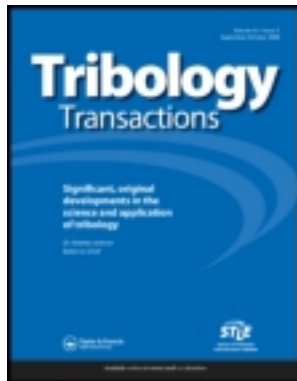


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### The Computer Simulation of Lubricated Cold Rolling Processes

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# The Computer Simulation of Lubricated Cold Rolling Processes<sup>©</sup>

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*A realistic friction model for lubricated cold rolling process with front and back tension is developed. The model combines a rigid-plasticity finite element code with a lubrication analysis. The rolls are assumed to be rigid and uniform deformation and isothermal conditions are assumed to prevail. The surface roughness effect on lubricant flow is included by using a average Reynolds equation. The active lubrication regime and appropriate friction factor can then be expressed in terms of internal interface variables in addition to the more traditional external variables. These internal variables include mean lubricant film thickness, workpiece roughness and roll roughness. The external variables are interface pressure and sliding speed. Numerical results using the coupled code such as roll separating force and roll torque under different rolling conditions are compared with experimental*

*investigation. The simulation results are in good agreement with experimental data.*

## KEYWORDS

Surface Roughness; Lubrication Analysis; Friction

## INTRODUCTION

The rolling of flat products is by far the most important metal forming processes. In most rolling processes, the sheet is passed through the roll gap, back and forth, while the roll gap is progressively reduced to achieve a desired sheet thickness reduction with each pass. Lubricants are used in metal rolling to act as coolant, to control friction between the roll and sheet and to protect the roll and strip surfaces. There is considerable interaction between the asperities on the roll and sheet surfaces, but there is also some hydrodynamic action of the lubricant. Too thick a lubricant film will result in a matt surface due to insufficient constraint by the roll surface to suppress free surface deformation of the sheet, while too thin a film might allow direct metal-to-metal contact

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## NOMENCLATURE

$a$	= roll radius
$h$	= film thickness
$h_1$	= film thickness at the inlet edge of the work zone
$n$	= strain hardening coefficient
$p$	= hydrodynamic pressure at the interface
$s$	= back tension stress
$t$	= front tension stress
$u_1$	= sheet velocity at the inlet
$u_2$	= sheet velocity at the outlet edge
$x$	= distance from the line joining roll centres
$x_1$	= distance to the inlet edge of the work zone
$x_2$	= distance to the outlet edge of the work zone
$y$	= sheet thickness
$y_1$	= inlet sheet thickness
$y_2$	= outlet sheet thickness
$U_1$	= surface velocity of the rolls

$U_2$	= surface velocity of the billet
$\bar{u}$	= average velocity of the surfaces
$V$	= rolling speed
$Y$	= material constant
$Y_0$	= yield strength of the sheet
$\gamma$	= pressure coefficient
$\bar{\sigma}$	= effective stress
$\mu$	= friction coefficient
$\eta$	= viscosity of the lubricant
$\eta_0$	= viscosity of the lubricant at ambient pressure
$\tau_f$	= friction stress
$\tau_h$	= hydrodynamic stress
$\sigma$	= composite surface roughness
$\sigma_1$	= surface roughness of the roll
$\sigma_2$	= surface roughness of the billet
$\phi_x$	= pressure flow factor
$\phi_s$	= shear flow factor

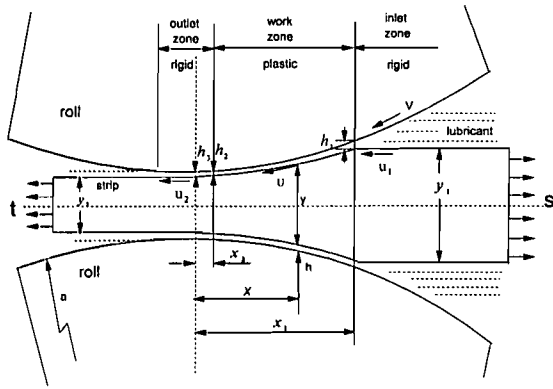


Fig. 1—Schematic representation of the rolling process.

which is responsible for roll wear, sheet surface scoring and other surface related defects. Improvements in sheet surface quality such as cleanliness, aesthetic appearance, and functional characteristics for secondary forming processes and productivity in industrial metal rolling require a good understanding of mechanics of this lubrication process (Wilson, 1978).

Due to the development of digital computers in the past few years, the finite element method has been extensively used for metal forming analysis and computer-based simulations have already proven themselves as valuable tools for product, process and tooling design. If such simulations are to provide accurate information, they must incorporate realistic models of sheet-roll friction. Modeling of metal rolling is at present restricted by the accuracy of the friction modeling. Most current simulations use relatively simple friction models such as Amontons-Coulomb constant coefficient of friction. A more realistic friction model should take account of the fundamental processes involved in friction and lubrication at the billet-roll interface.

Hsu and Wilson (1993) and Wilson, et al. (1995) developed a realistic friction model in lubricated sheet metal forming. The new model was coupled with a finite element code and applied to an axisymmetric stretch forming operation. Numerical results using the coupled codes showed excellent agreement with measured strain distribution over a range of operating condition. Hsu and Lee (1997a, 1997b) analyzed the simple upsetting operation by using the same strategy and the predictions from the model such as the distribution of the friction stress and normal pressure showed good correlation compared with the experimental measurements.

The present paper describes the development of a full film friction model for use in the computer simulation of lubricated cold rolling process. The influence of surface topography on lubricant flow in the thick film and thin film regime is included and the mean lubricant film thickness is calculated using hydrodynamic theory which includes roughness effects. The hydrodynamic shear stress is then expressed as a function of internal variables (mean lubricant film thickness, workpiece roughness and roll roughness) and external variables (interface pressure and sliding velocity). Although the asperity contact is relatively frequent in practice, the

friction stress caused by the plowing and adhesion effect in the mixed and boundary regime is not modeled in the current formulation. Several experimental data are used to check the validity of the proposed model. The behavior of forward slip and average friction coefficient during rolling are investigated. The effects of rolling speed, front and back tension, rolling oil viscosity and roll diameter are examined. The results of computer simulation agree well with the experimental data.

## LUBRICATION ANALYSIS

The process to be analyzed is shown in Fig. 1. The axes of these rolls are set horizontally, but one above the other, in the same vertical plane, and a billet is passed between them. The billet has a back tension stress,  $s$ , and a front tension stress,  $t$ , applied to it. Hydrodynamic lubrication between the billet and rolls may be characterized by Reynolds equation as inlet zone, work zone and outlet zone (Wilson and Walowit, 1971 and Wilson and Murch, 1976). In the inlet zone, the lubricant is drawn into the spaces between the billet and rolls by hydrodynamic wedge action. The lubricant film pressure rises rapidly until the billet yields at the inlet edge of the work zone. The billet deforms plastically in the work zone and the outlet zone in which the material again becomes rigid is followed as the interface contact pressure drops to ambient atmosphere.

If the roughness of the surface is significant compared with the mean lubricant film thickness, an average Reynolds equation which allows for the influence of roughness on lubricant flow should be used. Patir and Cheng (1978, 1979) derived the average Reynolds equation and can be expressed as

$$\frac{\partial}{\partial x} \left( \phi_x \frac{h^3}{12\eta} \frac{\partial p}{\partial x} \right) = \bar{U} \frac{\partial h}{\partial x} + \frac{U_1 - U_2}{2} \sigma \frac{\partial \phi_s}{\partial x} + \frac{\partial h}{\partial t} \quad [1]$$

where  $h$  is the mean film thickness,  $x$  is the distance along the film,  $U_1$  and  $U_2$  are the surface speeds and  $\bar{U}$  is the average speed. The composite surface roughness  $\sigma$  is defined as

$$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2} \quad [2]$$

where  $\sigma_1$  and  $\sigma_2$  are the surface roughness of the rolls and billet (Rq values).  $\phi_x$  and  $\phi_s$  are the pressure and shear flow factors which compensate for the effect of roughness and may be expressed as function of  $h$  and  $\sigma$ . If the roughness are isotropic the authors may use the simple expressions provided by Tripp (1983)

$$\phi_x = \frac{3}{2} \frac{\sigma}{h} \frac{\sigma_1^2 - \sigma_2^2}{\sigma^2} \quad [3]$$

$$\phi_s = \frac{3}{2} \frac{\sigma}{h} \frac{\sigma_1^2 - \sigma_2^2}{\sigma^2} \quad [4]$$

The viscosity of the lubricant  $\eta$  is assumed as Newtonian and

governed by

$$\eta = \eta_0 e^{\gamma p} \tag{5}$$

where  $\eta_0$  is the viscosity at atmospheric pressure and  $\gamma$  is the pressure coefficient of viscosity.

Since the lubricant film thickness is small compared with the reduction in billet thickness and the work zone is short compared with the circumference of the rolls, the film thickness in the inlet zone is assumed as

$$h = h_1 + \frac{x_1}{a}(x - x_1) \tag{6}$$

where  $x_1$  is the distance from the inlet edge of the work zone to the line joining roll centers and  $a$  is the roll radius. The boundary conditions are

$$\text{at } x = x_1, h = h_1, P = P_1 \text{ and } dp/dx = 0 \tag{7}$$

$$\text{at } x = \infty, h = \infty \text{ and } p = 0 \tag{8}$$

and  $p_1$  is the yielding stress of the billet. The inlet film analysis involves the integration of the average Reynolds Eq. [1] with the film thickness  $h$  given by Eq. [6] and viscosity  $\eta$  given by Eq. [5]. After applying the boundary conditions given in Eqs. [7] and [8], the governing equation of the inlet film thickness is expressed as

$$\frac{-1}{\gamma} (e^{\gamma p} - 1) = -\frac{a^2 \eta_0}{x_1} \int_{x_1}^x \frac{1}{\phi^2 h^3} \left[ 6(V+u)(h-A) + 6(V-u) \left( \frac{3\sigma_0 - 3\sigma}{2h} \right) \right] dx + \int_{x_1}^x \frac{1}{\phi^2 h^3} \left[ 6(V+u)(h-A) + 6(V-u) \left( \frac{3\sigma_0 - 3\sigma}{2h} \right) \right] dx \tag{9}$$

where  $u$  is the velocity of billet at inlet edge and

$$h_c = 10 \sigma \tag{10}$$

and

$$\sigma_0 = \sigma_1^2 - \sigma_2^2 \tag{11}$$

Equation [9] can be simplified by using integration by parts and rewritten as

$$\begin{aligned} e^{\gamma p} - 1 = & 6(V+u) \frac{a^2 \eta_0}{x_1} \left[ \frac{1}{\sqrt{6\sigma}} \left( \ln \frac{h - \sqrt{\frac{3}{2}} \sigma}{h - \sqrt{\frac{3}{2}} \sigma} - \ln \frac{h + \sqrt{\frac{3}{2}} \sigma}{h + \sqrt{\frac{3}{2}} \sigma} \right) + \frac{1}{3\sigma^2} \left( 2 \ln \frac{h - \sqrt{\frac{3}{2}} \sigma}{h - \sqrt{\frac{3}{2}} \sigma} - \ln \frac{h + \sqrt{\frac{3}{2}} \sigma}{h + \sqrt{\frac{3}{2}} \sigma} \right) \right] \\ & + \frac{9\sigma_0(V-u) a^2 \eta_0}{x_1} \left[ \frac{1}{3\sigma^2} \left( \frac{2}{h} - \frac{2}{h} \right) + \sqrt{\frac{2}{3}} \frac{1}{\sigma} \ln \frac{h - \sqrt{\frac{3}{2}} \sigma}{h - \sqrt{\frac{3}{2}} \sigma} - \sqrt{\frac{2}{3}} \frac{1}{\sigma} \ln \frac{h + \sqrt{\frac{3}{2}} \sigma}{h + \sqrt{\frac{3}{2}} \sigma} \right] + \frac{1}{3\sigma^2 h} \left( 2 \ln \frac{h - \sqrt{\frac{3}{2}} \sigma}{h - \sqrt{\frac{3}{2}} \sigma} - \ln \frac{h + \sqrt{\frac{3}{2}} \sigma}{h + \sqrt{\frac{3}{2}} \sigma} \right) \\ & + \frac{a^2 \eta_0}{x_1} \left[ 6(V+u) \left( \frac{1}{h} - \frac{A}{2h^2} \right) + 9\sigma_0(V-u) \left( \frac{1}{3h} + \frac{1}{2h^2 A} \right) \right] \end{aligned} \tag{12}$$

Thus the film thickness at the inlet edge of the work zone is determined by solving Eq. [12].

In the work zone the average Reynolds equation may be written

$$\phi_x \frac{h^3}{\eta} \frac{dp}{dx} = 6[(U+V)h - (u_1+V)h_1] + 9\sigma_0 \left[ (V-U) \frac{1}{h} - (V-u_1) \frac{1}{h_1} \right] \tag{13}$$

If work hardening is low, the pressure gradient in the work zone is small and can be neglected, the distribution of the film thickness in the work zone is calculated as

$$h = \frac{\left[ 6(u_1+V)h_1 + \frac{9\sigma_0}{h_1}(V-u_1) \right] + \sqrt{\left[ 6(u_1+V)h_1 + \frac{9\sigma_0}{h_1}(V-u_1) \right]^2 - 216\sigma_0(U+V)(V-U)}}{12(U+V)} \tag{14}$$

After the film thickness is determined, the friction stress between the rolls and billets is largely due to viscous shear and the hydrodynamic friction is expressed as

$$\tau_f = \tau_h = \phi_f \eta \frac{U_1 - U_2}{h} \tag{15}$$

and  $\phi_f$  is the shear stress factor which may be evaluated from the expression given by Patir and Cheng (1978)

$$\phi_f = \frac{35}{32} z \left\{ (1-z^2)^3 \ln \left( \frac{z+1}{z-1} \right) + \frac{z}{15} [66 + z^2(30z^2 - 80)] \right\} \tag{16}$$

where  $z$  is the non-dimensional film thickness defined by

$$z = \frac{h}{3\sigma} \tag{17}$$

## RESULTS AND DISCUSSIONS

A rigid-plasticity finite element code assuming plane-strain rolling is developed to couple the lubrication/friction models described above. The basic strategy for coupling the deformation and lubrication/friction models was to use external variables from the F.E.M. deformation model as inputs to the lubrication/friction model (Wilson, et al., 1995). The initial mesh is shown in Fig. 2(a) and only one-half of the billet is needed for the analysis due to the symmetry of the deformation. The number of elements is between 245 and 745 which depends on the arc of contact, the thickness of the billet and the ratio of reduction. The stress-strain relation used for computation is of the following type.

$$\bar{\sigma} = Y_0 (1 + A \bar{\epsilon}^n) \tag{18}$$

where  $n$  and  $A$  are constants and are determined from the curves given in the literature.

The simulation starts with the first two columns of elements fed into the roll gap as shown in Fig. 2(a). Then the billet moves forward incrementally and the velocity field obtained in one step is used as the initial approximation for the next step. The deformed mesh system after 15 to 20 steps is shown in Fig. 2(b) and a steady-state condition is obtained which the billet is drawn into the roll gap completely. The velocity field in the deforming material and external forces along the arc of contact were obtained. From these quantities, roll separating force, roll torque,

$y_1$ (mm)	0.254
$Y$ (MPa)	552
$s$ (MPa)	138
$t$ (MPa)	110
Reduction	0.2
$a$ (mm)	76.2
$V$ (m/s)	10.160
$\eta_0$ (Mpa.s)	6.9E-8
$\gamma$ (Mpa) <sup>-1</sup>	1.45E-2

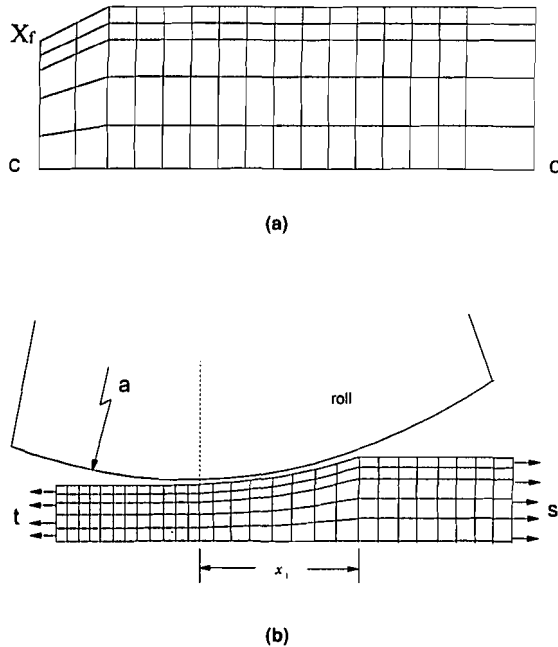


Fig. 2—Rolling process.  
 (a) Initial mesh system of the rolling process  
 (b) deformed mesh system of the rolling process

as well as stress and strain fields, are calculated.

The analysis of the lubricant film thickness starts as the steady-state is reached. The interface pressure at the edge of inlet as well as the property of lubricant and surface topography are substituted into Eq. [12] to calculate the film thickness at the inlet edge first. Secondly, the distribution of film thickness along the arc of the contact is determined by solving Eq. [14]. The resulting local friction stress is then calculated according Eq. [15] and pass back to the deformation model as the deformation proceeds.

Wilson and Walowit (1971) developed a hydrodynamic lubrication theory for sheet rolling with front and back tension and considered a numerical example to calculate the film thickness, pressure and friction stress in the typical rolling practice. The parameters are listed in Table 1. The perfect plasticity material model is adopted in the simulation which material constant  $A$  is assumed as 0. The comparisons between the results from the current simulation and their analysis are shown in Figs. 3 and 4. Figure 3 illustrates the distribution of the pressure and friction stress and the distribution of the film thickness is shown in Fig. 4. Since the lubrication model is based on the thick film regime in both results, they displayed a similar behavior close to each other

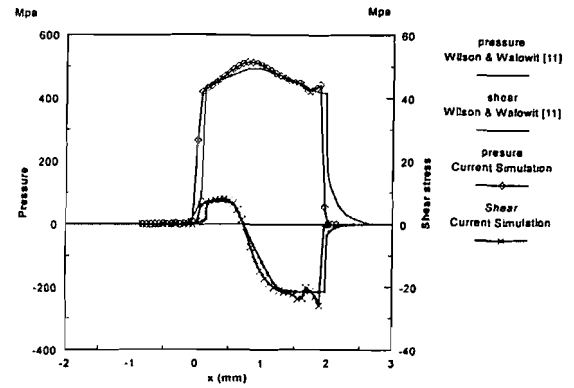


Fig. 3—The distribution of normal pressure and shear stress.

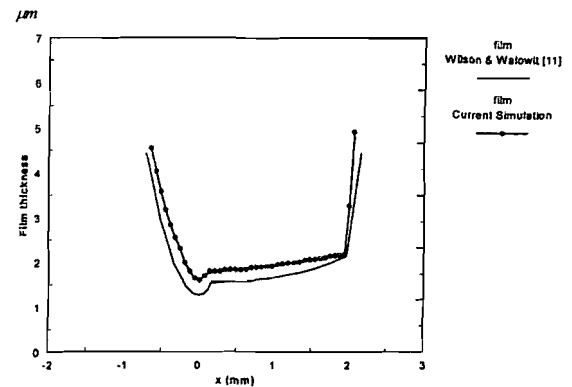


Fig. 4—The distribution of film thickness.

and only a slight difference is observed which is caused the implementation of the different deformation model.

A mathematical model for lubricated rolling operation in the full film regime was developed by Sa and Wilson (1994) and a series of tests were run at particular reductions with high viscosity lubricant over a wide range of speeds. In order to compare the current simulation with the experiments appropriately, the reduction and rolling speeds are chosen to ensure the existence of the full film regime. This can be decided by comparing predicted film thickness with surface roughness. The inlet film thickness generated by the polyphenyl ether calculated by the present model are plotted in Fig. 5 as a function of rolling speed for different reduction ratios. According to Wilson's (1978) definition, in the full film regime, which includes the thick film and thin film regime, the mean lubricant film thickness is at least three times the composite surface roughness. From Fig. 5 it is evident that full film conditions are only attained for rolling speeds above 0.2 m/s at both reductions. Otherwise the asperity contacts become dominant if the film thickness below three times the surface roughness. Thus it seems reasonable to predict the rolling phenomena by using the proposed model in full film regime and the parameters are listed in Table 2. The strip material was commercially pure aluminum 1100-H14 and the material constants  $A$  and  $n$  are 0.9 and 0.355, respectively.

$y_1$ (mm)	1.02
$Y$ (MPa)	100
Reduction	0.2 or 0.38
$a$ (mm)	51.0
$V$ (m/s)	0.3 - 1.0
$\eta_0$ (Mpa.s)	$2.0E-6$
$\gamma$ (Mpa) <sup>-1</sup>	$7.0E-2$

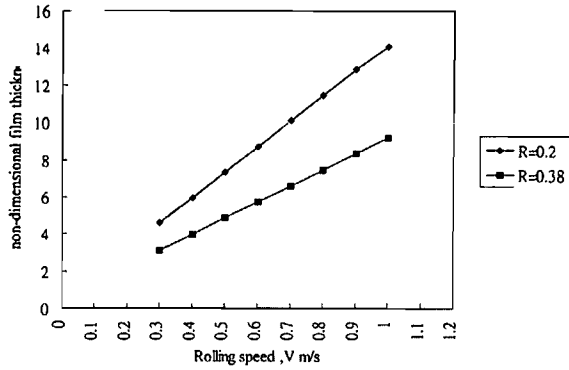


Fig. 5—The relationship between non-dimensional film thickness and rolling speed.

The predicted roll separating force, roll torques and outlet speed ratio for the polyphenyl ether case are compared with the measured values from Sa and Wilson (1994). Figure 6 is the comparison of roll separating force for the values of the reduction R of 0.2 and 0.38. The correlation between the roll force and rolling speed is not apparent and larger roll force is expected in the larger reduction ratio. The results of roll torques and outlet speed ratio are shown in Figs. 7 and 8, respectively. The same phenomena is observed in Fig. 7 as shown in Fig. 6. The outlet speed ratio is defined as the ratio between the speed measured from the exit of the outlet zone and the rolling speed. In general the quantitative predictions of the proposed model are very close to the experimental measurements, except at low rolling speed perhaps became a significant occurrence of mixed lubrication regime at this point.

**CONCLUSIONS**

The computer simulation of cold rolling processes successfully combines a rigid-plasticity finite element code with a realistic lubrication analysis and friction model. The surface roughness effect on lubricant flow is considered by using a average Reynolds equation. The local values of roughness and film thickness are then combined to decide the hydrodynamic friction stress at each location and time within the billet/rolls interface. The roll force, roll torques as well as the outlet velocity ratio are compared with the experimental investigation from earlier studies. The new model gives a accurate estimate of rolling phenomena in the full film regime due to the changes of rolling conditions such as rolling speed and reduction ratio..

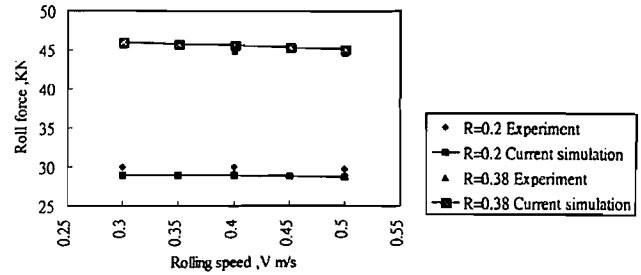


Fig. 6—The comparison of roll force between the experiment and current simulation.

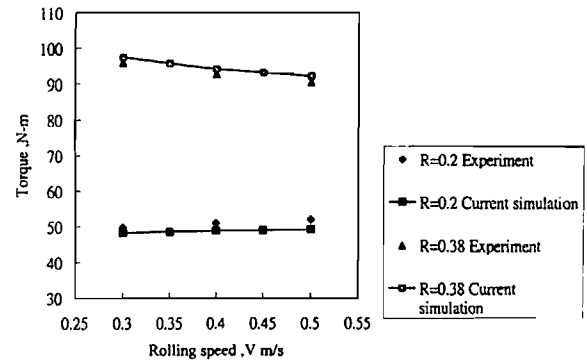


Fig. 7—The comparison of torque between the experiment and current simulation.

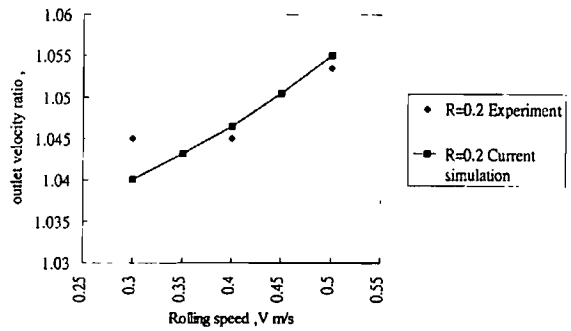


Fig. 8—The comparison of outlet velocity ratio between the experiment and current simulation.

In practical rolling operations, the mixed regime is undoubtedly of more practical importance than the thick film regime. The control of sheet surface roughness through contact with the rolls usually requires operation in the mixed regime. The extension of the current model to accommodate mixed regime and boundary regime formulation is currently underway. The effects of plastic heating and emulsion phenomena could also be included if needed. To use the concept of parallel process to reduce the penalty time involved in using the more complicated friction model will also guide metal forming tribologist in the presentation of their research in a manner which can be assimilated more easily by the community of computer simulation.

## ACKNOWLEDGMENTS

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