctuations reduce the effect of the inhomogeneities.

Two major theoretical simplifications are generally made. In majority of the works, the vortex matter is considered as an array of elastic lines [1]. This (London) approximation is generally valid far from the higher critical field $H_{c2}(T)$, when the vortex density is low. Critical current is interpreted as a current at which the Lorentz force on the vortex line system overpowers the pinning force. An alternative simplification to the vortex matter is valid far enough from the lower critical field $H_{c1}(T)$. At high vortex densities magnetic fields of many vortices overlap and the resulting magnetic inductance is nearly homogeneous and Ginzburg–Landau (GL) model at constant magnetic field can be used. It is usually supplemented by the so called lowest Landau level (LLL) approximation. Here one does not see a well separated vortices, but rather a distribution of the order parameter fields with zeroes of the order parameter marking the centers of "cores". In most of the cases, however, the dynamics of the vortex matter is described in two steps. First, the system is treated

* Corresponding author.
E-mail address: vortexbar@yahoo.com (B. Rosenstein).

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collectively in the "elastic medium" approximation, namely first elastic properties under external "forces" are determined and then random or permanent gradients of the external parameters act of this elastic medium. Elastic properties of the vortex matter therefore become essential in understanding thermodynamic and transport properties of the disordered vortex matter [2]. In addition, the shear modulus was measured recently in BSCCO superconductor using the AC response technique [3]. It demonstrates sharp decrease at on the melting line of the vortex lattice. In particular, detailed knowledge of the elasticity of the vortex lattice is required to understand the "peak effect" in critical current [4–6].

In this paper, we consider the vortex lattice not far from $H_{c2}(T)$ in the presence of small gradients of the critical temperature $T_c(r)$ using the GL approach. First, the inhomogeneity is considered as a perturbation. The changes in distribution of both the superfluid density and the supercurrent are obtained. The pinned state is described as a static state which carries a net current. Critical current for periodic pinning potential is obtained and general expressions for elastic moduli at long wavelength are derived. The value for the shear modulus near $H_{c2}(T)$ is larger than calculated before [7] restricting the strained system to LLL. We show that it is impossible to restrict the pinned system to LLL: contribution of the first Landau level (LL) is crucial for both pinning and elastic deformations. We then argue that the discontinuous peak effect in critical current which appears near $H_{c2}(T)$ line in low T_c strongly type II superconductors can be understood using the elastic moduli. Influence of thermal fluctuations is also considered and we find softening of the shear modulus in the vicinity of vortex lattice melting line. The softening of the shear modulus was directly measured recently [3]. We focus on a particular case of strongly type II superconductors for which the ratio $\kappa = \lambda/\xi$ is very large (for high T_c cuprates and most of the widely used and studied low T_c type II superconductors κ is ranging between 10 and 100). Qualitatively for large κ the compression and the tilt moduli are practically the same as those of magnetic field in vacuum (except the dispersion [7]) and the most important modulus id the shear which is much smaller (by a factor of $1/\kappa^2$).

Our starting point is the GL Gibbs energy of a superconductor in homogeneous magnetic field $\mathbf{H} = (0, 0, H)$:

$$G = \int \mathrm{d}\mathbf{r}\left[\frac{\hbar^2}{2m^*}|D_\alpha \Psi|^2 + \frac{\hbar^2}{2m_z^*}|D_z\Psi|^2 + a'|\Psi|^2 + \frac{b'}{2}|\Psi|^4 + \frac{(\mathbf{B}-\mathbf{H})^2}{8\pi}\right], \quad (1)$$

where $\alpha = x, y, m^*$ and $m_z^* = \gamma^2 m^*$ are effective masses in directions perpendicular and parallel to that of magnetic field and γ is the anisotropy parameter. We assume for simplicity $a' = a(T_c - T)$. It will be convenient to use $\xi^2 = \frac{\hbar^2}{2m^*\alpha T_c}$ as unit of length in the x–y plane, and $\xi_z = \gamma\xi$ in the field direction, $g_{\mathrm{GL}} = \frac{H_{c2}^2}{8\pi\kappa^2}$ as unit energy density and rescale the order parameter field $\Psi^2 = \frac{2\alpha T_c}{b'}\psi^2$. The upper critical field $H_{c2} = \frac{\Phi_0}{2\pi\xi^2}$, is a unit of magnetic field, so that $h = H/H_{c2}$ describes the external field and $b = B/H_{c2}$. In these units, the dimensionless GL energy takes a form:

$$g = \frac{G}{4\xi^2\xi_c g_{\mathrm{GL}}} = \int_r \left[\frac{1}{2}|D_i\psi|^2 - \left(\frac{1-t}{2}\right)|\psi(\mathbf{r})|^2 + \frac{1}{2}|\psi(\mathbf{r})|^4 + \frac{\kappa^2}{4}(\mathbf{b}-\mathbf{h})^2\right], \quad (2)$$

where $i = x, y, z$ and $D_i = \frac{\partial}{\partial x^i} - \mathrm{i}A_i$ are covariant derivatives.

"Force" acting on the vortex matter can be physically realized in a variety of ways [8]. Let us consider a superconductor with spatially inhomogeneous critical temperature $T_c(\mathbf{r})$. Within the framework of the GL theory it is described by a potential:

$$a' = \alpha[T - T_c(r)] = \alpha\{T - T_c[1 + U(r)]\}. \quad (3)$$

This model of generally used to describe the δT_c pinning [1]. This additional term naturally results in an elastic deformation of the vortex lattice. In the absence of the potential term, the solution is the Abrikosov lattice solution minimizing the functional Eq. (2) is given by a well defined expansion in two small parameters κ^{-2} and a_h [9]:

$$\psi_{\mathrm{mf}}(r) \simeq \sqrt{\frac{a_h}{\beta_A}}\left[\phi_0 + a_h\phi_c + \mathrm{O}(a_h^2)\right] + O(\kappa^{-2})$$
$$\mathbf{b}_{\mathrm{mf}}(r) \simeq \mathbf{h} + \kappa^{-2}\mathbf{b}_c + \mathrm{O}(\kappa^{-4}), \quad (4)$$

where $\mathbf{b}_c = -\mathbf{h}\frac{a_h}{h\beta_A}|\phi_0|^2 + \mathrm{O}(a_h^3)$ and ϕ_0 is the Abrikosov wave function for hexagonal lattice:

$$\phi_0 = 3^{1/8}h^{-1/2}\sum_{l=-\infty}^{\infty}\exp\left[\mathrm{i}hxy + .\mathrm{i}l\left(\frac{\pi l}{2} + 3^{1/4}\pi^{1/2}h^{1/2}x\right) - \frac{1}{2}(h^{1/2}y - 3^{1/4}\pi^{1/2}l)^2\right], \quad (5)$$

normalized to unit superfluid density. The correction ϕ_c contains higher LLs. The currents pattern is simple: vortices around positions of zeroes at $\mathbf{r_n} = (a(n_1 + n_2/2), 2\pi/an_2)$ with n_1, n_2 integers.

To first-order in U the correction to the wave function $\theta(r)$ can be expanded in LLs basis $\phi_{N\mathbf{k}}(\mathbf{r})$, where $\mathbf{k}$ is quasimomentum and N LL:

$$\theta(\mathbf{r}) = -b^{-1}a_h^{1/2}\beta_A^{-1/2}\sum_{N\mathbf{k}} U_{N\mathbf{k}}\phi_{N\mathbf{k}}(\mathbf{r}), \quad (6)$$

where $U_{N\mathbf{k}} = \int_{\mathbf{r}}\phi_{N\mathbf{k}}U(\mathbf{r})\phi_0$ and higher orders in a_h were neglected. The most important contribution to the current density $\mathbf{J}(\mathbf{r}) = \frac{i}{2}(\psi^*\mathbf{D}\psi - \psi\mathbf{D}\psi^*)$ comes from the first LL, since the covariant derivatives in $\mathbf{J}(\mathbf{r})$ contains one "rasing" operator, $-\mathrm{i}D_x\phi_0 = D_y\phi_0 = (2b)^{-1/2}\phi_1$:

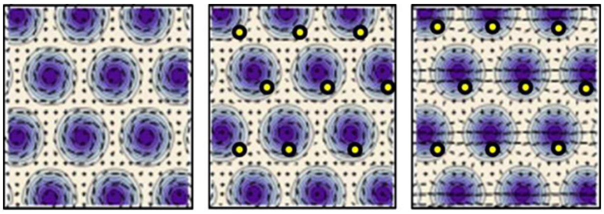

Fig. 1. Current distribution of the pinned vortex lattice. Pins are shown as blobs. Left: unperturbed current distribution. Center: the perturbed distribution by a localized periodic potential. Right: The current distribution with the unperturbed subtracted. Since the unperturbed configuration does not carry net current, the right picture demonstrate the persistent current.

$$\delta J_x = \frac{a_h}{\beta_A b}(2b)^{-1/2}\sum_{\mathbf{k}}\left[\phi^*_{1\mathbf{k}}(\mathbf{r})U_{1\mathbf{k}}\phi^*_0(\mathbf{r}) + cc\right]$$
$$\delta J_y = \frac{ia_h}{\beta_A b}(2b)^{-1/2}\sum_{k}\left[\left(\phi^*_{1\mathbf{k}}(\mathbf{r})U_{1\mathbf{k}}\phi^*_0(\mathbf{r})\right)^* - cc\right]. \tag{7}$$

As an example, let us consider potential which is periodic (matching field H). One observes (see Fig. 1) that when minima of the potential do not coincide with the zeroes of the order parameter, the correction to current integrated over a certain volume (the unit cell for example), $\delta\mathbf{I} = \int_{\mathbf{r}}\delta\mathbf{J}(\mathbf{r})$, is nonzero and can be defined as the Lorentz force, $\mathbf{F}_L = c^{-1}[\delta\mathbf{I}\times\mathbf{b}]$: $-F_{Ly} + iF_{Lx} = a_h\beta_A^{-1}(b/2)^{-1/2}U_{\mathbf{10}}$. The Lorentz force is balanced by "electric" pinning force, $\mathbf{F}_{pin} = \int_{\mathbf{r}}|\psi|^2\nabla U$ as can be explicitly seen. The current per vortex calculated that way cannot exceed a certain value, thus determining the critical current. The above consideration demonstrates that an equilibrium state in the presence of the pinning force inevitably has mesoscopic supercurrents and in view of Eq. (7) cannot be treated in framework of LLL only.

One can determine perturbatively in U the new positions of zeroes of the order parameters ("centers" of the vortices). Before the perturbation was applied they were located at $\mathbf{r_n} = (a(n_1 + n_2/2), 2\pi/an_2)$. New positions of the zeroes are found by demanding $\psi(\mathbf{r_n} + \mathbf{u_n}) = 0$. Since displacement $\mathbf{u_n}$ is first-order in U, expanding this relation to the first-order gives:

$$a_h^{1/2}\beta_A^{-1/2}\partial_\alpha\phi_0(\mathbf{r_n})u_{\alpha\mathbf{n}} = -\theta(\mathbf{r_n}), \tag{8}$$

determining all the displacements. If one considers a configuration possessing the hexagonal symmetry generally the elastic energy can be written as:

$$g_{el} = \frac{1}{2}\sum_{\mathbf{k}}\Big[c_{11}\left(k_x u_{0x} + k_y u_{0y}\right)^2 + c_{66}\left(k_x u_{0y} - k_y u_{0x}\right)^2 + c_{44}k_z^2\left(u_{0x}^2 + u_{0y}^2\right)\Big]. \tag{9}$$

The basic relation between stress and strain for potential $\partial_\alpha F_\beta = -v_{\beta\alpha} = c_{\alpha\beta\gamma\delta}\partial_\gamma u_\delta$ with $F_\beta = -\partial_\beta U$ one obtains the following expression for the shear modulus:

$$c_{66} = \frac{ha_h}{2\beta_A}, \tag{10}$$

in units of $H_{c2}^2\xi^2\xi_c/(2\pi)$.

In solid state, physics one can conveniently consider elastic stress as a force acting on pointlike atoms. However, from the earliest times electromagnetic fields can also be considered as a kind of "elastic medium". In particular, a constant uniform magnetic field H in vacuum has well defined compression and tilt moduli $C_{11} = C_{44} = \frac{H^2}{4\pi}$, while the shear modulus vanishes [1]. The elasticity is associated with field deformation within the volume containing the field. In electromagnetically, active media generally additional fields describing matter like magnetization in magnets, polarization vector in ferroelectrics, etc., contribute to elastic properties. Generally for any system of fields one can obtain an expression for elastic moduli in a same way one derives an expression for angular momentum (rotation modulus). Qualitative picture behind theoretical approach to elasticity was that magnetic field penetrates the material as a system of Abrikosov vortices (fluxons). Topological argument within the simplest GL model of a one component superconductor implies that if n elementary flux units Φ_0 penetrate the sample, then the order parameter ψ has exactly n zeroes. This determines unambiguously centers of vortices surrounded by normal cores of size of coherence length. When cores of the vortices are well separated, one can reduce the problem to the elasticity of a collection of linelike (without internal structure) objects. This description becomes problematic near the upper critical field $H_{c2}(T)$, when vortices are poorly separated and their internal structure is of importance. Moreover, there are important cases in physics when fluxons do not possess a core at all, see for example p-wave superconductors (similar to those in superfluid He_3). This does not imply that these materials do not have a well defined elastic properties. In these, cases one should not rely on location of zeroes, but rather return to the original field theoretical description.

A more powerful general approach to elasticity is geometrical in nature [12]. Elastic moduli describe the rigidity with respect to *local* translations. For our purposes it is sufficient to consider displacements in the plane perpendicular to external magnetic field $\mathbf{u}(r) = (u^\alpha, 0)$. The corresponding transformations of a scalar and a vector fields are:

$$\psi'(r) = \psi(r+u) \approx \psi(r) + u_\alpha\psi_\alpha,$$
$$A'_i(r) \approx A_i(r) + u^\beta A_{\alpha,\beta} + u^\beta_\alpha A_\beta, \tag{11}$$

where a short notation for derivatives, e.g., $u^\beta_j = \frac{\partial u^\beta}{\partial r^j}$ is used and $i = 1, 2, 3$. Considering the displacement, $u^{\alpha,\beta} = u_0^\alpha k^\beta$, and expanding in powers k, one observes that to order k^0 the contributions cancel. This is just the Goldstone theorem, which asserts that when a continuous symmetry (global translations in the present case) is spontaneously broken, there appears a "soft" mode. Terms linear in k vanish due to reflection symmetry of the Abrikosov lattice configuration, while the terms quadratic in k determine the elastic moduli.

Calculating the second functional derivatives and substituting the mean field solution of Eq. (4), one obtains

$$\delta g = \frac{1}{2\mathrm{vol}} \int_{\mathbf{k},\mathbf{r}} u^0_\alpha u^0_\beta k^i k^j \left[\kappa^2 \left(\delta_{ij} A_{1,\alpha} A_{1,\beta} + A_{\alpha,j} A_{i,\beta}\right) + \delta_{ij}\left(D^*_\beta \psi^* D_\alpha \psi + D_\beta \psi D^*_\alpha \psi^*\right)\right]. \tag{12}$$

Now we proceed to calculate the moduli in the Abrikosov lattice configuration neglecting thermal fluctuations. As mentioned in Introduction, the case of large κ is of special interest. It is natural therefore to expand a physical quantities in powers of κ^{-1}. Fortunately the expression for the most important shear modulus is a regular function of both κ^{-2} and the wave vector k, (the other two moduli are not [7]). The contribution to elastic moduli to the leading order in κ^{-2} comes solely from the magnetic energy term. Substituting $A_\alpha = \frac{1}{2} h \varepsilon_{\alpha\beta} r^b$ (symmetric gauge), one obtains that the contributions to the compression and the tilt moduli are equal,

$$c^0_{11} = c^0_{44} = \frac{h^2\kappa^2}{4}, \tag{13}$$

and consistent with the κ^2 term in expansion of Eq. (12) at $k = 0$. The contribution to the next to leading order in κ^{-2}, $c^1_{ij\alpha\beta}$ comes both from magnetic and the order parameter terms. The magnetic term is of order a^2_h:

$$c^{1A}_{ij\alpha\beta} = \frac{1}{\mathrm{vol}} \int_\mathrm{r} \left(\delta_{ij} A^0_{1,\alpha} A^c_{1,\beta} + A^c_{\alpha,j} A^0_{i,\beta} + (\alpha \leftrightarrow \beta)\right) = -\frac{h a_\mathrm{h}}{2\beta_\mathrm{A}}(1 + 2a_\mathrm{h} d_0)\delta_{\alpha i}\delta_{\beta j}.$$

The order parameter term (in Eq. (12)) contribution is proportional to $c^{1\psi}_{ij\alpha\beta} = \delta_{ij} s_{\alpha\beta}$

$$s_{\alpha\beta} = \frac{1}{\mathrm{vol}} \int_r \left(D^*_\beta \psi^*_\mathrm{mf} D_\alpha \psi_\mathrm{mf} + cc\right) = \delta_{\alpha\beta} s, \tag{14}$$

where $s = \frac{h a_\mathrm{h}}{2\beta_\mathrm{A}}(1 + 2a_\mathrm{h} d_0)$. The fact that symmetric tensor $s_{\alpha\beta}$ is proportional to $\delta_{\alpha\beta}$ follows from hexagonal symmetry of the mean field solution. One therefore has the following contributions to compression, tilt and shear moduli

$$c^1_{11} = d_0 \frac{a^2_\mathrm{h} h}{\beta_\mathrm{A}};\ c^1_{44} = c^1_{66} = \frac{h a_\mathrm{h}}{2\beta_\mathrm{A}}(1 + 2a_\mathrm{h} d_0). \tag{15}$$

Note that the order a_h contributions to the compression modulus from the magnetic term and the order parameter term cancel. Also the correction to the tilt modulus leads to the known result $c^0_{44} = \frac{hb\kappa^2}{4}$.

One observes that, while the compression and the tilt moduli are the same as in thermodynamically calculated [7], the value of the shear modulus is different. The thermodynamic and the LLL calculation gives near $H_{c2}(T)$ a value of $c_{66} \simeq \frac{0.24}{\beta^2_\mathrm{A}\kappa^2} a^2_\mathrm{h}$, smaller than in Eq. (15). We discuss this below.

The mean field result of the previous subsection is modified in strongly fluctuating superconductors like high T_c materials by thermal fluctuations. Generally it results is softening of elastic moduli. In the extreme case, when the system approaches the overheated crystal spinodal line the shear modulus vanishes making the crystalline state unstable. Below this temperature, however, the melting transition into the vortex liquid state takes place. Thermal fluctuations on the mesoscopic scale are accounted for by averaging over all the configurations of fields with corresponding Boltzmann factor. Generally for strongly type II materials fluctuations of the magnetic field are negligibly small. Since the compression and the tilt moduli remain finite even in the liquid state, the only important modulus is the shear. Therefore we concentrate on calculation of corrections to the mean field for the quantity

$$s_{\alpha\beta} = \left\langle \frac{1}{\mathrm{vol}} \int_r D_\alpha \psi D^*_\beta \psi^* + D^*_\alpha \psi^* D_\beta \psi \right\rangle_{th} = \frac{Z^{-1}}{\mathrm{vol}} \int_{\psi,\psi^*} \int_r \left\{D_\alpha \psi D^*_\beta \psi^* + D^*_\alpha \psi^* D_\beta \psi\right\} \mathrm{e}^{-\frac{G\{\psi\}}{T}}, \tag{16}$$

where $Z = \int_{\psi,\psi^*} \exp[-\frac{g\{\psi\}}{\pi^2\sqrt{2Gi}t}]$.

Assuming hexagonal symmetry, this symmetric tensor simplifies $s_{\alpha\beta} = \delta_{\alpha\beta} s$, so that $c_{66} = -\frac{1}{\mathrm{vol}} \int_r \langle \psi^* D^2 \psi \rangle_{th}$. When thermal fluctuations are not very large, one can apply the low temperature perturbation theory around the field solution $\psi = \psi_\mathrm{mf} + \chi$. In the leading order, in a_h one can neglect thermal fluctuations of the higher Landau harmonics, restricting the field to LLL: $D^2\psi_\mathrm{LLL} = -h\psi_\mathrm{LLL}$. In this case, the modulus and the Gibbs energy simplify significantly: $c_{66} = \frac{h}{2} \frac{1}{\mathrm{vol}} \int_r \langle |\psi|^2 \rangle_{th}$, $g_\mathrm{LLL} = \int_r [-a_\mathrm{h}|\psi|^2 + \frac{1}{2}|\psi|^4]$. Therefore within this approximation the shear modulus is proportional to superfluid density. In the low temperature, expansion it was calculated in [9]

$$c_{66} = \frac{h a_\mathrm{h}}{2\beta_\mathrm{A}} - \frac{0.55\sqrt{Gi}th^2}{a^{1/2}_\mathrm{h}}. \tag{17}$$

The softening intensifies upon approaching the mean field transition line $a_\mathrm{h} = 0$. However, well below this line the perturbation theory breaks down.

The shear modulus softening just below $H_{c2}(T)$ plays a crucial role in explaining the "peak effect" in the critical current [13]. The peak generally appears just before the "melting" of the Abrikosov lattice due to thermal fluctuations. Within the collective pinning theory [1], the critical current is estimated from the balance of the pining force on Larkin domain and the Lorentz force $J_\mathrm{c}B$. Size of the Larkin domains can be estimated via relevant elastic moduli leading to

$$J_\mathrm{c} = \frac{A}{b c_{44} c^2_{66}}. \tag{18}$$

The constant A is dependent on a_h and gets smaller near $H_{c2}(T)$, although the exact dependence is not known. However, since c_{66} of the thermodynamical argument is proportional to a^2_h, it was argued that one obtains a gradual increase in J_c approaching $H_{c2}(T)$ since "softening" of the vortex lattice overcomes decrease of the pinning force. This corresponds to an "old" view on the "peak effect", when this increase was thought to be followed by an abrupt jumps of the critical current to zero at the melting point (in practice might be smeared out by sample inhomogeneities). The recent view, supported by experiments in which Corbino geometry or width dependence were used to minimize or subtract the edge effects [4,5], attributes the peak to the amorphous homogeneous state. Critical current actually monotonically decreases with field and then jumps from a relatively low value in the crystalline state to a very high value in the vortex glass (this was noticed early on in [10]). Qualitatively this is due to the fact that it is easier to pin a disordered homogeneous state than a rigid crystalline one. The continuous rise of the critical current observed in numerous earlier experiments was caused by poor resolution due to overheating of the solid and overcooling of the homogeneous states. The critical current in the amorphous phase rapidly drops as $\sqrt{T-T^{g}}$,when temperature approaches the glass temperature [14] T^{g}. Thus traditional picture predicts a gradual increase with subsequent drop of the critical current, while modern picture predicts a sudden increase followed by a fast but continuous decrease. If one uses the larger value of the shear modulus obtained here, Eq. (10), one indeed obtains a monotonic decrease to a constant value since both pinning force and softening drop with similar rate in Eq. (18).

Using the GL theory under the assumption that the system under a stress remains constrained to LLL, Brandt derived [7] expressions for the softest modulus, the shear, of the vortex lattice. At large κ and near the mean field line C_{66} is proportional to a_{h}^{2}. The modulus is consistent with thermodynamic derivation in which lattice energies of different symmetries were compared. The expression for shear modulus and other moduli are used in numerous theoretical descriptions of phenomena as different as vortex lattice melting [11] and critical current [1,7] of the pinned lattice. As we demonstrate below the LLL assumption does not follow directly from the "thermodynamic" argument. The stress necessarily transforms the LLL equilibrium state into a state containing significant contribution of higher LLs near the $H_{c2}(T)$ line. Qualitatively it reflects the fact that shear for example changes the shape of the order parameter spatial distribution into sheared one (ellipsoidal one rather than round). The location of zeroes is exactly the same, but internal structure of the vortex becomes important.

The sheared state is not a ground state of the Abrikosov lattice with the same symmetry. The later is explicitly constructed in the symmetric gauge by Brandt [7]. Restricting the shear transformations to LLL, he effectively retained the notion of a single state for a given lattice symmetry. Physically it is equivalent to an assumption that the degrees of freedom related to shape of the vortices can "relax" to their positions with minimal energy. It would mean that the system returns to an LLL state upon this relaxation. There is a popular belief that all degrees of freedom can be divided in two sets: "slow" and "fast". "Slow" variables are the locations of vortices determined, for example, by the vortex center positions (where $\psi=0$), and "fast" variables which contain all the other degrees of freedom related to the shape of the vortices. While near H_{c1} one can argue that the internal degrees of freedom are very costly energetically, near H_{c2} this is not correct. To our knowledge, there are no works which establish in what field range the separation between two set of degrees of freedom becomes possible. The GL energy does not contain an evident small parameter or "energy gap" which allows such a separation. Correspondingly, in dynamics based on the time-dependent GL equation there is no separation of time relaxation of different degrees of freedom. Mathematically the shear transformation takes the ground state ϕ_0 out of the LLL sector since it does not commute with the "Hamiltonian" $-\frac{1}{2}D^2+\frac{h}{2}$. Although in our calculation magnetic induction deviates slightly from the external field h, the reason for a significant increase of the shear modulus compared to earlier estimates is not related to this.

To summarize, we considered elastic response of the vortex lattice to inhomogeneity near the second critical field $H_{c2}(T)$ using the GL approach and showed that in the pinned state the system is necessarily excited to states outside of the LLL. This reflects the deformation of the current distribution profile under stress. As a result the shear modulus is much larger (of order $1-T/T_c-H/H_{c2}$) than that found by considering minimal energies of configurations with symmetries corresponding to sheared lattice, leading to $(1-T/T_c-H/H_{c2})^2$ The obtained shear modulus leads to a monotonic decrease of the bulk contribution to the critical current in the crystalline phase before it discontinuously jumps to a much higher value in vortex glass. Such a behavior was obtained experimentally recently when the edge contributions were minimized.

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