Analysis and Design of Large Leaky-Mode Array Employing the Coupled-Mode Approach

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***Abstract*—This paper presents the coupled-mode approach to the analysis and design of a large leaky-mode array, in which an N-element microstrip array above a common ground plane supports N coupled leaky modes (leaky EH_1 modes) at the first higher order. Following a brief description of a conventional full-wave nonstandard eigenvalue problem for solving the complex propagation constants, we present the detailed formulation of the coupled-mode approach, clearly showing the transformation of the nonstandard eigenvalue problem into a standard one. Thus, all the eigenvalues (complex propagation constants) and eigenvectors (modal current distributions) are solved simultaneously, regardless of the size of the array (N). Two key issues pertinent to the successful implementation of the proposed coupled-mode approach are addressed: the determination of the coupling coefficients and the uniqueness of the isolated uncoupled leaky mode, which represents the leaky modal solution of a single microstrip, but is derived from a system of coupled leaky-mode solutions provided the coupled microstrips have equal width and spacing. Closed-form expressions for obtaining the coupling coefficients of orders two, three, and four are presented. Theoretical studies of closely coupled microstrip arrays of two, three, and four elements show that the magnitude of the coupling coefficient $C_{i,i+j}$ between elements i and $i+j$ decreases at the order of 10^{-j}. These theoretical case studies also lead to the same isolated uncoupled leaky-mode solution as predicted. Furthermore, the dispersion characteristics of the microstrip array at the first higher order obtained by the coupled-mode approach and the full-wave approach agree excellently for all the case studies. Error analyses indicated that at least two coupling coefficients ($C_{i,i+1}$ and $C_{i,i+2}$) are required for obtaining accurate complex propagation constants with rms errors less than 1% for most of the leaky region of the particular array under investigation. An example of applying the proposed coupled-mode approach for analyzing a corporate-fed leaky-mode array of eight elements is reported, revealing that only four out of the eight leaky modes are excited. The coupled-mode theory predicts the far-field radiation pattern in the main beam region in excellent agreement with the measured results.**

***Index Terms*—Coupled microstrip lines, coupled-mode approach, coupling coefficients, leaky-mode antenna, leaky modes.**

I. Introduction

MUTUAL coupling of guided modes has been an important phenomenon widely applied in various microwave circuits such as directional couplers and filters [1]–[6]. A system of parallel coupled microstrips placed on a substrate with an infinite ground plane, for example, supports N bound modes if N parallel lines are present above the common ground plane, where N is a positive integer. These bound modes are usually termed dominant or quasi-TEM modes although they are hybrid modes in nature, i.e., the superposition of TE and TM waves [7], [8]. The well-known bound dominant microstrip mode can be designated as *order zero*, followed by *first* (odd), *second* (even), *third* (odd), $\ldots$, *higher order* modes, possessing either odd or even field symmetries. Adopting this notation, the dominant modes of the N parallel lines are essentially the results of the N mutually coupled microstrip modes of *order zero*. This paper, however, departs from the conventional results and investigates the less known, but useful guiding properties of the coupled microstrip modes of *order one*, from which similar results can be obtained for microstrip modes of *order two, three*, $\ldots$, etc.

The open microstrip modes of order greater than zero belong to the class leaky modes. A single open microstrip has been known for supporting leaky modes in various ways [9]–[13], in which the first higher order microstrip mode leaks in the form of space wave or surface wave. Coupled leaky modes of order one, however, had been reported for a two- and a 20-element array, respectively, of which the leaky modes are tightly distributed in the dispersion characteristics [14]. When N, the number of elements, is increasingly larger, the solutions for leaky modes are more difficult to obtain by full-wave methods [15], [16], which result in the nonstandard eigenvalue problem [7]. Immediately after the complex propagation constant γ is obtained, the modal corresponding currents on each of the N coupled lines are readily known. Thus, each γ corresponds to a modal current vector, describing the modal current distributions of the N microstrips. Thus, for N coupled leaky modes of *order one*, there are N independent current vectors.

Section II of this paper transforms the nonstandard eigenvalue problem into a standard eigenvalue problem by adopting the coupled mode approach [17], [18] mingled with the rigorous full-wave formulation of the N parallel lines supporting maximally N coupled leaky modes, where N is also a positive integer. With this notation, γ becomes the complex eigenvalue λ and the modal current vector is the complex eigenvector $\bar{I}$ of the eigenvalue problem. This paper extensively expands the work of [14] by showing how to obtain the adjacent coupling coefficient and the coupling coefficients of nonadjacent lines in Section III. Substituting these coupling coefficients into the coupled-mode formulation described in Section II, the solutions for eigenvalue λ_i and eigenvectors I_i, for $i=1,2,3,\ldots$, and N, are obtained simultaneously, greatly reducing the time-consuming numerical efforts encountered by the well-known full-wave analyses [7], [19]. It is interesting to note that the coupled-mode theory had very often provided vivid physical insights for explaining the

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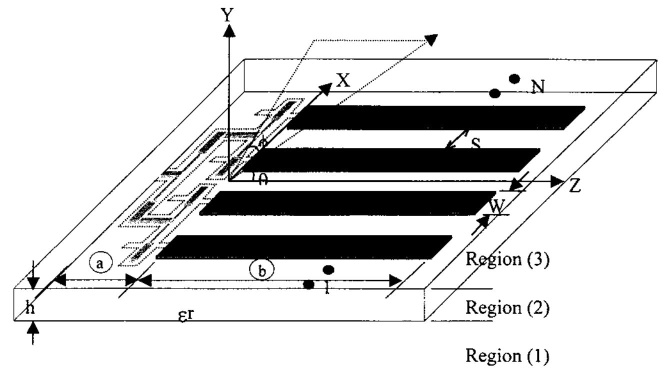

Fig. 1. Generic N-element microstrip leaky-mode antenna array, including a CPW corporate-fed network, CPW-to-slotline transitions, and slotlines to excite the higher order EH_1 modes of the microstrip lines [21]. ⓐ: CPW corporate-fed network followed by N CPW-to-slotline transitions on the backside of the substrate. ⓑ: Microstrip leaky-mode antennas with length of L, width of W, and gap of S on the top side of the substrate.

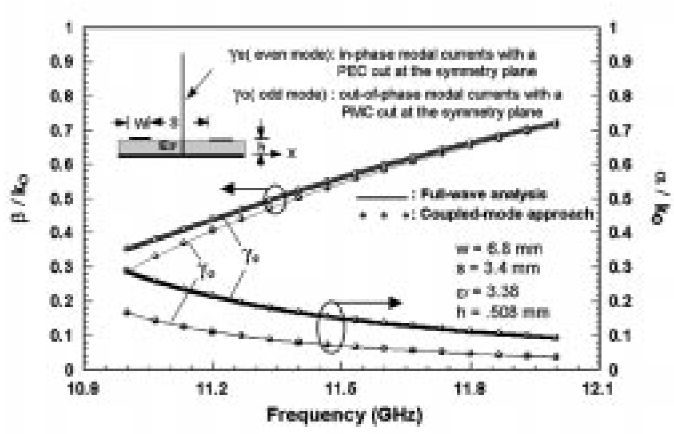

Fig. 2. Normalized dispersion characteristics of the symmetrical coupled microstrips at the higher order EH_1 mode. The data obtained by the full-wave nonstandard eigenvalue approach (in solid lines) and those by the standard eigenvalue approach (in circle symbols) agree excellently. The parameters for the coupled microstrips are also shown in the inset.

guiding properties of complex waves in a variety of waveguides [18], [20], [21].

One important application of this paper is to analyze the microstrip array from the leaky-mode perspective, as described in Section IV. As shown in Fig. 1, this array may consist of N coupled microstrip lines printed on the top surface and a coplanar waveguide (CPW) feeding network on the backside of the substrate. After a continuous wave (CW) signal source is applied to the CPW input end, the CPW T-junction splits the signal into two paths of equal amplitude and phase. Such power dividing sequence continues until, finally, each radiating element receives equal amount of power and phase. Followed by the CPW-to-slotline transitions, the uniformly divided signals are coupled to the slotlines to excite the EH_1 modes of the coupled microstrips [22]. Since microstrip lines can be placed closely to form a compact linear array, strong mutual couplings between N coupled lines will alter the dispersion characteristics of the leaky modes. Eventually, a large linear N-element array that simultaneously supports N coupled leaky modes is established. The quest for obtaining all the complex dispersion characteristics considering these mutual coupling effects is essential for the first-pass design of the array. Section V describes how to apply the novel technique to accurately predict the far-field radiation pattern of an experimental corporate-fed eight-element microstrip array [22], demonstrating excellent agreement in the main-lobe region between the simulated and measured results. Conclusions are finally made in Section VI.

II. Modal Analysis and Eigenvalue Problems

A. Full-Wave Approach: Nonstandard Eigenvalue Problem

When considering the microstrip array with fewer elements, e.g., two- or three-element, the leaky properties of the coupled microstrips can be characterized by invoking the rigorous full-wave integral-equation method using the Green's impedance function approach [16]. By applying the appropriate boundary conditions at the interfaces, the tangential electric-field components at the interface of each adjacent region can be expressed in terms of current distributions. One can then write

$$\begin{bmatrix} E_x^{(s)}(x,y \mid x') \\ E_z^{(s)}(x,y \mid x') \end{bmatrix} = \begin{bmatrix} G_{xx}^{(s)}(x,y \mid x') & G_{xz}^{(s)}(x,y \mid x') \\ G_{zx}^{(s)}(x,y \mid x') & G_{zz}^{(s)}(x,y \mid x') \end{bmatrix} \cdot \begin{bmatrix} J_{xm}^{(s)}(x') \\ J_{zm}^{(s)}(x') \end{bmatrix} \quad (1)$$

where $s = 1, 2,$ and 3 designates regions (1)-(3), respectively (see Fig. 1). Integrating (1) along all the metal strips, the general dyadic Green's impedance function $[Z]$ can be obtained as follows:

$$\begin{bmatrix} E_x^{(s)}(x,y) \\ E_z^{(s)}(x,y) \end{bmatrix} = \begin{bmatrix} Z_{xx}^{(s)}(x,y) & Z_{xz}^{(s)}(x,y) \\ Z_{zx}^{(s)}(x,y) & Z_{zz}^{(s)}(x,y) \end{bmatrix} \cdot \begin{bmatrix} \tilde{J}_{xm}^{(s)} \\ \tilde{J}_{zm}^{(s)} \end{bmatrix} \quad (2)$$

where

$$\tilde{J}_{xm}^{(s)} = \int J_{xm}^{(s)}(x') \cos\left(\frac{m\pi}{b}\right) x' \, dx' \quad \text{(3a)}$$

$$\tilde{J}_{zm}^{(s)} = \int J_{zm}^{(s)}(x') \sin\left(\frac{m\pi}{b}\right) x' \, dx'. \quad \text{(3b)}$$

Let the unknown current distributions $J_x^{(s)}$ and $J_z^{(s)}$ be expressed in terms of a complete set of basis functions. Following the Galerkin procedure to solve (2), a system of homogenous linear equation can be obtained. The nontrivial solution for the unknown current coefficients can be found by equating the determinant to zero. The roots γ $(\gamma = \alpha + j\beta)$ of the determinant are the propagation constants for microstrip modes of *order zero, one, two, three*, ..., etc.

Fig. 2 shows the normalized complex propagation constants of a two-element coupled microstrip of *order one*, namely, the EH_1 even mode (γ_{e}) and the EH_1 odd mode (γ_{o}), where k_0 is the free-space wavenumber equal to $2\pi/\lambda_0$ and λ_0 is the free-space wavelength. The inset of Fig. 2 also shows that the EH_1 even mode (odd mode) can be obtained by inserting an equivalent electric (magnetic) wall at the symmetry plane. Another view for distinguishing the two leaky modes is based on the eigenvectors of the modal current distributions [J_x of (3a)] on the strips, either in-phase for the EH_1 even mode or out-of-phase for the EH_1 odd mode. These in-phase and out-of-phase modal current distributions are two orthogonal eigenvectors of the two-element microstrip array from the perspective of the coupled-mode approach, which will become apparent in subsequent discussions within this paper.

B. Coupled-Mode Approach: Standard Eigenvalue Problem

By invoking the coupled-mode theory [23], [24], the transversal and longitudinal electric and magnetic fields can be expressed as follows:

$$\vec{E} = \vec{E}_t + E_z\hat{a}_z = \sum_{i=1}^{N} Z_i I_i(z)\vec{\phi}_t^i(x,y)e^{-\gamma_i z} + \sum_{i=1}^{N} Z_i I_i(z)\phi_z^i(x,y)\hat{a}_z e^{-\gamma_i z} \quad (4)$$

$$\vec{H} = \vec{H}_t + H_z\hat{a}_z = \sum_{i=1}^{N} I_i(z)\vec{\phi}_t^i(x,y)e^{-\gamma_i z} + \sum_{i=1}^{N} I_i(z)\phi_z^i(x,y)\hat{a}_z e^{-\gamma_i z}. \quad (5)$$

where N denotes the number of microstrips, γ_i represents the complex propagation constant of the ith microstrip in a single isolated condition, $\hat{a}_z$ is the unit vector in z (longitudinal) direction, $\vec{\phi}_t^i(x,y)$ and $\phi_z^i(x,y)$ are the normalized eigenfunction in the transversal (denoted by t) and longitudinal (denoted by z) directions, respectively, and $I_i(z)$ and Z_i represent the corresponding modal current and wave impedance, respectively. Substituting (4) and (5) into Maxwell's $\mathrm{curl}\vec{H}$ equation and taking the transverse part will lead to

$$\sum_{i=1}^{N}\left[\frac{dI_i(z)}{dz} - \gamma_i I_i(z)\right]\left[\hat{a}_z \times \vec{\phi}_t^i(x,y)\right] = -\sum_{i=1}^{N} I_i(z)\left[\nabla_t\phi_z^i(x,y)\times\hat{a}_z - jk_i\vec{\phi}_t^i(x,y)\right]. \quad (6)$$

Applying the orthogonality relationship to (6) allows us to obtain a system of linear first-order differential equations for the modal currents [23], [24] as follows:

$$\frac{dI_i(z)}{dz} = \gamma_i I_i(z) - \sum_{j=1}^{N} C_{i,j} I_j(z). \quad (7)$$

where $j = 1, 2, \ldots$, and N, but $j \neq i$. Essentially $C_{i,j}$, the mutual coupling coefficient between the ith element and the jth element, can be expressed as

$$C_{i,j} = \int_S \left[\nabla_t\phi_z^i(x,y)\times\vec{\phi}_t^{j*}(x,y)\right]\cdot\hat{a}_z ds - jk\int_S \vec{\phi}_t^i(x,y)\cdot\vec{\phi}_t^{j*}(x,y)ds. \quad (8)$$

Notice that the conversion of Maxwell's equations into the coupled-mode equations (7) is referred as the classical method. When dealing with the leaky modes, the transverse fields grow exponentially. Therefore, (8) is never used for computing coupling coefficients. Equation (7) is viewed phenomenologically [25]. Section III describes in detail how to obtain the coupling coefficients for the coupled-mode equation (7). Using a matrix notation, (7) is abbreviated as

$$\frac{d\bar{I}}{dz} = \overline{\overline{B}}\cdot\bar{I} \quad (9)$$

where $\overline{\overline{B}} = [\mathrm{diag}(\gamma_i) - [C_{i,j}]]$ and $\bar{I} = [I_1, I_2, \ldots, I_N]^t$. When all the coupled microstrips are in equal width, γ_i must be equal to γ, where γ is the complex propagation constant of the EH_1 leaky mode of a single microstrip. On the other hand, N coupled leaky-mode solutions should exist in the N-element array, denoted as $\lambda_1, \lambda_2, \ldots$, and λ_N. For a specific mode with complex propagation constant λ_i, the modal solution mandates

$$\frac{d\bar{I}}{dz} = \lambda_i\cdot\bar{I}. \quad (10)$$

Substituting (10) into (9) for $i = 1, 2, \ldots$, and N, we obtain

$$\left[\mathrm{diag}(\lambda) - \mathrm{diag}(\gamma_i) + [C_{i,j}]\right]\cdot\bar{I} = \bar{0}. \quad (11)$$

The source-free modal solutions require the nontrivial solutions for the modal current vector $\bar{I}(z)$. Therefore, (11) leads to a standard eigenvalue problem by solving

$$\det(\overline{\overline{A}}) = 0. \quad (12)$$

where $\overline{\overline{A}} = [\mathrm{diag}(\lambda) - \mathrm{diag}(\gamma_i) + [C_{i,j}]]$. The procedure for obtaining the complex propagation constants is transformed to a classical eigenvalue problem of which the eigenvalues (λ_i) are the complex propagation constants and the eigenvectors ($\bar{I}$) represent the modal currents flowing at the microstrips. The tedious root-finding process in the full-wave nonstandard eigenvalue problem is eliminated.

III. COUPLING COEFFICIENTS OF THE N PARALLEL COUPLED MICROSTRIPS

A. Formulas for the Coupling Coefficients

Combining (11) and (12) for $N = 2$, one may deduce the coupling coefficient of the two symmetrical microstrips at the higher order EH_1 leaky mode as

$$C_{1,2} = \frac{(\lambda_e - \lambda_o)}{2} \quad (13)$$

$$\lambda_e = \gamma + C_{1,2} \quad (14)$$

$$\lambda_o = \gamma - C_{1,2} \quad (15)$$

where $\lambda_{e(o)}$ denotes the even-mode (odd-mode) EH_1 solution. The corresponding normalized eigenvector $\bar{I}_e$ and $\bar{I}_o$ are, respectively,

$$\bar{I}_e = \left[\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right] \quad (16)$$

$$\bar{I}_o = \left[\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}}\right]. \quad (17)$$

$\bar{I}_{e(o)}$ implies that the leaky modal current distributions flowing at the symmetrical coupled microstrips are in-phase

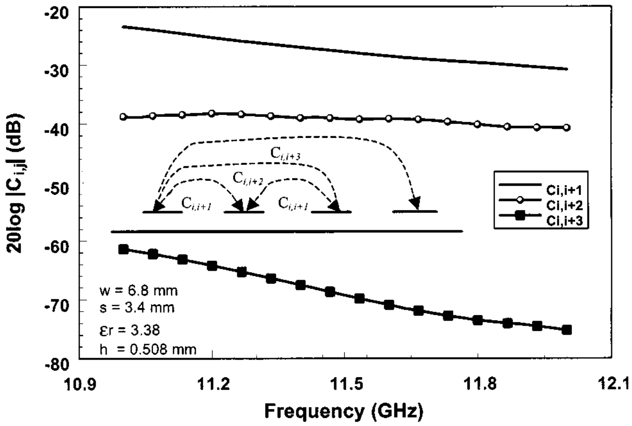

Fig. 3. Magnitude (in decibels) of the coupling coefficient $C_{i,i+j}$ obtained by the cases of $N = 2, 3,$ and 4, respectively.

(out-of-phase) with equal amplitude. Therefore, a perfect electric conductor (PEC) and a perfect magnetic conductor (PMC) can be inserted along the symmetry plane, as shown in the inset of Fig. 2, for the even and odd EH_1 modes, respectively. Applying the full-wave data already shown in Fig. 2, we obtain the coupling coefficient $C_{1,2}$ using (13). Applying the full-wave analyses again for a single identical isolated microstrip, the complex leaky-mode (EH_1) propagation constant γ is obtained. Substituting this value (γ) and the coupling coefficient ($C_{1,2}$) into (14) and (15), i.e., the complex propagation constants obtained by the standard eigenvalue problem [see (14) and (15)], we find excellent agreement between both approaches, as shown in Fig. 2.

We can generalize the above discussions for a two-element case study to a large coupled microstrip array. Given a system of N coupled microstrips of equal width (W) and spacing (S), we may rewrite (12) as

$$\det(\overline{\overline{A}}) = \lambda^N + a_{N-1}\lambda^{N-1} + \cdots + a_1\lambda^1 + a_0 = \prod_{i=1}^{N}(\lambda - \lambda_i). \tag{18}$$

where λ_i is the complex propagation constant of the ith leaky mode, for $i = 1$ to N. Expanding the determinant ($\det(\overline{\overline{A}})$), and comparing order by order at both sides of (18) for $N = 3$, we obtain the following equations for solving the unknown coupling coefficients of $C_{1,2}$ and $C_{1,3}$, representing the coupling coefficients of the adjacent elements and other-than-adjacent elements, respectively (see the inset of Fig. 3). Notably, $\gamma = \gamma_1 = \gamma_2 = \gamma_3$ and $C_{1,2} = C_{2,3}$ since each coupled microstrip has identical width (W) and is positioned at equal spacing (S). Thus, we obtain the following system of equations for solving $C_{1,2}$ and $C_{1,3}$:

$$\begin{cases} \gamma^3 - 2{C_{1,2}}^2 C_{1,3} - (2{C_{1,2}}^2 + {C_{1,3}}^2)\gamma = \lambda_1\lambda_2\lambda_3 \\ 3\gamma^2 - 2{C_{1,2}}^2 - {C_{1,3}}^2 = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3, \end{cases} \tag{19}$$

where $\gamma = (\lambda_1 + \lambda_2 + \lambda_3)/3$. Invoking the full-wave nonstandard eigenvalue approach for solving the solutions for λ_1, λ_2, and λ_3, respectively, we obtain the value of γ, which is the arithmetic mean of the three complex propagation constants of coupled EH_1 modes. Such observation can be generalized by the following expression:

$$\gamma = \sum_{i=1}^{N} \frac{\lambda_i}{N}. \tag{20}$$

Similarly, for $N = 4$, one may derive a system of equations as follows for solving the coupling coefficients $C_{1,2}$, $C_{1,3}$, and $C_{1,4}$, noticing that $C_{1,2} = C_{2,3} = C_{3,4}$, and $C_{1,3} = C_{2,4}$:

$$6\gamma^2 - 3C_{1,2}^2 - 2C_{1,3}^2 - C_{1,4}^2 = \lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_3 + \lambda_2\lambda_4 \tag{21a}$$

$$4\gamma^3 - 4C_{1,2}C_{1,3}C_{1,4} - 4C_{1,2}^2C_{1,3} - 2(3C_{1,2}^2 + 2C_{1,3}^2 + C_{1,4}^2) = (\lambda_1\lambda_2\lambda_3 + \lambda_1\lambda_2\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_2\lambda_3\lambda_4) \tag{21b}$$

$$\gamma^4 - (3C_{1,2}^2 + 2C_{1,3}^2 + C_{1,4}^2)\gamma^2 - 4(C_{1,2}C_{1,3}C_{1,4} + C_{1,2}^2C_{1,3})\gamma + C_{1,2}^4 + C_{1,3}^4 + C_{1,2}^2C_{1,4}^2 - 2C_{1,2}^2C_{1,3}^2 - 2C_{1,2}^3C_{1,4} - 2C_{1,2}C_{1,3}^2C_{1,4} = \lambda_1\lambda_2\lambda_3\lambda_4. \tag{21c}$$

B. Assessment of the Magnitudes of the Coupling Coefficients

Applying (13), (19), and (21), we obtain distinct coupling coefficients for $N = 2$, $N = 3$, and $N = 4$, respectively. Assume that a system of tightly coupled microstrips of width 6.8 mm ($W \cong 0.26\lambda_0$) and spacing 3.4 mm ($S \cong 0.13\lambda_0$) with number of elements equal to two, three, and four are under investigation. The magnitude of the coupling coefficients in decibel scale against frequencies of interests is plotted in Fig. 3 for $C_{1,2}$, $C_{1,3}$, and $C_{1,4}$, respectively. $C_{1,2}$ values of the coupling coefficients for the case study of $N = 2$, coincide with those for $N = 3$. $C_{1,3}$, one of data of the coupling coefficients for the case study of $N = 3$, also coincide with those for the case study of $N = 4$. Notice that the magnitude of $C_{1,3}$ ($C_{1,4}$) is one order of magnitude less than that of $C_{1,2}$ ($C_{1,3}$). This implies that the magnitude of $C_{i,i+j}$ decreases very rapidly in the order of (10^{-j}) as j increases, for $j = 1, 2, \ldots$ and ($N - 1$). When computing the eigenvalues (complex propagation constants) of a large array, the coupling coefficients obtained by assuming $N = 4$, i.e., $C_{i,j} = 0$ for $|i - j| > 3$, are adequate for practical accuracy.

C. Numerical Results and Validity Checks

The data shown in Fig. 2 have validated the eigenvalue approach for obtaining the complex propagation constants of the two-element symmetrical coupled microstrips. In this section, the confirmation of the proposed coupled-mode formulation is extended to the cases of $N = 3$ and 4. Since the microstrips are all identical and placed an equal distance apart, the solution for γ, i.e., the complex propagation constant of a single isolated microstrip of first higher order, must be the same for $N = 2, 3, \ldots$. N is an arbitrary positive integer. Fig. 4 plots the results obtained by (20), the arithmetic mean of the coupled complex propagation constants, showing excellent agreement for all frequencies of interests in the cases of $N = 2, 3,$ and 4. Notice that the full-wave solution for a single microstrip line is almost indistinguishable from the plots obtained by the coupled-mode approach. Such finding is important for conjecturing that the particular coupled microstrips system can be viewed as

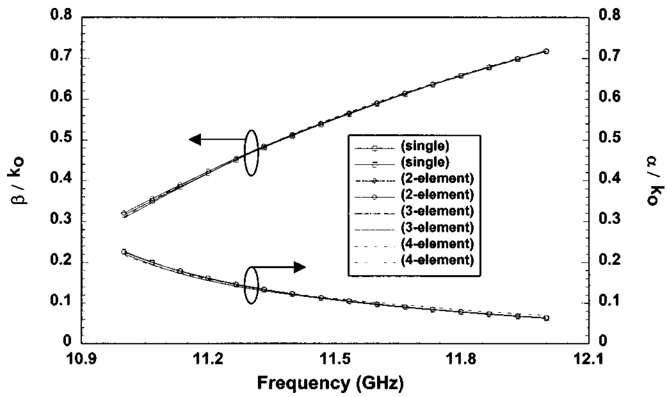

Fig. 4. Plots of the complex propagation constant of a single isolated microstrip at first higher order obtained by (20) using the data of the coupled mode solutions of the two-, three-, and four-element coupled microstrips of identical width and spacing.

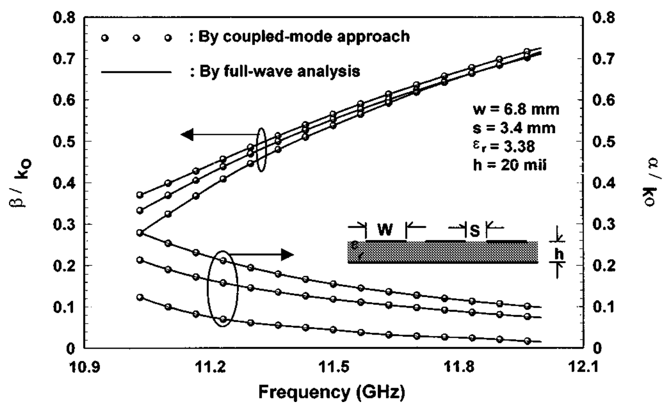

Fig. 5. Validity of the proposed coupled-mode approach is confirmed by studying a three-element array; data obtained by the full-wave analysis agree excellently with those by the coupled-mode approach considering all the coupling coefficients.

the mode coupling of a single leaky mode (EH_1) at each microstrip.

Figs. 5 and 6 validate the coupled-mode approach that transforms the conventional full-wave nonstandard eigenvalue problem into a standard eigenvalue problem for $N = 3$ and $N = 4$, respectively. The three (four) sets of data of the complex propagation constants obtained by the coupled-mode approach are virtually indistinguishable from those by using full-wave analyses, as shown in Fig. 6 for the entire leakage regime of interests. The coupled-mode approach is not only accurate for obtaining the modal solution, but also explicit for providing physical insights on the coupled lines behavior in terms of their corresponding propagation constants (eigenvalues) and modal current distributions (eigenvectors).

D. Error Analysis for Cases with Fewer Number of Coupling Coefficients

Section III-C shows the nearly error-free solutions of the coupled leaky EH_1 modes obtained by the coupled-mode approach provided that all coupling coefficients are considered. This section investigates the percentage of rms errors if the term of coupling coefficients is intentionally reduced. Given a system of a coupled four-element microstrip array, as studied in the previous section, the coupling coefficients of the particular case

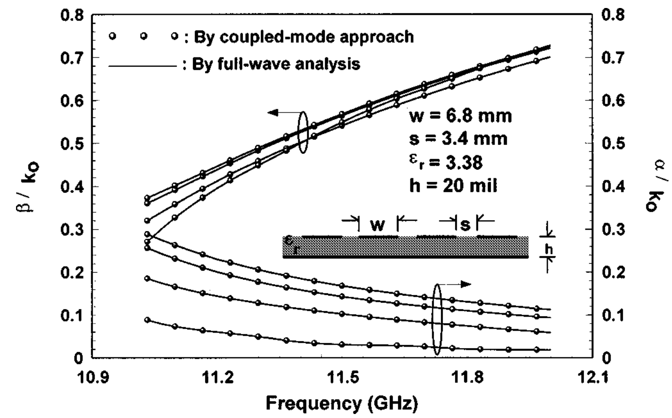

Fig. 6. Validity of the proposed coupled-mode approach is confirmed by studying a four-element array; data obtained by the full-wave analysis agree excellently with those by the coupled-mode approach considering all the coupling coefficients.

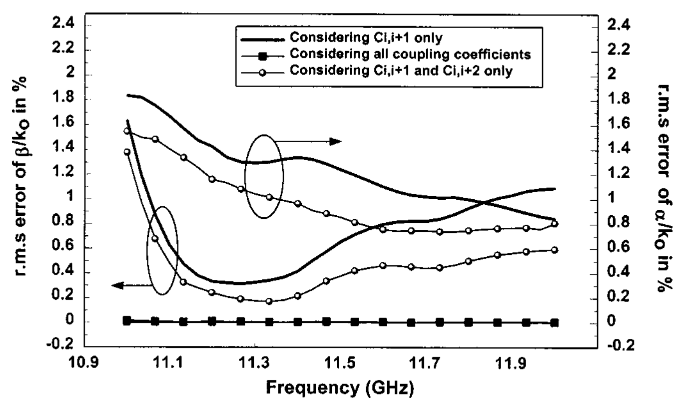

Fig. 7. Percentage of the rms error defined by (22) for the four-element array using one term ($C_{i,i+1}$), two terms ($C_{i,i+1}$ and $C_{i,i+2}$), and all coupling coefficients, respectively.

study contain the following terms $C_{1,2}$, $C_{1,3}$, and $C_{1,4}$. Meanwhile, the percentages of rms errors for the normalized phase constant and normalized attenuation constant are defined as follows:

$$\%\beta_{\text{rms}} = \sqrt{\frac{\sum_{i=1}^{N} |(\beta_i/k_0)_{\text{coupled-mode}} - (\beta_i/k_0)_{\text{full-wave}}|^2}{N}} \times 100\% \tag{22a}$$

$$\%\alpha_{\text{rms}} = \sqrt{\frac{\sum_{i=1}^{N} |(\alpha_i/k_0)_{\text{coupled-mode}} - (\alpha_i/k_0)_{\text{full-wave}}|^2}{N}} \times 100\%. \tag{22b}$$

The results of the coupled-mode approach assuming one term ($C_{1,2}$), two terms ($C_{1,2}$ and $C_{1,3}$), and three terms ($C_{1,2}$, $C_{1,3}$, and $C_{1,4}$) are plotted in Fig. 7, which shows the nearly error-free solutions for $\%\beta_{\text{rms}}$ and $\%\alpha_{\text{rms}}$, when all coupling coefficients ($C_{1,2}$, $C_{1,3}$, and $C_{1,4}$) are considered. The worst rms errors occur at the lowest frequency near 11.0 GHz on the left-hand side of Fig. 7, where 1.7% (1.86%) and 1.4% (1.55%) errors are observed for $\%\beta_{\text{rms}}$ ($\%\alpha_{\text{rms}}$) when coupling coefficients of one and two terms are considered, respectively. For a typical leaky-wave antenna design of this study, the normalized attenuation constant is usually chosen to be about 0.1 [21]. Corresponding the frequency of operation of the presented coupled microstrips is near 11.7 GHz (see Figs. 5 or 6). Notice that the two-term approximation results in a significant reduction in $\%\beta_{\rm rms}$ against one-term approximation, i.e., from 0.96% to 0.48% near 11.7 GHz. For practical use of the coupled-mode approach, at least two terms of coupling coefficients are recommended for obtaining solutions for all the coupled leaky modes. What follows is an array design example described in the following section, in which we apply three coupled coefficients to ensure accuracy.

IV. Microstrip Leaky-Mode Array Design Subject to Arbitrary Excitations—Theory

Given a microstrip leaky-mode array, the radiation fields are characterized by the modal current distributions (eigenvectors), $I_i(z)$, for $i = 1, 2, \ldots$, and N. Considering an arbitrary input current excitations $\bar{i}^{\rm inc}(0)$ at the incident plane, where $z = 0$, (11) can be modified as

$$\bar{\bar{A}} \cdot \bar{I}(z) = \bar{i}^{\rm inc}(0) \tag{23}$$

Notice that the square matrix $\bar{\bar{A}}$ is diagonalizable. If $\lambda_1, \lambda_2, \ldots$, and λ_N are the eigenvalues of $\bar{\bar{A}}$ and $\bar{I}^{(1)}, \bar{I}^{(2)}, \ldots$, and $\bar{I}^{(N)}$ are the associated linearly independent eigenvectors, the matrix $\bar{\bar{P}}$, formed by applying $\bar{I}^{(j)}$ as the jth column, can be used to diagonalize $\bar{\bar{A}}$. That is,

$$\bar{\bar{A}} = \bar{\bar{P}} \cdot \bar{\bar{D}} \cdot \bar{\bar{P}}^{-1} \tag{24}$$

where $\bar{\bar{D}}$ is the diagonal matrix in the form of $\mathrm{diag}[e^{\lambda_i z}]$ and $\bar{\bar{P}}^{-1}$ is the inverse matrix of $\bar{\bar{P}}$. Upon substituting (24) into (23), one derives the excited current vector

$$\bar{I}(z) = \bar{\bar{P}} \cdot \bar{\bar{D}}^{-1} \cdot \bar{\Omega} = \sum_{i=1}^{N} \Omega_i e^{-\lambda_i z} \bar{i}^{\rm inc} \tag{25}$$

and the coefficient vector

$$\bar{\Omega} = \bar{\bar{P}}^{-1} \cdot \bar{i}^{\rm inc}(0). \tag{26}$$

Given a known arbitrary signal $\bar{i}^{\rm inc}(0)$, the coefficient vector $\bar{\Omega}$ can be obtained by (26) and, therefore, the resultant current distributions $\bar{I}(z)$ over the entire array are fully determined by (25). Applying the Fourier transform to the equivalent current sources on aperture, we obtain the far-zone electric field contributed by the jth microstrip of length L [26], [27] as follows:

$$\begin{aligned} E_r &\cong E_\theta \cong 0 \\ E_\phi^j &\cong -jE_o \frac{k_0 h e^{-jkr}}{\pi r} \\ &\quad \cdot \left\{ \sin\theta \left[\frac{\sin(Y)}{Y}\right] \sum_{i=1}^{N} \Omega_i P_{ij} \left[\frac{e^{Z_i L} - 1}{Z_i}\right] \right\} \\ &\quad \cdot \cos\left(\frac{k_0 w \sin\theta\cos\phi}{2}\right) \end{aligned} \tag{27}$$

where

$$\begin{aligned} Y &= k_0 h \sin\theta \sin\phi \\ Z_i &= j(k_0\cos\theta - \beta_i) - \alpha_i \\ k_0 &= \frac{2\pi}{\lambda_0} \ (\lambda_0 \text{ is the free-space wavelength}) \end{aligned}$$

where β_i and α_i are the propagation constant and attenuation constant of the ith coupled leaky mode, respectively. The total far-zone electric field E_ϕ^T is the superposition of the N-element microstrip leaky-mode array written as

$$E_\phi^T = \sum_{j=1}^{N} E_\phi^j \exp(jk_0 x_j \sin\theta\cos\phi) \tag{28}$$

where x_j is the center location of jth microstrip in the x-direction. Equations (27) and (28) explicitly relate the excited modal currents against the known current source $\bar{i}^{\rm inc}(0)$ to the far-field radiation pattern.

V. Example of a Corporate-Fed Microstrip Leaky-Mode Array Design

Fig. 1 shows a typical corporate-fed microstrip leaky-mode array. A proof-of-concept design of an eight-element array with $W = 3.304$ mm, $L = 115$ mm, and $S = 13.38$ mm was fabricated on a 25-mil-thick Duroid 6010 substrate with relative dielectric constant 10.2. Careful designs were exercised for obtaining the input voltage standing wave ratio (VSWR) less than 1.2 at microstrip input feed and isolation better than -17 dB at divider outputs (made of a CPW T-junction) at the operation center frequency. Therefore, the mutual coupling due to the feeding network, as shown in Fig. 1, is minimized and can be neglected when a signal of a CW source is applied to the input port of the array during the measurement. The coupled-mode approach will explicitly show what coupled leaky modes contribute the radiation subject to a particular form of excitations. An eight-element leaky-mode array should have eight sets of eigenvectors $\bar{I}^{(i)}$, for $i = 1, 2, \ldots$, and 8. Each represents the modal currents of a specific leaky mode with respect to an eigenvalue λ_i (the complex propagation constant), as shown in Table I. Since the corporate-fed network results in signals of equal amplitude and phase reaching each microstrip, the incoming current source vector $\bar{i}^{\rm inc}$ can be expressed as $1/\sqrt{8}[1, 1, 1, 1, 1, 1, 1, 1]^{\rm T}$. By invoking (26), we obtain the coefficient vector $\bar{\Omega}$, as tabulated in Table II, which explicitly shows that leaky modes numbered 2, 4, 6, and 8 are not excited. Only four leaky modes numbered 1, 3, 5, and 7 are presented in the corporate-fed leaky-mode array. The leaky mode denoted as λ_1 determinates the corporate-fed array since the magnitude of Ω_1 is much greater than that of Ω_3, Ω_5, and Ω_7. Substituting the computed coefficient vector $\bar{\Omega}$ and the eigenvalues of the excited leaky modes into (27) and (28), we obtain the far-field radiation as shown in Fig. 8, which also plots the measured radiation pattern and that obtained by the unit-cell approach described in [9] and [22]. Notice that the H-plane (y–z-plane) far-field patterns are normalized and plotted at the observation angle off broadside from 25° to 65° for close comparison. Although the unit-cell approach shows

TABLE I
EIGENVECTORS $\bar{I}^{(j)}$ AND EIGENVALUES (COMPLEX PROPAGATION CONSTANTS) OF AN EIGHT-ELEMENT CORPORATE-FED ARRAY ($W = 3.304$ mm, $S = 13.38$ mm, $h = 25$ mil, $\varepsilon = 10.2$, AND f = 12.16 GHz

Ele.No / $\bar{I}^{(i)}$	Element 1	Element 2	Element 3	Element 4	Element 5	Element 6	Element 7	Element 8	Eigen-values ($\gamma_i=\alpha_i+j\beta_i$)
$I^{(1)}$	0.16∠0°	0.30∠0°	0.41∠0°	0.46∠0°	0.46∠0°	0.41∠0°	0.30∠0°	0.16∠0°	.153+j0.710
$I^{(2)}$	0.30∠78°	0.46∠78°	0.41∠78°	0.16∠78°	0.16∠-102°	0.41∠-102°	0.46∠-102°	0.30∠-102°	.146+j0.733
$I^{(3)}$	0.41∠-102°	0.41∠-102°	0.00∠0°	0.41∠78°	0.41∠78°	0.00∠0°	0.41∠-102°	0.41∠-102°	.159+j0.687
$I^{(4)}$	0.46∠-23°	0.16∠-23°	0.41∠157°	0.30∠157°	0.30∠-23°	0.41∠-23°	0.16∠157°	0.46∠157°	.140+j0.756
$I^{(5)}$	0.46∠78°	0.16∠-102°	0.41∠-102°	0.30∠78°	0.30∠78°	0.41∠-102°	0.16∠-102°	0.46∠78°	.164+j0.669
$I^{(6)}$	0.16∠-102°	0.30∠78°	0.41∠-102°	0.46∠78°	0.46∠-102°	0.41∠78°	0.30∠-102°	0.16∠78°	.134+j0.773
$I^{(7)}$	0.30∠-43°	0.46∠137°	0.41∠-43°	0.16∠137°	0.16∠137°	0.41∠-43°	0.46∠113°	0.30∠-43°	.167+j0.658
$I^{(8)}$	0.41∠-160°	0.41∠20°	0.00∠0°	0.41∠-160°	0.41∠20°	0.00∠0°	0.41∠-160°	0.41∠20°	.131+j0.785

TABLE II
COEFFICIENT VECTOR [Ω] OF THE PARTICULAR EIGHT-ELEMENT ARRAY UNDER STUDY

i	1	2	3	4	5	6	7	8
Ω_i	0.943∠0°	0.000∠0°	0.29∠106°	0.00∠0°	0.14∠-74°	0.00∠0°	0.06∠-72°	0.00∠0°

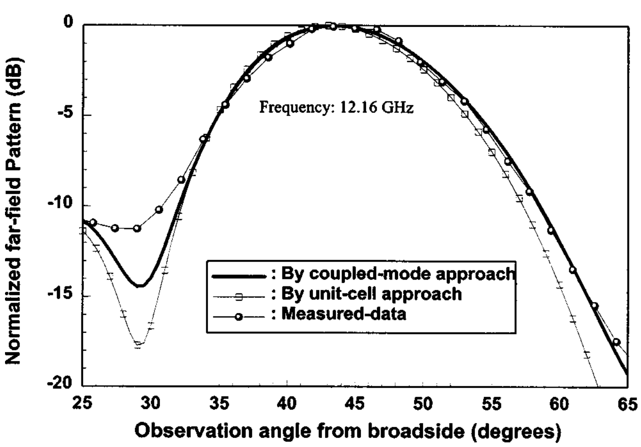

Fig. 8. An almost replica of the measured radiation pattern in the main beam from 33° to 65° is obtained by the coupled mode analyses of the eight-element leaky-mode array. The well-known unit-cell approach [9], [21], however, predicts a narrower bandwidth with a slight shift in the beam-pointing angle.

close agreement with the measured data, the coupled-mode approach, which accounts for all the contributing coupled leaky modes, offer further improvement. An excellent agreement is observed for the radiation pattern in the main beam region from 33° to 65° at the H-plane cut, between the measured data and those obtained by the coupled-mode approach. Importantly, the mutual coupling affects the array on the radiation pattern by slightly shifting the beam pointing direction from 43.2° to 44.7° and increasing 3-dB beamwidth from 13.5° to 14.7°. The latter effect can be attributed to the contribution of the excited leaky modes, possessing slightly different phase propagation constants. This effect cannot be predicted by the unit-cell approach (a single leaky-mode analysis) but, however, can be accurately predicted by using the coupled-mode approach (four excited leaky-mode analyses).

VI. CONCLUSION

This paper has presented and validated the coupled-mode approach for the analysis and design of the large leaky-mode array from the perspective of mode coupling of the leaky modes of higher order. Detailed formulation is derived to transform the well-known nonstandard eigenvalue problem using the full-wave approach to the standard eigenvalue problem by the coupled-mode approach, which simultaneously obtains all the eigenvalues (complex propagation constants) and eigenvectors (modal current distributions) regardless of the size of the array (N). Consequently the proposed coupled-mode approach greatly reduces the tedious root-searching process of the nonstandard eigenvalue problem, which may become unmanageable for a large array problem. Closed-form equations for obtaining the coupling coefficients inhering in the microstrip array are derived and the theoretical results indicate that the magnitude of the coupling coefficient $C_{i,i+j}$ between elements i and $i+j$ decreases at the order of 10^{-j} as j increases. Extensive study for validating the new approach is conducted both numerically and experimentally, clearly indicating that at least two coupling coefficients are required for obtaining accurate complex propagation constants with rms errors less than 1% for most leaky region of the particular array under investigation. Finally, the investigation of a corporate-fed eight-element array using the coupled-mode approach explicitly shows which leaky modes are excited at what levels and demonstrates that the effect of mutual coupling on the array performance can be accurately determined.

REFERENCES

[1] G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*. Norwood, Mass.: Artech House, 1980.

[2] E. M. T. Jones and J. T. Bolljahn, "Coupled-strip-transmission line filters and directional couplers," *IRE Trans. Microwave Theory Tech.*, vol. MTT-4, pp. 78–81, Apr. 1956.
[3] S. B. Cohn, "Parallel-coupled transmission-line-resonator filters," *IRE Trans. Microwave Theory Tech.*, vol. MTT-6, pp. 223–231, Apr. 1958.
[4] H. A. Haus and C. G. Fonstad, "Three waveguide coupler for improved sampling and filtering," *IEEE J. Quantum Electron.*, vol. QE-17, pp. 2321–2325, Dec. 1981.
[5] C. Yeh and H. F. Taylor, "Contradirectional frequency-selective couplers for guided-wave optics," *Appl. Opt.*, vol. 19, pp. 2848–2855, 1980.
[6] M. Zhang and E. Garmire, "Single-mode fiber-film directional coupler," *J. Lightwave Technol.*, vol. LT-5, pp. 260–267, Feb. 1987.
[7] T. Itoh, "Spectral domain immittance approach for dispersion characteristics of generalized printed transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 733–736, July 1980.
[8] V. K. Tripathi and H. Lee, "Spectral-domain computation of characteristic impedances and multiple ports coupled microstrip," *IEEE Trans. Microwave Theory Tech.*, vol. 37, pp. 215–221, Jan. 1989.
[9] A. A. Oliner and K. S. Lee, "The nature of the leakage from higher modes on microstrip line," in *IEEE MTT-S Int. Microwave Symp. Dig.*, 1983, pp. 721–726.
[10] H. Shigesawa, M. Tsuji, and A. A. Oliner, "Simultaneous propagation of both bound and leaky dominant modes on printed-circuit lines: A new general effect," *IEEE Trans. Microwave Theory Tech.*, vol. 43, pp. 3007–3019, Dec. 1995.
[11] L. Carin and N. K. Das, "Leaky waves on broadside-coupled microstrip," *IEEE Trans. Microwave Theory Tech.*, vol. 40, pp. 58–66, Jan. 1992.
[12] D. P. Nyquist, J. S. Bagby, C. H. Lee, and Y. Yuan, "Identification of propagation regimes on integrated microstrip transmission line," *IEEE Trans. Microwave Theory Tech.*, vol. 41, pp. 1887–1893, Nov. 1993.
[13] D. R. Jackson, D. P. Nyquist, F. Mesa, and C. Di Nallo, "The role of the SDP in the excitation of leaky modes on printed-circuit lines," in *URSI Proc. Int. Electromag. Theory Symp.*, Thessaloniki, Greece, May 1998, pp. 494–496.
[14] C. -K. C. Tzuang and C. -N. Hu, "The mutual coupling effects in large microstrip leaky-mode array," in *IEEE MTT-S Int. Microwave Symp. Dig.*, 1998, pp. 1791–1793.
[15] C. -H. Lee and C. -C. Tu, "Dispersion and leakage analysis of coupled microstrip transmission lines," in *Proc. Asia–Pacific Microwave Conf.*, vol. 1/5, 1993, pp. 66–69.
[16] W. -T. Lo and C. -K. C. Tzuang, "A new full-wave integral equation method for the analysis of coplanar strip circuit using mixed-potentials eigenfunction expansion technique," in *IEEE MTT-S Int. Microwave Symp. Dig.*, 1993, pp. 1539–1542.
[17] J. R. Pierce, "Coupling of modes of propagation," *J. Appl. Phys.*, vol. 25, no. 2, pp. 179–183, Feb. 1954.
[18] C. -K. C. Tzuang and J. -M. Lin, "On the mode-coupling formation of complex modes in a nonreciprocal finline," *IEEE Trans. Microwave Theory Tech.*, vol. 41, pp. 1400–1408, Aug. 1993.
[19] R. H. Jansen, "The spectral-domain approach for microwave integrated circuits," *IEEE Trans. Microwave Tech.*, vol. MTT-33, pp. 1043–1056, Oct. 1985.
[20] H. Shigesawa, M. Tsuji, P. Lampariello, F. Frezza, and A. A. Oliner, "Coupling between different leaky-mode types in stub-loaded leaky waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. 42, pp. 1548–1560, Aug. 1994.
[21] L. Carin, G. W. Slade, and K. J. Webb, "Mode coupling and leakage effects in finite-size printed interconnects," *IEEE Trans. Microwave Theory Tech.*, vol. 46, pp. 450–457, May 1998.
[22] C. -N. Hu and C. -K. C. Tzuang, "Microstrip leaky-mode antenna array," *IEEE Trans. Antennas Propagat.*, vol. 45, pp. 1698–1699, Nov. 1997.
[23] H. A. Haus and W. Huang, "Coupled-mode theory," *Proc. IEEE*, vol. 79, pp. 1505–1518, Oct. 1991.
[24] A. Hardy and W. Streifer, "Coupled-mode theory of parallel waveguides," *J. Lightware Technol.*, vol. LT-13, pp. 1135–1146, Oct. 1985.
[25] H.-C. Huang, *Microwave Approach to Highly Irregular Fiber Optics*. New York: Wiley, 1998, ch. 1.
[26] C. A. Balanis, *Antenna Theory Analysis and Design*. New York: Wiley, 1982, ch. 12, pp. 621–630.
[27] G. -J. Chou and C. -K. C. Tzuang, "An integrated quasi-planar leaky-wave antenna," *IEEE Trans. Antenna Propagat.*, vol. 44, pp. 1078–1085, Aug. 1996.

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